MULTI-PERIOD SCENARIO GENERATION TO SUPPORT PORTFOLIO OPTIMIZATION

by

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ABSTRACT OF THE DISSERTATION

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Stochastic Programming (SP) models are widely used for real life problems involving uncertainty. The random nature of problem parameters is modeled via discrete scenarios, which makes the scenario generation process very critical to the success of the overall approach. In this study we consider a portfolio management problem and propose two scenario generation algorithms and a SP model to support investment decisions. The main objective of the scenario generation algorithms is to infer representative probability values to be assigned to the scenario realizations sampled from historical data.

The first algorithm assigns the probabilities by using similarity scores, assigning higher probabilities to the scenarios with data paths that are relatively similar to historical paths, where similarity scores are computed by means of distance measures. We first implement

this approach using the weighted Euclidean distance (WED). We also propose a new distance measure to obtain similarity scores as an alternative to WED.

The second scenario generation algorithm is based on the combination of moment-matching technique and the Exponential Generalized Auto-Regressive Conditional Heteroskedasticity (EGARCH) model. Scenario probabilities are assigned such that the first four moments of the sampled returns are fit to target moments through a linear programming model, where the second target moments are set to be conditional on the past scenarios on the scenario tree using the EGARCH model. An additional set of constraints are proposed to increase robustness.

The generated scenarios become input to the SP model to restructure the existing portfolio such that the expected final wealth is maximized and the risk exposure is controlled through constraining Conditional Value-at-Risk at each decision epoch on the scenario tree. We finally propose a generic approach to reduce potential losses and implement it on a logistic regression framework.

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1. INTRODUCTION and LITERATURE REVIEW

The objective of this study is to present a stochastic programming (SP) based framework to support financial portfolio management decisions with a focus on the scenario generation phase. The concept of portfolio optimization has attracted many researchers and become a major research area in finance due to the challenging nature of the problem and large demand for solid solutions. Before elaborating on our approach, we briefly discuss some other approaches employed for portfolio optimization. Some of the prominent approaches used for this purpose are:

- Mean-Variance Model and its variations.
- Stochastic Control.
- Monte Carlo Methods.

1.1. Mean-Variance Approach

The first significant mathematical treatment of portfolio optimization is the mean-variance (MV) model developed by Markowitz (1952). In this Nobel prized study Markowitz presents a quadratic programming model where, given a fixed level of return, the objective is to minimize the portfolio risk, which is represented by the standard deviation of the portfolio return. The objective might be also given as the difference between the mean return of the portfolio and the variance multiplied with a constant that controls the level of risk aversion. Out of all possible portfolio configurations the MV model yields the ones that are on the *efficient frontier*, which represents the optimum portfolios obtained by setting different levels of risk aversion.

The MV model has the assumption of normally distributed asset returns, which is not always valid due to the fat tailed characteristic of most financial data. Other shortcomings of the original model are the penalization of the positive deviations over the mean portfolio return; single-periodicity; and the exclusion of several restrictions faced in real life (e.g., transaction costs). After Markowitz, portfolio optimization attracted significant attention from researchers with the purpose to improve the MV model and explore new methodologies that would address the problem for different settings.

For example, Markowitz (1959) proposed the *semi-variance* as the risk measure for the portfolio optimization problem instead of the variance, which is defined as $S = E[\min(0, R - c)^2]$, where R is the portfolio return and c is equal to E[R] or some other constant. Apparently, the semi-variance is a risk measure where only the negative deviations from the mean are penalized.

For instance, Konno and Yamazaki (1991) use a different approach with the mean absolute deviation (MAD) as the risk measure disregarding the quadratic objective function. They show that the MAD model can generate solutions faster than the original MV model. Zenios and Kang (1993) also propose an MAD model to manage the portfolios of mortgage-backed securities.

Benati and Rizzi (2007) present an extension of the Markowitz model where variance is replaced by Value-at-Risk (VaR), which can be simply defined as the α -quantile of the portfolio return distribution. Specifically, VaR is the maximum loss a portfolio can incur that can not be exceeded with the confidence level of 1-α (see Section 2.4.3.4 for more discussion on VaR).

Li and Ng (2000), and references therein, are among the studies extending the MV model to a multi-period setting. Li and Ng (2000) provide an analytical solution for the multi-period problem where the mean and variance of the final wealth are in the objective function. Moreover, they provide an analytical expression for the resulting efficient frontier.

Ogryczak and Ruszczynski (1999) utilize a relatively different approach of stochastic dominance (Fishburn (1964)) instead of a typical mean-risk approach in order to compare alternative outcomes on random variables. They consider standard semi-deviation and absolute semi-deviation and provide conditions that make the mean-risk approach consistent with second order stochastic dominance. In another study, Ogryczak (2000) develops a linear goal programming model with a multi-criteria setting. Multiple linear functions of portfolio return are considered as a weighted sum in order to find portfolios that are optimum with respect to several different risk-averse preferences.

Instead of using merely mean and variance in the objective function, Parpas and Rustem (2006) include all first four moments of the portfolio return into the objective function. Considering a minimization problem, skewness of the return would have a negative coefficient in the objective function since positive skewness is desirable due to making high returns possible. Kurtosis is included with a positive coefficient, since it implies a fat tailed distribution, which may lead to high risk exposure.

1.2. Stochastic Control

After Markowitz's single period model, many studies directed attention on the differences between the single period and multi-period model results. A very important research area in portfolio optimization after Markowitz has been developing financial models using stochastic control techniques. These are studies using dynamic stochastic control to obtain closed form solutions for the multi-period/continuous time portfolio management and option pricing. Merton (1969) is the first significant article treating the portfolio optimization as a lifetime consumption and investment problem assuming the asset return rates are generated by a Wiener process and this study can be regarded as the starting point of continuous-time finance literature. Another seminal study, interestingly published in the same year, is Samuelson (1969), where the author achieves a similar analysis in discrete time. Another immediate important study is Hakansson (1970), which provides optimum consumption investment for a class of utility functions.

A review of the stochastic control literature on portfolio optimization reveals that usually controlled stochastic processes are used for modeling the asset returns and wealth of the decision makers are obtained as stochastic differential equations. Dynamic programming and martingale optimality conditions are among the tools used to solve the resulting portfolio optimization problem.

The advantage of these approaches is the simple closed-form solutions obtained under the required assumptions (e.g., no transaction costs). As the assumptions are relaxed and additional constraints are included the problem becomes complicated to solve analytically. Therefore, the practical applicability of the original studies is limited since real life problems contain transaction costs and many other constraints.

In order to obtain more realistic models, researchers extended the original studies. For instance, Davis and Norman (1990) and Taksar et al. (1998) added the transaction cost to continuous time modeling considering two assets. Chellathuraia and Draviamb (2007), and references therein, provide models that consider fixed and proportional transaction costs within a stochastic control framework.

Some of the other noteworthy studies in this area are Cox and Huang (1989), Karatzas (1989), Richardson (1989), Chow (1993), and Kohlmann and Zhou (2000).

1.3. Monte Carlo Methods

Different than the alternative approaches, Monte Carlo Methods (MCMs) are highly computational tools. Instead of pure theoretical modeling, MCMs are employed mostly to evaluate/improve an existing model or a decision rule. In this context, the two main subjects where MCMs are used are

- *Evaluating portfolio risk*: The performance of an existing portfolio for future periods could be evaluated by simulating the underlying random variables (e.g., asset prices). This may involve generating a high number of paths for asset prices after which the decision maker can see how his asset allocation would behave in different future scenarios. Specifically, histograms can be built to represent the future distribution of the portfolio return and different risk measures are evaluated to improve the existing asset allocation.

- *Option pricing*: The value of an option is strictly dependent on an underlying asset, mostly a stock. Generating paths for asset price enables the decision maker to observe possible values the option can have at the horizon and therefore value the option at the time of the simulation.

Despite the ease in modeling, Monte Carlo simulation can be computationally demanding due to the high number of scenarios to be generated. Another shortcoming is the possible high variance for the resulting estimates, which is handled via variance reduction techniques (see Glasserman (2003) for more details on MCMs).

1.4. Stochastic Programming (SP)

The basic characteristics of the SP approach are:

- Optimization is achieved under uncertainty.
- Probability distributions are known or estimated through discrete scenario sets. Once the scenarios are generated, the decision maker solves the deterministic version of the original stochastic program.
- A typical objective might be the maximization of the expected value of wealth (or utility function) and minimization of some risk measures.

- SP models are generally two-stage or multi-stage models with recourse. This results in two types of variables to be used in the model**:**
    - *1st stage variables*: These variables are scenario independent and determine the decisions made immediately after the model run. In other words they state the set of actions the decision maker should take considering the set of future scenarios. For example, in a supply chain problem, the first stage variables might be used to select which production plants will be established at the time of network design.
    - *2nd stage (i.e., tth stage* or *time staged) variables*: Different than the first group, these variables are scenario dependent and they determine which action should take place at a particular scenario and a time period given the set of actions taken and outcomes realized in the previous stages. In other words, they take the corrective action after the first stage decisions are made (i.e., *recourse variables*).

The advantage of using SP models is the ability to merge a high number of state variables with various objective functions and complex real life constraints. In comparison to the other approaches, SP comes with more flexibility since

- The decision maker can consider more than two assets.
- It is easy to include transaction costs into the model.
- It is possible consider a wide range of objective functions and risk measures.
- There is no restriction on the distribution of asset returns.

- Other real life restrictions can be incorporated into the model easily. These may include:
    - Liability flows.
    - Taxation rules.
    - Legal and regulatory issues.
    - Company-driven restrictions.

The main shortcomings of SP might be listed as follow:

- The size of the model can grow exponentially as the number of stages and variables increase, which bring in computational challenges.
- It is difficult for practitioners to develop and implement such complicated models.
- The results of SP models are harder to interpret.

Even though the roots of SP go back to Dantzig (1955), the application to the portfolio optimization domain has not gained acceleration until the recent technological advancements in computing power. The most successful applications of SP are implemented as Asset and Liability Management (ALM) tools. ALM is a generalization of asset allocation problems where various cash flows are also incorporated in the model. Therefore, ALM problems are faced mostly by pension funds and insurance companies. Mulvey (1996a) lists Towers Perrin, State Farm Insurance, Falcon Asset Management, Frank Russell, and Unilever among the companies implementing multistage SP models for their ALM systems.

One of the earliest studies proposing an SP approach to the financial domain is Bradley and Crane (1972). The model presented in this article considers a portfolio of fixed income securities and provide the basic structure that will be used by many following studies in the same area.

The first significant real life application of the SP approach is Kusy and Ziemba (1986). They develop a comprehensive multi-period SP model and implement the model as an ALM framework for Vancouver City Savings Credit Union. Their model is intended to better manage the uncertainty in deposits to and withdrawals from accounts. A shortcoming of their model is that the scenario tree in not updated as the horizon moves into the future.

One of the noteworthy applications on this domain is presented in Carino et al. (1994), which introduces the SP based approach implemented for a Japanese Insurance company. Compared to other models focusing on a specific problem or a set of assets, the difference of this study is that it constitutes the entire dynamic ALM framework of the insurance company. The details of the model are given in Carino et al. (1998) and Carino and Ziemba (1998). They propose and implement a multi-stage linear SP model covering the decisions of the company for the following five years over a scenario tree of 256 scenarios at the horizon. They discuss three different scenario generation approaches to be used for the software. Another important study immediately following Carino et al. (1994) and leading the way to a successful real life application is by Dert (1995), who develops an ALM model to be used by Dutch pension funds.

Mulvey et al. (2000) provide another success story of the SP approach, which presents the SP based ALM system developed for Towers Perrin-Tillinghast for its insurance and pension plan services (see also Mulvey (1996b) and Mulvey and Thorlacius (1998)). The model relates the liabilities and asset returns to structural economic factors. In fact, the generation of scenarios is a multi-layer process, where the top layer is the generation of interest rates and price inflation. Then the other parameters such as fixed income returns, stock returns, and wage inflation are generated through sets of stochastic differential equations at different layers.

Eckstein and Hiller (1993) is also one of the earliest applications of the SP approach with particular focus on the computational side. They apply a relatively simple SP model to a portfolio of fixed income securities. In an accompanying paper, Dantzig and Infanger (1993) present a multi-period stochastic linear programming model for multi-period portfolio optimization and addresses efficient methodologies to solve the resulting SP model. They utilize importance sampling in order to restrict the scenario tree size for computational concerns. Consigli and Dempster (1998) provide a generic SP model to handle the ALM issues for a pension fund.

Golub et al. (1995) present a successful application to money management, where they propose a two stage SP model to manage a portfolio of mortgage backed securities. Interest rate scenarios are generated using the binomial lattice model of Black et al. (1990) and the cash outflows are generated by using the prepayment method described in Kang and Zenios (1992). A similar study is Zenios et al. (1998) where they present a multi-stage dynamic model for the management of fixed income portfolios. The interest rate scenarios in this study are also generated through Monte Carlo simulations based on the binomial lattice model of Black et al. (1990). They implement the model in different settings for two different problems, bond index tracking and money management with mortgage-backed securities.

A relatively recent study is Ziemba and Zhao (2001), which presents an asset allocation problem with multiple risky assets and a risk-free asset. Different than the previous studies, they consider the worst case payoff as the risk measure. Asset return scenarios in this study are generated through auto-regressive models estimated on quarterly index data (i.e., the Salomon Brothers bond index and S&P 500 stock index). Their benchmarking analysis reveals that the SP based model outperforms the mean-variance approach.

Fleten et al. (2002) compare the performance of a multi-stage SP model to that of a static fixed mix within the same problem context where a two asset portfolio is considered. They resort to expert opinion to obtain the statistical features of the asset returns and then achieve moment matching over a discrete set of scenarios so that the recommended statistical features are preserved.

The interested reader is referred to Ziemba and Mulvey (1998) and Censor and Zenios (1997) for more information and references on the topic.

1.5. Scenario Generation

Most SP models build decisions by approximating the future uncertainty through a finite set of scenarios having a discrete probability distribution; therefore, the success of SP models directly depends on the effectiveness of the design stage for scenarios. Special effort is required to decide on the number of scenarios (i.e., topology of the scenario tree) and assign individual probabilities for each scenario regarding the random processes in the model.

As stated in Ziemba and Zhao (2001), modeling future asset returns can be categorized into two broad approaches. The first one is the *adaptive expectations* approach where future asset returns are generated by relating them to past observations of the explanatory variables. This approach can be regarded to be a technical analysis. Alternatively, a *rational expectations* approach generates scenarios using forecasts produced by conceptual macroeconomic models where expert expectations are used. The studies in this group are based on fundamental analysis.

We next discuss some of the approaches that can be used for scenario generation.

1.5.1. Bootstrapping Historical Data

Bootstrapping historical data is the simplest scenario generation method since it does not require a mathematical effort. This method is basically sampling random periods from the historical data and using the returns realized in those periods as scenarios for the current model. If there are multiple assets in question, all returns from the selected period are recorded into a vector to preserve the correlation among assets. In addition other statistical features of the historical distributions are preserved.

In one of the methods used in Carino and Ziemba (1998), they assume that scenarios are independent across periods and generate scenario by sampling the historical data. Then they employ an algorithm to adjust the variance so that the resulting distribution for each random parameter has the same mean and variance obtained from historical data.

1.5.2. Brownian Motion (Wiener Process)

This continuous-time stochastic process was originally defined to describe the movement of particles in fluids. It was also used to characterize some stochastic processes having specific features. In fact, the typical discrete random walk converges to a Wiener process as the time step gets infinitesimal. A stochastic *continuous* process W_t is a Wiener process if $W_0 = 0$ and it has independent increments such that $W_t - W_\tau$ is normally distributed with mean 0 and variance $t-\tau$ where $0 < \tau < t$.

After Merton (1969), Wiener process was frequently utilized to model the behavior of stock price to be used for portfolio optimization and option pricing. In fact, in these models a Wiener process is used to drive the stochastic portion of the prices process such the resulting logarithms of returns follow a Wiener process (i.e., normally distributed given a time interval). Paths of stock prices can be generated via Monte Carlo simulations to build portfolios or evaluate risk exposure of a given portfolio. For instance, the stochastic differential equations used in Mulvey and Thorlacius (1998) to generate asset return scenarios are driven by correlated Wiener processes. The Wiener process is usually utilized by continuous-time financial models (see Duffie and Richardson (1991) and references therein among many others).

1.5.3. Vector-Auto-Regression (VAR)

Auto-regressive (AR) models are univariate econometric models that are used to capture the behavior of time series data and make forecasts. Let X_t be the variable in question. Then AR(p) (i.e., AR of degree p) expresses X_t in terms of its lags as $X_t=\beta_0 + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \ldots + \beta_p X_{t-p} + \varepsilon_t$ where ε stands for the error term with zero mean. VAR is the generalization of the AR model, which covers multiple time series and their dependencies on each other. Different than the AR models, VAR models relate each variable to the lags of the other variables in addition to their own lags.

Suppose that $\mathbf{Y}_t$ is the vector of all n variables in question. Then VAR(p) is expressed in matrix form as $\mathbf{Y}_t=\mathbf{C} + \mathbf{D}_1\mathbf{Y}_{t-1} + \mathbf{D}_2\mathbf{Y}_{t-2} + \ldots + \mathbf{D}_p\mathbf{Y}_{t-p} + \mathbf{E}_t$ where $\mathbf{C}$ is an nx1 constant vector, $\mathbf{D}_1 \ldots \mathbf{D}_p$ are nxn matrices, and $\mathbf{E}_t$ is the nx1 vector of error terms with zero mean. The selection of the degree of VAR (i.e., length of time lag) and calibrating the parameters are critical for the success of VAR.

Boender (1997), Dert (1995), Kouwenberg (2001) and Ziemba and Zhao (2001) are some of the studies using a VAR model to generate scenarios for asset returns and other random parameters.

1.5.4. Moment Matching

This method could be based on an optimization model or some other heuristic where the objective is to obtain a set of scenarios (i.e., realizations of random parameters and probabilities corresponding to each realization) such that the resulting moments, usually the first four, are close or equal to the moments obtained from historical data or some other target set by the decision maker (see Hoyland and Wallace (2001)).

A potential shortcoming is non-convexity since it may prevent finding a perfect match for the moments due to the high probability of obtaining a locally optimum solution. Even though the global optimum can not be found, it can be still acceptable to have statistical features that are sufficiently good approximations to the target moments from a SP point of view. The nonlinearity in this approach can be eliminated by linearizing the objective function such that it is a weighted sum of absolute deviations from the target moments instead of squared deviations.

Ji et al. (2005) reports a comparison between linear and nonlinear versions. Gulpinar et al. (2004), Fleten et al. (2002), and Parpas and Rustem (2006) are some of the studies using moment matching technique in the scenario generation phase.

1.5.5. Factor Models

Factor modeling is about relating the evolution of a random variable to some other independent factors. Suppose that we interested in the return of an asset. Then possible factors could be

- return on some other individual asset.
- return on a stock or bond index.
- macro-economic variables such as interest rate, GDP, inflation, currency exchange rates, unemployment, etc.
- some benchmark portfolio or fund.

where the choice of the factors completely depends on the belief of decision maker. A factor model can be simply stated as $X = \beta_0 + \beta_1 f_1 + \beta_2 f_2 + \ldots + \beta_m f_m + \varepsilon$ where X is the variable in question and $f_1 \ldots f_m$ are the m factors that are assumed to explain the value of X.

Carino and Ziemba (1998) report three different scenario generation approaches in their study for the Japanese insurance company and one of the approaches is based on a factor model to build correlation across assets and periods, where the factors are interest rate, overall equity market behavior, and exchange rate. Dantzig and Infanger (1993) provide another study utilizing factor modeling.

1.5.6. Binomial Lattice

Being first developed by Black et al. (1990), this method is generally used to generate possible scenarios for short term interest rates considering the current long term interest rates and corresponding volatilities. Therefore, it is mainly used by the studies which focus on portfolios of fixed income securities such as bonds.

Bertocchi et al. (2006) and Bertocchi and Dupacuva (2001) utilize this model over the Italian Bond market whereas Beltratti et al. (1999) consider international bond markets. Golub et al. (1995) and Zenios et al. (1998) are among the other studies using lattice structures to generate scenarios for fixed income portfolios.

1.5.7. Expert Judgment

Experts of various domains can have anticipations and beliefs on possible moves on market indicators, indices, or assets, which might be different than those suggested by the historical data. These beliefs may provide a base to construct scenarios either directly by setting values and probabilities or indirectly by modifying the probability distributions suggested by the historical data. In other words, expert judgment may be input directly as a complete scenario or indirectly (or partially) through the specification of some statistical features of the random process.

One of the three methods employed by Carino and Ziemba (1998) is the direct utilization of user input involving expert judgment, which enables the decision makers to build scenario trees with maximum flexibility. Fleten et al. (2002) also utilizes expert judgment to specify target moments for the moment matching phase of their scenario generation.

1.6. Research Objectives

Our first objective is to search for new ways of handling the uncertainty for the portfolio optimization problem, which will be achieved through developing new scenario generation methods. Another objective is to build an optimization model to obtain optimum portfolios considering different risk exposures within a multi-period setting and analyze the performance of the proposed approach through different experiments.

We briefly discuss these objectives next. The details of the methodology are given in Chapter 2.

1.6.1. Scenario Generation Methodology

Considering the significance of scenario generation, two scenario generation algorithms are proposed with the motivation of exploring a new and acceptable methodology to handle uncertainty.

The first scenario generation algorithm is based on reducing the historical data set into a smaller but more relevant set such that the similarity between the current situation and the historical data set is increased. This reduction is achieved using similarity scores, which are computed via distance measures. A new distance measure is developed and used in conjunction with Euclidean distance to compute the similarity scores. The same distance measures are used to assign probabilities to the scenarios according to the similarity between the scenarios and the past behavior of the data in the reduced historical data set.

The second scenario generation algorithm is based on the moment matching technique discussed in Section 1.5.4., which aims to find a discrete probability distribution for the scenarios such that the first four moments of the generated scenarios are closely matched with the historical data. Originally, Hoyland and Wallace (2001) present this optimization based approach to match general statistical features. They build a nonlinear model where both security prices and corresponding probabilities are decision variables. A shortcoming of that model is the high level of nonlinearity. We consider a linear program and take security returns and interest rates as given parameters (generated before the matching). In addition, we incorporate state dependency of variance via the well-known Exponential - Generalized Auto Regressive Conditional Heteroskedasticity (EGARCH) model, which, to the best of out knowledge, is included into a scenario tree construction for the first time. Instead of node-by-node optimization, we consider solving a single optimization model that optimizes the entire scenario tree at once.

1.6.2. Portfolio Optimization Model

Once the scenario tree is constructed, the next step is to input the scenario tree into an SP model in order to obtain the investment decisions. The advantage of SP models is the ability to consider various objective functions and constraints. Therefore, SP based studies focus on different various measures of reward and risk where the measure of reward is usually expected final wealth. Utility functions are used to combine the risk and reward into a single objective. In our SP model, we build the objective function as the maximization of the expected final wealth. We control the risk exposure by limiting the conditional Value-at-Risk (CVaR) within linear constraints. Changing the limiting parameters in these constraints would yield investment strategies for investors having different risk averseness. Different than the studies in literature, we employ risk control dynamically at each node of the scenario tree.

Even though our focus is on the expected shortfall (i.e., CVaR) we provide a generic model that can cover two other measures. The first one is the severity of the worst case scenario, which could be improved by a *maximin* approach (i.e., maximizing the wealth for the worst case scenario). The second one is to decrease the fluctuation of portfolio returns over the time-span of the scenario tree. This corresponds to the minimization of expected deviations among consecutive periods.

In summary, we aim to build a model which will be used to rebalance the current portfolio on hand such that the expected final wealth maximized over the planning horizon subject to the control on risk exposure (NOTE: The term *rebalancing* is usually used in conjunction with the *fixed-mix* rule to refer the trading process for restoring the constant mix. In this report, it refers to executing the buy/sell decisions suggested by the SP model).

The following is a list of the data categories that can be input to the portfolio optimization framework.

- *Asset Prices/Returns:* It is assumed that the current portfolio is a collection of a finite number of financial assets (e.g., stocks, bonds, cash, etc.). It should be noted that the term ‘asset’ may refer to ‘asset class’ since it is common in practice for some investors to invest in asset classes instead of individual assets. The data in this category are of two types:

- *Historical data*: This set of data includes the historical return data for the universal set of assets and interest rates.
- *Future scenarios*: This set of data is generated by the two scenario generation algorithms discussed in Chapter 2. This is the most critical data set since it determines how well the model captures the uncertainty associated with the future asset returns.

- *Cash inflows/outflows*: Some assets generate positive cash flows to the investor (e.g., dividend payments of stocks) whereas some investors may have pre-scheduled or random cash outflows (e.g., liability flows) throughout the planning horizon.

- *External and Internal Parameters*: This group includes parameters those are external (e.g., transaction costs, interest rates) and those set by the investor (e.g., upper and lower bounds on individual asset positions, confidence level and limits for CVaR, objective weights, etc.).

1.7. Organization of the Text

We present our methodological approach in Chapter 2, where Section 2.2 presents the scenario generation algorithms and Section 2.3 provides the details of the optimization model. In Chapter 3, we explain about the implementation process and present computational results and sensitivity analysis. Chapter 4 presents a general approach to be used incorporated into any trading strategy and aims to increase the performance from a return/risk point of view. Chapter 5 provides a summary of conclusions, research contributions, and possible extensions of the study.

2. METHODOLOGICAL APPROACH

In this chapter, a multi-period portfolio management framework is presented, which consists of two scenario generation algorithms that provide input parameters and a stochastic programming (SP) model that uses those input parameters and builds a robust portfolio. Sections 2.2 and 2.3 present the scenario generation algorithms and the SP model is given in Section 2.4.

2.1. A Brief Introduction to Scenarios

The uncertainty associated with parameters can be modeled through continuous density functions or some discrete probability distributions, the first being not quite practical due to the structure of SP models. As a common practice, a discrete set of possible outcomes (i.e., scenarios) are created with individual realization probabilities as the input to most SP models. Caution is needed when creating this set since the number of scenarios may increase dramatically as the number of parameters and time periods (for multi-period models) increase. The trade-off between the computational power required due to the problem size and the ability of the model to sufficiently capture real-life uncertainty has been a major issue for the SP approach, especially in the past when high levels of computational power were not as available as today. Even though the technological progress keeps loosening the trade-off, developing scenario generation algorithms is still an appealing research area.

What is exactly meant by the term *scenario*? Each scenario corresponds to a realization of a random variable (e.g., asset returns, interest rates, cash flows, etc.) over the planning horizon and has an associated realization probability with it. A multi-period scenario is represented by a set of consecutively ordered single-period scenarios, which can be illustrated by a scenario tree of four different scenarios as in Figure 1, where the first scenario (i.e., s_1) is the dashed path highlighted by red lines.

The point where $t=0$ usually represents the current time where the decision is made considering the possible four scenarios, s_1, s_2, s_3, and s_4. Each scenario is a realization of random asset returns over the next three periods. After branching in the initial time periods, returns follow a different path into the future. It might be early but worth to note at this point that the decisions for different scenarios can not be different from each other until the period where the corresponding scenarios start differentiating from each other. This rule prevents models that have a scenario tree as the input from exploiting the future information within the tree and is established by the introduction of *non-anticipativity* constraints.

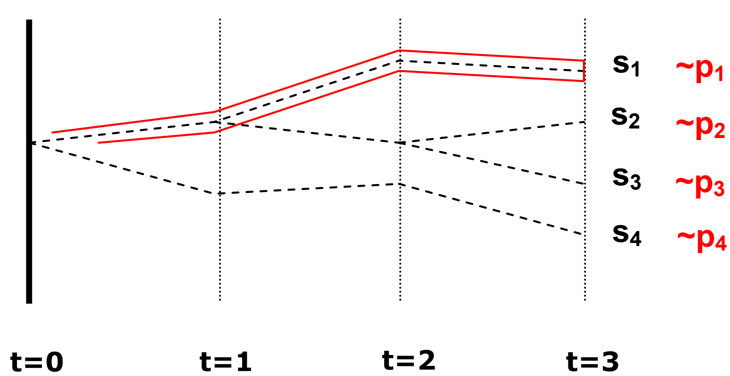

Figure 1. Scenario tree for 3-period planning horizon

For instance, the return realizations s_1, s_2, and s_3 are the same in the first period but we observe variations in the following periods. Since the latter periods have not been realized yet at t=1, these two scenarios must be equivalent in terms of the decisions made (e.g., buy/sell decisions) in these particular scenarios. Let z_{ts} denote the security holdings for scenarios s at time period t. Then, for the scenarios in Figure 1, (1) and (2) are the two non-anticipativity constraints where we force the positions to be equal to each other in the first period for s_1, s_2, and s_3.

$$z_{1S1} = z_{1S2} \tag{1}$$

$$z_{1S2} = z_{1S3} \tag{2}$$

In this section, we propose two scenario generation algorithms. The first one is based on moment matching whereas the second relies on similarity scores.

In this study the asset returns (i.e., the realization component of the scenarios) are generated by sampling the historical data. However, depending on the type of the algorithm the process of sampling slightly varies, which will be specified in the detailed descriptions of the algorithms.

The generated asset returns are configured on a scenario tree, where we assume that

- It is a *symmetric tree*, where the number of branches emanating from each node is the same for all nodes at the same stage.
- The number of branches emanating from each node decreases for the latter periods.

A sample tree under these assumptions is given in Figure 2. Once such a tree is configured with the generated asset returns, the algorithms, which will be presented in Sections 2.2 and 2.3 assign the probability distributions given the discrete sets of realizations for asset returns at each node except the leaf nodes. Note that, if the decision making process involve multiple assets, then each node (i.e., one period scenario) corresponds to a vector of asset returns and a single probability attached to it.

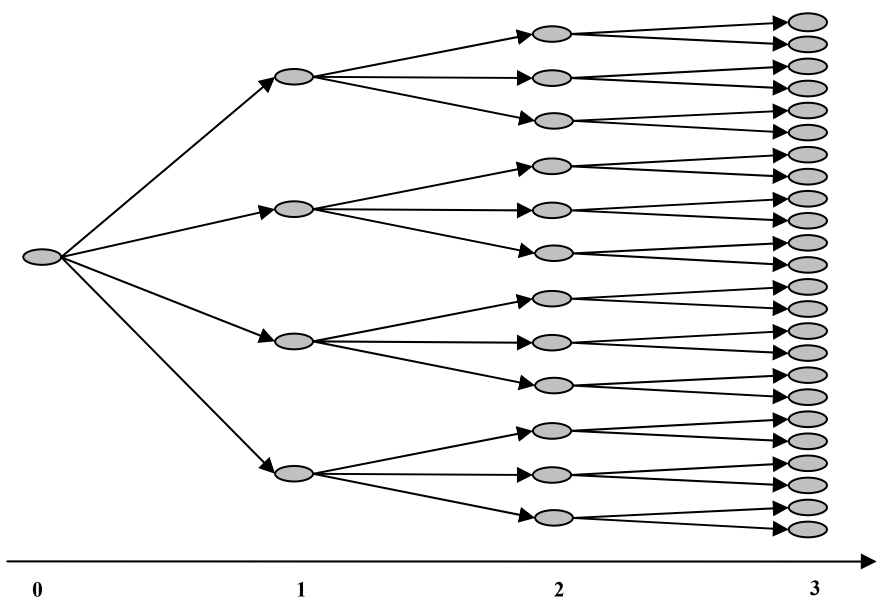

Figure 2. A symmetric scenario tree for T=3

2.2. Probability Assignment through Similarity Scores

In this section, we propose two scenario generation algorithms that rely on the evaluation of similarity between past and current behavior of the asset returns.

2.2.1. Basics of the Approach

Before providing the details, we first present the proposed approach as a combination of two main steps:

Reduction of the Historical Data Set: In this step we extract the most relevant part of the historical data set to be used in the next step. Consider the most recent pattern of length θ in the time series ending with the last observation in our data set. We seek *similar* patterns observed in the past having the same length. This provides us with the part of the history in which a sequence of realizations occurred that are similar to the most recent pattern in the data set.

Comparison of Past and Generated Scenario Set: We now turn our focus to what really had happened one period after those *similar* patterns occurred in the past (obtained at the previous step) and to the scenarios we have just generated for one period ahead. The one-period-ahead past realizations are believed to give us insight about how likely the current scenarios are to happen one period later. As a scenario gets more similar to the selected past realizations, it is assigned higher probabilities of occurrence.

In order to better illustrate the approach, consider node J shown in Figure 3, where N_{t+1} scenarios (i.e., nodes) were generated. In order to assign the probabilities for the N_{t+1} scenarios, the last n data points (i.e., the most recent path shown in red with length $n-1$) are considered in order to relate the current state to historical outcomes. The algorithm first searches within the historical data to find out the paths of the same length that are similar to the current path ending with node J, which is achieved through the similarity scores of the individual historical paths. Then we turn our focus to the $(n+1)^{th}$ data points in those historical paths and obtain the similarity scores of the current N_{t+1} scenarios on hand with respect to them. The higher the similarity of a scenario is, the higher the probability of that scenario becomes.

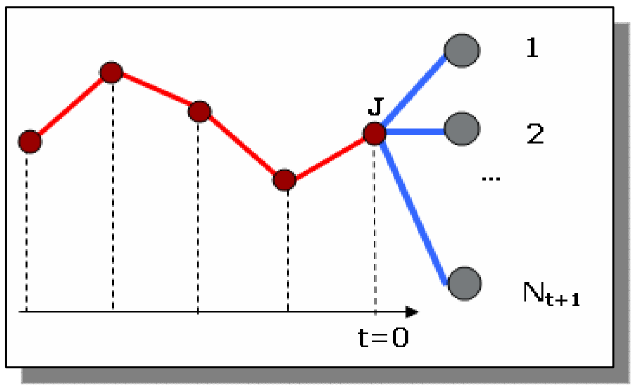

Figure 3. Illustration of one-period scenario generation

The aforementioned similarity scores between any two paths are computed via the overall distance between those two paths in question. In the presented algorithms, the similarity score is inversely proportional to the distance.

The two algorithms presented in Sections 2.2 and 2.3 are similar since they are built upon the same idea briefly provided above. The type of time series and distance measures used in the algorithms form the main differences. The first one utilizes the well-known Euclidean distance measure. We propose and utilize a new distance measure in the second algorithm, the *UD Distance*, which evaluates the paths with respect to their sequence of up and down movements.

2.2.2. Scenario Generation Based on Weighted Euclidean Distance

In this algorithm we are assumed to have the historical *arithmetic* returns for all *risky* assets given by

$$return_t = \frac{price_t}{price_{t-1}} - 1 \qquad (3).$$

The weighted Euclidean distance, which can be simply computed as in (4) between two vectors **X** and **Y**, is employed in order to measure both the distance between the most recent and the historical paths. The distance between the one-period-ahead realizations of the most recent path and historical paths is computed as the squared difference as described below in detail.

$$D_{EU}(\mathrm{X},\mathrm{Y}) = \sqrt{\sum_{i=1}^{n} w_i (X_i - Y_i)^2} \qquad (4)$$

2.2.2.1. Algorithm Description: Alg-1A

Suppose that we have *m* risky assets and the last *TH* (i.e., size of historical data set) historical returns for each of these assets and risk-free asset. Then, we can build the historical data matrix *H* as follows:

$$H = \begin{bmatrix} r_{11} & r_{12} & r_{13} & \cdots & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & & & \cdots \\ r_{31} & \cdots & & & & \\ \cdots & & & & & \\ \cdots & & & & & \cdots \\ r_{TH1} & \cdots & & & \cdots & r_{THm} \end{bmatrix} \qquad (5)$$

where r_{ti} is the return of asset i in period t.

Another initial step is to decide on a scenario tree topology (i.e., the horizon and the number of scenarios emanating at each period) such as one given in Figure 2 and set the source node of the tree as *current source node* (CSN). Then the algorithm can be formally described as follow:

Step 1 : For each risky asset, generate the corresponding number of asset returns by sampling from H on a random basis.

Step 2 : Consider last θ return vectors for all assets. This implies a θxm matrix formed by the last θ rows of H. Denote this matrix by A.

Step 3 : Let K^l be the θxm matrix formed by the rows l, l+1, l+2..., l+θ-1 of H.

For l =1 to TH-θ, compute

$$SK^{l}{}_{EU} = \left(\sum_{i=1}^{m} \sum_{t=1}^{\theta} w_{ti} \left(K_{ti}{}^{l} - A_{ti} \right)^2 \right)^{-1} \tag{6}$$

where $SK^l{}_{EU}$ is the similarity score given by the multiplicative inverse of the sum of the squared weighted Euclidean distances over all assets. Also note that $K^l{}_{ti}$ and A_{ti} denote the t^{th} elements within the i^{th} columns of K^l and A matrices, respectively. The parameters w_{ti} are the weights assigned to the Euclidean similarity for period t within the path.

Step 4 : Rank the K^l matrices according to their SK^l and select the top C matrices where C is a user-specified scalar (C < TH-θ). If K^l is selected, then let R^l be the $(l+\theta)^{th}$ row of H. Form a Cxm matrix with the C row vectors (R^l) and denote it by R.

Step 5 : For each return scenario s emanating from the CSN, compute the similarity score SS_{EU}^{s}, where

$$SS_{EU}^{s} = \frac{1}{\sum_{i=1}^{m}\sum_{c=1}^{C}\left(S^{s}{}_{i} - R_{ci}\right)^{2}} \tag{7}$$

It should be noted that a scenario corresponds to a vector of returns for m assets and is denoted by S^{s}.

Step 6 : Normalize SS_{EU}^{s} to obtain p_s for the scenarios emanating from CSN.

Step 7 : If all nodes excluding the leaf nodes have already been assigned as a CSN, STOP. Otherwise, set the node below the CSN as the new CSN. If the CSN is already the bottom, set the node at the top of the next time period's nodes as the new CSN. Go to Step 8.

Step 8 : Update the matrix H to include the newly generated scenario realizations. Update the matrix A such that A is the new θxm matrix with the most recent θ return vectors including the CSN. Go to Step 1.

2.2.3. Scenario Generation Based on the New UD Distance

In this algorithm we introduce new distance measure that is intended to capture the similarities of directional movements of the time series data in a different way.

2.2.3.1. UD Distance Measure

When financial time series data is in question, it may not be sufficient to rely solely on the absolute distances between individual data points to measure the overall similarity between two paths. Consider Figure 4, where three distinct paths, **X, Y** and **Z**, are plotted.

It is obvious that **X** and **Y** have a very low Euclidean distance and therefore higher similarity since the individual values are similar. However, the directional movement of the data is quite different between **X** and **Y**. In fact, the illustration implies a strong negative correlation between the processes driving **X** and **Y** (*Note that the paths discussed in this section represent the same asset at different time periods).*

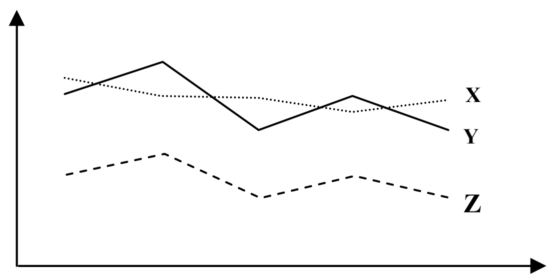

Figure 4. Illustration for three arbitrary paths, ***X, Y*** and ***Z***.

In addition to the difference among data values, the serial movements of the values (consecutive ups and downs) are also worth to be taken into account when comparing two paths. Path **Z** in Figure 4 is apparently much more similar to path **Y** with respect to their patterns; however, the comparison between these two paths may suffer due to high Euclidean distance since the individual observations are far apart.

Developing a distance measure that will decrease the distance as the two paths have more common ups and downs may still result in low distance for some dissimilar paths. The paths **W** and **Z** in Figure 5 have three common ups and downs (highlighted in gray) out of five possible outcomes. The number of common ups and downs would imply a 60% similarity between **W** and **Z** but these patterns obviously have a high dissimilarity. We believe that the distance measure in this case should result in high distance if the common ups and downs occur in separate locations within the path and low distance if they occur without interruption in between.

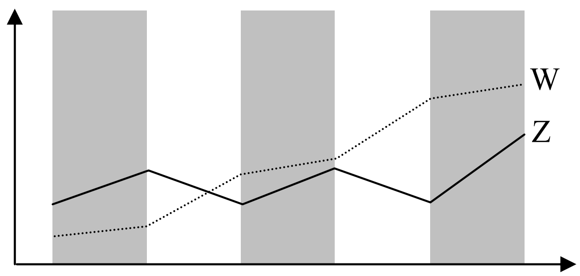

Figure 5. Illustration for two arbitrary paths, **W** and **Z.**

Suppose now that we have two paths, **U** and **V** in Figure 6, having the same number of *consecutive* ups and downs that are common with a benchmark path, **Z**. The objective of developing this distance measure is to extract from the universe of historical data those paths which have low distance, or say more similar, to the most recent path (**Z**) we have at time *t* and relate the one-step-ahead *forecast* of the data process for **Z** to the one-step-ahead *realization* of historical path that is similar to **Z**. Therefore, considering a particular historical path, the most recent behavior within that path is more important than the ones at the beginning. This approach will also provide more flexibility when choosing the lag length. Therefore, we conclude that in our problem setting the distance between **V** and **Z** must be lower than the distance between **U** and **Z**. In other words, **V** must be more similar to **Z** than **U**.

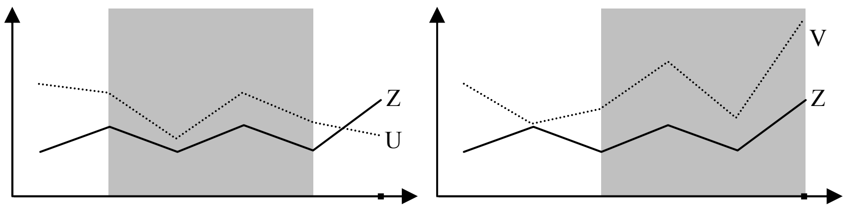

Figure 6. Illustrations for three arbitrary paths, **U**, **V**, and **Z.**

We first focus on comparing the direction of one period moves before defining the distance measure. If the directions of moves are the same then the resulting distance must be low; otherwise it must be higher to reflect the level of opposition. The possible situations for the one-period move directions of two return paths, **X** and **Y**, and corresponding distances can be listed as in Table 1.

Table 1. Comparison of one period directions

$X_i \rightarrow X_{i+1}$	$Y_i \rightarrow Y_{i+1}$	*Distance*
↑	↑	0
↓	↓	0
→	→	0
↑	→	1
→	↑	1
↓	→	1
→	↓	1
↑	↓	2
↓	↑	2

In order to define the distance measure, we define a function that describes the mapping in Table 1. Let $f : \Re^4 \rightarrow \Re$ and defined as

$$f(x_1, x_2, x_3, x_4) = \Big|\, |\text{sgn}(x_2 - x_1) + \text{sgn}(x_4 - x_3)| - 2 \Big|$$

$$+ 2 \max\{|\text{sgn}(x_2 - x_1)|, |\text{sgn}(x_4 - x_3)|\} - 2 \qquad (8)$$

Considering that X_i is the i^{th} data point of path **X**, we observe that $f(X_i, X_{i+1}, Y_i, Y_{i+1})$=0 if the paths **X** and **Y** move in the same direction after the i^{th} position and $f(X_i, X_{i+1}, Y_i, Y_{i+1})$=2 if they move in the opposite direction. If at least one of them remains unchanged, then $f(X_i, X_{i+1}, Y_i, Y_{i+1})$=1.

Let two return paths be represented by vectors **X** and **Y** both with n elements. Then the *UD* distance between these two paths is given by

$$D_{UD}(X, Y) = \max_{1 \le i \le n-1} \{c_i f(X_i, X_{i+1}, Y_i, Y_{i+1})\} \qquad (9)$$

where c_i's are positive scalars. In order to give higher importance for more recent periods, we use c_i values such that $c_{i+1} / c_i > 2$ (the maximum value f can get is 2) and $c_1 > 1$.

Under the light of this discussion, *UD* is a new distance measure that evaluates the course of increments and decrements along two separate same-sized data paths. It assigns high values of distance if any of the following is true.

1. The two paths do not have (or have few) common ups and downs.

2. Common ups and downs do not occur at consecutive positions.

3. The two paths have opposite moves at recent steps.

In addition to the comparison between the patterns to reduce the historical data set (i.e., selecting the most similar historical paths), the logic behind *UD* distance measure is also used to compare the generated one-step-ahead scenarios with the one-step-ahead realizations of the selected historical patterns. Just like the similarity scores for the scenarios in Alg-1A are computed using the squared Euclidean distance in *Step 5* as given by (6), the similarity scores for the scenarios in this algorithm will be computed by a simplified version of the *UD* distance measure utilizing function *f*.

2.2.3.2. Algorithm Description: Alg-1B

Now we can describe the second algorithm, Alg-1B, which is quite similar to Alg-1A with following two differences:

1. The distances measure used is *UD* distance instead of the Euclidean.
2. The input data are the historical *prices*, instead of historical *returns*.

Working on price data instead of return data is an issue when it comes to historical sampling since the price data is known to be nonstationary. Therefore, we prefer to *detrend* the historical price. Next we describe the algorithm formally:

Suppose we have the historical data matrix H as follows:

$$H = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \dots & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & & & \dots \\ x_{31} & \dots & & & & \\ \dots & & & & & \\ \dots & & & & & \dots \\ x_{TH1} & \dots & & & \dots & x_{THm} \end{bmatrix} \tag{10}$$

where x_{ti} is the price of asset i in period t.

Step 1 : For i =1 to m;

Detrend the price series data (i.e., modify H such that $H_{ti} = x_{ti} - a_i - b_i t$ where a_i and b_i are linear regression constant and coefficient for asset i, respectively with t, the historical time periods, being the independent variable). Generate the *detrended* price scenarios emanating from the CSN by randomly sampling from H.

Step 2 : Consider last θ detrended price vectors for all assets. This implies a θxm matrix formed by the last θ rows of H. Denote this matrix by A.

Step 3 : Let K^l be the θxm matrix formed by the rows l, l+1, l+2..., $l+\theta$-1 of H.

For l =1 to TH-θ;

$$SK^l = \left(\max\left\{ 1, \sum_{i=1}^{m} D_{UD}(K_i^{\,l}, A_i) \right\} \right)^{-1} \tag{11}$$

where SK^l is the similarity score computed via the UD distance measure. Also note that K^l_i and A_i denote the i^{th} column vectors of K^l and A matrices, respectively.

Step 4 : Rank the K^l matrices according to their SK^l and select the top C matrices where C is a user-specified scalar (C < TH-θ). If K^l is selected, then let R^l be the 2xm matrix formed by the $(l+\theta-1)^{\text{th}}$ and $(l+\theta)^{\text{th}}$ rows of H. Form a 2Cxm matrix with the C matrices (i.e., R^l) and denote it by R.

Step 5 : For each price scenario s emanating from the CSN, compute the similarity score SS^s_{UD}, where

$$SS^s_{UD} = \frac{1}{\sum_{i=1}^{m}\sum_{c=1}^{C}\left(\tau_1 f(H_{THi}, S^s_i, R_{(2c-1)i}, R_{2ci}) + \tau_2\right)} \tag{12}$$

In (12), H_{THi} is the most recent price data for asset i and S^s_i is the detrended price scenario for asset i. The constants τ_1 and τ_2 can be selected such that $\tau_1 f(\bullet) + \tau_2 > 0$ in order to prevent a possible division by zero (recall that $f(\bullet) \in \{0,1,2\}$).

Step 6 : Normalize SS^s_{UD} to obtain p_s for the scenarios emanating from CSN.

Step 7 : If all nodes excluding the leaf nodes are assigned probabilities, STOP. Add the trend back to the detrended price scenarios to obtain the actual price scenarios and use (3) to obtain the return scenarios to be used in the SP model. Otherwise, set the node below the CSN as the new CSN. If the CSN is already the bottom, set the node at the top of the next time period's nodes as the new CSN. Go to Step 8.

Step 8 : Set H back to the original historical price data matrix such that now it also includes the most recently generated price scenarios. Update the matrix A such that A is the new θxm matrix with the most recent θ price vectors including the new CSN. Go to Step 1.

The idea behind both Alg-1A and Alg-1B can be illustrated as in Figure 7.

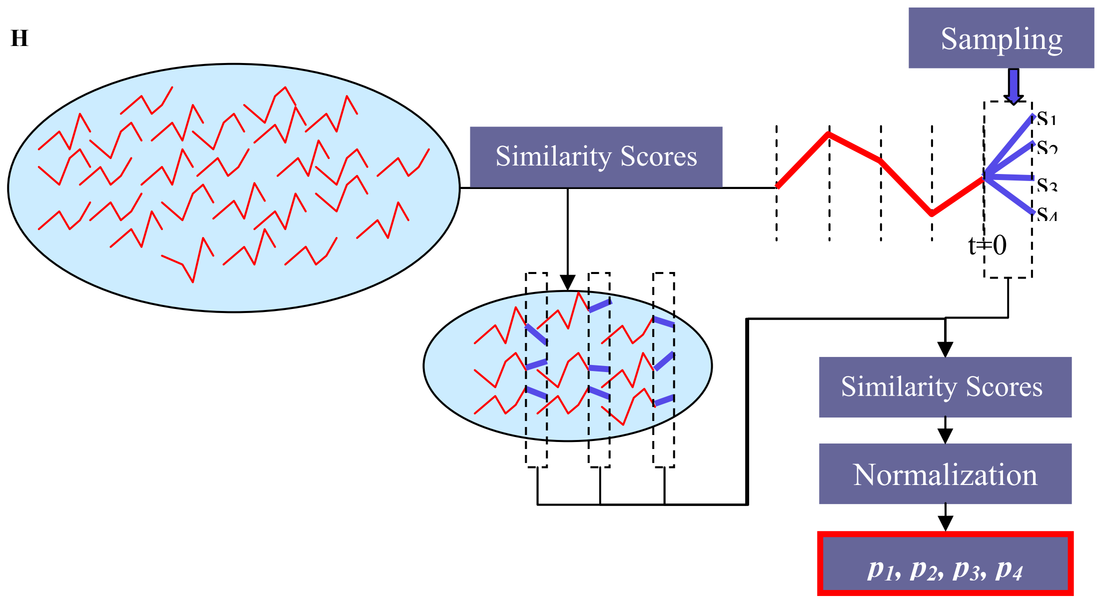

Figure 7. Illustration for Alg-1A and Alg-1B.

2.3. Probability Assignment through Moment Matching and Heteroskedasticity

Hoyland and Wallace (2001) use moment matching to assign probabilities to scenarios such that some statistical features of the generated scenarios comply with the statistical features of the historical data. In this study, the nonlinear optimization model given by (13) and (14) is solved taking the asset return scenarios and the corresponding probabilities as decision variables.

$$\min_{x,p} \sum_{i \in ST} w_i (f_i(x,p) - SV_i)^2 \tag{13}$$

$$s.t. \qquad \sum_{s}^{S} p_s = 1, \qquad p_s \geq 0 \tag{14}$$

In this model, x is the vector of returns whereas p is the vector of scenario probabilities. The set of statistical features is represented by ST and $f_i(x, p)$ and SV_i represents the i^{th} statistical feature obtained from generated data and the historical data, respectively. Moment matching approach is also employed by Gulpinar et al. (2004) where it is applied either sequentially on each node (i.e., scenario) or globally on the whole scenario tree, the first resulting in a number of small nonlinear optimization problems while the second yields one but a large nonlinear optimization problem.

Our approach is based on the same setup; however, we incorporate a *volatility clustering* approach to model the state dependency of variances and covariances via Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH). In addition, we present an additional set of constraints to improve performance of the SP model to be run at the final step. Third, we consider a linear version of the optimization model for moment matching. Another difference is that we generate asset returns first and take only the probability values as decision variables in optimization instead of optimizing both return and probability simultaneously because the latter leads to a nonlinear model.

2.3.1. Autoregressive Conditional Heteroskedasticity (ARCH)

The literature on financial time series includes empirical evidence for the fact that most financial time series data have volatility clusters at different time periods. In other words, the variance of data (e.g., stock returns, interest rates, etc.) is not constant over time. In fact, time periods with high (low) variance are followed by time periods with high (low) variance. This persistence of volatility in financial data was formally modeled first by Engle (1982). In his seminal paper, Engle presents his original ARCH (Autoregressive Conditionally Heteroskedastic) model that relates the variance conditionally to the previous periods' residuals.

Consider the following process on excess returns, R_t:

$$R_t = a + \varepsilon_t \tag{15}$$

Then ARCH(q) model assumes

$$\varepsilon_t = \sigma_t z_t \tag{16}$$

$$z_t \sim i.i.d.\ ,\ z_t \sim N(0,1) \tag{17}$$

$$\sigma_t^{\ 2} = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2 \tag{18}$$

where $\alpha_0 > 0$ and $\alpha_i \geq 0$ to make sure that conditional variance is positive.

Bollerslev (1986) proposed an extension of this ARCH model, Generalized ARCH (GARCH). In a GARCH setting, variance is conditioned on the previous periods' variances in addition to their residuals. The conditional variance equation for a GARCH(p,q) model is given in (19) where the inequalities $\alpha_0 > 0$, $\alpha_i \geq 0$, and $\beta_i \geq 0$ must hold in order to ensure having positive conditional variances.

$$\sigma_t^{\ 2} = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \ldots + \beta_p \sigma_{t-p}^2 \tag{19}$$

Upon being introduced by Engle (1982) and generalized by Bollerslev (1986), ARCH (GARCH) type models have been a popular approach for modeling stochastic volatility in a discrete time setting. However, a shortcoming of the GARCH models is that only the *magnitudes* of the shocks (innovations) are considered to affect to conditional volatility. Many researchers and practitioners on the other hand believe that stock returns are negatively correlated with stock return volatilities. Black (1976) is a very early study that addresses this issue calling it the *leverage effect*, which can be stated as: A drop in the value of a stock increases the financial leverage, which makes the stock riskier and thus increases the volatility. The original GARCH model was unable to capture the fact that bad news (negative shocks) tend to increase volatility whereas, good news (positive shocks) have a lowering effect.

2.3.2. EGARCH

Different extensions of GARCH models were proposed to have a more flexible conditional volatility modeling that captures the aforementioned asymmetric volatility behavior. Among the proposed extensions, the most popular ones are Exponential GARCH (EGARCH) of Nelson (1991), GJR-GARCH of Glosten et al. (1993), Asymmetric GARCH (AGARCH) of Engle and Ng (1993), the threshold GARCH (TGARCH) model of Zakoian (1994), and quadratic GARCH (QGARCH) of Sentana (1995). Bera and Higgins (1993), Bollerslev (1992), and Poon and Granger (2003) are some of the papers providing reviews of those models.

Having the asymmetric volatility as a major weakness of GARCH, another issue for most GARCH models is the non-negativity constraints on α_i ($i=0...q$) and β_i ($i=1...p$) for the purpose of obtaining positive σ_t^2 for all t in all cases. These constraints make the estimation of GARCH models more difficult and create issues regarding the persistence of volatility over time. In order to overcome these issue Nelson (1991) introduced the exponential GARCH (EGARCH). A frequently used version of EGARCH(p,q) is given in (20).

$$\ln\sigma_t^2 = \omega + \sum_{i=1}^{p}\alpha_i \ln\sigma_{t-i}^2 + \sum_{j=1}^{q}\beta_j \frac{\varepsilon_{t-j}}{\sigma_{t-j}} + \sum_{j=1}^{q}\gamma_j\left(\frac{\left|\varepsilon_{t-j}\right|}{\sigma_{t-j}} - E\left(\frac{\left|\varepsilon_{t-j}\right|}{\sigma_{t-j}}\right)\right) \qquad (20)$$

Since $z_t \sim i.i.d.$, $z_t \sim N(0,1)$, $\varepsilon_t = \sigma_t z_t$, and $E(|z_t|) = \sqrt{2/\pi}$, (10) can be reduced to

$$\ln\sigma_t^2 = \omega + \sum_{i=1}^{p}\alpha_i \ln\sigma_{t-i}^2 + \sum_{j=1}^{q}\beta_j z_{t-j} + \sum_{j=1}^{q}\gamma_j\left(\left|z_{t-j}\right| - \sqrt{2/\pi}\right) \qquad (21)$$

Review of the relevant literature shows that using first order lags (i.e., $p = q = 1$) sufficiently captures the heteroskedastic behavior of the most financial time series data. Therefore, we will consider the EGARCH(1,1) model given as

$$\ln\sigma_t^2 = \omega + \alpha\ln\sigma_{t-1}^2 + \beta z_{t-1} + \gamma\left(\left|z_{t-1}\right| - \sqrt{2/\pi}\right) \qquad (22)$$

In contrast with most of the other ARCH models, the restrictions on the parameters (ω, α, β, and γ) are removed since their values have no effect on the sign of σ_t^2. The utilization of the logarithms ensures always having a positive conditional variance. A careful analysis of (22) reveals that σ_t^2 is now a function of both the *sign* and the *magnitude* of z_t, which enables the model to capture asymmetric volatility. Through the last term in the expression, EGARCH captures the magnitude effect of the shocks such that high (low) volatility periods are followed by high (low) volatility periods (i.e., volatility clustering).

The reason to select EGARCH to incorporate in our scenario generation algorithm is its well known capability to capture the heteroskedastic behavior of financial time series data and cope well with asymmetric volatility. Relevant literature agrees on the superiority of the asymmetric GARCH models over regular GARCH models and even though there is not a single and 100% agreed-upon model, EGARCH turns out to be one of the most popular (asymmetric) GARCH models. Some papers include experimental studies testing different GARCH models and supporting EGARCH, such as Awartani and Corradi (2005) and Bali (2007).

2.3.3. A New Set of Constraints for Probabilities

We introduce a new set of constraints for the usual moment matching model. The motivation behind this approach is to partially remove a potential shortcoming of the usual moment matching method that can be realized when multi-period models are involved.

Suppose that we generate a symmetric scenario tree for the SP model and let nn_t denote the number of single-period scenarios (i.e., nodes) emanating from a source node at time

t-1 where t=1..T and T is the length of the horizon. For example, considering the symmetric scenario tree in Figure 2, one would have nn_1 =4, nn_2 =3, and nn_3 =2. We consider matching the moments for every single scenario set. For instance, the tree in Figure 1 leads to 17 moment matching processes, which equals the number of nodes in the tree except the *leaf* nodes.

Every moment matching process assigns probabilities to the corresponding set of nodes, some of which can be zero as a result of the optimization. Considering the tree in Figure 1, one would have 24 *three-period* scenarios at the horizon and the final probabilities for these scenarios are computed by multiplying the probabilities of preceding *one-period* scenarios. A low probability assigned in one of the preceding nodes leads to a low probability for the three-period scenario. In fact, if a node is assigned a probability of zero, then the rest of the tree emanating from that node is ignored by the SP model since the following three-period scenarios will assume a zero probability. For example, suppose that one of the four one-period scenarios for the first period of the tree in Figure 2 is assigned a zero probability; then 6 three-period scenarios emanating from that node will be disregarded by the SP model. In general, $\prod_{j=t+1}^{T} nn_j$ scenarios will be disregarded at the horizon when a one-period scenario assumes a zero probability at time t.

Even though zero probability scenarios are statistically acceptable to get close to the specified moments, this would produce an unwanted probability distribution at the horizon, where the expected wealth is computed. We believe that as the number of scenarios with zero (or close to zero) probabilities increase; the SP model will input fewer *T-period* scenarios, which might worsen the robustness of decisions led by the SP model. Therefore, *we propose setting lower-bounds for the probabilities during the moment matching process*. The decision on the levels of bounds is an open question and higher bounds will obviously lead to large deviations from the target moments; however, these parameters can be computationally calibrated by back-testing given a set of data.

To illustrate the issue, we present the probability values we obtained during a test case using a symmetric tree where T =2, nn_1 =30, and nn_2 =15. It is observed from Table 2 (See Appendix A) that only 25 of the 450 scenarios have nonzero probability with accuracy to four decimal places.

2.3.4. Algorithm Description: Alg-2

We now describe some notation for the simplicity of the presentation. Suppose that ND represents the set of all nodes in the symmetric scenario tree excluding the leaf nodes. Suppose also that ND^n is the set of nodes that emanate from a particular node n. Once the scenario realizations are generated and the target moments are set, the problem is to assign probabilities for all the scenarios except the source node of the tree.

At this point there are two possible approaches to match the moments, first being to solve a large scale optimization model that will assign probabilities to all scenarios in a single run. We prefer the alternative approach, which is based on solving one optimization model for each of the nodes in ND so that every set of scenarios is individually considered. Recalling the notation described in previous sections, the number of optimization models to be solved can be simply computed as $1+\sum_{t=1}^{T}\prod_{j=1}^{t} nn_j$.

Using the previous notation, *i* is used as the index for risky assets, where we have *m* risky assets. The algorithm can be stated as follow:

Step 0 : Generate asset return realizations for all nodes in the tree by randomly sampling the historical arithmetic returns (i.e., *H*).

Step 1 : Estimate the third and fourth central moments, M_{i3} and M_{i4} (i=1..m), for all risky assets. Estimate the correlation matrix *R* among risky assets.

Step 2 : Estimate the *univariate* EGARCH(1,1) models for *each* risky asset to obtain parameters $a, \omega, \alpha, \beta,$ and γ by maximum log-likelihood method.

Step 3 : Using the constant-mean model and the scenario realizations, obtain the error terms ε for each asset and node $n \in ND$.

Step 4 : Using the estimated EGARCH(1,1) model and the error terms, predict for all risky assets the conditional variances M_{i2}^{n} for each $n \in ND$.

Step 5 : Using the correlation matrix R and the conditional variances, compute the conditional covariance matrix Σ^{n} for each $n \in ND$ (See the explanation below for the constant correlation model).

Step 6 : Solve the optimization problem (23)-(30) to find the optimum probabilities *for each* $n \in ND$ such that the weighted sum of distances to the target moments are minimized.

Min

$$\sum_{i=1}^{m}\left(\sum_{k=1}^{4} w_{ik}\,(m_{ik}^{+}+m_{ik}^{-})+\sum_{i<l} w_{il}\,(c_{il}^{+}+c_{il}^{-})\right) \tag{23}$$

st

$$\sum_{s\in ND^n} p_s = 1 \tag{24}$$

$$\sum_{s\in ND^n} r_{is}\,p_s + m_{i1}^{+} - m_{i1}^{-} = a_i \qquad \forall i \tag{25}$$

$$\sum_{s\in ND^n} (r_{is}-a_i)^2\,p_s + m_{i2}^{+} - m_{i2}^{-} = M_{i2}^{n} \qquad \forall i \tag{26}$$

$$\sum_{s\in ND^n} (r_{is}-a_i)^k\,p_s + m_{ik}^{+} - m_{ik}^{-} = M_{ik} \qquad \forall i; k=3,4 \tag{27}$$

$$\sum_{s\in ND^n} (r_{is}-a_i)(r_{ls}-a_l)\,p_s + c_{il}^{+} - c_{il}^{-} = \Sigma_{il}^{n} \qquad \forall i; i<l \tag{28}$$

$$p_s \ge LB^n \qquad s\in ND^n \tag{29}$$

$$p_s, m_{ik}^{+}, m_{ik}^{-}, c_{il}^{+}, c_{il}^{-} \ge 0, \qquad s\in ND^n; \forall i, i<l, k=1..4 \tag{30}$$

In (23)-(30), w_{ik} and w_{il} are the coefficients that capture the relative importance of the moments. The variables denoted by m_{ik} and c_{il} are employed to compute the deviations.

The second moments are time-dependent and are computed conditionally on the previous periods' realizations (i.e., scenarios) using EGARCH at every node throughout the scenario tree.

The correlation between assets has been addressed by researchers through Multivariate-GARCH (MGARCH) models (for a recent and comprehensive review on MGARCH type modeling, see Bauwens et. al. (2006)). Even though these models are successful in capturing the conditional correlation among risky assets, an important shortcoming is the high number of parameters to be estimated, which brings in further computational concerns. Therefore, researchers proposed various versions of MGARCH models that provide less flexibility but come with simplicity. One of these studies is the Constant Conditional Correlation GARCH (CCC-GARCH) proposed by Bollerslev (1990). This approach decreases the number of parameters to be estimated significantly by assuming that the correlations among assets are constant over time. In CCC-GARCH, the conditional variances are computed via some univariate GARCH model (e.g., EGARCH) and then the covariance matrix is constructed using the initial correlations and conditional variances.

Let D_n be the mxm diagonal matrix formed by M_{i2}^{n} values where i=1..m. According to the CCC-GARCH model, the conditional covariance matrix for node n, is computed as $\Sigma^{n} = D_n R D_n$. We use the elements of this covariance matrix in (28) in order to match covariance.

Constraint (29) puts a lower bound on the probability values for each node as discussed in Section 2.3.3. The value of the parameter LB^{n} is subject empirical tests and can be computationally calibrated over a series of back-testing procedures. Even though we can not claim a specific value, it is immediately realized that $LB^{n} < 1/\left|ND^{n}\right|$ for all n.

Therefore LB^n can be written as $LB^n = lb^n \times 1/\left|ND^n\right|$ so that lb^n can be calibrated by experimenting with values of $lb^n \in (0,1)$.

2.4. Stochastic Programming Model

This section presents the mathematical notation and the details of the optimization model, which is a multi-stage SP model. The outcomes that are of interest are the *first stage variables* representing the rebalancing decisions (i.e., how much to buy/sell each asset at the time of running the model). Assuming that the investor continues investing over the discrete time periods, the process of managing the portfolio is based on updating the data sets and the initial portfolio and re-running the model at the beginning of each period (See Figure 8).

2.4.1. Parameters

The main parameters used in the SP model can be grouped as:

- *Asset return scenarios and scenario probabilities*: These one period rates and probabilities are obtained via the scenario generation algorithms presented in Sections 2.2 and 2.3. They are the most critical parameter set since the model captures the real life uncertainty through these parameters.
- *Economic parameters*: Parameters such as transaction costs, interest rates and initial security prices can be considered in this group depending on the implemented investment scheme.

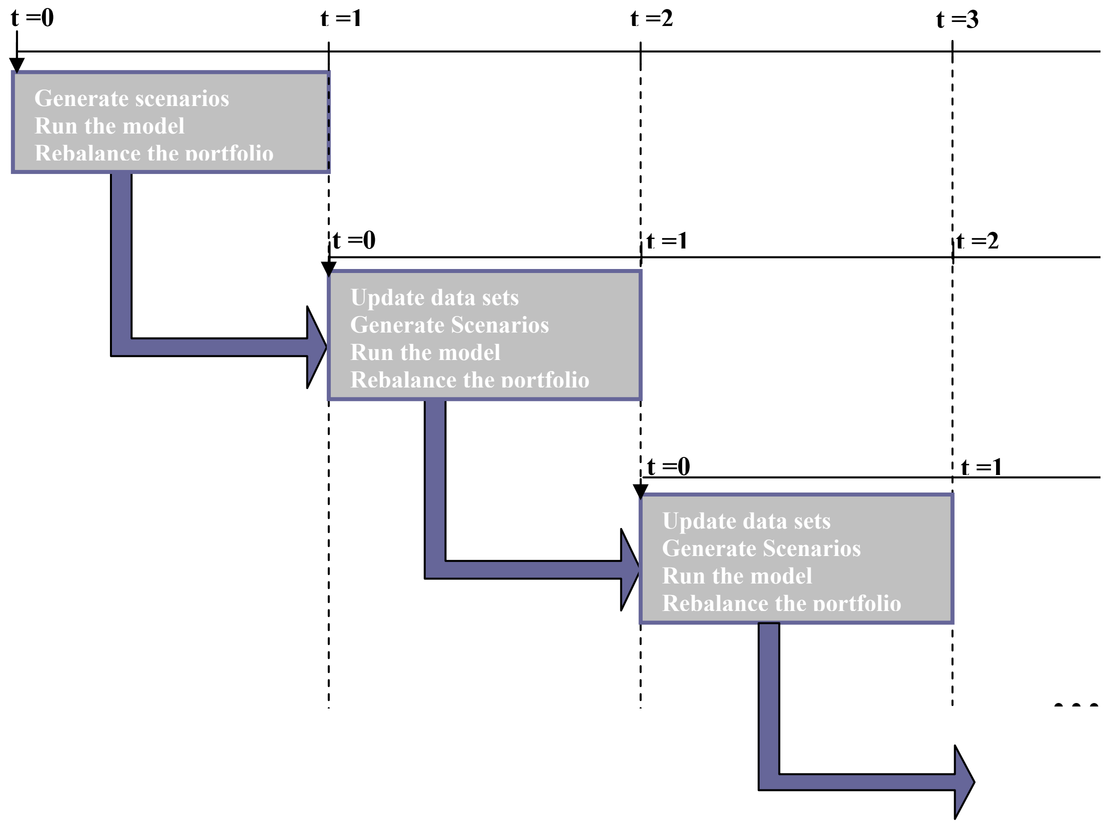

Figure 8. Portfolio management process

- *User-defined parameters*: Some parameters are set by the investor according to preferences or restrictions. Experimenting with these parameters may give valuable insight about how the model will behave in different environments (e.g., parameters controlling the risk averseness and exposure, statistical confidence levels, weights for different objectives, upper/lower bounds on asset positions, etc.).
- *Cash flows*: This group may contain dividend payments and possible liability obligations depending on the investment scheme.

In this chapter we provide a general *deterministic* version of the SP model built for this study.

NOTE: For simplicity of the presentation, the variables that apply to all risky assets are given as vectors shown in bold. Index i to will be appended later to represent the individual elements of the vector.

Sets:

s : Scenarios

t : Time periods

I : Set of monetary assets available ($|I|=m$)

N_T : Set of scenarios for $t=T$ (T is the horizon length)

Parameters:

y : Risk-free asset (cash) available at $t = 0$ (in monetary units)

$\mathbf{w}$: Vector for the amount of assets in the current portfolio (in monetary units)

$\mathbf{P_{ts}}$: Vector of security prices

rb_{ts} : One period risk-free investment rate

$\mathbf{r_{ts}}$: Vector of one period returns for securities

L_{ts} : Liabilities or cash outflows

$\mathbf{c_{ts}}$: Vector of cash flows generated by securities (per unit share)

p_s : Probabilities of a scenario at $t = T$

ε : Unit transaction cost

ub, lb : Vectors of upper and lower bounds for securities (weights in the portfolio)

δ : Spread between borrowing and lending rates

β_k : Weight of the k^{th} objective in the objective function

α : Confidence level for CVaR.

LCVaR: Limit for CVaR

2.4.2. Variables

The variables in this model are of two types:

- *Variables for decisions*: In this set are the variables regarding the portfolio rebalancing and cash flow decisions. In other words, the decisions of how much to sell/buy each of the securities; how much to invest in risk-free asset; and how much to borrow/lend are made through these variables.
- *Other variables*: These variables are implied by the investment decisions such as expected wealth, auxiliary variables, etc.

Since the optimization will be achieved within an SP model, we group the variables as first stage and t^{th} stage variables.

2.4.2.1. First stage variables

These are the variables that will be actually practiced by the decision maker. They are time and scenario independent variables giving information on the optimum rebalancing decisions at $t = 0$ after the optimization model is run. The multi-period portfolio management framework proposed in this study is built on the successive running of the model at the beginning of each period and practicing the first stage variables immediately

(See Figure 8). These first stage variables can be regarded to have a time index as $t = 0$. The variables used in our model are:

$\mathbf{xb_0}$: Vector of amounts of securities bought (in monetary units)

$\mathbf{xs_0}$: Vector of amounts of securities sold (in monetary units)

$\mathbf{z_0}$: Vector of amounts of securities held after purchase and selling (in monetary units)

y_0 : Amount of cash invested in risk-free asset

b_0 : Amount of cash borrowed

2.4.2.2. Variables for stage *t*

Different than the first stage variables, variables for stage t are recourse variables which determine the corrective actions as future information is realized over the scenario tree after the first stage decisions are made. They are not actually practiced by the decision maker. The t^{th} stage variables used in our model are:

$\mathbf{xb_{ts}}$: Vector of amounts of securities bought (in monetary units)

$\mathbf{xs_{ts}}$: Vector of amounts of securities sold (in monetary units)

$\mathbf{z_{ts}}$: Vector of amounts of securities held after rebalancing (in monetary units)

y_{ts} : Amount of cash invested in risk-free asset

b_{ts} : Amount of cash borrowed

2.4.2.3. Other Variables

These variables are used to get aggregate information on the outcomes of all investment decisions. This information might be used to achieve different types of performance analysis (see Sections 2.4.3.5-2.4.3.8 for further information on these variables and their indices).

W_s : Wealth at the end of the horizon

W_t : Wealth at intermediate periods

Dev_t^+ : Expected positive deviation

Dev_t^- : Expected negative deviation

EW : Expected final wealth

MW_n : The variable used to limit the worst case scenario wealth.

INS : The variable used to measure the instability.

u_{nj}, ζ_n : Auxiliary variables used for CVaR constraints

2.4.3. Deterministic Version of the Stochastic Programming Model

The model presented in this section is the deterministic version of the stochastic programming model built for the optimization step of the multi-period portfolio management framework.

2.4.3.1. Initialization and Restrictions

As mentioned in Section 2.4.2.1, first stage variables are time and scenario independent. These first stage parameters must be linked to time dependent variables at $t = 0$. In other words, the time dependent variables are initialized to their first stage counterparts. Variables for sales/purchases and security holdings are initialized in (31)-(33). The cash-related initializations are done through (34) and (35).

$$\mathbf{xb_{0s}} = \mathbf{xb_0} \qquad \forall s \qquad (31)$$

$$\mathbf{xs_{0s}} = \mathbf{xs_0} \qquad \forall s \qquad (32)$$

$$\mathbf{z_{0s}} = \mathbf{z_0} \qquad \forall s \qquad (33)$$

$$y_{0s} = y_0 \qquad \forall s \qquad (34)$$

$$b_{0s} = b_0 \qquad \forall s \qquad (35)$$

A very typical set of restrictions that might be posed by investors are the lower and upper bounds for the weights of particular assets in their portfolios. These might be personal, institutional or legal restrictions. Constraints (36) and (37) assure that the weights are within the allowed limits.

$$z_{tsi} \leq ub_i \sum_{i=1}^{m} z_{tsi} \qquad \forall s,i,t \qquad (36)$$

$$z_{tsi} \geq lb_i \sum_{i=1}^{m} z_{tsi} \qquad \forall s,i,t \qquad (37)$$

Another reasonable restriction that must be included in order to have a practically feasible solution is to prevent last time borrowing. In addition, no trading must take place at the very end of the planning horizon. Considering the planning horizon of *T*, any transaction will have to be closed at the end in order to evaluate the true performance of the portfolio. These restrictions can be imposed by the constraints in (38).

$$b_{Ts} = 0 \;;\; \mathbf{xb_{Ts} = 0};\; \mathbf{xs_{Ts} = 0} \qquad \forall s \qquad (38)$$

Short-selling (i.e., the trading strategy where the seller does not have the stock when he sells it with the expectation of a price fall during the delivery of the asset) prevention and other sign restrictions are satisfied by (39).

$$\mathbf{xb_{ts}}, \mathbf{xs_{ts}}, \mathbf{z_{ts}}, y_{ts}, b_{ts} \geq 0 \qquad \forall s,t \qquad (39)$$

2.4.3.2. Asset/Cash Balance

Variables in consecutive periods are connected through asset balance equations. Eq. (40) captures the relation between the current portfolio and the rebalanced portfolio using the first stage variables related to sell/purchase decisions. Eq. (41) captures the same relation between other time periods and scenarios.

$$\mathbf{w\text{-} xs_0 + xb_0 = z_0} \qquad (40)$$

$$z_{(t\text{-}1)si}(1+r_{tsi}) - xs_{tsi} + xb_{tsi} = z_{tsi} \qquad \forall s,i,t = 1..T \qquad (41)$$

Cash balance equations are given by (42) and (43). Eq. (42) builds the cash flow balance at t = 0, whereas (43) stands for the latter periods. In (43), the first term on the left is the investment in the risk-free asset (cash) and the second is the proceeds from dividend payments. The cash obtained from security sales are in the third term with applied transaction cost. The last term on the left is the borrowed cash, if any. The summation of these four terms is equal to the sum of outflows for cash investment, security purchase, liability obligation, and the pay-back for the previous periods borrowing, respectively. In the last equation (44) in this group, we compute the final wealth for each scenario for use in portfolio performance analysis.

$$y + b_0 + (1-\varepsilon)\sum_{i=1}^{m} xs_{0i} - (1+\varepsilon)\sum_{i=1}^{m} xb_{0i} = y_0 \tag{42}$$

$$y_{(t-1)s}(1+rb_{ts}) + \sum_{i=1}^{m}(c_{tsi}z_{tsi}/P_{tsi}) + (1-\varepsilon)\sum_{i=1}^{m} xs_{tsi} + b_{ts}$$

$$= y_{ts} + (1+\varepsilon)\sum_{i=1}^{m} xb_{tsi} + L_{ts} + b_{(t-1)s}(1+rb_{ts}+\delta) \quad \forall s, t = 1..T \tag{43}$$

$$W_s = y_{Ts} + \sum_{i=1}^{m} z_{Tsi} - b_{(T-1)s}(1+rb_{Ts}+\delta) \qquad \forall s \tag{44}$$

2.4.3.3. Non-Anticipativity

Non-anticipativity constraints are given in (45)-(49). These constraints make sure that the scenarios with common history up to time *t* (s and s' in (45)-(49)) will have the same values for all decision variables up to time *t*. In other words, exploitation of the future information (see Section 2.1. for the illustration) is prevented. Construction of these constraints strictly depends on the scenario tree topology and needs specific attention.

$$\mathbf{xb_{ts}} = \mathbf{xb_{ts'}} \qquad \forall t \tag{45}$$

$$\mathbf{xs_{ts}} = \mathbf{xs_{ts'}} \qquad \forall t \tag{46}$$

$$\mathbf{z_{ts}} = \mathbf{z_{ts'}} \qquad \forall t \tag{47}$$

$$y_{ts} = y_{ts'} \qquad \forall t \tag{48}$$

$$b_{ts} = b_{ts'} \qquad \forall t \tag{49}$$

2.4.3.4. CVaR

Any investment management scheme must consider the risk exposure led by the investment decisions. Risk management is a broad subject that has applications in numerous domains, financial risk being one of the most commonly researched ones.

Financial risk, depending on the context and type of the investment, may take different names such as capital risk, credit risk, interest rate risk, liquidity risk, default risk, market risk, etc. Even though they have different names and definitions, all refer in some way to monetary loss. The financial industry created plenty of instruments in order to hedge financial risk (e.g., derivatives) and there is a very broad literature addressing financial risk in different contexts, which would be beyond the scope of this study to summarize here. We can briefly state our approach as the measurement and minimization of the downside risk of the portfolio.

A very common risk measure used for portfolio management is Value-at-Risk (VaR), which can be defined as the maximum loss of a portfolio that can not be exceeded given a confidence level. VaR is defined over a fixed time interval, which is the time between the initial investment and the final valuation of the portfolio. Figure 9 illustrates VaR over a histogram of portfolio loss, given a confidence level of β.

Despite its high popularity, VaR has received many critics. First, VaR does not provide any information regarding the severity of loss. Second, it is not a coherent risk measure according to the criteria proposed by Artzner et al. (1999). For instance VaR of two instruments can be greater than the sum of individual VaRs. Non-convexity is another issue raised by VaR critics (Uryasev (2000)).

Conditional Value-at-Risk (CVaR) is a risk measure that addresses the aforementioned issues and provides some other implementation-related advantages. Minimizing CVaR leads to solutions with low VaR and in fact they both result in the same decisions when the return-loss distribution is normal (Uryasev (2000)). Verbally, it is the expected loss, given that the loss exceeds VaR. In other words, it is the weighted average of VaR and losses exceeding VaR. Therefore, $CVaR \geq VaR.$ strictly holds (See Figure 9).

We incorporate into our model a discrete version of CVaR as a constraint set. Here is a brief definition of CVaR:

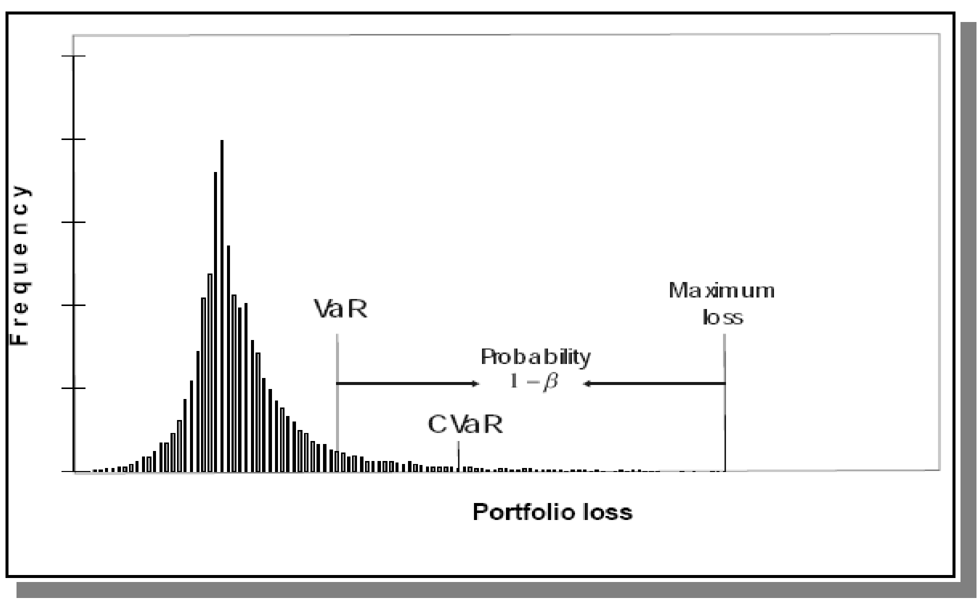

Figure 9. Illustration of VaR and CVaR (Uryasev (2000)).

Apart from the notation presented previously in this text, let *f(x,y)* be the loss function associated with decision vector *x* and random vector *y*. Supposing that *y* has a density, the cumulative function of this loss function is

$$\psi(x,\zeta) = \int_{f(x,y)\le\xi} p(y)dy \tag{50}$$

Given the confidence level α,

$$VaR_\alpha = \xi_\alpha(x) = \min\{\xi \in \Re : \psi(x,\zeta) \ge \alpha\} \tag{51}$$

$$CVaR_\alpha = \phi_\alpha(x) = (1-\alpha)^{-1} \int_{f(x,y)\ge\zeta_\alpha(x)} f(x,y)p(y)dy \tag{52}$$

Define

$$F_\alpha(x,\zeta) = \zeta + (1-\alpha)^{-1}\int [f(x,y)-\zeta]^+ p(y)dy \tag{53}$$

Then $CVaR_\alpha$ is obtained as

$$\phi_\alpha(x) = \min_{\zeta\in\Re} F_\alpha(x,\zeta) \tag{54}$$

If y has discrete scenarios with probabilities π_j instead of a continuous distribution, then (53) can be approximated as

$$\tilde{F}_\alpha(x,\zeta) = \zeta + (1-\alpha)^{-1}\sum_{j=1}^{J}\pi_j[f(x,y)-\zeta]^+ \tag{55}$$

Then, using auxiliary variables u_j, CVaR is restricted above by w through the constraints (56) and (57).

$$\zeta + (1-\alpha)^{-1}\sum_{j=1}^{J}\pi_j u_j \le w \tag{56}$$

$$u_j \ge f(x, y_j) - \zeta, \qquad u_j \ge 0, \qquad \forall j \tag{57}$$

Suppose that an investor defines his loss function as the difference between the initial portfolio value and the final wealth, which is obtained through discrete scenarios and denoted by W_s. Therefore, given the confidence level of α and upper limit *LCVaR*, constraints (58)-(60) impose control CVaR.

$$\zeta + (1-\alpha)^{-1}\sum_{s\in N_T} p_s u_s \le LCVaR \tag{58}$$

$$u_s \ge \sum_{i=1}^{m} w_i + y - W_s - \zeta \qquad \forall s \tag{59}$$

$$u_s \ge 0 \qquad \forall s \tag{60}$$

It should be noted that the actually realized CVaR may not be strictly controlled by *LCVaR* since, constraints (58)-(60), as all other constraints in the model, assume that the inputted scenario tree reflects the true future uncertainty (i.e., the true scenario tree). Since the generated scenario tree is an approximation, constraints (58)-(60) provide an approximate risk control.

2.4.3.5. Worst Case Scenario

Investors benefit from robustness especially when out-of-sample scenarios come true. It should be noted that not all out-of-sample scenarios are critical in terms of robustness since some of them may be captured to some extent by the in-sample scenarios. Plus, robustness can not remedy all problems caused by an out-of-sample scenario but can decrease the realized *severity* according to the structure of the scenario.

SP models can not include all possible scenarios due to complexity issues but approximate the actual universal set of scenarios in order to achieve optimization under uncertainty. Therefore, depending on the scenario generation process, there might be a tradeoff between the robustness and computational complexity. Given a fixed number of scenarios, an SP model based approach may increase the robustness by including some worst-case scenarios and optimizing the objective function for those unexpected scenarios. The worst case scenario results in the minimum objective value (for a maximization problem); therefore, maximizing the minimum possible outcome may improve the robustness of the decisions.

2.4.3.6. Dynamic Risk Control

It is expected that an investor would be interested in the performance of his investment strategy solely at the end of the planning horizon, therefore tolerate losses in the intermediate periods. However, when multi-period models are involved in decision making, measuring and controlling the risk of an investing strategy *merely* at the end of the horizon may lead to unwanted results. If is risk control is achieved only to the end of the horizon and a multi-period model with a scenario tree is utilized, the model would result in decisions that may lead to deviations from the targeted risk exposure in the former periods as long as performance constraints at the end of the horizon are not violated.

The issue can be understood better by considering the investment process in Figure 8. Even though the investor has a multi-period model, he will re-run the model at the beginning of each period and exercise the first stage decisions provided by the model. Therefore, the performance of the resulting trading will be highly dependent on the first stage decisions. If risk is not controlled within the first period, then the first stage variables will assume values such that the objective at the end will be optimized, which may lead to high risk exposure for the first period. Repetition of the investment process eventually creates high risk exposure in the long run even if it is controlled in the last period of the model.

In order to decrease the deviation from the desired risk level, the proposed model constrains CVaR for each node of the scenario tree, other than the leaf nodes. In other words, risk is controlled by limiting CVaR at each decision epoch considering the one period ahead risk exposure and, in turn, spreading risk control over the whole scenario tree.

The can be illustrated via a simple case. Suppose that ω_n is a decision vector that corresponds to node n and ρ is a risk measure such that $\rho(\omega_n)$ is the risk exposure caused by decision vector ω_n. Let RL_n be the risk limit to be imposed on the decisions for node n. Consider the simple scenario tree in Figure 10 which is used to solve a two-period problem. Then the constraints given by (61) provide a more powerful risk control over the scenario tree than the case where risk control is accomplished merely at the horizon over the leaf nodes represented by smaller circles.

$$\rho(w_n) \le RL_n \quad ; n = 1..11 \tag{61}$$

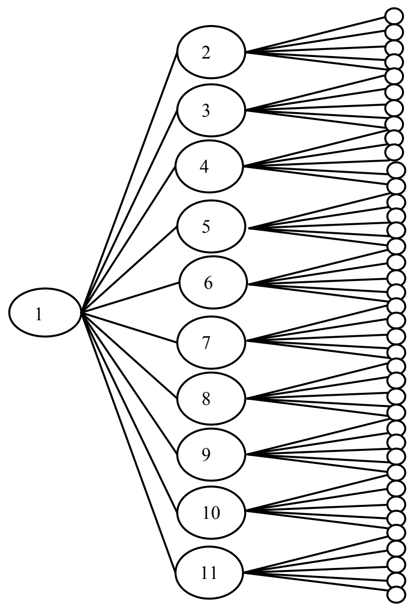

Figure 10. A simple scenario tree with two periods

Recalling *ND* represents the set of all nodes in the scenario tree for the proposed SP model excluding the leaf nodes and that ND^n is the set of nodes that emanate from node *n*, the idea of dynamic risk control can be combined with the linear CVaR constraints as in (62)-(64) to be incorporated into the SP model. Note that we will use index *n* instead of *s* for simplicity.

$$\zeta_n + (1-\alpha)^{-1} \sum_{j \in ND^n} p_{nj} u_{nj} \le LCVaR \qquad \forall n \in ND \tag{62}$$

$$u_{nj} \ge \sum_{i=1}^{m} z_{tni} + y_{tn} - \sum_{i=1}^{m} z_{tni}(1+r_{t+1ji}) - y_{tn}(1+rb_{t+1j}) - \zeta_n \qquad \forall n \in ND, \forall j \in ND^n \tag{63}$$

$$u_{nj} \geq 0 \qquad \forall n \in ND, \forall j \in ND^n \quad (64)$$

In (62)-(64), t represents the time period that node n implies in the tree and p_{nj} is the probability of the scenario represented by node j, which emanates from node n such that $\sum_{j \in ND^n} p_{nj} = 1$ for $\forall n \in ND$. $LCVAR$ is the same for all nodes; however, different values can be used to obtain different risk limits for different periods and/or different nodes, which might bring in further flexibilities in risk control. We build the loss function for CVaR to be the one-period-ahead loss given by $\sum_{i=1}^{m} z_{tni} + y_{tn} - \sum_{i=1}^{m} z_{tni}(1 + r_{t+1ji}) - y_{tn}(1 + rb_{t+1j})$.

The same setting can be used for other risk measures such as maximizing the wealth for the worst possible scenario after each node. For each node n, denote by MW_n the minimum wealth resulting from the decision vector of node n, which can be placed in the objective for maximization and constrained above by the wealth for each scenario emanating from node n as in (65).

$$\sum_{i=1}^{m} z_{tni}(1 + r_{t+1ji}) + y_{tn}(1 + rb_{t+1j}) \geq MW_n \qquad \forall n \in ND, \forall j \in ND^n \quad (65)$$

2.4.3.7. Stability over the Horizon

Another performance measure that might be of interested is the fluctuation of the portfolio value over time. Given a fixed horizon, less fluctuating portfolio returns over oscillating portfolio returns if the final wealth levels do not differ significantly might be preferable. It should be noted that this measure obviously has overlapping characteristics with other risk measures such as *variance* or *volatility*. However, direct minimization of these measures implies nonlinearity. The stability can be improved by minimizing the expected deviations between the portfolio returns of consecutive periods within a linear approach as in (66)-(69) where *INS* denotes an average measure for instability.

$$W_t = \sum_{s \in N_T} p_s \left(\sum_{i=1}^{m} z_{tsi} + y_{ts} \right) \qquad t = 1...T \qquad (66)$$

$$W_t = W_{t-1} + Dev_t^+ - Dev_t^- \qquad t = 1...T \qquad (67)$$

$$INS = \frac{1}{T} \sum_{t=1}^{T} \left(Dev_t^+ + Dev_t^- \right) \qquad (68)$$

$$W_0 = \sum_{i=1}^{m} w_i + y; \quad Dev_t^+, Dev_t^- \geq 0 \qquad t = 1...T \qquad (69)$$

2.4.3.8. Objective

Like many other portfolio management models, maximizing the expected return of the portfolio at the end of the horizon is the major objective. In order to be able to consider the expected return in the objective function, (70) is included in the constraint set.

$$EW = \sum_{s \in N_T} p_s W_s \qquad (70)$$

Following the discussions in Sections 2.4.3.5 and 2.4.3.7, a *general* objective function can be written as:

$$\beta_1 EW + \beta_2 \sum_{n \in ND} MW_n - \beta_3 INS \qquad (71)$$

where β_1, β_2, and β_3 are the weights preferred by the investor that measure the relative importance of expected final wealth, worst case scenario improvement, and stability, respectively. Therefore, a comprehensive SP model can be given by

Max (70)

Subject to (31)-(49) and (62)-(70).

3. COMPUTATIONAL RESULTS and SENSITIVITY ANALYSIS

In this chapter we present the results of the implementation of the proposed approach, which includes three scenario generation algorithms (Alg-1A, Alg-1B, and Alg-2) and a Stochastic Programming (SP) model. Section 3.1 presents the computational results obtained via Alg-1A and Alg-1B (i.e., the algorithms based on similarity scores) and Section 3.2 presents the results led by Alg-2 (i.e., the algorithm based on moment matching and heteroskedasticity).

3.1. Algorithm-1A and Algorithm-1B

Even though Algorithm-1A (Alg-1A) and Algorithm-1A (Alg-1B) can be implemented for problems with multiple risky assets, in this section we assume that the decision maker has two investment options in total, one risky and one risk-free asset. Results are obtained through both single-period and multi-period investing strategies and analyzed in terms of their risk-return profiles.

3.1.1. Setup for Computations

In this section, some details regarding the implementation process such as the input data and selection of some parameters are provided.

3.1.1.1 Data

The risky asset is represented by the index S&P 500, which is a commonly used indicator of the stock market. The time unit for the decisions is assumed to be one week. Different than some SP approaches, the time period remain constant over the different stages in the planning horizon. In other words, stages of the SP model correspond to consecutive one-week periods.

The historical data for S&P 500 covers 301 index values for each week between 10/15/2001 and 7/16/2007, which is illustrated in Figure 11. Applying (3), we obtain 300 weekly arithmetic return values for the index. The *average weekly return* over this time frame turns out to be 0.1368%.

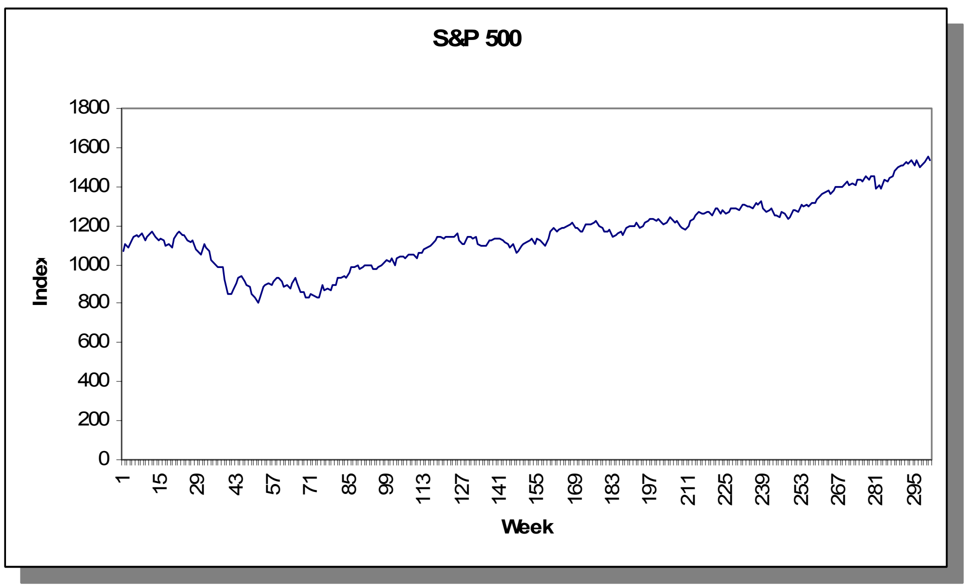

Figure 11. S&P 500 index on a weekly basis.

We assume that the short term interest rate is fixed and represents the risk-free asset. In addition, the length of planning horizon will be short enough (few weeks) to support this assumption. The *weekly* risk-free rate is assumed to be 0.04%.

3.1.1.2. Parameters

Both the scenario generation algorithms and the general SP model presented earlier include many different parameters. However, we implemented a simplified version of the approach presented in Chapter 2. In what follows is the discussion about the important parameters in the model:

SP MODEL:

- *LCVAR*: This parameter is used in the SP model in order to control CVaR by imposing an upper limit. Trying different values for *LCVAR* would give reliable information on the behavior of the model with respect to changing risk limits. Considering the loss function embedded in (63) with an initial wealth of \$1,000, we used various *LCVAR* values starting from 10 in different experiments.
- α: Confidence level for CVaR is 0.90.
- β_k: For the computations in this report, only the expected final wealth is considered. Therefore, β_1=1, β_2=0, β_3=0.
- ε: Unit transaction cost is taken to be \$0.001 unless specified as different values.
- *ub, lb*: No limitation is imposed on security weights in the portfolio.
- δ: Borrowing is not allowed for the implementation.
- L_{ts}: It is assumed that there are no liabilities.

- c_{ts}: We assume that there are no dividend payments from the risky asset (i.e., S&P 500). Therefore c_{ts} and P_{ts} are not considered for the model presented in this chapter.

SCENARIO GENERATION:

- *T*: The planning horizon is 4 weeks in this study.
- θ: The length of return paths is selected to be three weeks, which is specified with four consecutive data points.
- *Scenario Tree Topology*: Different numbers of branches per node are used in this study. For all computations the number of scenarios in the first period is 40. In the remaining periods, this number is 4, 2, and 2, respectively. Therefore we create a total of 40x4x2x2=640 scenarios at the horizon. Having more branches per node at the initial periods is a common practice in a tree structure used for SP models. This is partly because the uncertainty in the first period plays a more crucial role than the following periods. Another reason is that, since the scenarios are built upon the previous scenarios, such a structure guarantees to have high number of scenarios at each period. The resulting scenario tree that is used in the computations is partly illustrated in Figure 12.

3.1.1.3. Implementation

The scenario generation algorithms are implemented in MATLAB version 7.1. The input for MATLAB is the set of index values for 301 consecutive weeks. Then the algorithms are run and the outputs are written on different MS-EXCEL sheets. The outputs are return scenarios together with their corresponding probabilities. In addition, the sets for non-anticipativity constraints and some other sets and parameters are also printed in specific locations in the workbook.

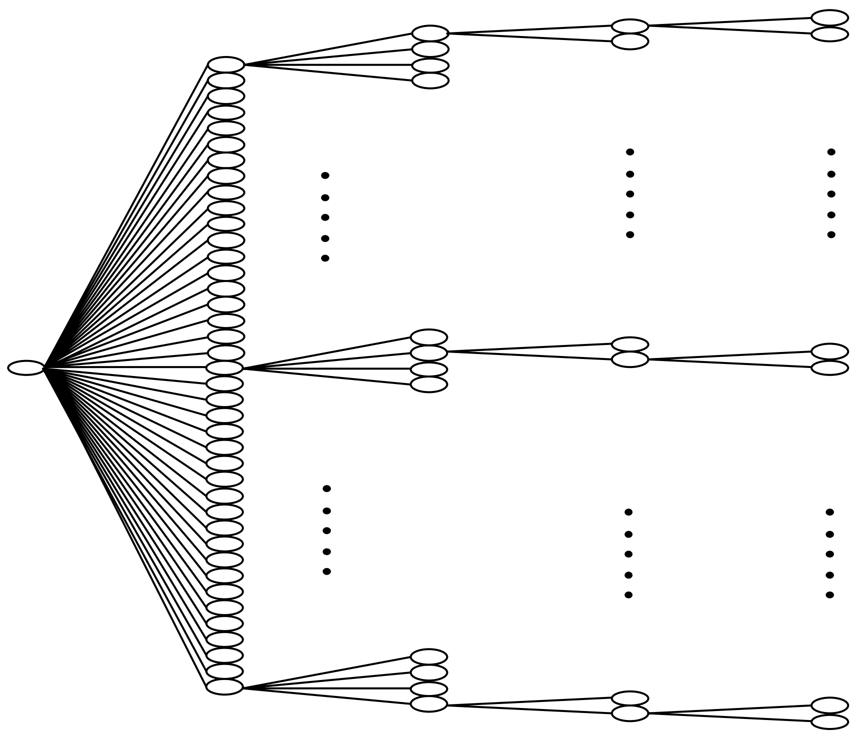

Figure 12. Scenario tree used in the implemented model.

The SP model is implemented in GAMS. After the scenario generation process is complete the GAMS model is run to read the inputs created by MATLAB and the resulting SP model is solved by CPLEX solver. The computational time for scenario generation is relatively higher being around 1 minute, most of which is caused by the communication between MATLAB and MS-EXCEL, whereas a single run of optimization takes around 6 seconds (Figure 13 provides a rough sketch of the overall process for one time period).

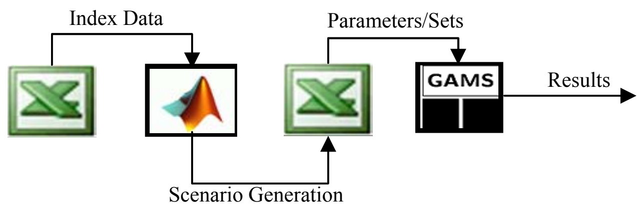

Figure 13. Rough sketch of the computation process

3.1.2. Single Period Analysis

The presented portfolio management framework is evaluated through both single-period and multi-period investment points of view. In a single-period investment setting, the investor makes the asset allocation decision at a specific time and realizes the results of his decisions after one period where the only concern is the one-period performance of the portfolio decisions and reinvesting the resulting portfolio is not considered.

NOTE: The focus of the implementation effort for the whole methodological approach presented in Chapter 2 is investment in a *multi-period* sense. Therefore, for the single-period analysis, we implement and present the results obtained via *only Alg-1A*.

3.1.2.1. In-Sample Testing & Methodology

In this section we present the results of the model obtained by computing the future values of investment decisions through in-sample return scenarios.

3.1.2.1.1. Obtaining Investment Decisions

The process of obtaining investment decisions (i.e., asset allocation) can be summarized as follow:

Step 0: Let *TH* be 200 and the initial wealth be $1,000.

Step 1: Take the first *TH* historical returns and generate the scenario tree with T = 4 starting at $t = TH$.

Step 2: Solve the SP model for the generated scenarios keeping the resulting investment decisions.

Step3: Initialize wealth back to $1,000 cash. If TH<299 then set TH=TH+1 and go to *Step 1*; otherwise, stop.

Considering the historical data set contains 300 weekly returns, this methodology requires the repetition of the process in Figure 13 for 100 times. Since we have two alternative assets, the output of this process is a 100x2 matrix where the i^{th} row is the allocation of funds among two assets at the beginning of $(i+199)^{\text{th}}$ period. It should be noted that these correspond to the first stage variables of the stochastic program (i.e., z_0 and y_0).

The next step is to compute the value of this portfolio one period after the asset allocation decision is made using the *in-sample* return scenarios. To do this, we randomly select 100 return values from the historical dataset of the first 200 returns and match them with the 100 different allocations obtained at the previous step. Suppose that z_0 and y_0 are the amounts of money invested on the risky and risk-free asset at some time t and r_{t+1} (a randomly selected in-sample scenario for the risky asset) and rb_{t+1} (i.e., risk-free rate of 0.04%) are the corresponding rate of returns for the next period. Then the portfolio value after one period is computed as

$$W_{t+1} = z_0(1 + r_{t+1}) + y_0(1 + rb_{t+1}) \qquad (72)$$

After these computations, we can add a third column to the asset allocation matrix which will contain the realized portfolio value after one period, which we can hereafter call *decisions-table for* convenience.

3.1.2.1.2. Experimentation with Various Parameters

The presented framework is composed of numerous parameters, which makes it tedious to carry over the computations using several values for all parameters. For the purposes of this section we focused on computations with different values of only *LCVAR* from the SP model. The different LCVAR values would allow the user to obtain a return/risk curve for the whole strategy.

We carried over the computation process mentioned in Section 3.1.2.1.1 for $LCVAR \in \{10,15,20\}$; therefore we obtained 3 different decisions-tables (See Tables 3-5 in Appendix B). An immediate observation from these tables is that the average amount allocated to the risky asset as *LCVAR* obtains higher values (See Figure 14). This is an expected result since increasing the risk limit leads to higher risk exposure through higher allocation on the risky asset.

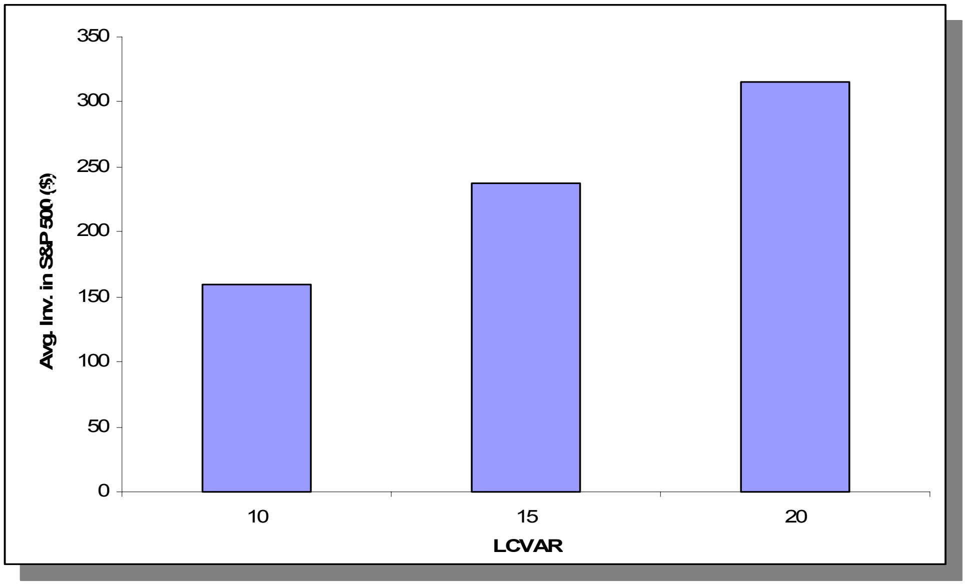

Figure 14. Average amount allocated to the risky asset (Alg-1A).

3.1.2.1.3. Performance Analysis and Benchmarking

As mentioned at the beginning of this chapter, we will analyze the proposed portfolio management framework in terms of the resulting return vs. risk profile. In order to decrease the potential bias from one outcome, we will aggregate the results over all 100 in-sample scenarios.

We use two measures of observed risk, *variance* and *average shortfall*. Even though the proposed SP model does not constrain variance, it is conjectured that control over shortfall through *LCVAR* will have a minimizing effect on variance.

Suppose that we have a decisions-table part of which is given in Table 6. Then the average return, variance, and average shortfall are computed as in (73)-(75).

$$\overline{R} = \frac{1}{n}\sum_{t=1}^{n} R_t = \frac{1}{n}\sum_{t=1}^{n}(W_t - 1{,}000)/1{,}000 \tag{73}$$

$$V = Var(R) \tag{74}$$

$$\overline{S} = \frac{1}{n}\sum_{t=1}^{n}\max(1{,}000 - W_t, 0) \tag{75}$$

Table 6. *W* column of a *sample* decisions-table .

t	W
1	1023
2	1022
3	989
4	1045
5	1019
6	1003
7	990
8	995
…	…
n	1013

The measures specified in (73)-(75) are applied to Tables 3-5 so we obtain three measures (i.e., average return, variance, and average shortfall) for each. In order to decrease the bias, we repeat the same process for three additional in-sample sets each including 100 randomly selected historical return scenarios. The resulting return/risk profiles are provided in Tables 7 and 8.

Table 7. Return-Variance profiles obtained from different samples (Alg-1A)

	Sample 1		**Sample 2**		**Sample 3**		**Sample 4**	
LCVAR	Avg. Ret.	VAR	Avg. Ret.	VAR	Avg. Ret.	VAR	Avg. Ret.	VAR
10	1.66	39.89	1.13	22.41	1.04	23.15	1.33	26.13
15	2.05	87.64	1.25	49.20	1.11	50.77	1.51	57.66
20	2.42	153.78	1.37	86.33	1.17	89.11	1.71	101.15

Table 8. Return-Shortfall profiles obtained from different samples (Alg-1A)

	Sample 1		Sample 2		Sample 3		Sample 4	
LCVAR	Avg. Ret.	Avg. Short.	Avg. Ret.	Avg. Short.	Avg. Ret.	Avg. Short.	Avg. Ret.	Avg. Short.
10	1.66	0.91	1.13	0.89	1.04	0.96	1.33	0.86
15	2.05	1.39	1.25	1.39	1.11	1.51	1.51	1.38
20	2.42	1.88	1.37	1.90	1.17	2.06	1.71	1.88

Table 9. Return-Variance profiles obtained from different samples (RS)

	Sample 1		Sample 2		Sample 3		Sample 4	
SP/Cash	Avg. Ret.	VAR	Avg. Ret.	VAR	Avg. Ret.	VAR	Avg. Ret.	VAR
0.2	1.04	20.82	0.40	12.52	0.57	16.10	1.01	12.37
0.4	1.68	83.26	0.41	50.08	0.74	64.40	1.62	49.46
0.6	2.32	187.34	0.41	112.68	0.91	144.89	2.24	111.29
0.8	2.96	333.06	0.42	200.32	1.08	257.58	2.85	197.85

Table 10. Return-Variance profiles obtained from different samples (RS)

	Sample 1		Sample 2		Sample 3		Sample 4	
SP/Cash	Avg. Ret.	Avg. Short.	Avg. Ret.	Avg. Short.	Avg. Ret.	Avg. Short.	Avg. Ret.	Avg. Short.
0.2	1.04	1.23	0.40	1.18	0.57	1.23	1.01	0.90
0.4	1.68	2.58	0.41	2.53	0.74	2.62	1.62	1.95
0.6	2.32	3.94	0.41	3.88	0.91	4.02	2.24	3.00
0.8	2.96	5.30	0.42	5.24	1.08	5.41	2.85	4.05

For benchmarking purposes, now we include a third strategy into our performance analysis in order to see any possible gains over the simplest single-period strategy. Since single-period investing is in question, a typical portfolio strategy is nothing but deciding on the ratio of wealth to be invested on the risky asset (risk-free asset). Let us denote with *RS* the strategy of deciding on a ratio and allocating the funds according to the ratio. In order to obtain a risk/return profile for *RS*, we consider different ratios for the risk-free asset in the portfolio; being 0.2, 0.4, 0.6, and 0.8.

Since the investment decisions are already known for this strategy, it only remains to create a decisions-table to compute the W column using the same in-sample return scenarios for all 100 time periods across four different scenario sets. The return/risk profiles obtained by this strategy are given in Tables 9 and 10. As an expected result, we observe that increasing the amount allocated to the risky asset leads to higher risk exposure through higher variances and average shortfalls.

In order to compare Alg-1A with the RS strategy, we plot for each sample the average return versus variance and the average return versus average shortfall. Therefore, we obtain 8 graphs where the approaches are compared with each other. Figures 15 and 16 provide the plots obtained from Sample 1. Alg-1A clearly dominates RS since it provides higher returns for the same level of risk exposure regardless the type of the risk is measure. The results obtained from Samples 2-4 are given in Figures 17-22 in Appendix B. The dominance of Alg-1A is strongly supported by the results obtained from all of these samples (i.e., Samples 2, 3, and 4) when average shortfall is considered as the risk measure. Alg-1A is dominated by RS only in Sample 4 when variance is considered to be the risk measure.

3.1.2.2. Out-of-Sample Testing

Out-of-sample scenarios are the actually observed rate of returns for S&P 500 for the periods 201 through 300. In other words the results in this section reflect the true single-period performance of the proposed portfolio management if it had been applied in periods 201 through 300.

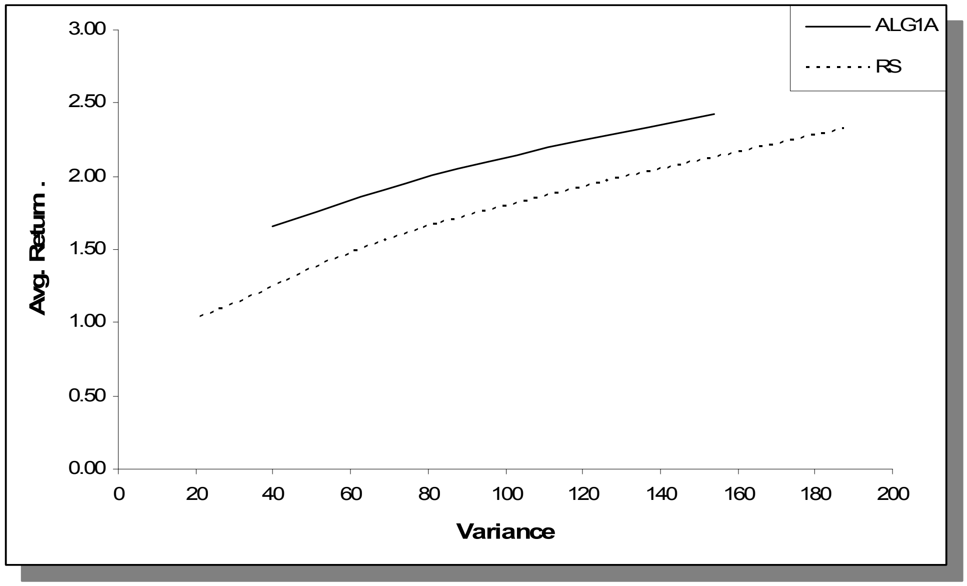

Figure 15. Comparison of Alg-1A and RS over Sample-1 (Variance)

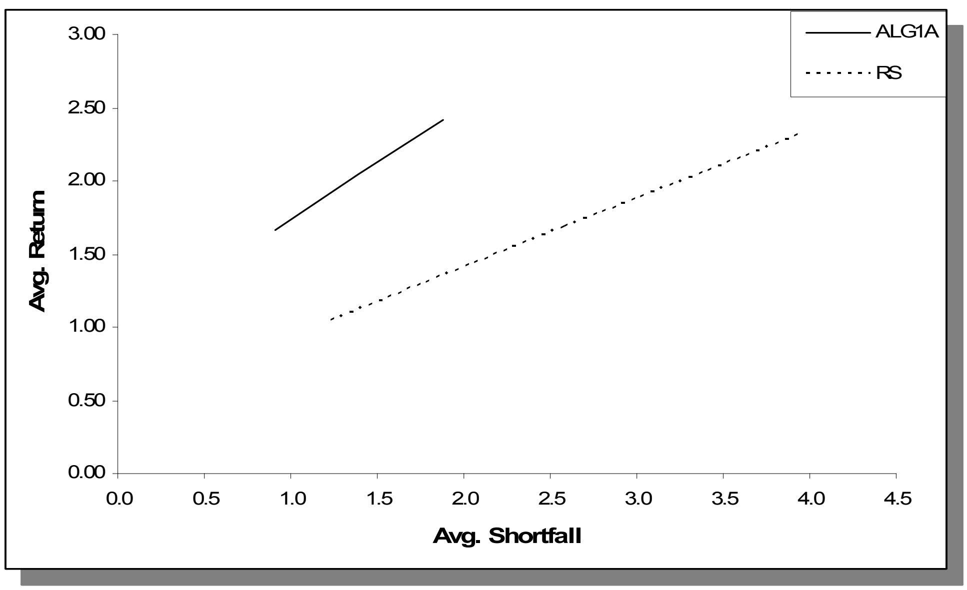

Figure 16. Comparison of Alg-1A and RS over Sample-1 (Avg. Shortfall)

In order to provide a compact presentation, we will not explain about the methodology used to obtain the aggregate results since it is the same as explained in Sections 3.1.2.1.1.-3.1.2.1.3. The only difference here is the set of return scenarios used to obtain the statistics about the model performance. Instead of four different in-sample scenario sets, now we have only one set of return scenarios which consists of *actual* returns of S&P 500 in periods 201 through 300.

Considering one out of sample scenario set and different investment decisions for $LCVAR \in \{10,15,20\}$ we again obtain three different decisions-tables (see Tables 11-13 in Appendix B).

As in in-sample testing, we obtained average returns and corresponding variances using the W_t values in Tables 11-13. The same process is also repeated for the RS strategy with the amounts invested in risky asset being \$200, \$400, \$600, and \$800. The results are given in Tables 14 and 15 and plotted in Figures 23 and 24.

Table 14. Return/risk profiles of Alg-1A (Single Period, Out-of-Sample)

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
10	0.63	10.76	0.57
15	0.75	23.59	0.91
20	0.86	41.41	1.25

Table 15. Return/risk profiles of RS (Single Period, Out-of-Sample)

SP/Cash	Avg. Ret.	VAR	Avg. Shortfall
0.2	0.60	8.01	0.86
0.4	0.80	32.05	1.89
0.6	1.00	72.11	2.93
0.8	1.19	128.19	3.97

We observe that Alg-1A still enables the investor to experience more efficient portfolios in terms of a mean/variance perspective; however, the level of domination is not significant. From our experiments using both in-sample and out-of-sample scenarios and weekly S&P index data covering periods 10/15/2001 and 7/16/2007, it can be concluded that Alg-1A produces a trading strategy that provides *slightly* more efficient portfolios (i.e., higher average return (lower variance) given a fixed level of variance (average return)) when compared with the RS strategy given a single-period investment approach.

Considering the average shortfall as the risk measure, Figure 24 reveals that Alg-1A dominates the *RS* strategy with a more significant difference when compared to the variance-based benchmarking. Variance is a commonly used measure for risk; however, it penalizes the negative deviations as well as positive deviations. For this reason, we use another measure to control risk, CVAR, which controls only negative deviations. The reason for poor performance of Alg-1A in a mean/variance setting (Figure 23) is that no constraint is imposed in the SP model to control variance. Instead, CVAR is constrained, which leads lower shortfalls on the average and superior performance in terms of Average Return/Average Shortfall trade-off. Therefore, from our experiments using both in-sample and out-of-sample scenarios and weekly S&P index data covering periods 10/15/2001 and 7/16/2007, it can be concluded that Alg-1A produces a trading strategy that provides more efficient portfolios (i.e., higher average return (lower average shortfall) given a fixed level of average shortfall (average return)) when compared with the RS strategy.

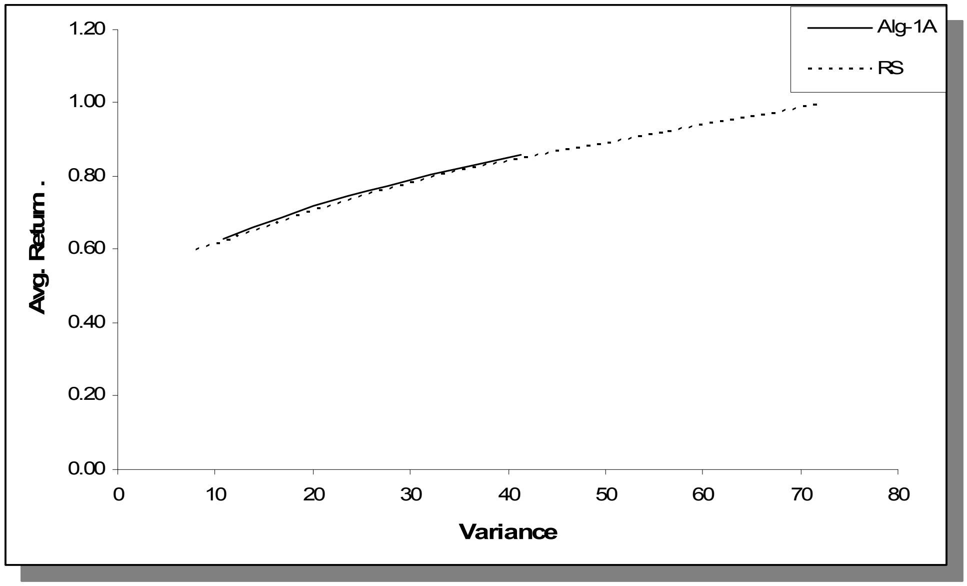

Figure 23. Comparison of Alg-1A and RS (Out-of-Sample, Variance)

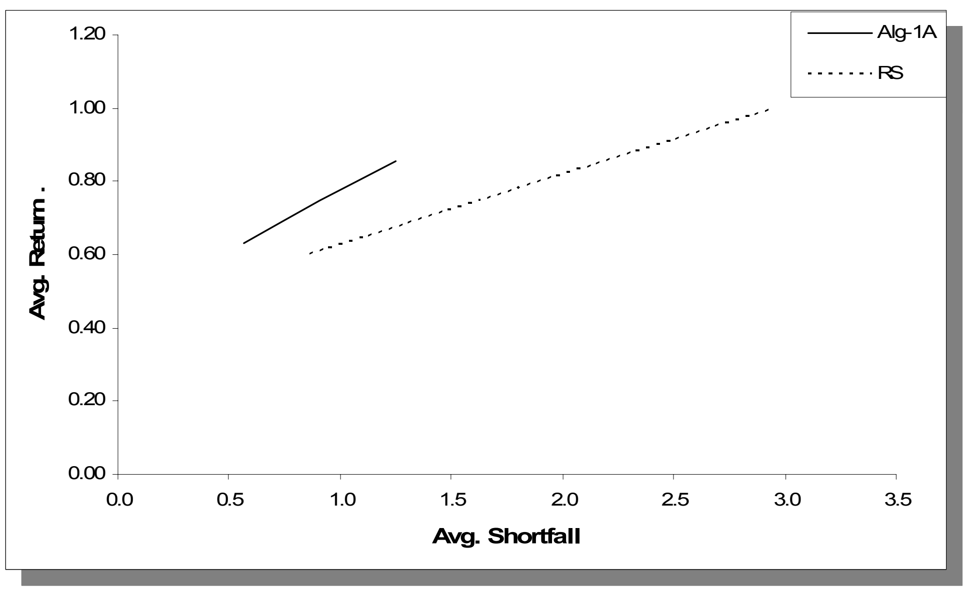

Figure 24. Comparison of Alg-1A and RS (Out-of-Sample, Avg. Shortfall)

3.1.3. Multi-Period Analysis

In this section the proposed similarity-scores-based approach is implemented through a multi-period investment perspective. Different than the single period setting, the investment decisions made at consecutive periods are not totally independent from each other. The decision made in period t may have effects on decisions in periods $\tau>t$. The resulting portfolio after the decisions made in period t becomes the initial portfolio for period $t+1$ after the actual returns are realized. In other words, the total wealth of the investor is assumed to be repetitively reinvested upon realizing the returns at consecutive periods.

In this section, in addition to Alg-1A we present the results also for Alg-1B, the algorithm based on the UD distance measure.

3.1.3.1 Implementation

The methodology applied for multi-period analysis is mostly the same with that of single period analysis. The major difference in implementation is that once the returns are realized at a specific period, the resulting portfolio is reinvested by running the whole approach (scenario generation and solving the SP model) for the next period such that the initial portfolio is set to the portfolio obtained from the previous period after the actual returns are realized and applied to the portfolio.

In order to convert the computation process into a multi-period setting, a few modifications are made. Figure 25 illustrates this modification in the computation process.

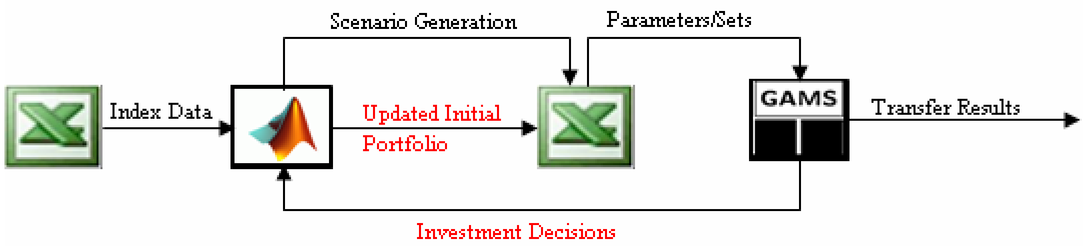

Figure 25. The sketch of the modified computation process

The process of obtaining investment decisions can be formally summarized as follows (Note that the methodology is the same as the single-period case except *Step 3*):

Step 0: Let *TH* be 200 and the initial wealth is $1,000.

Step 1: Take the first *TH* historical returns and generate the scenario tree of length $T = 4$ starting at $t=TH$.

Step 2: Solve the SP model for the generated scenarios keeping the investment decision.

Step3: Update the portfolio using the investment decisions and applying the actual returns for $t=TH+1$. If $TH < 299$ then go to *Step 1*; otherwise stop.

The performance measures are obtained via (76)-(78) where W_0=1,000.

$$\bar{R} = \frac{1}{100}\sum_{t=1}^{100}(W_t - W_{t-1})/1{,}000 \tag{76}$$

$$V = Var(R) \tag{77}$$

$$\bar{S} = \frac{1}{100}\sum_{t=1}^{100}\max(W_{t-1} - W_t, 0) \tag{78}$$

3.1.3.2. Performance Analysis

In multi-period analysis, we expand the interval for risk exposure by setting $LCVAR \in \{5,15,25,35\}$ instead of $LCVAR \in \{10,15,20\}$ as in single-period analysis. (See Tables 16 and 17 in Appendix B for the decisions obtained from both algorithms corresponding $LCVAR$ = 35). The return/risk profiles of the both algorithms are given in Table 18, where variance is taken as the risk measure and in Table 19, where average shortfall is the risk measure. As seen from these tables, the increase in *LCVAR* leads to increase in all three performance measures.

3.1.3.3. Benchmarking

We consider the benchmarking process within two different approaches. In the first one, we compare the proposed approach (i.e., Alg-1A or Alg-1B and the SP model) with another portfolio management technique. In the other approach, we compare the performance of the proposed scenario generation algorithms with two other scenario generation algorithms.

Table 18. Avg. Return/Variance profiles of Alg-1A and Alg-1B (Multi-Period)

	Alg-1A		Alg-1B	
LCVAR	Avg. Ret.	VAR	Avg. Ret.	VAR
5	0.77	5.24	0.93	44.15
15	1.63	48.60	1.51	60.11
25	2.31	129.73	1.92	84.62
35	2.43	172.00	2.20	111.91

Table 19. Avg. Return/Avg. Shortfall profiles of Alg-1A and Alg-1B (Multi-Period)

	Alg-1A		Alg-1B	
LCVAR	Avg. Ret.	Avg. Shortfall	Avg. Ret.	Avg. Shortfall
5	0.77	0.46	0.93	0.97
15	1.63	1.62	1.51	1.48
25	2.31	2.79	1.92	1.99
35	2.43	3.30	2.20	2.42

3.1.3.3.1 Benchmarking with the Buy & Hold Strategy

In this section, we compare the proposed approach with the *Buy and Hold* (B&H). B&H is a commonly used investing strategy and can be regarded as a good approximation to the well known *fixed-mix* rule in our case because the time unit is one week, which leads to lower returns, and in turn, lower deviations from a fixed-mix of asset allocation. Note that the deviations may increase as the time span of the test period gets larger as in our case (i.e., 100 weeks).

In order to obtain the average return/variance profile of the B&H strategy, decisions-tables are created by starting a certain amount of cash allocated in risky asset and the rest being invested in the risk-free asset. As the rule of the strategy, no trade takes place in the following periods. An obvious advantage of this strategy is that no transaction costs are incurred in the periods except the initial one.

Similar to the single period case, we initialize the allocation on the risky asset as $200, $400, $600, and $800. The resulting performance measures are shown in Table 20.

Table 20. Return/Risk profile of B&H Strategy (Multi-Period)

SP/Cash	Avg. Ret.	VAR	Avg. Shortfall
0.2	0.78	8.35	0.80
0.4	1.15	33.39	1.75
0.6	1.53	75.13	2.71
0.8	1.91	133.57	3.67

The performance measures tabulated in Tables 18-20 are plotted in Figures 26 and 27. Figure 26 compares the alternative strategies by considering variance as the risk measure, where we observe that the proposed strategy, when Alg-1A is used during the scenario generation phase, provides more efficient portfolios in terms of a mean/variance perspective. Moreover, the level of domination is significant over the whole interval of risk exposure. Alg-1B also provides more efficient portfolios when compared to B&H strategy, especially in the high risk region. Alg-1B performs relatively worse in the low risk region; however, *it should be noted that the proposed approach controls the shortfall as the risk measure instead of variance*. A more compact comparison in terms of Avg. Return/Variance relation is given in Figure 28, which provides plots of the Sharpe Ratios obtained from alternative strategies as computed in (79), where r_f is the risk free rate.

$$SharpeRatio = \frac{\overline{R} - r_f}{\sqrt{V}} \tag{79}$$

Figure 27 provides the comparison where we consider the average shortfall as the risk measure, *which is the actual measure targeted within the proposed approach*. We observe in Figure 27 that the proposed approach dominates the B&H strategy regardless of the algorithms used in the scenario generation phase. In other words, the repetitive usage of the proposed approach with continuous reinvesting yields higher returns (lower average shortfall) than the B&H strategy given a fixed level of average shortfall (average return).

Therefore, from our experiments using weekly S&P index data covering periods 10/15/2001 and 7/16/2007, it can be concluded that the *proposed approach* produces a trading strategy that provides more efficient portfolios (i.e., higher average return (lower risk) given a fixed level of risk (average return)) when compared with the B&H strategy within a multi-period investment approach. Both algorithms lead to superior results when average shortfall is assumed to be the risk measure whereas a slight underperformance is observed only in the low risk measure for Alg-1B when variance is considered to be the risk measure.

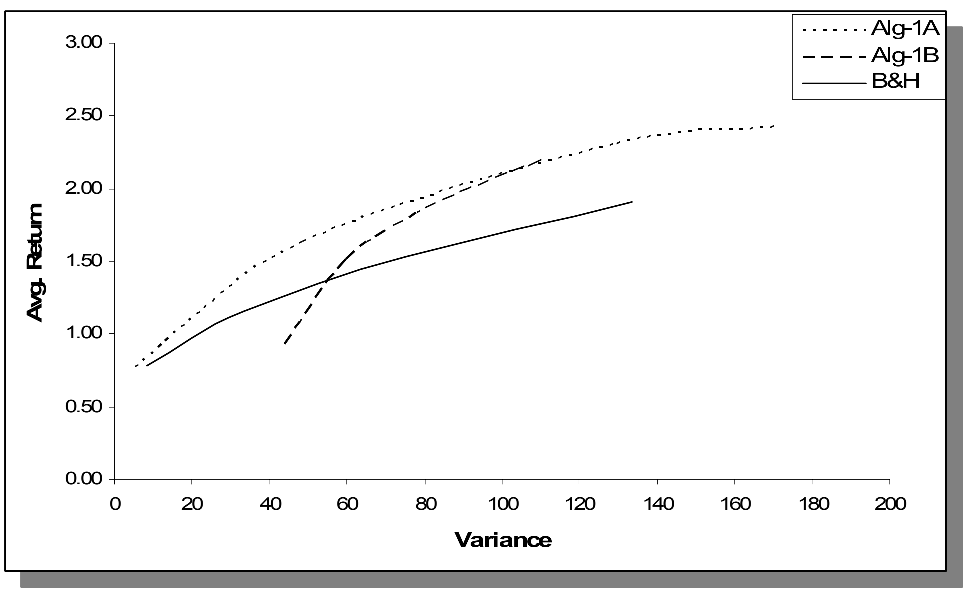

Figure 26. Comparison of Alg-1A, Alg-1B and B&H (Multi-Period, Variance)

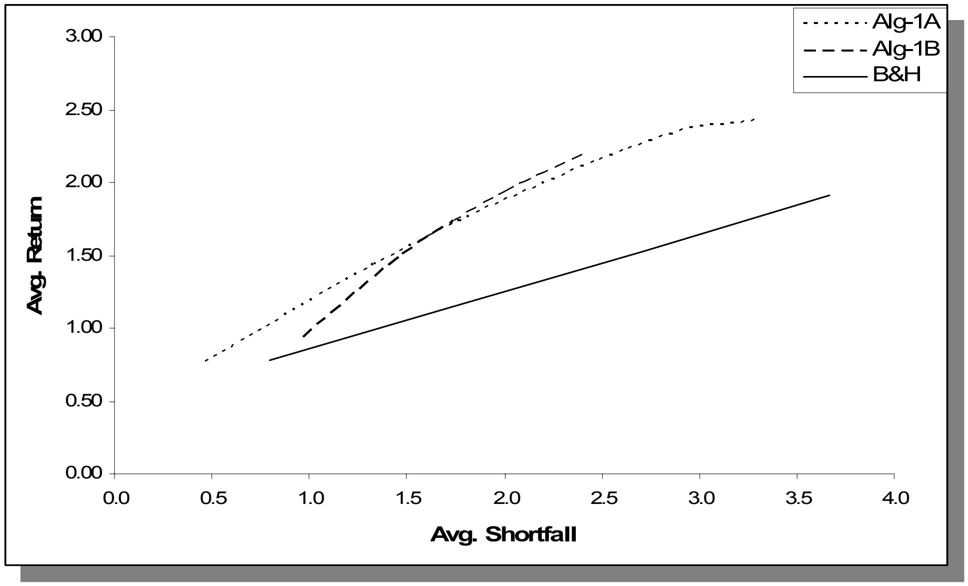

Figure 27. Comparison of Alg-1A, Alg-1B and B&H (Multi-Period, Avg. Shortfall)

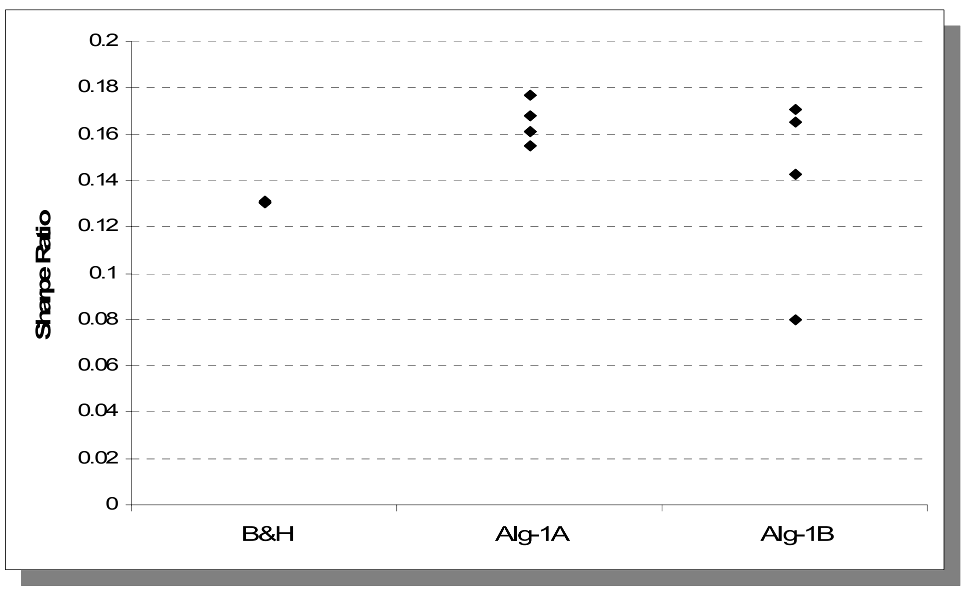

Figure 28. Sharpe Ratios obtained via Alg-1A, Alg-1B and B&H.

3.1.3.3.2 Benchmarking with other Scenario Generation Methods

In this section, our focus is on comparing the performance of the proposed scenario generation algorithms with other scenario generation algorithms. The objective is to assess the performance led merely by the scenario generation algorithms by considering the SP model as separate piece. In other words we try to differentiate between the benefits contributed by the algorithms and those coming from the usage of the SP model.

In order to achieve this objective, we run additional experiments where we replace Alg-1A and Alg-1B with alternative scenario generation algorithms but utilize the same proposed SP model used in previous computations. The two tools we will consider are:

- Geometric Brownian Motion (GBM)
- Auto-Regressive Model (AR)

In order to generate scenarios using GBM, we assume that the price of the risky asset, say x_t, follows the process given by the stochastic differential equation in (80)

$$dx_t = \mu x_t dt + \sigma x_t dW_t \tag{80}$$

where μ is the drift, σ is the volatility of the stochastic process and W_t is the Wiener process. The solution for equation (80) is

$$x_t = x_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right) \tag{81}$$

Therefore,

$$\ln\left(\frac{x_t}{x_0}\right) = \left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t \tag{82}$$

which implies that $\ln(x_t / x_0) \sim N((\mu - \sigma^2 / 2)t, \sigma^2 t)$. The Wiener process has independent increments; therefore, for the consecutive price values we have $\ln(x_t / x_{t-1}) \sim N((\mu - \sigma^2 / 2), \sigma^2)$. Considering the dataset containing the logarithms of consecutive price ratios, we can estimate drift and volatility parameters corresponding to the GBM given in (80).

Once we estimated the drift and volatility parameters, we consider a scenario tree having the same topology used by Alg-1A and Alg-1B. Then, for each node except the leaf nodes (i.e., CSN), we draw randomly from $N((\mu - \sigma^2 / 2), \sigma^2)$ to obtain a discrete set of price values with the cardinality being equal to the number of nodes emanating from the CSN. Due to the randomness in sampling, the resulting scenario probabilities are all equal to each other for a given CSN and they add up to 1 for each CSN.

The parameters for GBM are estimated repetitively at the beginning of each period and the SP model is run at each period after the scenario tree is constructed. The initial portfolios are continuously updated to obtain the multi-period results for the SP approach fed by GBM.

In addition to GBM, we considered an AR model of degree two for benchmarking purposes. Let r_t denote the return of the risky asset (e.g., index value) for period t. Then we estimate the AR(2) process in (83).

$$r_t = \beta_0 + \beta_1 r_{t-1} + \beta_2 r_{t-2} + \varepsilon \tag{83}$$

Similar to the methodology we used in GBM; once the estimation is accomplished we consider a scenario tree having the same topology used by Alg-1A and Alg-1B. We compute the residuals for all possible historical periods using the estimated equation and obtain a set of residuals to sample from.

Then, for each node except the leaf nodes (i.e., CSN), we draw randomly from the historical residuals and add them to the AR(2) equation implied for that period to obtain a discrete set of return scenarios with the cardinality being equal to the number of nodes emanating from the CSN. Due to the randomness in sampling, the resulting scenario probabilities are all equal to each other for a given CSN and they add up to 1 for each CSN.

Considering the 100 consecutive test periods, we estimate the parameters of GBM and AR(2) at the beginning of each period so that we always use the most up-to-date models in the scenario generation process. We next provide the estimations only for the first test period (i.e., t = 200) since it would not be reasonable to provide the estimations for all periods.

Regarding the GBM at t = 200, μ is estimated to be 0.000903 whereas σ is 0.020865 considering the weekly S&P 500 data. Therefore for this time period, we use $x_{t+1} = x_t \exp(0.000686 + 0.020865 W_1)$ for the generation of price scenarios (and then return scenarios) where W_1 is drawn from $N(0,1)$.

For the same period, the AR(2) equation estimated through ordinary least squares (OLS) method is given in (84).

$$r_t = 0.000798 + 0.092857 r_{t-1} + 0.022419 r_{t-2} + \varepsilon \tag{84}$$

Using the AR(2) and GBM in the scenario generation phase in place of Alg-1A and Alg-1B, we obtain the performance measures given in Tables 21 and 22 (See Tables 23 and 24 in Appendix B for a sample decisions-table obtained via each of these alternative methods).

Table 21. Return/Risk profile obtained via AR(2) (Multi-Period)

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
10	0.75	6.81	0.56
20	1.11	27.46	1.25
30	1.44	61.04	1.96
40	1.66	85.29	2.35
60	1.71	97.39	2.55

Table 22. Return/Risk profile obtained via GBM (Multi-Period)

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
10	0.74	13.42	0.89
20	1.07	54.98	1.95
30	1.43	115.30	2.87
40	1.50	160.74	3.47
60	1.56	169.21	3.57

Figures 29 and 30 provide the comparison between the proposed algorithms and AR(2). The results are quite similar to ones obtained from the benchmarking process against the B&H strategy. We observe that the proposed strategy, when Alg-1A is used during the scenario generation phase, provides more efficient portfolios in terms of a mean/variance perspective than the strategy where we use AR(2) during the scenario generation phase instead of Alg-1A. Alg-1B also provides more efficient portfolios when compared to AR(2) strategy, especially in the high risk region. Alg-1B performs relatively worse in the low risk region. Figure 33 provides the corresponding Sharpe Ratios where we observe that AR(2) is mostly outperformed by the proposed algorithms.

We observe in Figure 30 that the strategy of using *any* of the proposed algorithms during the scenario generation phase dominate the strategy led by using AR(2) for scenario generation when we consider the average shortfall as the risk measure, *which is the actual measure targeted within the proposed approach*.

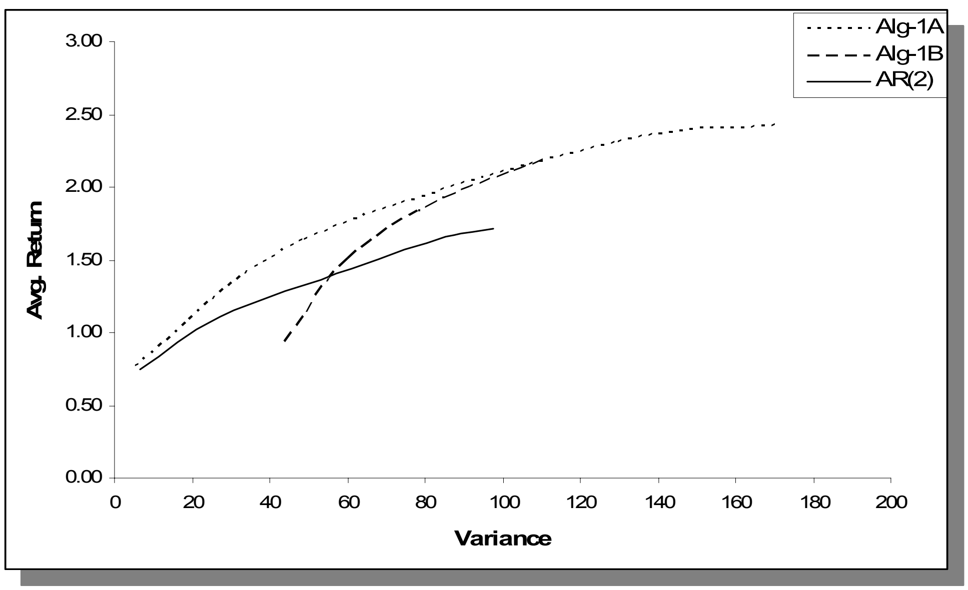

Figure 29. Comparison of Alg-1A, Alg-1B and AR(2) (Multi-Period, Variance)

Figures 31 and 32 provide the comparison between the proposed algorithms and GBM. We observe that the proposed strategy, *regardless of either Alg-1A or Alg-1B is used during the scenario generation phase*, provides more efficient portfolios in terms of a mean/variance perspective than the strategy where we use GBM during the scenario generation phase instead of the proposed algorithms. The Sharpe Ratios given in Figure 33 are clearly in favor of the proposed algorithms.

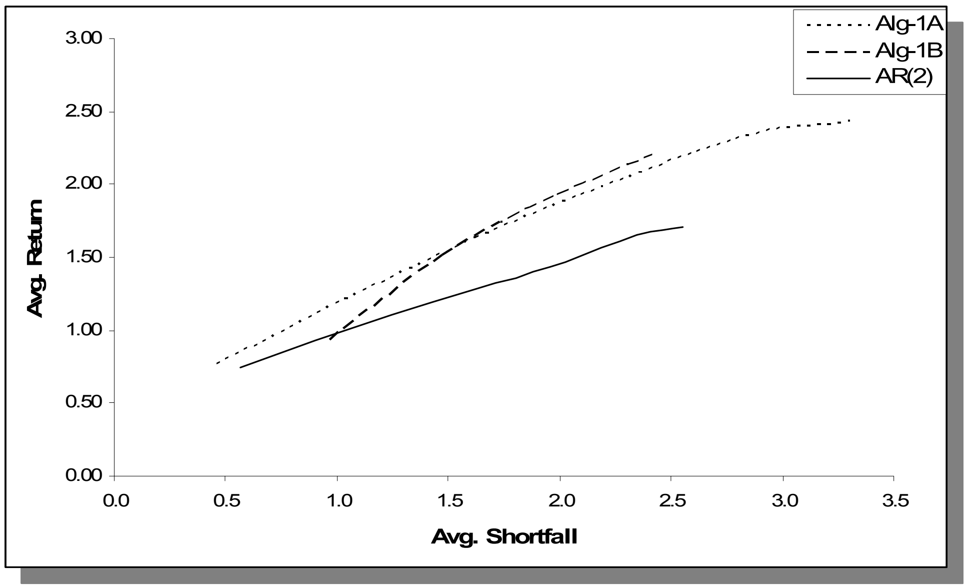

Figure 30. Comparison of Alg-1A, Alg-1B and AR(2) (Multi-Period, Avg. Shortfall)

We come to the same conclusion when we consider the average shortfall as the risk measure. Figure 32 illustrates the results showing that the strategy of using *any* of the proposed algorithms during the scenario generation phase dominate the strategy led by using GBM for scenario generation.

The distinction between the performances can be attributed merely to the proposed scenario generation algorithms since *the same* SP model is used to generate trading strategies for Alg-1A, Alg-1B,AR(2), and GBM. Therefore, from our experiments using weekly S&P index data covering periods 10/15/2001 and 7/16/2007, it can be concluded that within a multi-period investment scheme, the *proposed scenario generation algorithms* produce trading strategies that provide more efficient portfolios (i.e., higher average return (lower risk) given a fixed level of risk (average return)) when compared with the strategies led by using GBM or AR(2) during the scenario generation phase.

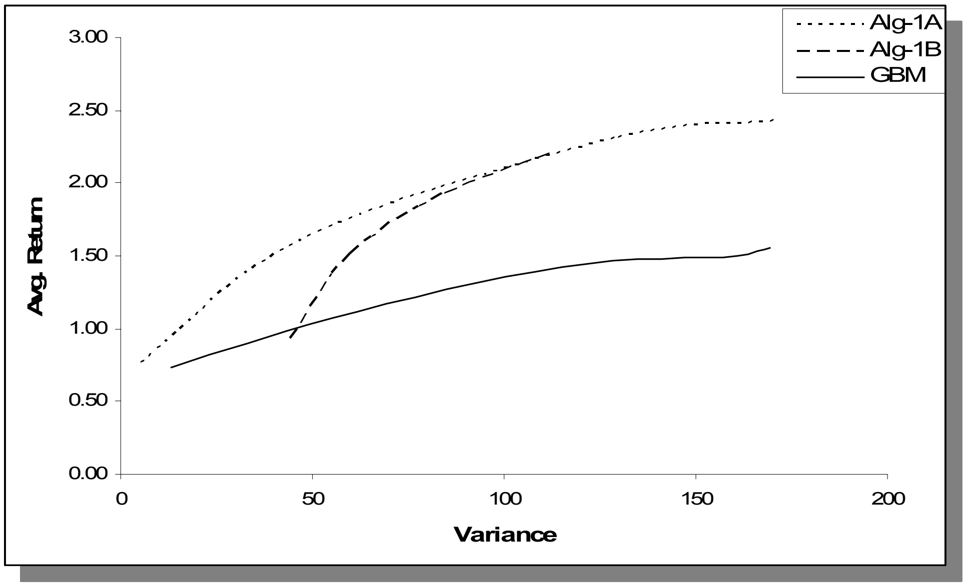

Figure 31. Comparison of Alg-1A, Alg-1B and GBM (Multi-Period, Variance)

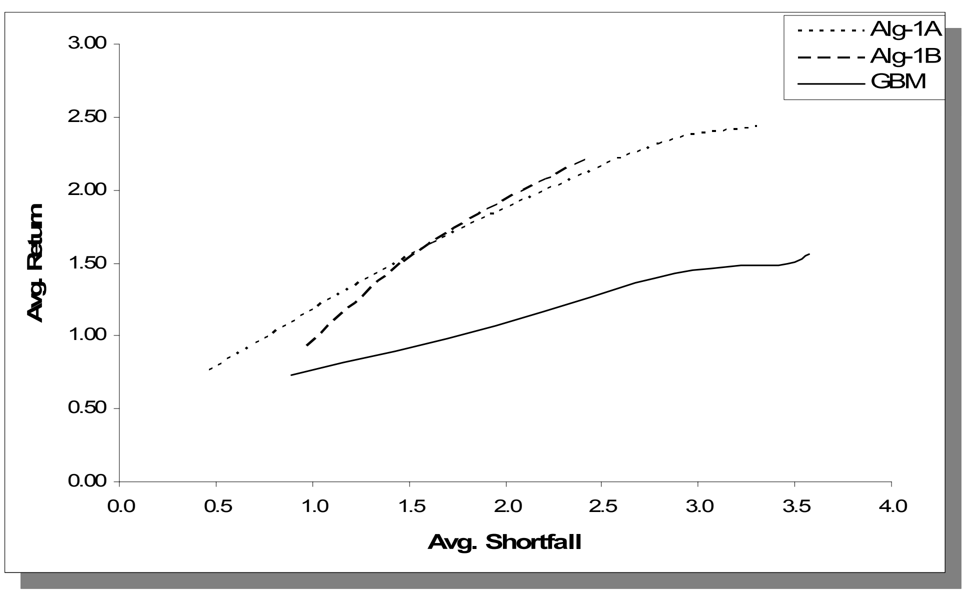

Figure 32. Comparison of Alg-1A, Alg-1B and GBM (Multi-Period, Avg. Shortfall)

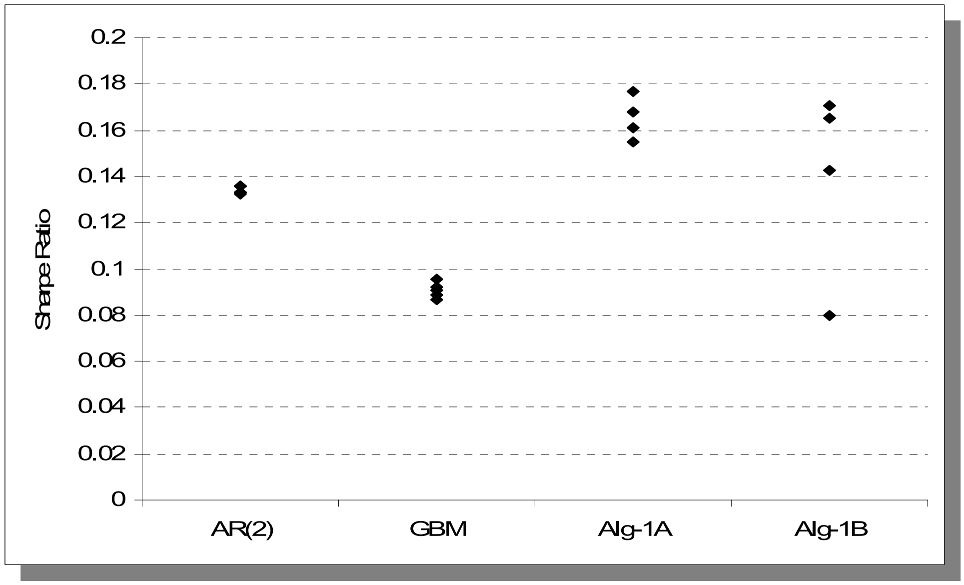

Figure 33. Sharpe Ratios obtained via Alg-1A, Alg-1B, AR(2) and GBM.

3.1.4. Sensitivity Analysis

In this section, we present the outcomes obtained by implementing the proposed approach with different parameters and input data. Our objective is to observe how the proposed strategy responds to slight changes in the model and evaluate its performance over the alternative approaches within different settings.

3.1.4.1. Sensitivity to Weights used in Euclidean Distance

Recall from the definition of Alg-1A that the parameters w_{ti} are the weights assigned to the Euclidean similarity for period t within the selected path of asset i. For the computations presented so far, the same weights were used for all assets. In addition, the latter periods were assumed to have more weight. Therefore, we used the same $\mathbf{w}$ = [0.1 0.1 0.2 0.2] vector for all computations.

In order to see the effect of using different weights and justify our assumption to have higher weights for more recent periods, we re-implemented Alg-1A with three other different **w** vectors. The first two assign higher weights to initial periods, whereas the last one sets weights to the recent periods that are even higher than initial experiments:

- $\mathbf{w}^1$ = [0.6 0.4 0.2 0.1]
- $\mathbf{w}^2$ = [0.2 0.2 0.1 0.1]
- $\mathbf{w}^3$ = [0.1 0.2 0.4 0.6]

The performance measures obtained using $\mathbf{w}^1$, $\mathbf{w}^2$, and $\mathbf{w}^3$ instead of **w** are given in Tables 25-27 (due to space concerns, we provide only the decisions-table for $\mathbf{w}^2$, Table 28 in Appendix B, obtained by setting *LCVAR*=35). The corresponding return/risk profiles are given in Figures 34 and 35 to provide a comparison with the base case, **w**.

Table 25. Return/Risk profile obtained via $\mathbf{w}^1$.

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
5	0.66	7.47	0.70
15	1.17	71.99	2.48
25	1.74	170.97	3.84
35	1.92	188.39	4.08

Table 26. Return/Risk profile obtained via $\mathbf{w}^2$.

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
5	0.72	6.28	0.59
15	1.36	61.41	2.12
25	1.87	152.77	3.45
35	2.00	178.02	3.80

Table 27. Return/Risk profile obtained via $\mathbf{w}^3$.

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
5	0.87	5.07	0.42
15	1.81	49.21	1.58
25	2.78	121.73	2.51
35	3.00	160.50	2.97

Given the weekly S&P index data covering periods 10/15/2001 and 7/16/2007, the first observation from Figures 34 and 35 is that the performance of the proposed approach, when Alg-1A is used for scenario generation, is significantly sensitive to the weights used for computing the Euclidean distances. This is an expected result since the changes in weights have a direct impact on the similarity scores and in turn the scenario probabilities, which will eventually change the investment decisions implied by the SP model.

The second observation is that the change in the performance supports our initial approach of assigning higher weights to more recent periods. In fact, there is a consistent improvement in the performance as more weight is shifted towards the latter periods. This can be observed clearly from the Sharpe Ratios depicted in Figure 36.

The two observations above remain valid regardless of the risk measure we consider, variance or average shortfall. A critical side note is that the performance of the proposed approach reported in Section 3.1.3 for Alg-1A could be significantly higher by setting $\mathbf{w} = \mathbf{w}^3$.

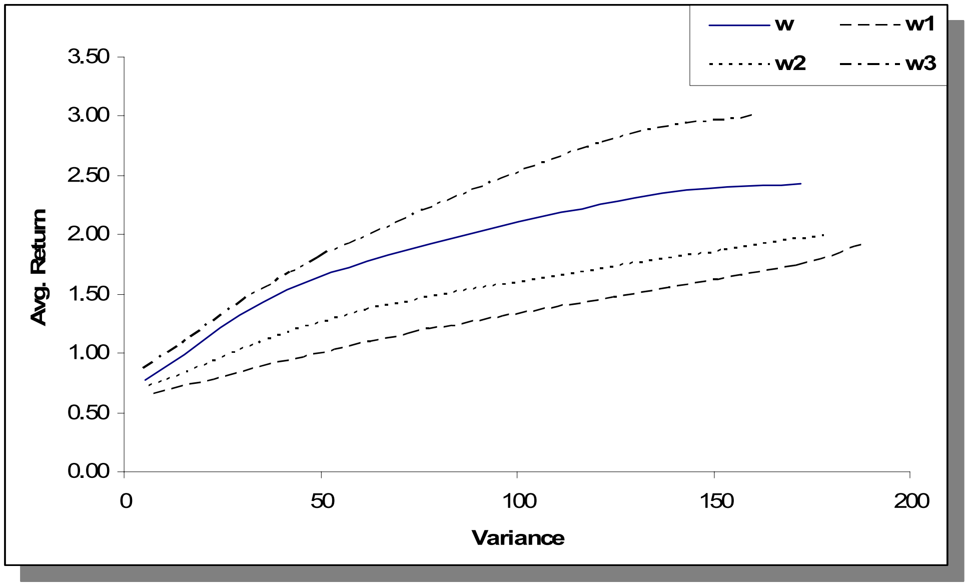

Figure 34. Sensitivity to Euclidean weights (Variance)

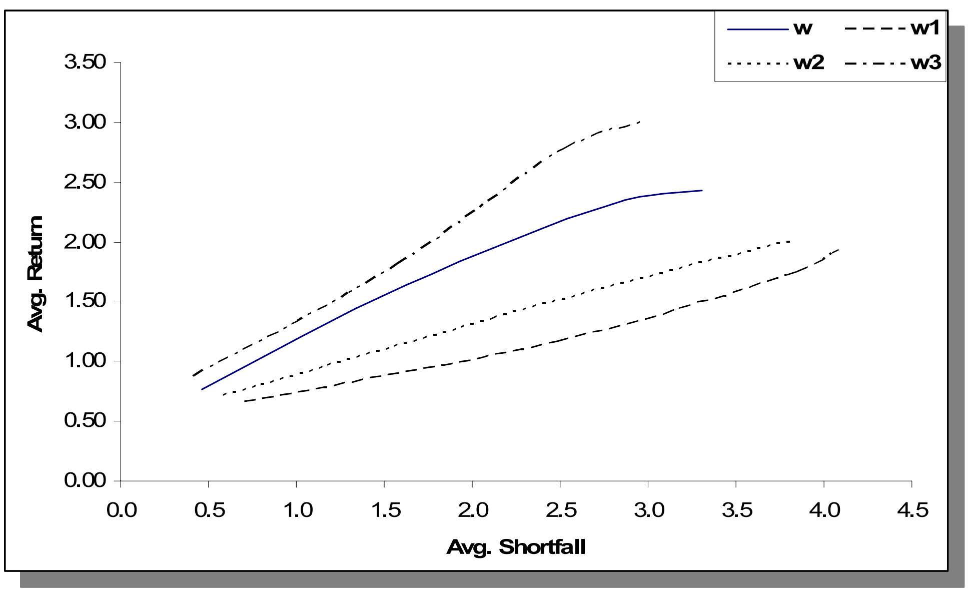

Figure 35. Sensitivity to Euclidean weights (Avg. Shortfall)

3.1.4.2. Sensitivity to Time Unit

In our previous experiments, the investor is assumed to make investment decisions at the beginning of each week and has no interest in the intra-week fluctuation in asset values.

However, high-frequency trading is also common in the financial industry shifting the focus from weeks, months, and quarters to a finer time grid where day-to-day and intra-day course of financial data is critical to make frequent investment decisions.

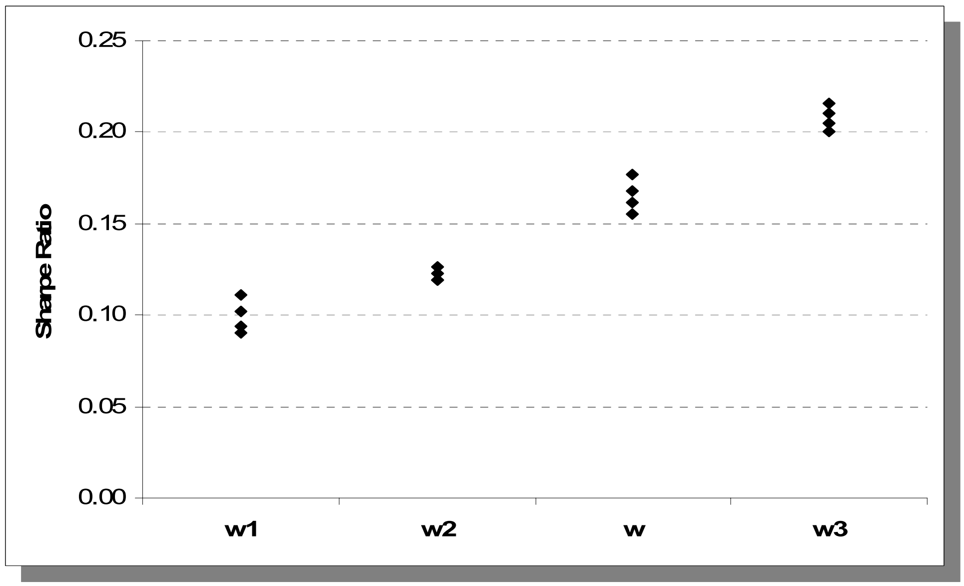

Figure 36. Sharpe Rations led by different Euclidean weights.

In this section we assume that the investor makes investment decisions on a daily basis. For our first experiment, similar to the computations presented earlier, we consider a time frame of 401 index values such that the historical data is composed of 400 arithmetic daily returns. Using the same methodology of Sections 3.1.2 and 3.1.3, we initially take the first 300 of the 400 data points to train the algorithms and increase the training set cardinality by one after decisions made for the corresponding test period. In other words, the first 300 return values are used to train the algorithms when t = 301, whereas the first 399 return values are used for training when t = 400. The initial data set is the daily S&P 500 index values starting from 04/14/2003 - 11/12/2004.

Switching from weekly data to daily data implies a change in the financial time series data fed into the model; therefore the following experiments are expected to yield valuable information on *the sensitivity of the proposed approach to the input data* and its performance with respect to several alternative approaches.

For the computations presented in this section, all parameters are kept the same as in the previous sections except the risk-free rate since the time unit is shortened. We now assume that the investor can invest at a risk-free rate of 0.005%. Similar to Section 3.1.3, alternative scenario generation methods (i.e., AR(2) and GBM) and an alternative investment approach (i.e., B& H) are estimated for benchmarking purposes using the aforementioned daily data set.

Tables 29 and 30 provide the performance measures obtained by using the proposed approach with Alg-1A and Alg-1B, whereas Tables 31 and 32 provide the results led by alternative scenario generation methods, AR(2) and GBM, respectively. Figures 37 and 38 plot the return/risk profiles given in Tables 29-32.

Table 29. Return/Risk profile obtained via Alg-1A

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
2	0.121	1.250	0.337
6	0.274	11.015	1.037
10	0.362	28.403	1.703
14	0.498	35.055	1.852

Table 30. Return/Risk profile obtained via Alg-1B

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
2	0.157	10.335	0.505
10	0.319	20.600	1.215
18	0.368	29.758	1.667
30	0.342	39.006	2.060

Table 31. Return/Risk profile obtained via AR(2)

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
2	-0.016	1.042	0.391
6	-0.138	9.526	1.227
10	-0.208	24.745	1.978
14	-0.127	37.486	2.368

Table 32. Return/Risk profile obtained via GBM

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
2	0.051	1.118	0.366
6	0.061	10.268	1.150
10	0.073	27.714	1.903
14	0.089	38.955	2.266

The results depicted in Figures 37 and 38 are in favor of the proposed approach. The return risk profiles generated by both Alg-1A and Alg-1B are superior to the alternative approaches used for this experiment regardless the risk measure considered. Among the alternative tools used for scenario generation, GBM leads to more efficient portfolio when compared to AR(2).

Similar to Section 3.1.3, we now compare the proposed approach with the alternative investment approach, B&H. Table 33 provides the return/risk profile obtained by this strategy (i.e., allocating different amounts of the initial wealth to S&P 500 and not trading for the next 100 periods). Figures 39 and 40 compare B&H to the proposed algorithms, where we observe that B&H is outperformed by both Alg-1A and Alg-1B.

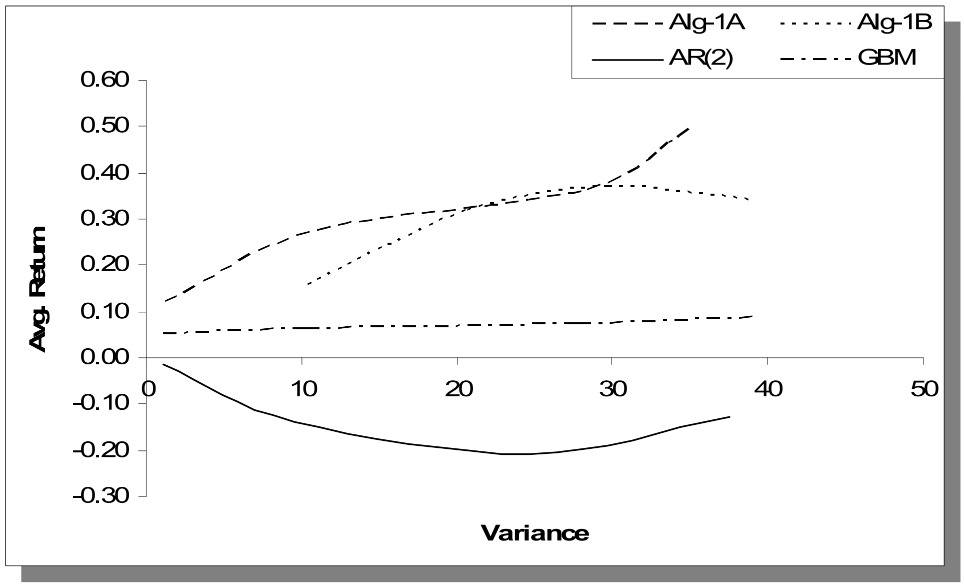

Figure 37. Comparison of Alg-1A, Alg-1B, AR(2) and GBM (Daily, Variance)

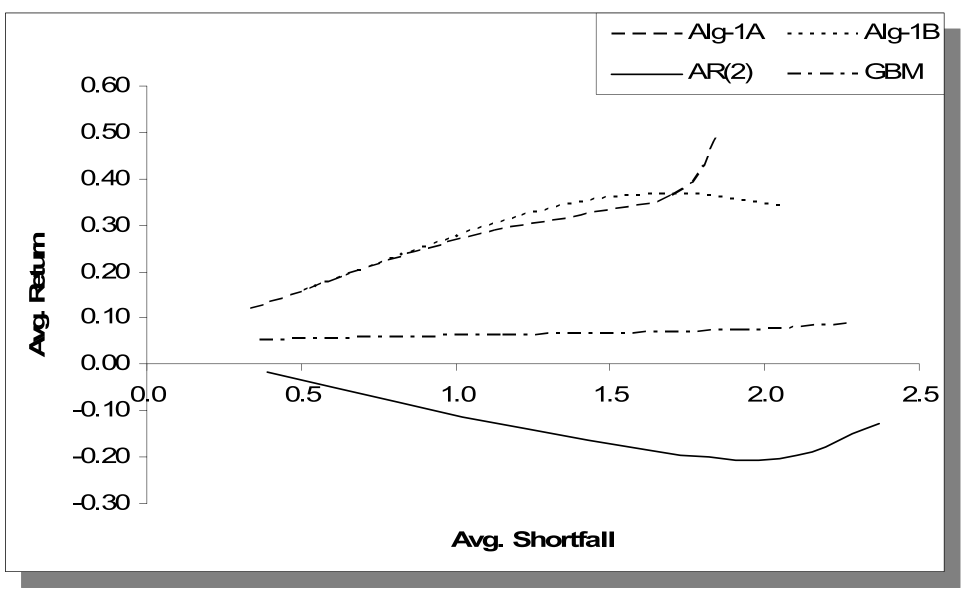

Figure 38. Comparison of Alg-1A, Alg-1B, AR(2) and GBM (Daily, Avg. Shortfall)

Table 33. Return/Risk profile obtained via B&H

SP/Cash	Avg. Ret.	VAR	Avg. Shortfall
0.20	0.107	1.829	0.475
0.35	0.150	5.602	0.847
0.50	0.192	11.433	1.219
0.65	0.235	19.322	1.591
0.80	0.277	29.269	1.963

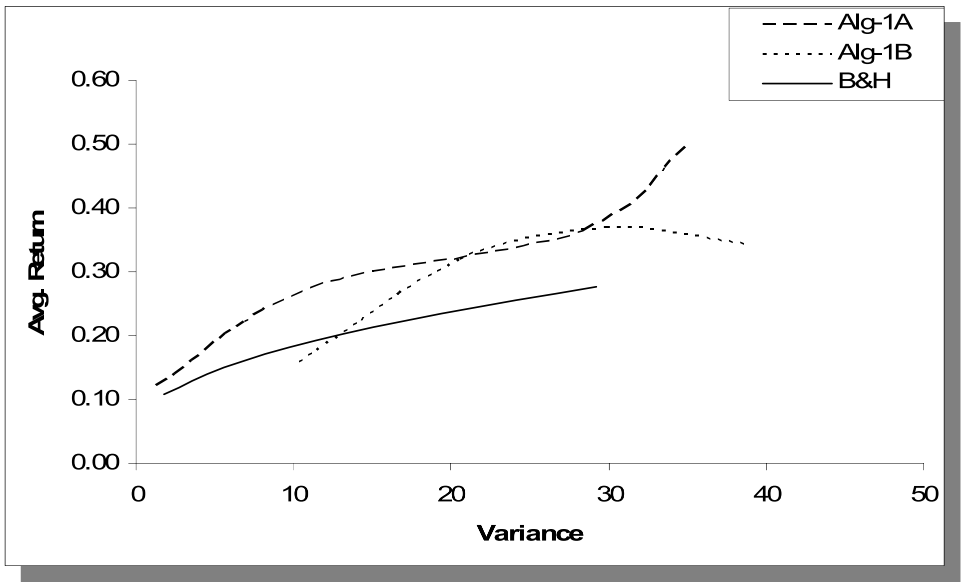

Figure 39. Comparison of Alg-1A, Alg-1B, and B&H (Daily, Variance)

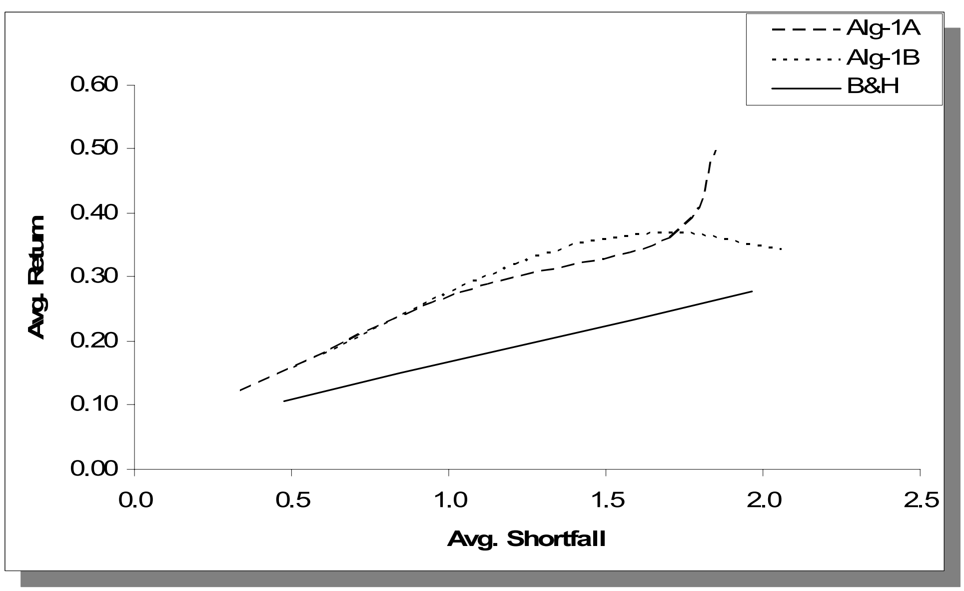

Figure 40. Comparison of Alg-1A, Alg-1B, and B&H (Daily, Avg. Shortfall)

3.1.4.3. Sensitivity to Time Window

As observed from Section 3.1.4.2, the performance of the proposed approach and the alternative ones are sensitive to time unit and in turn the input data, even though the conclusion from the benchmarking process did not change significantly favoring the proposed algorithms.

The sensitivity to input data requires further experiments to be able to provide a better judgment over the alternative approaches. Therefore, we now consider multiple time windows each including an exclusive test set of 100 days but overlapping training sets of various lengths. Each time window is created by adding the following 100 days' data to the previous time window starting from the initial 400 data points studied in the previous section (i.e., Section 3.1.4.2). Recalling that the original time window covers the data between 4/14/2003 to 11/12/2004 and denoting it by TW1, the next four time windows can be defined as the daily S&P 500 index values between the following dates (see Figure 41 for an illustration):

1. TW1: 04/14/2003 - 11/12/2004
2. TW2: 04/14/2003 - 04/08/2005
3. TW3: 04/14/2003 - 08/30/2005
4. TW4: 04/14/2003 - 01/24/2006
5. TW5: 04/14/2003 - 06/16/2006

Note that each time window starts at the same day spanning 100 days further into the future than the previous time window. For all time windows, the last 100 days are used for testing the trading strategies.

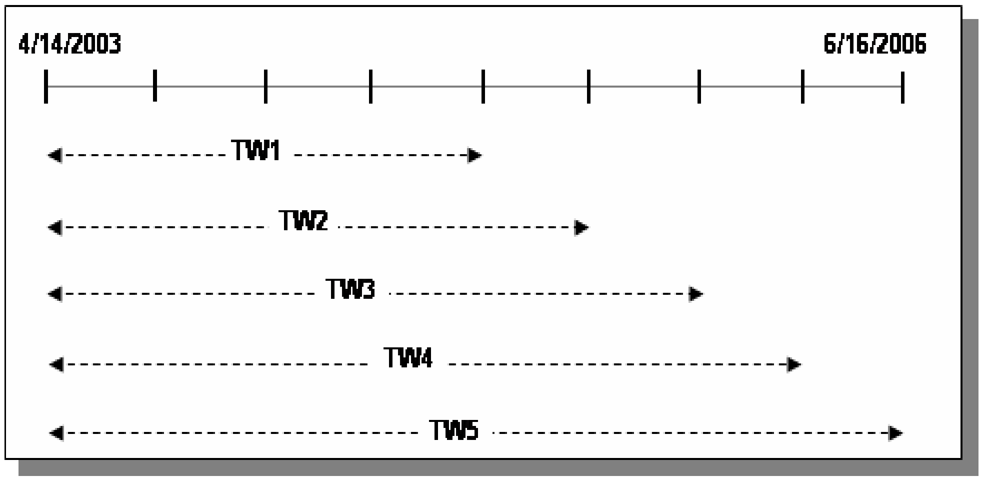

Figure 41. Time windows used for sensitivity analysis. Each bar represents 100 days.

All five approaches (i.e., Alg-1A, Alg-1B, AR(2), GBM, and B&H) are implemented over the four new time windows. Keeping the previous format of presenting the results, we obtained 20 tables, each corresponding to one of the five strategies over one of the four new time windows (See Tables 34-37 in Appendix B).

Similar to the results presented in the former sections, the conclusions for different time windows led by measuring risk by average shortfall are mostly in accordance with the conclusions led by measuring risk by variance. Therefore, instead of focusing on each time window separately, we prefer to report the only Sharpe Ratios, which provide a brief summary of the benchmarking process. Similar to Section 3.1.3, we provide a compact the comparison of the alternative approaches by plotting the Sharpe Ratios obtained from the points that build the Avg. Return vs. Variance curves (e.g., Figure 37, Figure 39, etc.). In other words we obtain a single Sharpe Ratio by setting a specific value for the parameter controlling risk (i.e., LCVAR for scenario generation algorithms and S&P500/Cash ratio for the B&H strategy) and implementing the corresponding method for the 100 consecutive test periods.

Five different levels of risk exposures are set for each of the approaches for each of the five time windows, TW1-TW5, leading to 125 Sharpe Ratios as given in Table 38 (see Appendix B) and plotted in Figure 42.

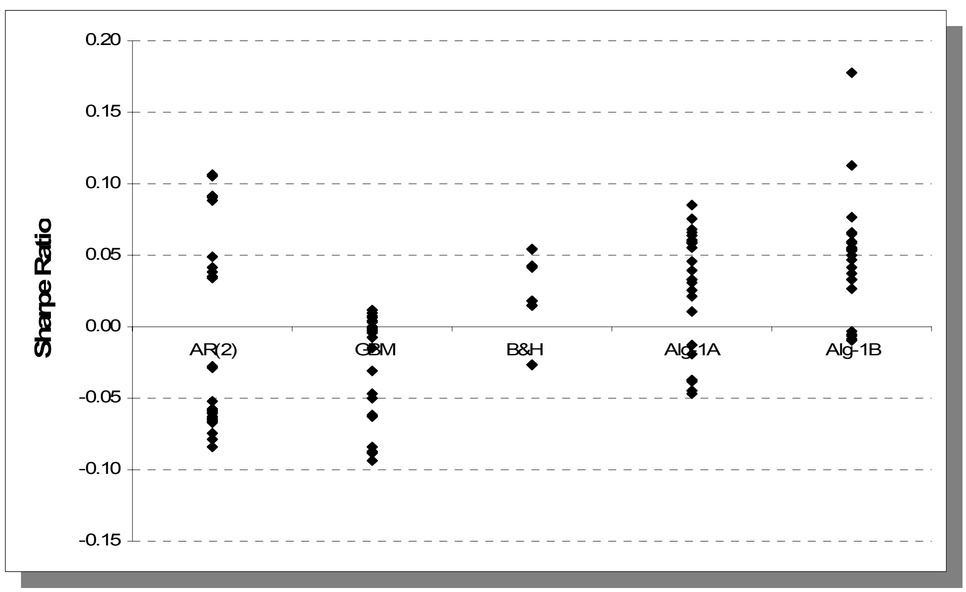

Figure 42. Sharpe Ratios led by alternative approaches over all time windows.

The performance of the alternative approaches exhibit high variation over different time windows, leading to large intervals for the Sharpe Ratios. However, Figure 42 reveals that Alg-1A and Alg-1B have relatively higher Sharpe Ratios compared to the alternative approaches. In fact, this relation is observed very clearly in Figure 43 where the average Sharpe Ratios are plotted for each of the methods in question.

The points in Figure 43 reveal that AR(2) and GBM are both outperformed by B&H, Alg-1A, and Alg-1B. Among these three, both Alg-1A and Alg-1B outperform B&H where Alg-1B has a higher score than Alg-1A considering merely the proposed algorithms. Therefore, considering the five different time windows (i.e., TW1-TW5) over the daily S&P 500 index values between 04/14/2003 - 06/16/2006 and the results presented in Tables 34-37 and Figures 42-43, it can be concluded that within a multi-period investment scheme, the proposed approach produces trading strategies that provide more efficient portfolios when compared with the strategies led by AR(2), GBM, and B&H.

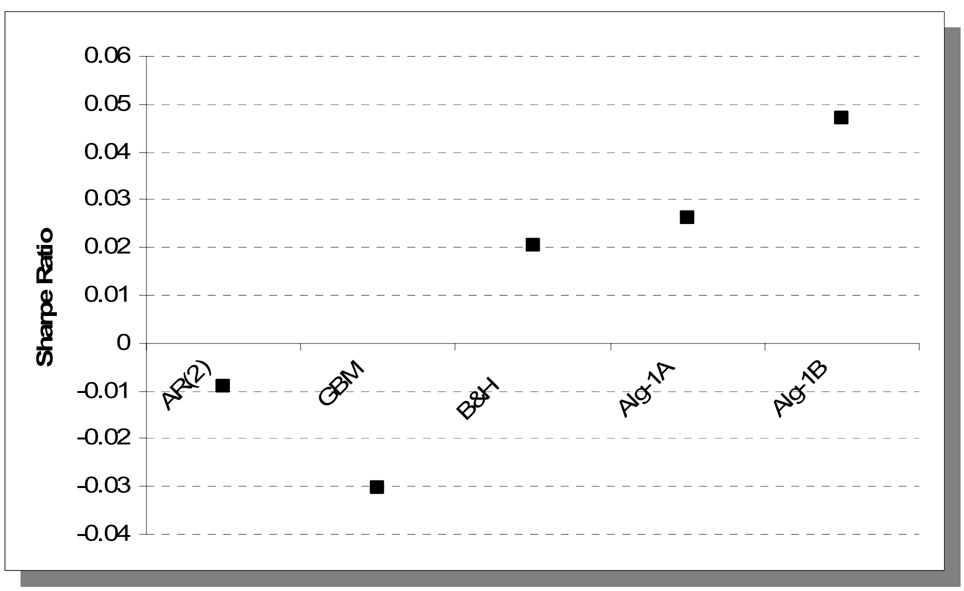

Figure 43. Average Sharpe Ratios led by alternative approaches.

3.2. Algorithm-2

After the analysis of Alg-1A and Alg-1B, we turn our focus to the computational analysis of Algorithm-2 (Alg-2). Different than Section 3.1, we now assume that the decision maker has three investment options in total, two risky assets and one risk-free asset. Another difference over Section 3.1 is that for all computations we consider only the multi-period investing scheme. In other words, we analyze the risk-return profiles assuming that the investor reinvests at each period his/her portfolio resulting from the previous period.

3.2.1. Setup for Computations

In this section, some details regarding the implementation process such as the input data and selection of some parameters are provided. Note that most of the implementation process is similar to Section 3.1.

3.2.1.1 Data

In addition to the S&P 500, we consider the Russell 2000 stock index to represent the second risky asset. The Russell 2000 is an index covering the stocks of small-cap companies in US equity market. The time unit for the decisions is again assumed to be one week for the initial experiments.

The historical data covers 801 index values for S&P 500 and the Russell 2000 from each week between 1/2/1990 and 5/9/2005, which is illustrated in Figure 44. Applying (3), we obtain 800 weekly arithmetic return values to be used in training and testing Alg-2 and alternative approaches. Basic statistics for these indices over the aforementioned period are given in Table 39. Similar to Section 3.1, the weekly risk-free rate is assumed to be fixed at 0.04%.

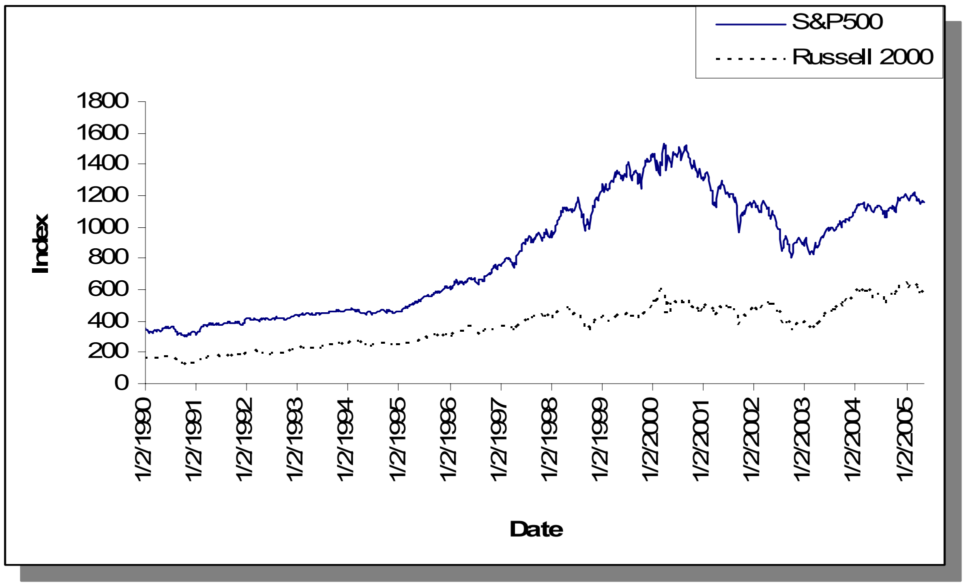

Figure 44. Weekly S&P 500 and Russell 2000 indices.

Table 39. Basic statistics of returns for S&P 500 and Russell 2000

Stat.	S&P500	Russell 2000
Mean Return	0.001712	0.001854
Variance	0.000453	0.000616
Skewness	-0.261194	-0.778697
Kurtosis	2.380273	5.162751

3.2.1.2. Parameters

Most parameters are assigned the same values as the ones used for Alg-1A and Alg-1B, especially those used within the SP model. As for the scenario generation, there are differences on two parameters:

- *T*: The planning horizon is assumed to be 2 weeks for the implementation purposes of Alg-2.
- *Scenario Tree Topology*: Unless mentioned differently, the number of scenarios in the first period is 30 where 15 scenarios are built after each scenario in the first period. Therefore a total of 30x15=450 scenarios are created at the horizon.

3.2.1.3. Implementation

We consider only a multi-period setting for the implementation of Alg-2. The same process used for the multi-period analysis of Alg-1A and Alg-1B is employed to obtain the results presented in the following sections (see Figure 25 for the illustration of the implementation process). The last 100 periods out of the 800 are used for testing whereas the initial 700 periods are used merely to train the alternative approaches. Similar to Section 3.1, as the alternative strategies are implemented for any period $t_1 \in \{701..800\}$, all the data corresponding to the periods $t < t_1$ are used for training.

3.2.1.4. Initial Analysis

The proposed methodology is based on the assumption that the data contains heteroskedasticity. In order to support this assumption, we compute the sample auto-correlation functions (ACF) on *squared returns* for lags 1, 2, and 3 of both indices. The results given in Figures 45 and 46 show that the auto-correlation is significant for both indices for lag 1.

The presence of heteroskedasticity can be justified also by the Ljung-Box test (Ljung and Box (1978)), where the null hypothesis is that the time series has no autocorrelation. The *squared residuals* obtained from the constant mean model are used as the input time series to test this null hypothesis. The resulting *Q-statistic* given in Table 40 is chi-square distributed with confidence level 0.95 and degrees of freedom being equal to the number of lags, which is equal to 1 since the proposed methodology is based on EGARCH(1,1). We observe from Table 40 that *p*-values are almost zero and *Q-statistics* are much larger than the $\chi^2_{0.95,1} = 3.8415$; therefore, we reject the null hypothesis.

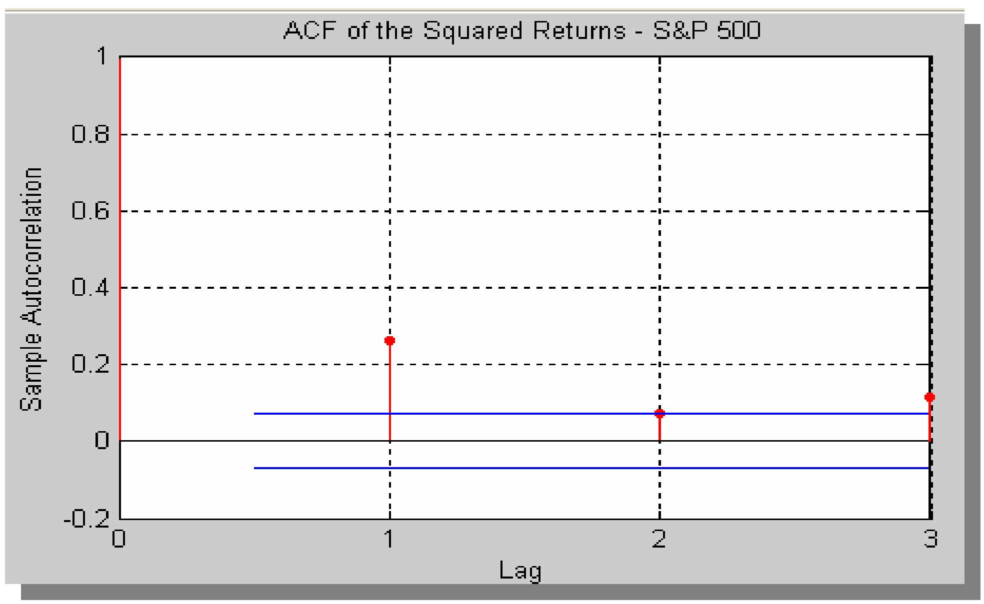

Figure 45. Sample ACF for S&P 500

Table 40. L-B Test for S&P 500 and Russell 2000 for lag 1.

Index	p-value	Q-Stat	Critical Value
S&P 500	0.84e-13	55.7091	3.8415
Russell 2000	0.17e-06	27.4013	3.8415

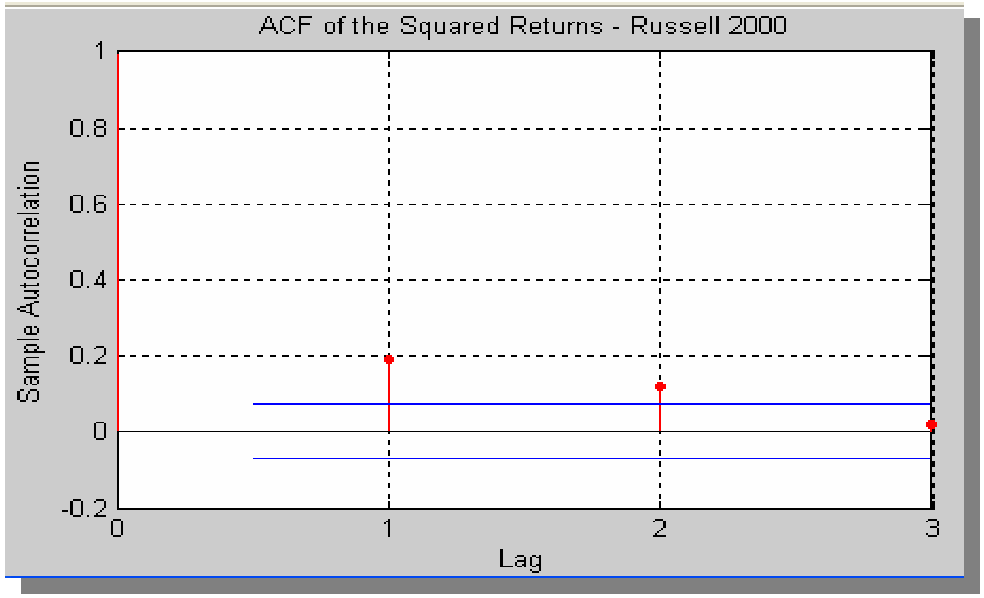

Figure 46. Sample ACF for Russell 2000

3.2.2. Effects of Constraints on Scenario Probabilities

The results obtained by setting no constraints on scenario probabilities (see Section 2.3.3) are provided in Table 41 whereas Tables 42 and 43 provide those obtained by setting relatively lower values for the parameter lb^n. These return/risk profiles are plotted in Figures 47 and 48, which support the argument on setting lower-bounds for probability values in moment matching process. When $lb^n = 0.1, \forall n$, we observe a decrease in risk given a fixed level of return. Moreover, the case $lb^n = 0.2$ yields a trading strategy that is improved further over the entire curve.

Table 41. Return/Risk profile obtained via Alg-2 ($lb^n = 0$)

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
10	0.898	38.882	1.975
20	1.391	157.606	4.173
30	1.829	359.853	6.418
40	2.369	542.350	7.900
50	2.835	676.307	8.748
70	2.971	857.193	10.028

Table 42. Return/Risk profile obtained via Alg-2 ($lb^n = 0.1$)

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
10	0.903	37.947	1.950
20	1.383	155.225	4.145
30	1.858	346.997	6.296
40	2.397	542.002	7.899
50	2.836	675.048	8.765
70	2.970	853.100	10.012

Table 43. Return/Risk profile obtained via Alg-2 ($lb^n = 0.2$)

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
10	0.922	35.039	1.866
20	1.422	142.764	3.961
30	1.897	323.150	6.070
40	2.501	517.796	7.707
50	2.850	670.966	8.769
70	2.980	855.991	10.054

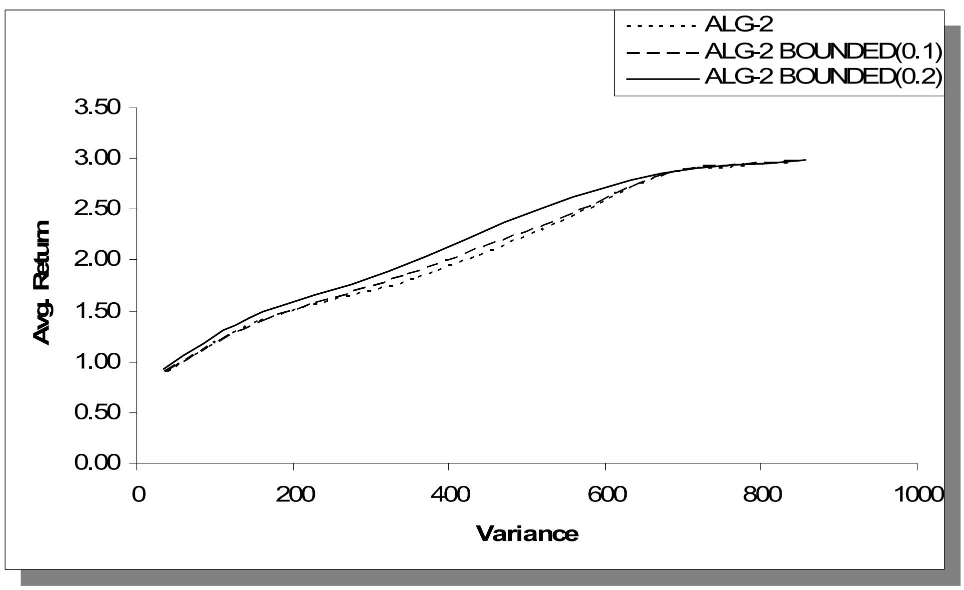

Figure 47. Results for Alg-2 where $lb^n \in \{0,0.1,0.2\}$ (Weekly, Variance)

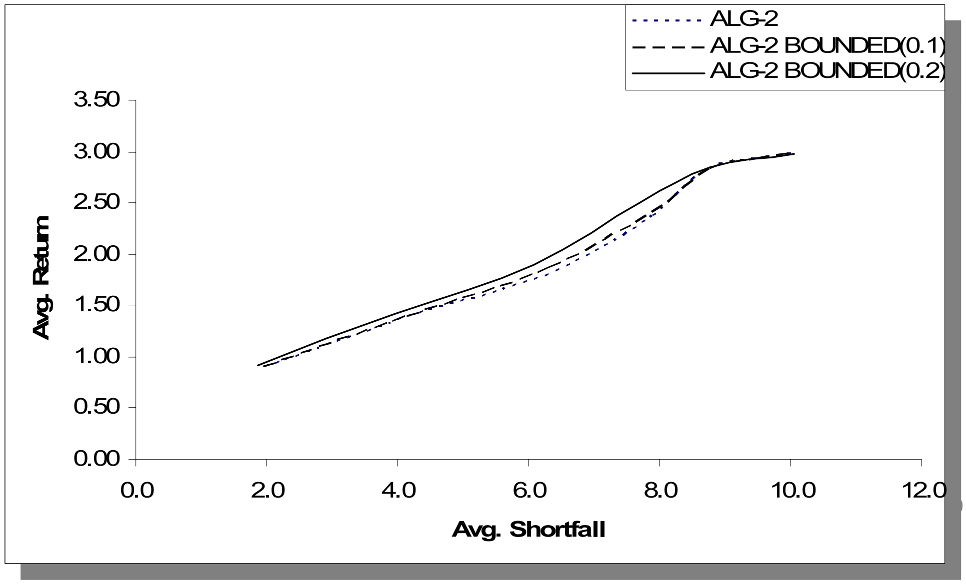

Figure 48. Results for Alg-2 where $lb^n \in \{0,0.1,0.2\}$ (Weekly, Avg. Shortfall)

However, setting relative higher values for lb^n does not lead to comprehensively improved strategies of the unbounded case. As seen Tables 44 and 45 and plotted in Figures 49 and 50, the cases where lb^n=0.4 and lb^n=0.6 may worsen the performance in various intervals. This is not unexpected since setting higher bounds lead to higher deviations from the target moments in the moment matching process. Conditioned on the current data and parameter sets, we conclude that the proposed methodology performs better when lb^n=0.2. Note that for another investment problem with different types of investment options and the time unit for decisions, further experiments should be carried over to obtain a value of lb^n calibrated for the specific investment problem.

Table 44. Return/Risk profile obtained via Alg-2 ($lb^n = 0.4$)

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
10	0.924	35.444	1.857
20	1.413	145.110	3.960
30	1.923	326.008	6.019
40	2.322	503.592	7.536
50	2.611	680.961	8.855
70	2.892	846.331	10.012

Table 45. Return/Risk profile obtained via Alg-2 ($lb^n = 0.6$)

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
10	0.911	31.472	1.722
20	1.353	124.623	3.620
30	1.833	284.270	5.570
40	2.015	456.990	7.162
50	2.310	654.007	8.729
70	2.757	865.807	10.145

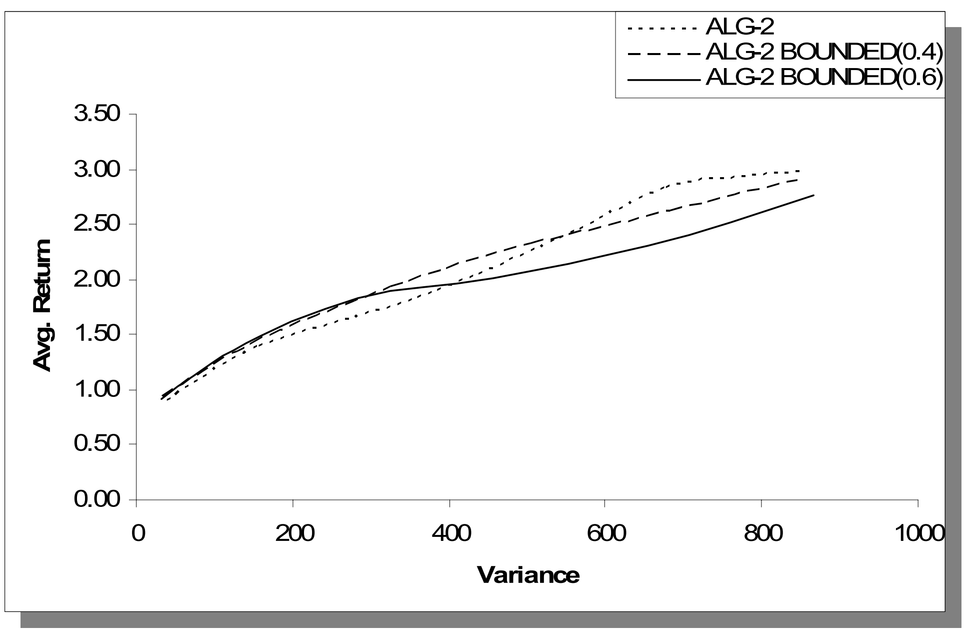

Figure 49. Results for Alg-2 where $lb^n \in \{0,0.4,0.6\}$ (Weekly, Variance)

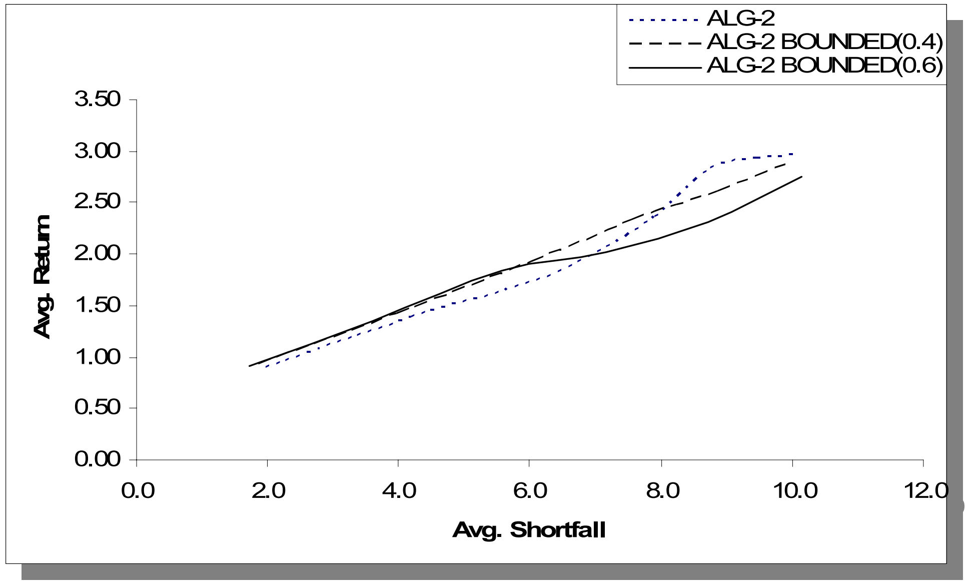

Figure 50. Results for Alg-2 where $lb^n \in \{0,0.4,0.6\}$ (Weekly, Avg. Shortfall)

3.2.3. Effects of E-GARCH

We now turn our focus on the effects of considering heteroskedasticity in scenario generation. We consider the case with no probability constraints and the concept of conditional variance is omitted for this comparative analysis; therefore, the historical variance and covariance are set as the targets in the moment matching process. In other words, all computations related to EGARCH and CCC-GARCH are ignored and variances and covariance matrix are assumed to be state-independent. Table 46 provides the resulting return/risk profile, which is plotted in Figures 51 and 52. The outcome strongly supports the proposed approach of including heteroskedastic models into the scenario generation process.

Table 46. Return/Risk profile obtained via Alg-2 (Fixed Variance)

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
10	0.518	20.521	1.345
20	0.622	84.722	2.890
30	0.726	192.641	4.435
40	0.878	310.137	5.706
50	0.968	421.162	6.731

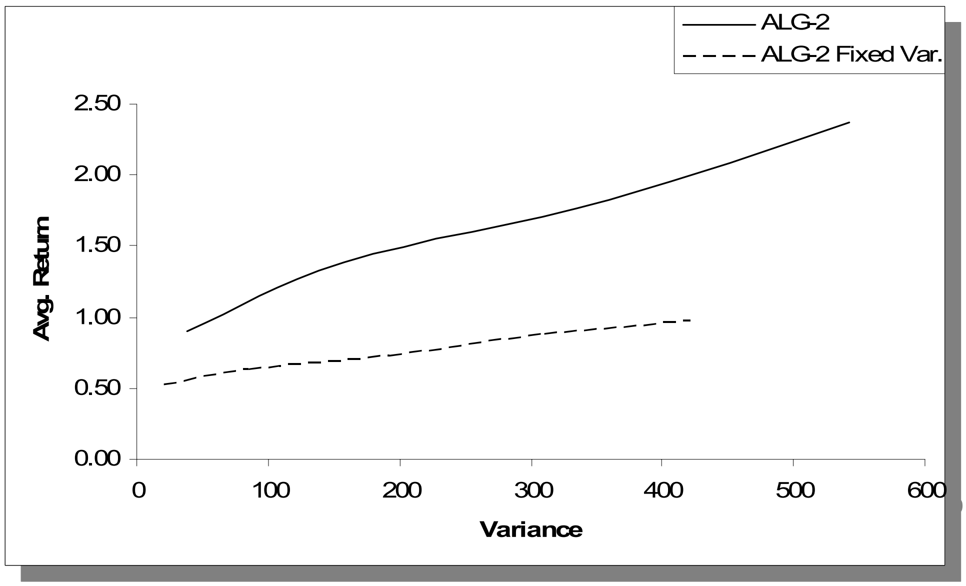

Figure 51. Results for Alg-2 where variance is state independent (Weekly, Variance)

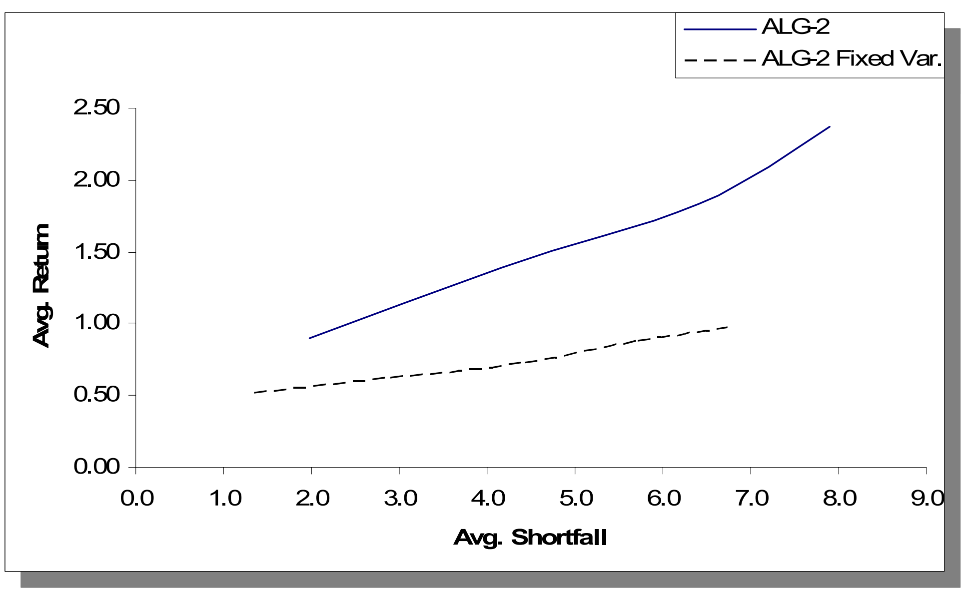

Figure 52. Results for Alg-2 where variance is state independent (Weekly, Avg.Shortfall)

3.2.4. Benchmarking with B&H and Markowitz's Model

We now consider two other investment strategies in order to evaluate the performance of the proposed approach against the alternatives. The first alternative is the simple B&H strategy mentioned in Section 3.1, where different percentages of the initial wealth are allocated to the risky assets and no trade occurs thereafter. The percentages are set as in Table 47 to represent different levels of risk averseness.

Table 47. Portfolio weights for B&H strategy

Cash	SP 500	RUS 2000
0.70	0.20	0.10
0.35	0.35	0.30
0.10	0.30	0.60
0.05	0.25	0.70
0.00	0.05	0.95

The second alternative is the popular mean-variance (MV) model developed by Markowitz (1952), which is a quadratic programming model where, given a fixed level of return, the objective is to minimize the portfolio risk, which is represented by the variance of the portfolio return. The objective might be also given as the difference between the mean return of the portfolio and the variance multiplied with a constant (i.e., risk aversion). We implemented the version given in (85)-(87),

$$\min \quad \frac{1}{2} x^T \Sigma x \tag{85}$$

$$\text{s.t.} \quad x^T \mu = \mu_{level} \tag{86}$$

$$x^T \mathbf{1} = 1 \tag{87}$$

where x is the column vector containing nonnegative portfolio weights for each asset; μ is the column vector of expected returns for each asset; Σ is the covariance matrix for asset returns; and **1** represents the vector of ones. Setting different values for expected portfolio return, μ_{level}, we obtain portfolio allocations for four different risk exposure levels. Note that the covariance matrix Σ and the expected return vector μ are computed for each test period using all past returns before that period. The resulting portfolio weights are then used to compute the performance measures.

The performance measures of the alternative approaches are given in Tables 48-49 and plotted in Figures 53-54 together with the measures obtained via Alg-2. These return/risk profiles in these figures reveal that the approach with Alg-2 provides more efficient portfolios than the B&H and MV strategies. In fact the difference gets more significant as the risk averseness is decreased (i.e., risk exposure is increased). This conclusion stays the same regardless the type of the risk measure, variance or average shortfall, we consider.

Table 48. Return/Risk profile obtained via B&H

Cash %	Avg. Ret.	VAR	Avg. Shortfall
70%	0.904	37.998	2.003
35%	1.587	212.181	4.982
10%	2.274	528.358	7.960
5%	2.462	640.678	8.776
0%	2.840	904.396	10.433

Table 49. Return/Risk profile obtained via MV

Risk Level	Avg. Ret.	VAR	Avg. Shortfall
1	0.774	25.543	1.631
2	1.128	108.437	3.557
3	1.469	258.565	5.608
4	2.692	926.533	10.691

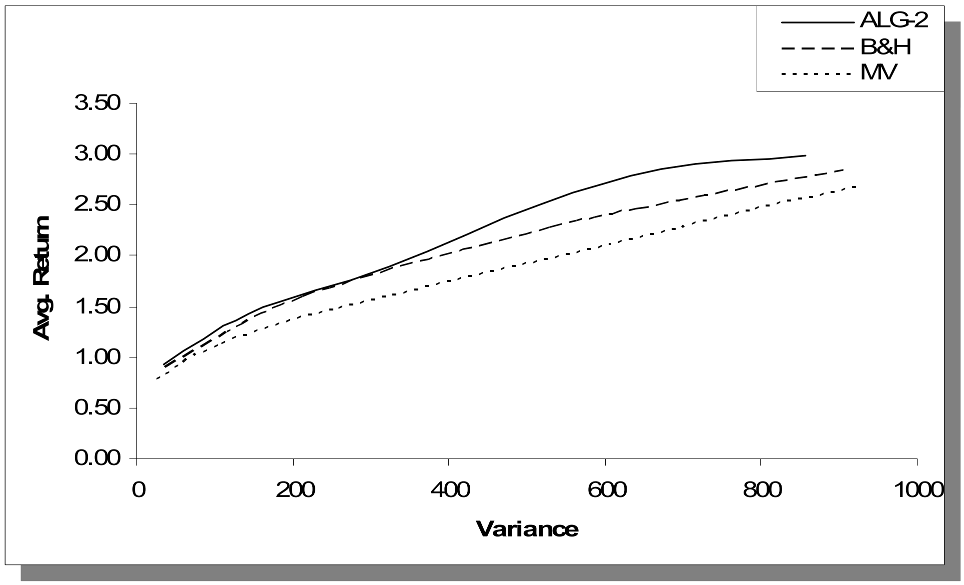

Figure 53. Comparison of Alg-2, B&H, and MV (Weekly, Variance)

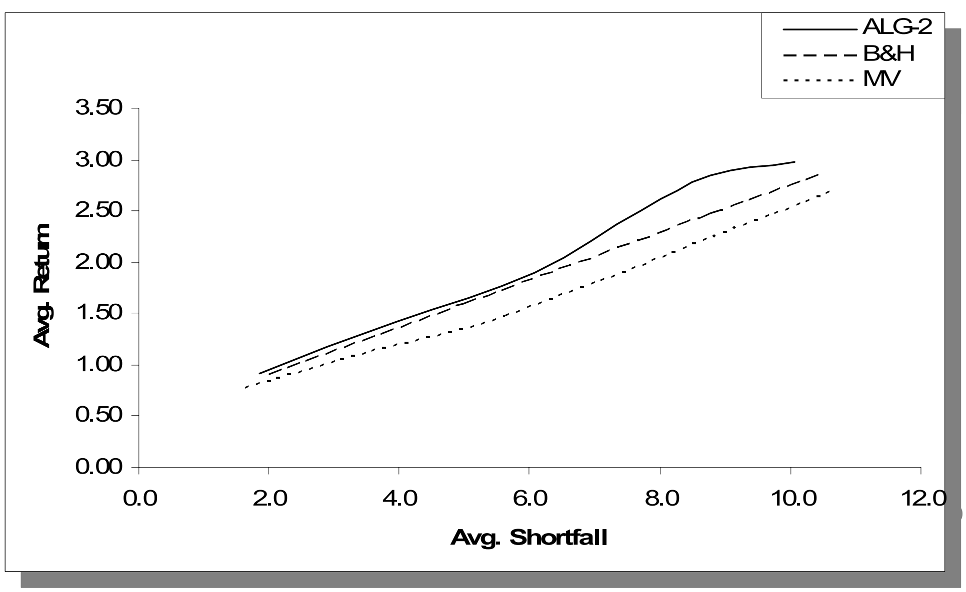

Figure 54. Comparison of Alg-2, B&H, and MV (Weekly, Avg. Shortfall)

3.2.5. Benchmarking with Vector Auto Regressive Model

The comparative assessment is now achieved by considering an alternative scenario generation method, Vector Auto Regression of degree 2 (VAR(2)). For this analysis, the proposed framework implemented in the aforementioned way; however, the proposed scenario generation algorithm is now replaced by VAR(2) estimated on index returns leaving the optimization stage unchanged with the same SP model.

Let X and Y denote the return series for S&P 500 and Russell 2000, respectively. Then, for each time period, the VAR(2) model given by in (88)-(89) is estimated using ordinary least squares method. Similar to the benchmarking presented in Section 3.1 and in Ziemba and Zhao (2001), we create a set, $\in$, of residuals ε^1 and ε^2 after the estimation, which represents the joint distribution for the error terms. Then we populate the scenario tree using equations (88) and (89) where the residuals are randomly sampled from $\in$. Scenario probabilities are equal given a source node. As mentioned in Section 3.2.1.3, this process is repeated consecutively for the last 100 time periods of the data set given a fixed *LCVAR*. Results for different *LCVAR* values are obtained to represent the return/risk profile of the approach with VAR(2), which are given Table 50.

$$X_t = c_1 + a_1 X_{t-1} + a_2 X_{t-2} + a_3 Y_{t-1} + a_4 Y_{t-2} + \varepsilon_t^1 \tag{88}$$

$$Y_t = c_2 + b_1 Y_{t-1} + b_2 Y_{t-2} + b_3 X_{t-1} + b_4 X_{t-2} + \varepsilon_t^2 \tag{89}$$

Figures 55 and 56 plot the return/risk profile obtained by using VAR(2) in the scenario generation process comparing it against Alg-2. The proposed scenario generation scheme produces higher returns than VAR(2) given a fixed level of variance. The same conclusion is valid for the higher risk region when average shortfall is considered. Another observation from the decisions-table obtain via VAR(2) is that the SP model results in a trading strategy where portfolio turnover is very high across periods, which might make it more sensitive to transaction cost. In Figures 57 and 58, risk/return curves obtained by setting transaction to 0.4% instead of 0.1% are shown (See Table 51). As expected, the VAR(2) curve shifted downwards whereas the proposed methodology is observed to be more robust with respect to an increase in the transaction cost.

Table 50. Return/Risk profile obtained via VAR(2)

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
10	0.891	39.233	1.815
20	1.411	155.537	3.670
40	1.672	344.397	5.768
90	1.727	406.291	6.389

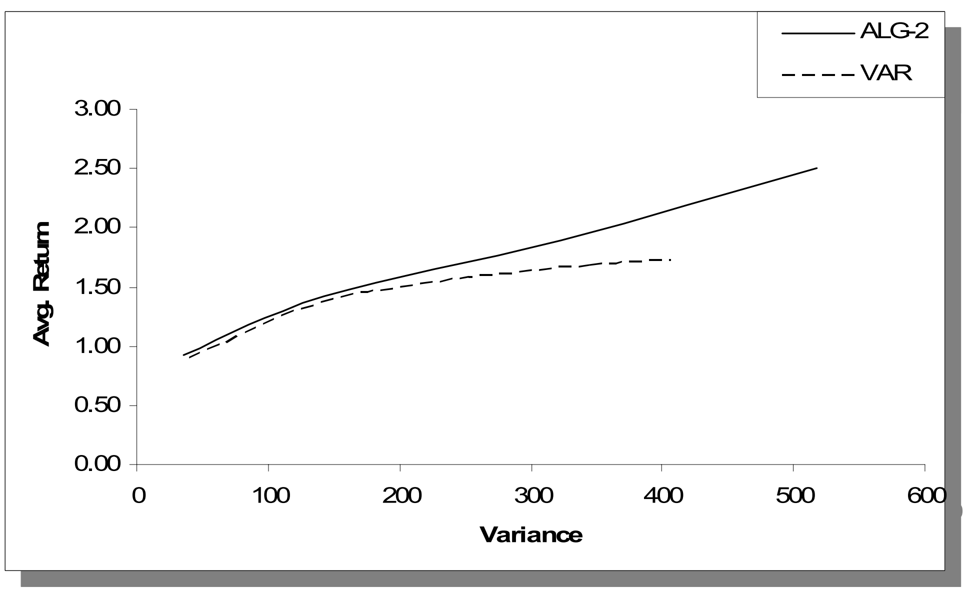

Figure 55. Comparison of Alg-2 and VAR(2) (Weekly, Variance)

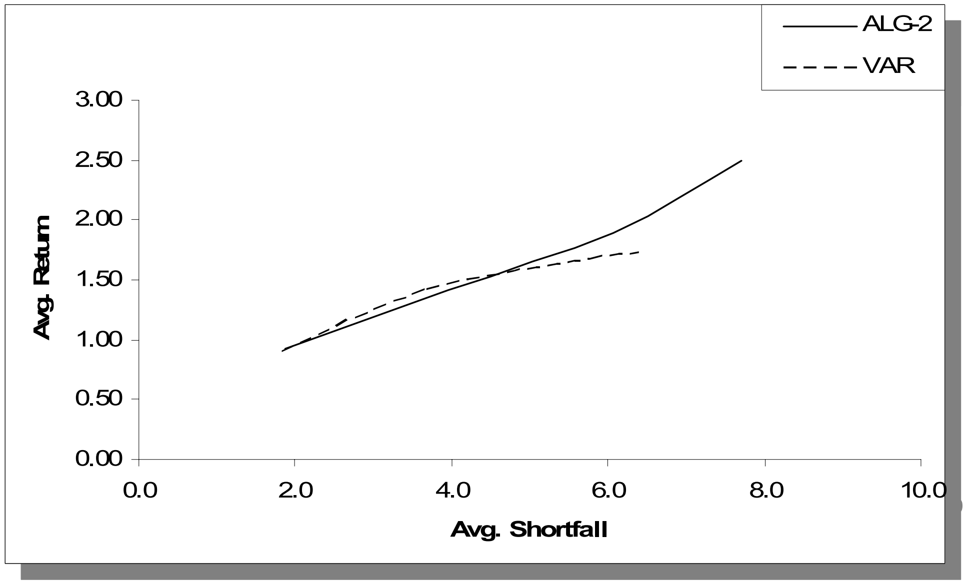

Figure 56. Comparison of Alg-2 and VAR(2) (Weekly, Avg. Shortfall)

Table 51. Return/Risk profile obtained via VAR(2) (Tran.=0.4%)

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
10	0.836	36.353	1.769
20	1.163	150.882	3.821
40	1.363	340.909	5.922
80	1.396	424.663	6.703

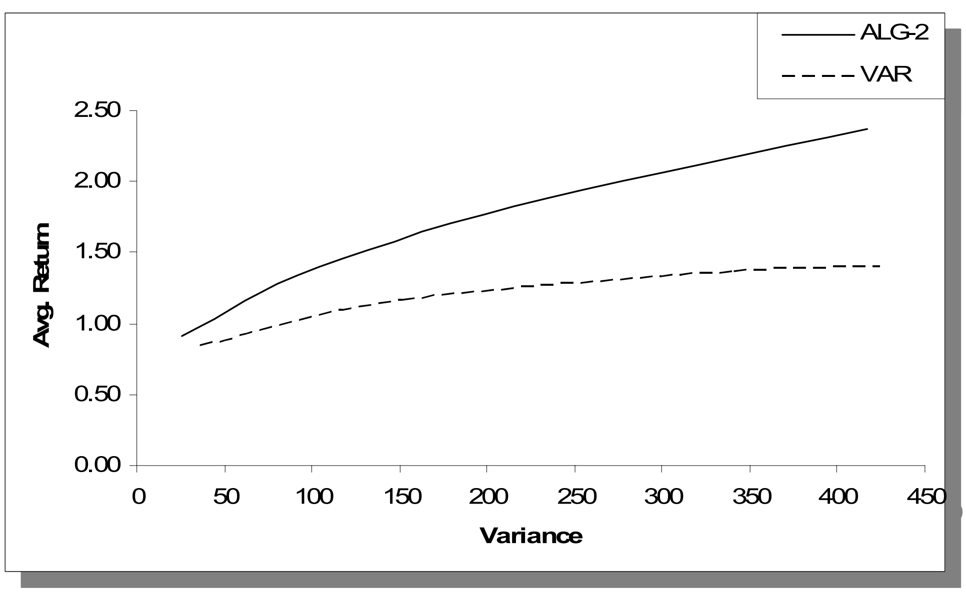

Figure 57. Comparison of Alg-2 and VAR(2) (Weekly, Variance, Tran=0.4%)

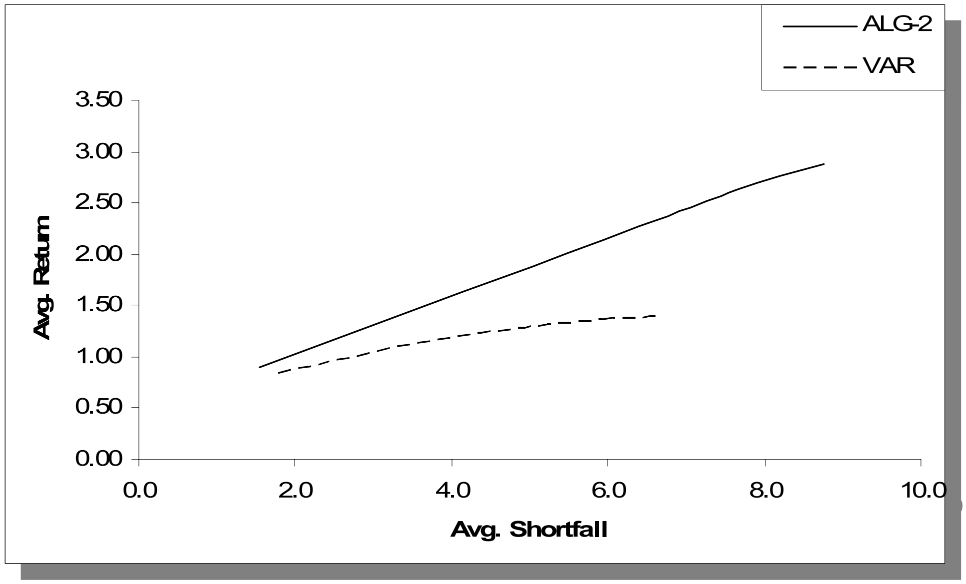

Figure 58. Comparison of Alg-2 and VAR(2) (Weekly, Avg. Shortfall, Tran=0.4%)

3.2.6. Sensitivity Analysis

Similar to Section 3.1.4, we now present the outcomes obtained by implementing the proposed approach with different parameters and input data. Our objective is to observe how the proposed strategy responds to slight changes in the model and evaluate its performance over the alternative approaches within different settings.

3.2.6.1. Sensitivity to Weights used in Moment Matching

Recalling the definition of Alg-2, we use different weights (i.e., w_{il} and w_{ik}) for deviations from different target moments in the objective function (23). For the computations presented in this section, we use the same weight vector for all assets. Let **we** denote the 1x5 row vector of these weights where the first four elements represent the weights used for the first four moments, whereas the fifth element is the weight used for the second co-moment (i.e., target covariance). The base case we used to obtain the aforementioned results gives priority to the first moment, followed by the second moment, fourth moment and the third moment.

Specifically **we** is set to [0.31 0.23 0.08 0.15 0.23], which is denoted by $\mathbf{we^0}$. For sensitivity analysis purposes we now consider the following four weight vectors each putting extra weight in one of the four moments (Notice that the maximums are underlined):

- $\mathbf{we^1}$ = [<u>0.56</u> 0.17 0.06 0.06 0.17]
- $\mathbf{we^2}$ = [0.12 <u>0.40</u> 0.04 0.04 <u>0.40</u>]
- $\mathbf{we^3}$ = [0.07 0.07 <u>0.71</u> 0.07 0.07]
- $\mathbf{we^4}$ = [0.07 0.07 0.07 <u>0.71</u> 0.07]

The performance measures obtained via using $\mathbf{we^1}$ – $\mathbf{we^4}$ are provided in Table 52 (see Appendix C) and plotted in Figures 59 and 60. The return/risk profile does not seem to be quite sensitive with respect to the weights used for the deviations from different target moments. This is not unexpected since the scenario tree used in this computation has 30 branches in the first period and 15 branches after each first-period scenario (See Section 3.2.1.2), which yields a high number of decision variables (i.e., scenario probabilities) to fit a given set of target moments. This scenario tree topology will be denoted by a row vector form, [30 15].

An alternative approach would be doing the same experiment with a different scenario tree having a lower number of branches and therefore providing few decision variables to fit the target moments. We expect to obtain higher sensitivity within such a setting. In order to illustrate this argument, we obtain the return/risk profiles obtained by using a scenario tree topology of [8 4] resulting in 32 scenarios at the horizon (See Table 53 in Appendix C).

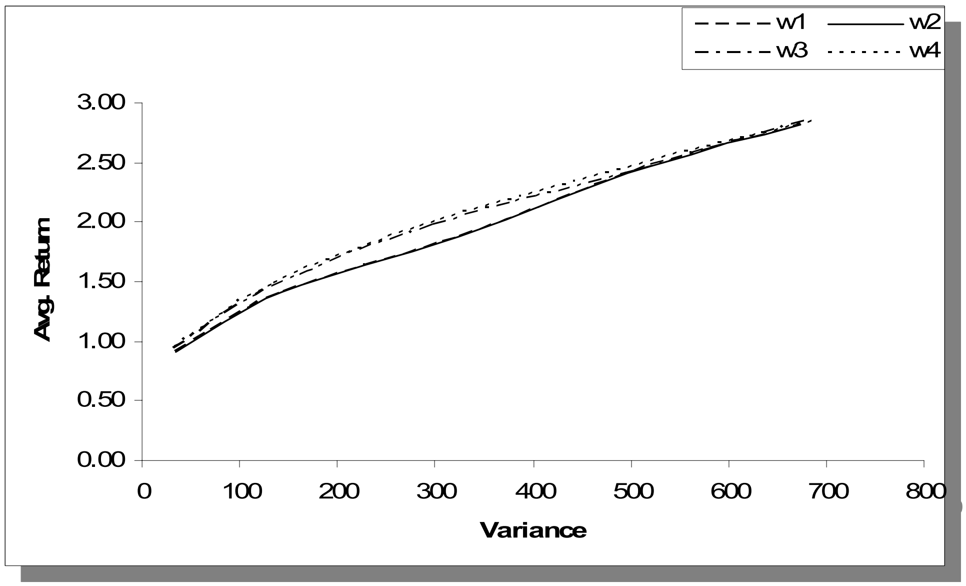

Figure 59. Comparison of weight vectors **we^1, we^2, we^3, we^4** (Variance)

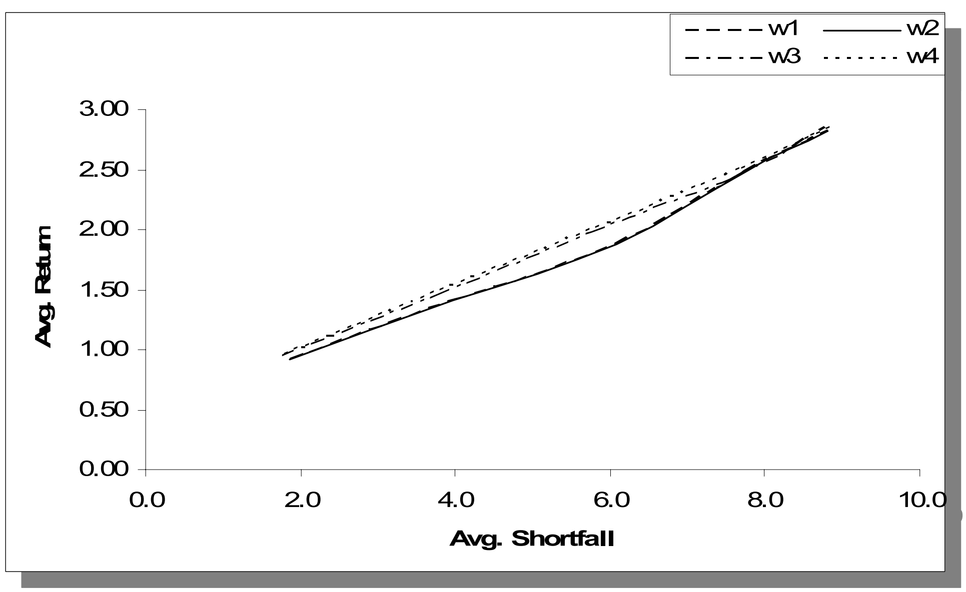

Figure 60. Comparison of weight vectors **we^1, we^2, we^3, we^4** (Avg. Shortfall)

The results obtained by using a [8 4] scenario tree are plotted in Figures 61-62. The outcome supports our expectation of a higher sensitivity when fewer scenarios are used at each period during the moment matching process. The return/risk curves are clearly apart from each other when compared to the curves in Figure 59-60, which implies the performance of the proposed approach becomes more sensitive to the weights assigned to deviation from target moments as the number of scenarios is decreased. We also present the Sharpe Ratios obtained from these two experiments (see Table 54 in Appendix C). The values for [8 4] topology have more variation than the values led by [30 15] topology, which can be better observed from the sample variances plotted in Figure 63.

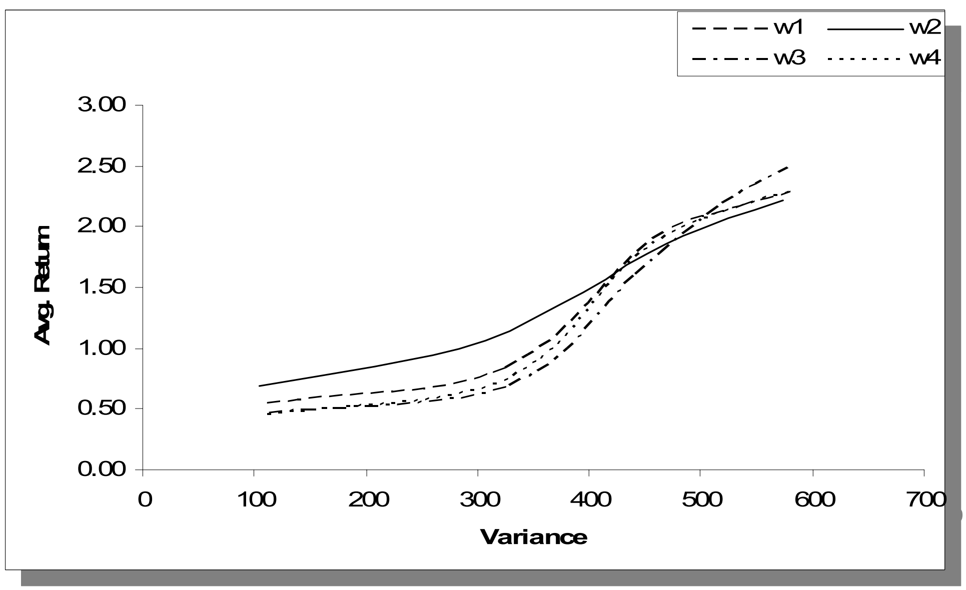

Figure 61. Comparison of weight vectors **we**[1], **we**[2], **we**[3], **we**[4] ([8 4], Variance)

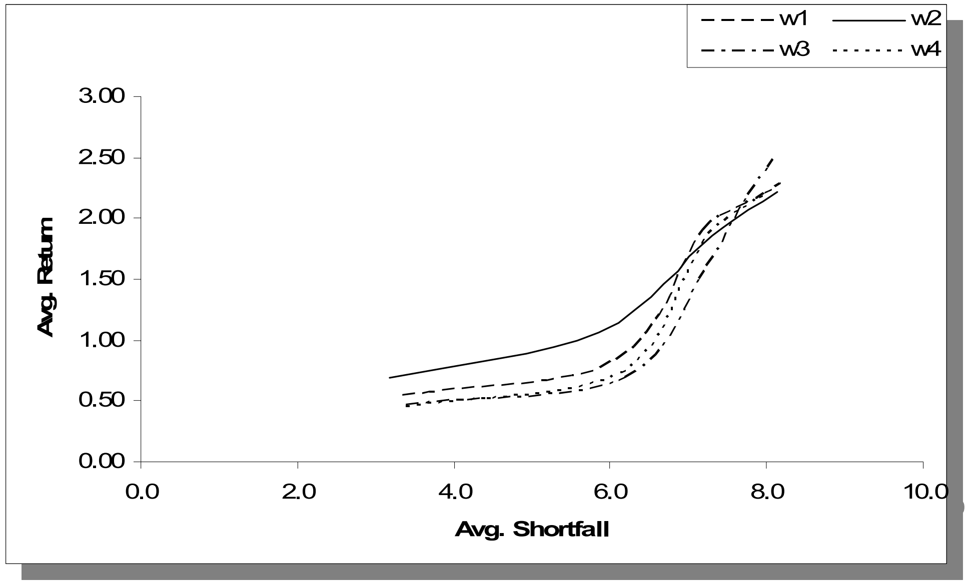

Figure 62. Comparison of weight vectors **we**^{1}**, we**^{2}**, we**^{3}**, we**^{4} ([8 4], Avg. Shortfall)

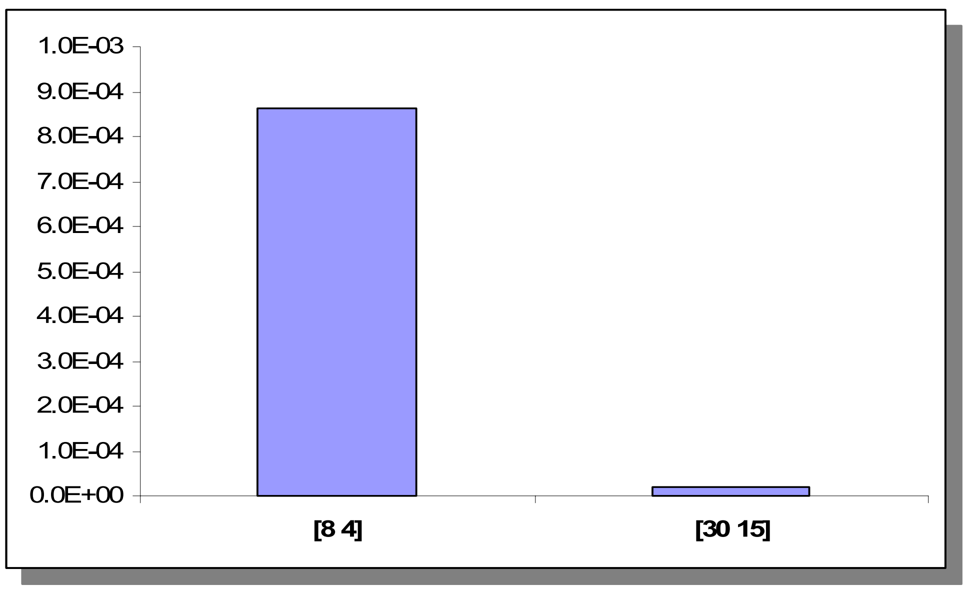

Figure 63. Sample variance for values in Table 54.

3.2.6.2. Sensitivity to the Number of Scenarios

As mentioned in Chapters 1 and 2, a major concern with SP models is the increase in problem size as the number of scenarios is increased, which may lead to significant increases in computation times. This constitutes a trade-off since a comprehensive scenario tree including a high number of scenarios would have more capability to capture a wide range of different discrete probability distributions for random variables involved in the model.

As mentioned in Section 3.2.6.1, we obtained results for two different scenario tree topologies, [8 4] and [30 15], having 32 and 450 scenarios at the horizon, which provides us with the opportunity to compare the performance of the proposed approach when low and high number of scenarios are used.

Noting that the conclusions are the same for both risk measures, we turn our focus to the Sharpe Ratios provided in Table 54 (see Appendix C), where the results of the experiments carried over with different weight vectors (i.e., $\mathbf{we}^0$-$\mathbf{we}^4$) are tabulated for two different scenario topologies. These values are plotted in Figure 64 and averages are compared in Figure 65. The immediate observation is that the performance is highly sensitive to the scenario tree topology. In addition, note that the topology [30 15] leads to significantly higher Sharpe Ratios when compared to [8 4]. This result confirms the validity of the trade-off between the problem size and the resulting power of modeling uncertainty, which in turn affects the performance of the SP-based approach.

3.2.6.3. Sensitivity to Time Unit and Time Windows

Similar to Section 3.1.4.2, we now change the dataset we use for our experiments such that the investment decisions are assumed to be made on a daily basis. In addition to evaluating the *sensitivity of the proposed approach with Alg-2 to the input data* and its performance with respect to several alternative approaches, we also aim to test the validity of some of the aforementioned conclusions when a daily-investment scheme is in question.

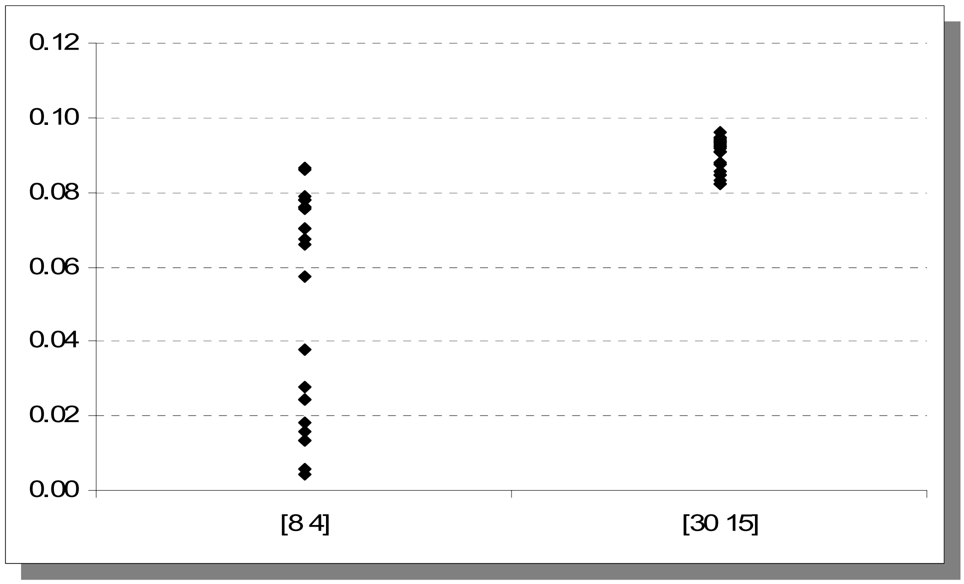

Figure 64. Sharpe Ratios obtained via different scenario tree topologies.

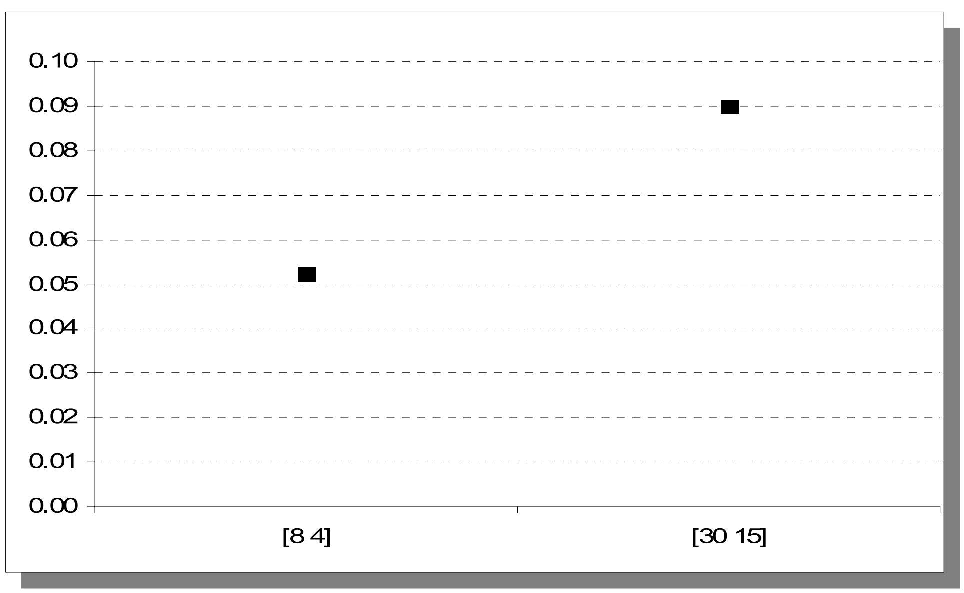

Figure 65. Average Sharpe Ratios obtained via different scenario tree topologies.

3.2.6.3.1. Effects of Constraints on Scenario Probabilities

The analysis presented in Section 3.2.2 is now carried over the daily data set, which covers the daily S&P 500 and Russell 2000 index values starting in the range 04/14/2003 - 11/12/2004. Note that this corresponds to the time window denoted by TW1 in Figure 41.

The objective is to see the effect of posing constraints on scenario probabilities during the moment matching process and calibrating the lower bound, lb^n, according to a daily setting. Keeping other parameters constant, four different experiments were done over TW1 such that lb^n is consecutively set to 0, 0.05, 0.10, and $0.20\,\forall n$. Tables 55-58 provide the resulting return / risk profiles which are plotted in Figures 66-67.

Table 55. Return / risk profile obtained when lb^n=0

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
2	0.1034	2.1962	0.5452
6	0.2228	19.8575	1.6823
10	0.3396	53.2657	2.7706
14	0.4551	84.9398	3.5074
24	0.7101	115.5461	4.0620
32	0.7103	115.5502	4.0620

Table 56. Return / risk profile obtained when lb^n=0.05

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
2	0.1072	2.0265	0.5162
6	0.2286	18.5740	1.6088
10	0.3924	49.2092	2.6141
14	0.5307	85.1486	3.4721
24	0.7103	115.5502	4.0620
32	0.7103	115.5502	4.0620

Table 57. Return / risk profile obtained when lb^n =0.10

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
2	0.0940	2.3001	0.5585
6	0.1973	20.6736	1.7170
10	0.3535	54.1148	2.7757
14	0.5214	88.1326	3.5369
24	0.7103	115.5502	4.0620
32	0.7103	115.5502	4.0620

Table 58. Return / risk profile obtained when lb^n =0.20

LCVAR	Avg. Ret.	VAR	Avg. Shortfall
2	0.1116	2.5382	0.5694
6	0.2468	22.9670	1.7566
10	0.3798	56.9178	2.8140
14	0.5055	87.7278	3.5351
24	0.7097	115.5365	4.0620
32	0.7103	115.5502	4.0620

As expected, the performance of Alg-2 increases when a scenario probabilities bounded below; however, the gain in performance is undermined when larger values are employed for bounding. This observation is similar to the one observed in Section 3.2.2; however, we now observe that the maximum performance is obtained when probabilities is bounded below by 0.05(number of scenarios)$^{-1}$ instead of 0.20(number of scenarios)$^{-1}$.

In order to illustrate the effect of constraints on probabilities, we now repeat the same analysis for four new time windows TW2, TW3, TW4, and TW5. These are the same time windows used in Section 3.1.4.3 and illustrated in Figure 41.

The performance measures obtained from these 16 experiments (i.e., four different bounding parameters over four different time windows) are given in Tables 59-62 in Appendix C. We present here the 120 (i.e., six different risk levels for each of the five time windows and for each of the four different values of lb^n) Sharpe Ratios obtained from all five time windows, TW1-TW5 (see Table 63 in Appendix C).

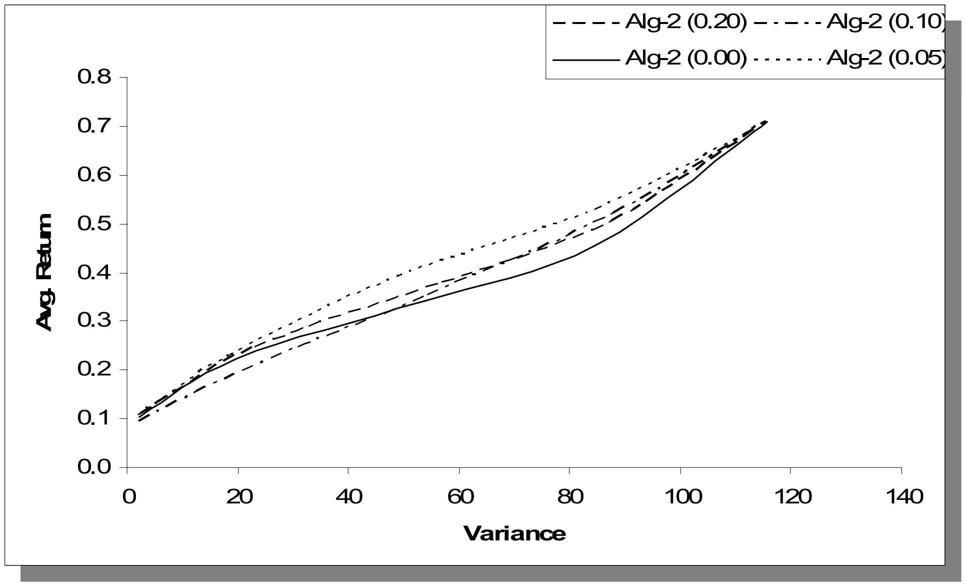

Figure 66. Results for Alg-2 where $lb^n \in \{0,0.05,0.10,0.20\}$ (TW1, Variance)

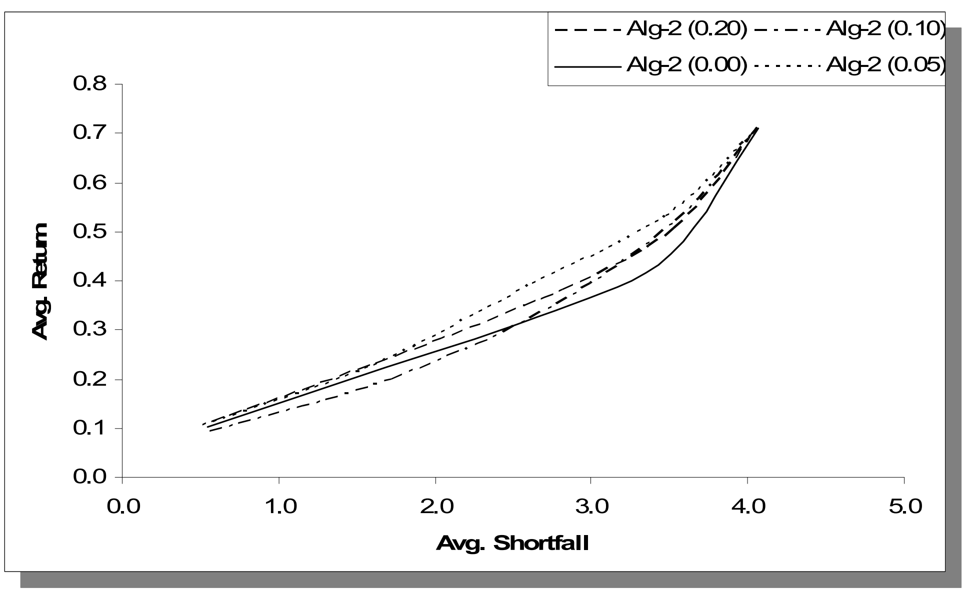

Figure 67. Results for Alg-2 where $lb^n \in \{0,0.05,0.10,0.20\}$ (TW1, Avg. Shortfall)

The average Sharpe Ratios are plotted in Figure 68. Even though they are close to each other, we observe that the performance of Alg-2 is maximized when lb^n=0.05, given the possible values of 0, 0.05, 0.10, and 0.20.

Different than the results led by weekly data where the performance is maximized when lb^n=0.20, we now observe that a smaller lower bound parameter, lb^n=0.05, leads to a better performance. Recalling that higher bounds would lead to higher deviations from target moments, the results on daily data imply that the deviations from target moments are penalized quicker than the weekly data. This can be attributed to the typical characteristics of the high-frequency financial data such as skewness and fat tails.

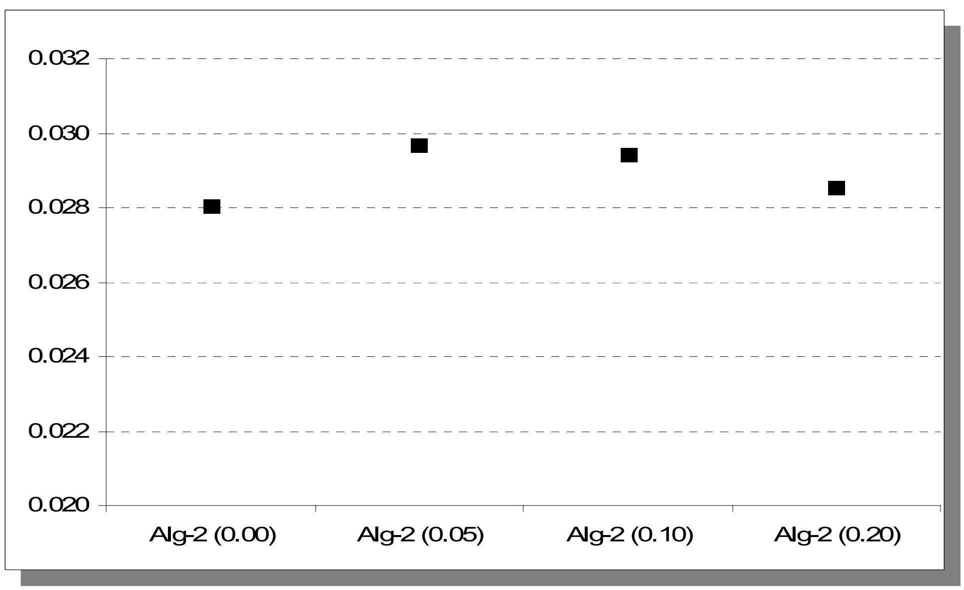

Figure 68. Comparison of average Sharpe Ratios led by different lb^n values

3.2.6.3.2. Effects of E-GARCH

In Section 3.2.3, we presented results obtained by removing E-GARCH modeling from the algorithm and using the historical unconditional variance and covariance values for moment matching. In addition, *the lower bounds on scenario probabilities had been removed* in order to better observe the effect contributed merely by using heteroskedastic modeling. The return/risk profiles obtained in Section 3.2.3 supported the idea of using E-GARCH.

Our objective is now to extend that the same analysis into the daily data framework. The five different time windows introduced in Section 3.1.4.3 and used in Section 3.2.6.3.1 (i.e., TW1-TW5) are considered for this analysis. Table 64 in Appendix C provides the performance measures obtained by using state independent variances and covariance matrix. Note that lb^n =0 for both cases.

Table 65 (see Appendix C) provides the Sharpe Ratios obtained by fixing the second moments and those obtained by Alg-2. The average values are plotted in Figure 69 where we observe that the usage of E-GARCH model within the scenario generation yields a higher Sharpe Ratios on the average. This comparison supports the conclusion in Section 3.2.3.

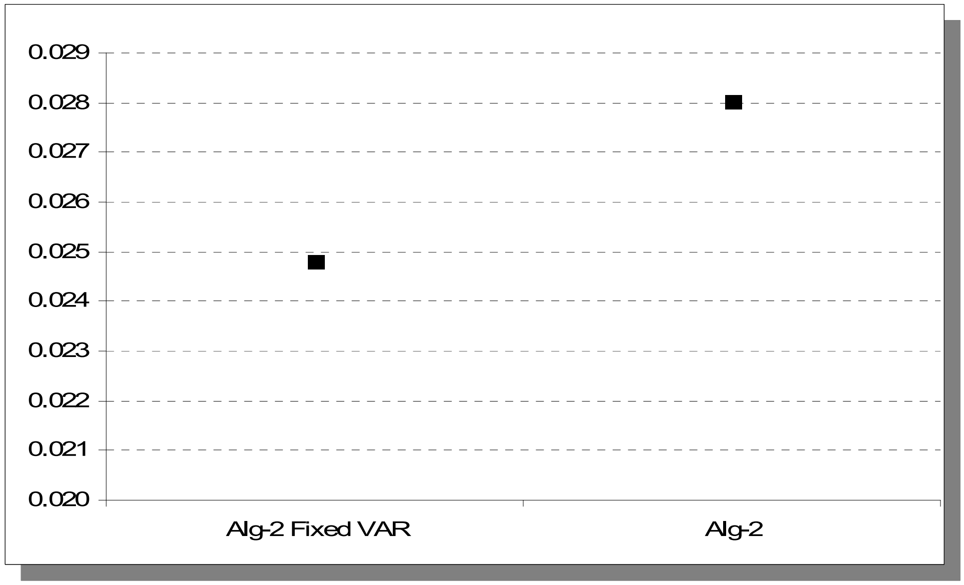

Figure 69. Comparison of average Sharpe Ratios

3.2.6.3.3. Sensitivity to Weights used in Moment Matching

We now analyze the sensitivity of the performance of Alg-2 with respect to the weights used in the objective function of moment matching model given the daily investment scheme.

In addition to the base vector $\mathbf{we}^{0}$, four different weight vectors introduced in Section 3.2.6.1 (i.e., $\mathbf{we}^{1}$ - $\mathbf{we}^{4}$) are considered in this analysis. The scenario tree topology is set to [30 15] leading to 450 scenarios at the horizon. Time windows TW1-TW5 are used to train and test the approach, which yielded the performance measures provided in Tables 66-71 (see Appendix C) and Sharpe Ratios plotted in Figure 70. The scatter diagram in Figure 70 does not reveal a significant sensitivity of the performance with respect to weight vectors. The average values depicted in Figure 71 illustrates the low level of sensitivity, where observe that the performance does not significantly change over weight vectors. Even though the differences are low, **we**^{4} yields a slightly higher average Sharpe Ratio among the vectors considered. This can be attributed the fat-tail characteristic of the high-frequency data since **we**^{4} puts more weight on the fourth moment.

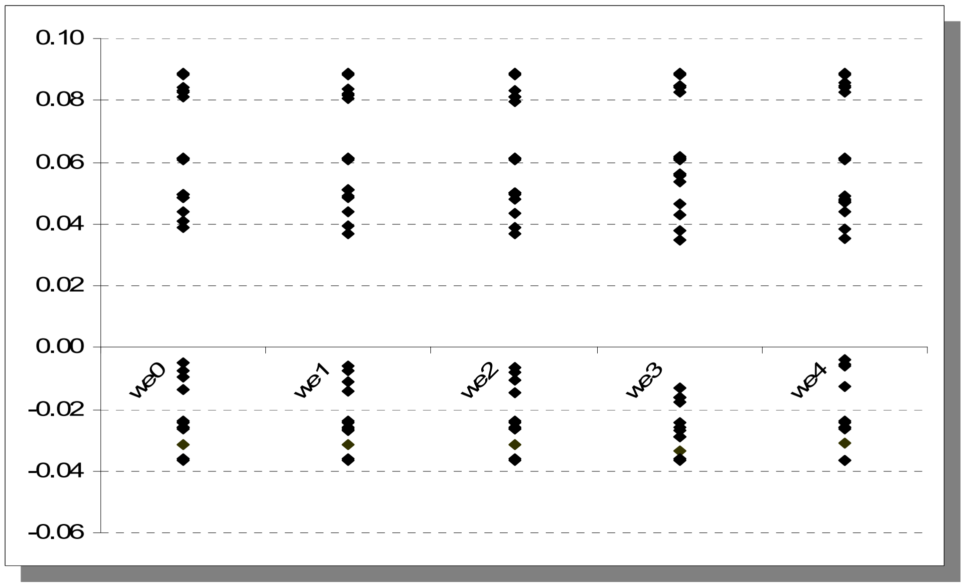

Figure 70. Sharpe Ratios obtained via different weight vectors.

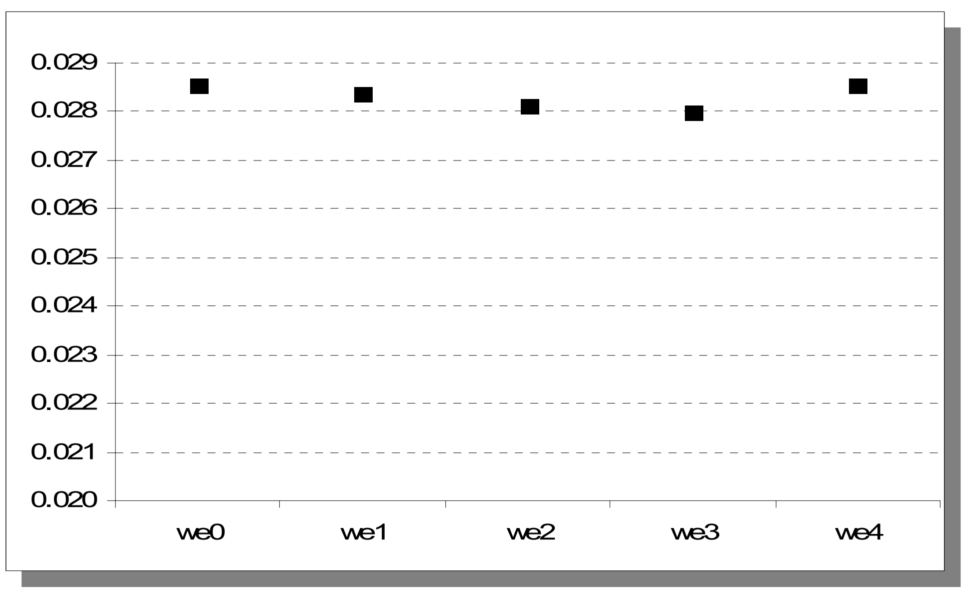

Figure 71. Comparison of average Sharpe Ratios led by different weight vectors.

3.2.6.3.4. Sensitivity to the Number of Scenarios

As mentioned Section 3.2.6.2, the topology of the scenario tree is critical to the success of SP based approaches. Recalling the experiments carried over considering the weekly data the increase in the number of scenarios has been shown to increase the performance of the proposed approach.

In this section we also consider the [80 40] topology in addition to [8 4] and [30 15] in order to see the affect of further increasing the number of scenarios. Therefore the analysis will cover the models with 32, 450, and 3200 scenarios at the horizon. Similar to the previous sections the analysis in this section also covers the time windows TW1-TW5.

The results obtained by using the [8 4] and [80 40] topologies are provided in Tables 72-73 (see Appendix C). We present in Table 74 (see Appendix C) all the Sharpe Ratios obtained from the experiments with all topologies and in Figure 72 the resulting average Sharpe Ratios. As expected, we obtain an increase in the average Sharpe Ratio as we use larger scenario trees with more scenarios at all periods. This observation supports the conclusion in Section 3.2.6.2 and is in accordance with the common trade-off regarding the problem sizes in SP models.

3.2.6.3.5. Benchmarking with B&H and Markowitz's Model

In this section we repeat the analysis presented in Section 3.2.4 considering the daily index data over time windows TW1-TW5. Regarding the B&H strategy, the asset allocations corresponding to different risk levels are as given in Table 63 (see Appendix C). The Markowitz's MV model given by (85)-(87) is implemented with four different target expected returns implying four different risk exposure levels.

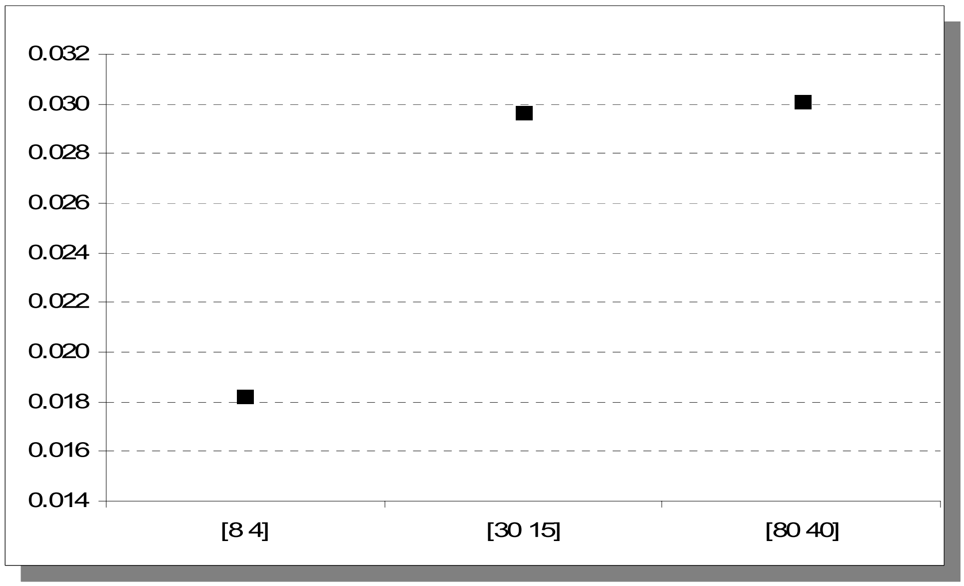

Figure 72. Comparison of average Sharpe Ratios led by different tree topologies.

The performance measures led by B&H and MV strategies over time windows TW1-TW5 are given in Tables 75-76 (See Appendix C). For benchmarking purposes we consider two versions of the proposed approach. The first is the base case with the scenario tree topology [30 15] whereas the second version has the scenario tree topology of [80 40].

Noting that the conclusions are the same regardless whether average shortfall or variance is considered as the risk measure, we present here the compact comparative analysis using the Sharpe Ratios obtained from alternative approaches. Figure 73 provides the scatter diagram and Figure 74 provides the average values. We observe from Figure 74 that the sample average of the Sharpe Ratios obtained from different experiments over TW1-TW5 gets its highest value when the proposed approach is implemented with the scenario tree topology [80 40]. When we decrease the number of scenarios (i.e., use [30 15] topology), the proposed approach is slightly outperformed by the strategy led by MV approach. Among all, the B&H strategy is outperformed by all of the approaches.

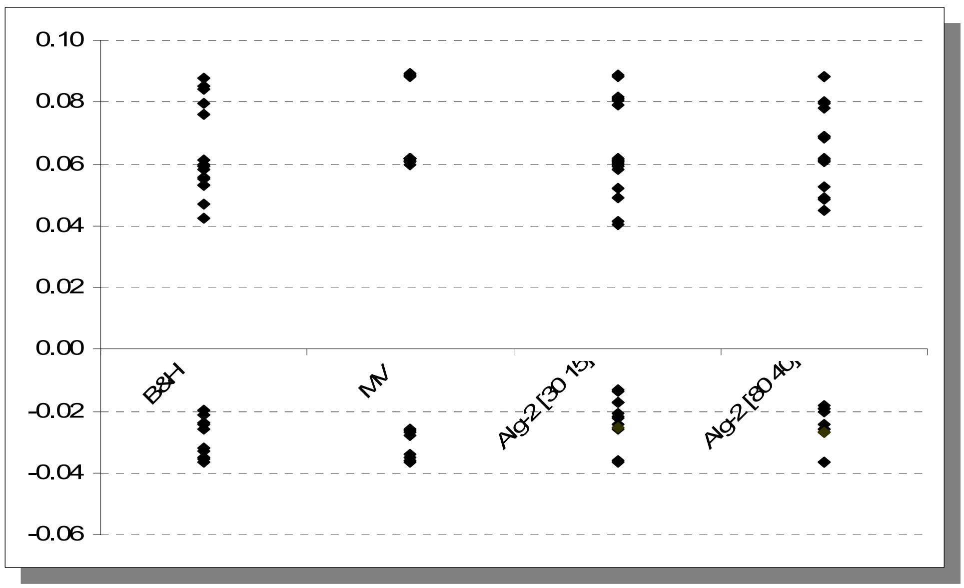

Figure 73. Sharpe Ratios obtained via alternative approaches.

3.2.6.3.6. Benchmarking with Vector Auto Regressive Model

We now repeat the analysis presented in Section 3.2.5 by estimating (88)-(89) over the daily index data over time windows TW1-TW5 and using the resulting model for scenario generation. The performance measures led by VAR(2) are given in Table 77 (See Appendix C). Similar to Section 3.2.6.3.5, we consider two versions of the proposed approach with two different the scenario tree topologies, [30 15] and [80 40].

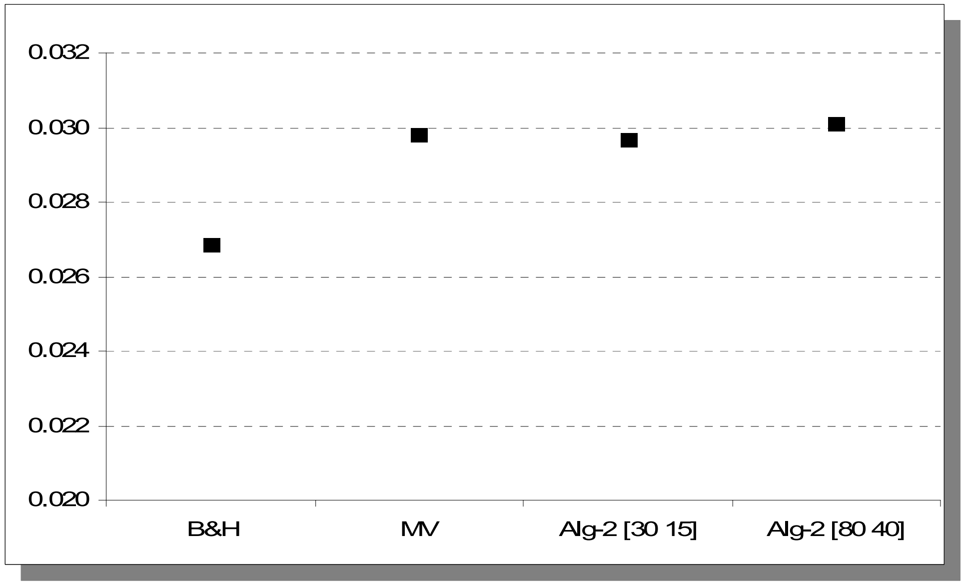

Figure 74. Average Sharpe Ratio values obtained via alternative approaches.

Figure 75 provides the scatter diagram and Figure 76 provides the average values. We observe from Figure 76 that Alg-2 significantly outperforms VAR(2) when the sample averages of the Sharpe Ratios obtained from different experiments over TW1-TW5 are considered. The conclusion is not sensitive to the scenario tree topology considered for implementation.

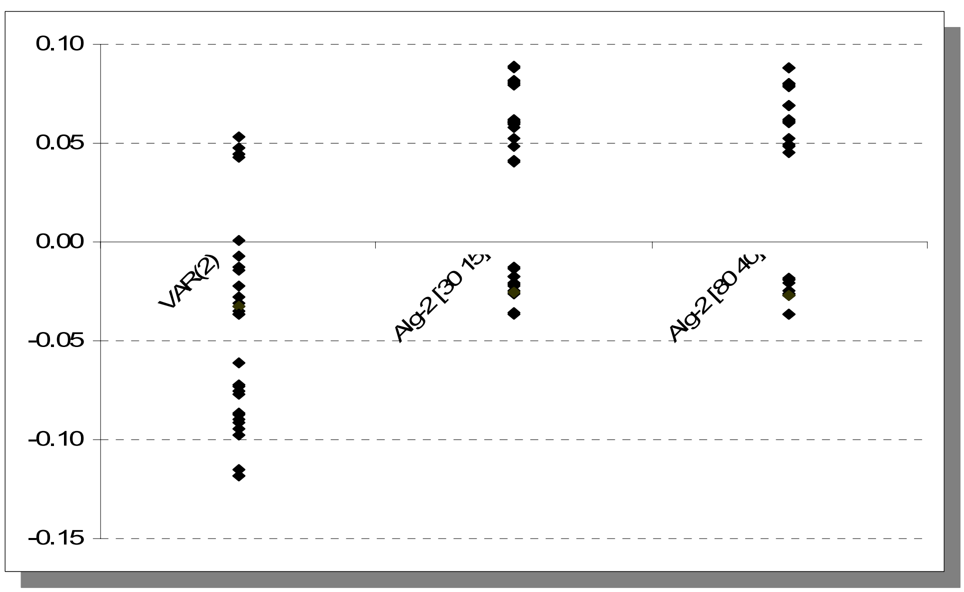

Figure 75. Sharpe Ratios obtained via alternative scenario generation methods.

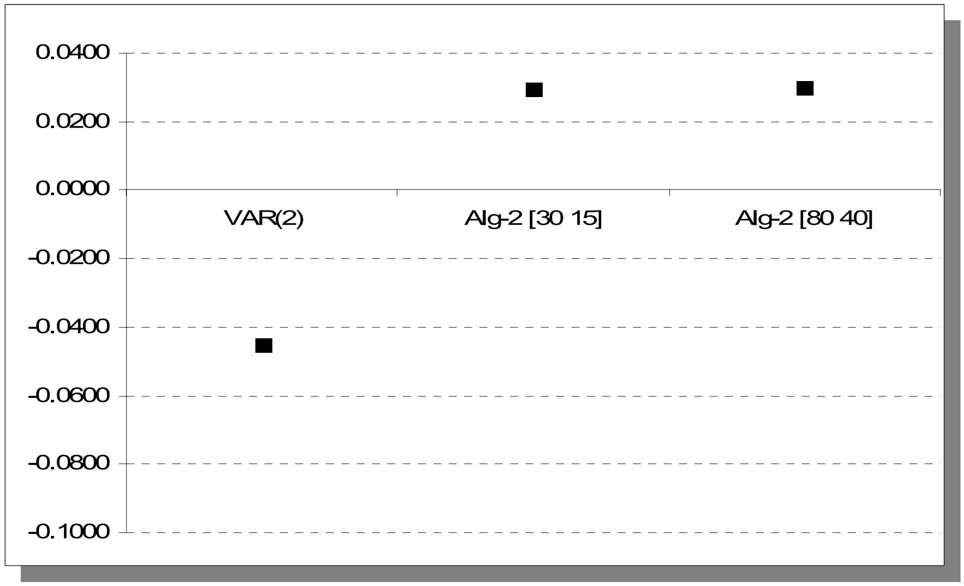

Figure 76. Average Sharpe Ratios obtained via alternative scenario generation methods.

4. A GENERAL APPROACH for LOSS REDUCTION

In this chapter, we address the process of modifying investment decisions led by a given trading strategy in accordance with the state of the market. In particular, we consider the state of the market in terms of volatility of the observed time series. Our objective is to examine an approach to identify potential ways to take corrective actions on a trading strategy such that the original return/risk profile is enhanced.

4.1. Market State

The state of the market can be related to various indicators such as GDP, unemployment, stock indices, etc. The usual classification distinguishes time intervals as periods where the economy is either in recession or not. Clustering time intervals in this manner may involve analyzing measures, such as returns and volatilities of stock market indices, interest rates, etc.

Markov switching models are broadly used in order to analyze the breaking points in the economy and make forecasts on market states given some observed variables. For instance, in his influential paper Hamilton (1989) proposes a methodology to obtain probabilistic inference about the unobservable regime changes in an observed time series data. In this study, it is assumed that the transition between the states is governed by a Markov process where the states indicate whether there is a negative (recessionary) or positive (non-recessionary) growth rate. State transition probabilities are estimated through an iterative procedure, which are utilized to draw inference regarding the turning points in the business cycle. There are many studies based on the methodology of

Hamilton (1989), such as Chu et. al. (1996), Yilmazkuday and Akay (2008), Chen (2008), Angelidis and Tessaromatis (2009), and references therein.

Li (2007) adopts the Markov switching ARCH model of Hamilton and Susmel (1994) in order to determine the states of the economy in terms of volatility. Fukuda (2009) analyzes the financial time series considering switching between probability distributions. Alternative approaches are evaluated where the observed data set is split into segments and different distributions (i.e., distribution of the innovations for the autoregressive model) are fit to each segment.

4.2. An Approach Based on Logistic Regression

The approach presented here does not aim to classify the past time intervals to be recessionary or not. In addition, we do not try to infer probabilities for the state of the market for the future periods. Our analysis focuses on the behavior of the trading strategy with respect to the volatility of the asset returns. The goal is to infer the probability of incurring a significant loss due to the decisions suggested by the trading strategy.

In order to achieve this goal we utilize logistic regression, which is a type of regression used to predict the occurrence of an event given a set of independent explanatory variables. Logistic regression fits data on a logistic curve expressed as in (90) and illustrated as in Figure 77.

$$L(Y) = \frac{1}{1+e^{-Y}} \tag{90}$$

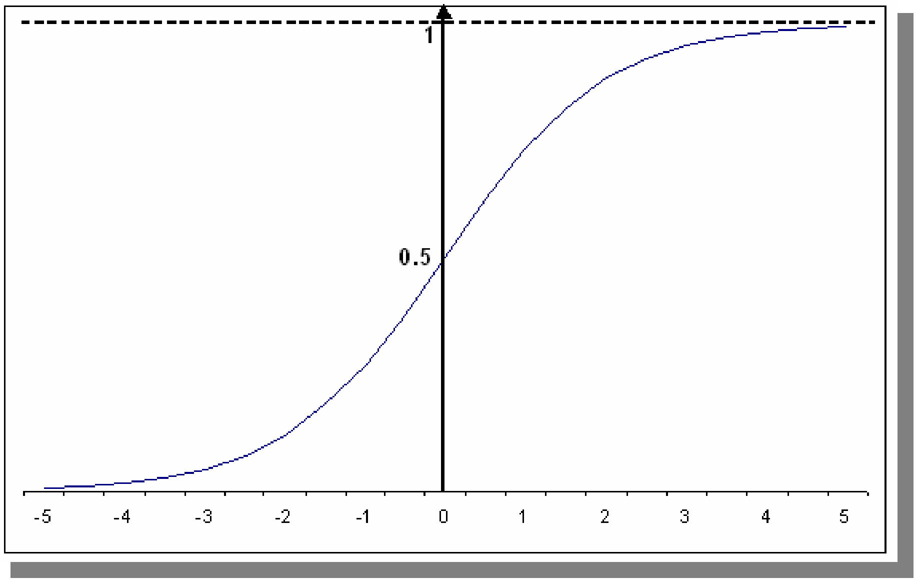

Figure 77. Logistic Curve for interval [-5, 5]

Logistic function takes values between 0 and 1 regardless the value of the independent variable. This property makes it a useful tool to predict the probability of an event given the explanatory variable(s). The variable Y in (90) is called *logit* and it is expressed as the linear combination of independent variables as in (91), where we assume that there are k independent variables denoted by $X_1 \ldots X_k$.

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k \tag{91}$$

In this approach we consider a *binary dependent* event, which can be described as the occurrence of the following two events *at the same time*

- The trading strategy suggests investment on a specific asset that exceeds some constant threshold, *TSD*. The parameter *TSD* can be considered as the weight of the asset in portfolio or an amount measured in terms of some monetary value.
- The return on that specific asset for the *following* period becomes negative.

The occurrence of these two events implies a significant loss that is led by the original trading strategy. The approach is based on estimating the probability of incurring such significant losses and updating the investment decision with the intention to decrease possibly high shortfalls.

This approach focuses on the evolution of time series data in conjunction with the actions led by the trading strategy. If the constant threshold *TSD* is set to 0, the resulting method would not be different than estimating the probability of having a negative return on a specific asset. However, setting a non-zero *TSD* value could enable the decision maker to track the decisions suggested by the trading strategy that produce a harmful effect for the portfolio. If the strategy does not invest in a specific asset, which will have a negative return in the following period, then no loss will be incurred. Therefore, the focus is not on whether the returns will be negative but on whether there will be a significant loss.

We are interested in relating the performance of a given trading strategy to the market volatility. Volatility is one of the most important characteristics defining the behavior of financial time series data. It is critical for the models used in option pricing, portfolio optimization, and many other models involving risk management.

Volatility is known to be a random variable rather than being constant over time. This concept is addressed by numerous stochastic volatility models (including ARCH-type models in discrete time) and a complete review of this subject is beyond the scope of our study. As for the general approach presented in this section, we are interested in using a *proxy* for volatility rather than predicting its value for the following periods.

Since volatility is derived from variance of the returns, we shift our focus to variance of returns. Different measures are used to obtain a proxy of return variance. We can list some common measures used to serve as a proxy of instantaneous return variance and volatility as follow:

- Squared return : r_t^2
- Absolute return : $|r_t|$
- Realized volatility : $\sqrt{\sum_{i=1}^{n} r_{t,i}^2}$ where $r_{t,i}^2$ denotes the returns obtained at n equal time intervals obtained within period t (e.g. intraday data).
- Implied volatility : This measure is obtained using market prices of options and an option pricing formula such as Black-Scholes. It gives insight about the expectations of the market.

For instance, Li (2007) uses squared return whereas Gavrishchaka and Ganguli (2002) consider the absolute returns. Some of the studies using realized return are Jacquier and Marcus (2001), Micciche et al. (2002), and Chu et al. (1996).

We consider two different measures, denoted by *M1* and *M2*, as proxies to the market volatility, which will serve as explanatory variables in two different logistic regressions where the constants will be 0. These are:

M1: *Absolute value of the product of consecutive returns*.

M2: *Sum of squared returns*.

Specifically, returns for the first three lags are considered. Therefore, the logistic regression models can be stated as

$$P_{i,t} = \frac{1}{1+e^{-Y_{i,t-1}}} \tag{92}$$

$$Y_{i,t-1} = \beta_i \left| r_{i,t-1} r_{i,t-2} r_{i,t-3} \right| \qquad (M1) \tag{93}$$

$$Y_{i,t-1} = \beta_i (r_{i,t-1}^2 + r_{i,t-2}^2 + r_{i,t-3}^2) \qquad (M2) \tag{94}$$

where $P_{i,t}$ denotes the probability of incurring a loss in period *t* due to investment in risky asset *i* that exceeds *TSD* given the past three return values (i.e., $r_{i,t-1}, r_{i,t-2}, r_{i,t-3}$) and proxy type (i.e., *M1 or M2*). The resulting $P_{i,t}$ value is used to update the investment decision suggested by the original trading strategy for period *t*. The approach is based on decreasing the amount of investment in asset *i* depending on the value of $P_{i,t}$ and transferring the corresponding funds to the risk-free asset. In fact, the amount of transfer becomes higher as $P_{i,t}$ gets larger values.

We estimate the regression model by (92) and (93) or (92) and (94) by dividing the set of *training periods* originally used by the trading strategy into two. The first set of periods, denoted by TR_1, is utilized for the usual training of the trading strategy. In other words, this set of data is used to estimate any parameters that belong to the trading strategy. For instance, consider Alg-2 presented in Chapter 2. This set of data is used to estimate merely the parameters of the EGARCH. The second set of data, denoted by TR_2, is used to estimate the regression parameters in (92)-(94) given the investment decisions and corresponding losses, if any, led by the trading strategy. The following periods are used for testing purposes, denoted by *Test*). See Section 4.3 for more details on the implementation process.

Suppose that $z_{i,t}$ and y_t denote the amount of investment on risky asset *i* and risk-free asset, respectively, suggested by the trading strategy at the beginning of period *t* (i.e., *before realizing* the return for period *t*). Also denote by $z^*_{i,t}$ the updated investment on risky asset *i*. Given the preferred proxy for volatility as *M1* (note that consideration of *M2* will only change the equation labels in the description below) and the time window as being the combination of TR_1, TR_2 and *Test*, the proposed approach can be outlined by the following steps *for each risky asset i*:

Step 0 : Estimate the parameters that belong to the trading strategy using the data in TR_1.

Step 1 : For each *t* in TR_2;

Execute the trading strategy and obtain $z_{i,t}$. Then assign $P_{i,t}$ as:

$$P_{i,t} = \begin{cases} 1 & \text{if } z_{i,t} > TSD \text{ and } r_{i,t} < 0 \\ 0 & \text{otherwise} \end{cases} \tag{95}$$

Let *REG* denote the set of observations obtained.

Step 2 : Estimate the logistic regression model (92)-(93) using the data set *REG*. Compute $P_{i,t}$ via (92)-(93).

Step 3 : For period *t* in *Test*, execute the trading strategy and obtain $z_{i,t}$. Add this investment decision into *REG* using (95) (Note that this step is included here for description purposes. The corresponding observation is utilized in period $t+1$, after the return for period t is realized).

Step 4 : Update the allocation on risky asset i as

$$z^*_{i,t} = (1\text{-}P_{i,t})\, z_{i,t} \tag{96}$$

Step 5 : Update the allocation on risk-free asset as given in (97). This essentially involves adding the funds obtained by the update (i.e., reduction) on risky asset positions given in (96); adding any transaction costs led originally by the trading strategy; and subtracting the transaction costs implied by (96).

$$\begin{aligned} y^*_t = y_t \; + P_{i,t} z_{i,t} + \varepsilon \left| z^*_{i,t-1}(1 + r_{i,t-1}) - z_{i,t} \right| \\ - \varepsilon \left| z^*_{i,t-1}(1 + r_{i,t-1}) - z^*_{i,t} \right| \end{aligned} \tag{97}$$

Step 6 : Repeat Steps 2-5 for all $t \in Test$.

4.3. Computational Results

The approach presented in Section 4.2 is based on a general idea and can be implemented in conjunction with any methodology mentioned in this study. For implementation purposes of this section, we consider only Alg-2. The input data contains the daily time windows, TW1-TW5, which are the same time windows used to obtain results in Chapter 3.

Each time window contains a set of time periods for training and another set for testing the trading strategy, which includes 100 consecutive time periods as mentioned in Chapter 3. In order to implement the decision rule we split the training data set into two as described in Section 4.2. Specifically we take the last 100 periods of the training set to obtain the observations, TR_2, to be used for estimating (92)-(94) (see Figure 78 for the illustration involving TW1 as an example, which is composed of 400 consecutive days).

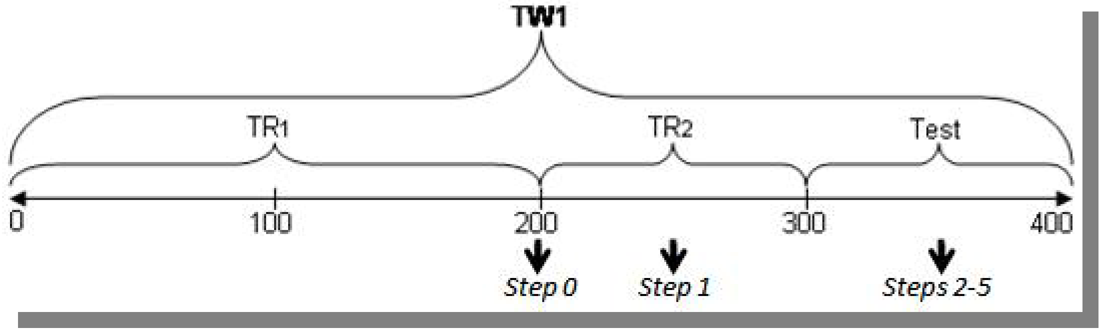

Figure 78. Illustration for the segmentation of time windows.

The critical parameter of the proposed approach is *TSD*, which is used to determine the value of the binary dependent variable in the regression. It specifies the threshold for the investment on an individual asset such that only the investments exceeding *TSD* are eligible to be considered as significant losses. Therefore, low *TSD* values are expected to modify the original trading strategy (i.e., Alg-2 combined with the multi-stage SP model) in a more conservative way than high values of *TSD*.

We consider four different values for *TSD* (i.e., \$200, \$400, \$600, \$800 noting that the initial portfolio value is \$1000), two different measures (i.e., *M1*, *M2*), five time windows (i.e., TW1-TW5), and six different risk tolerances (i.e., various *LCVAR* values). Similar to the implementation process presented in Chapter 3, the approach presented in this chapter required extensive computation. Considering the aforementioned setting, the combination of Alg-2 and the SP model needed to be run for 4x2x5x2x100x6=48,000 times and the logistic regression model needed to be estimated for 4x2x5x100x6=24,000 times.

The results are aggregated over time windows and risk tolerances leading to eight different trading strategies (i.e., four *TSD* values and two volatility measures). Similar to the previous analysis in Chapter 3, benchmarking is achieved by considering the Sharpe Ratios led by alternative strategies. Figures 79 and 80 provide the scatter plot of the individual Sharpe Ratios obtained from each time window and risk tolerance by implementing the proposed approach considering *M1* and *M2*, respectively whereas Figure 81 provides a comparison of the average Sharpe Ratios (see Tables 78-85 in Appendix D for the performance measures for each of the eight problem setups).

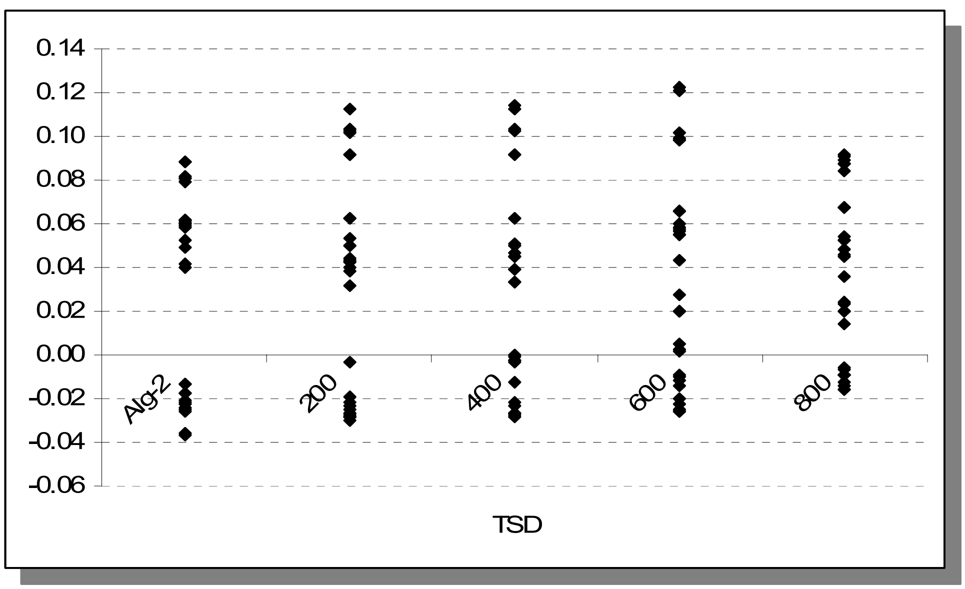

Figure 79. Sharpe Ratios obtained via different *TSD* values with *M1*.

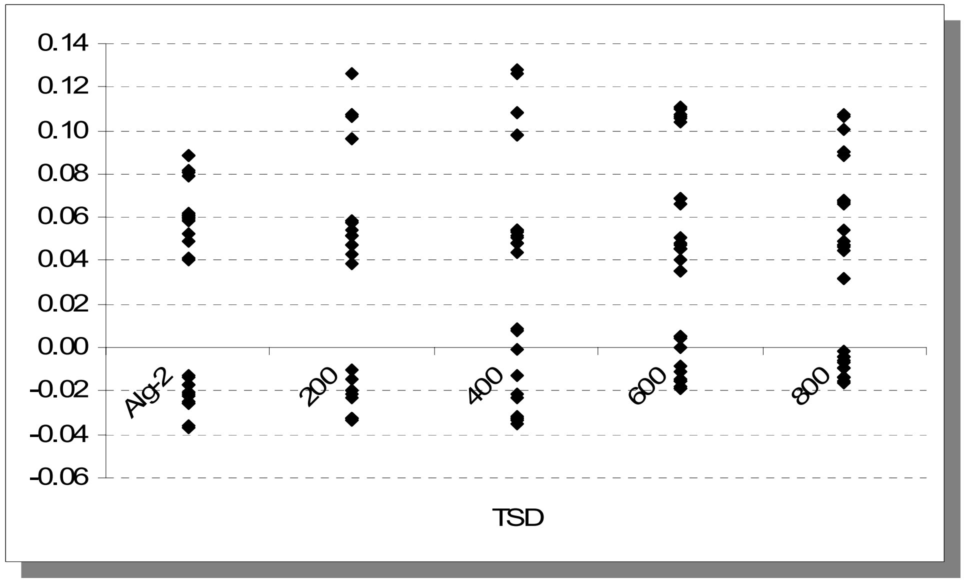

Figure 80. Sharpe Ratios obtained via different *TSD* values with *M2*.

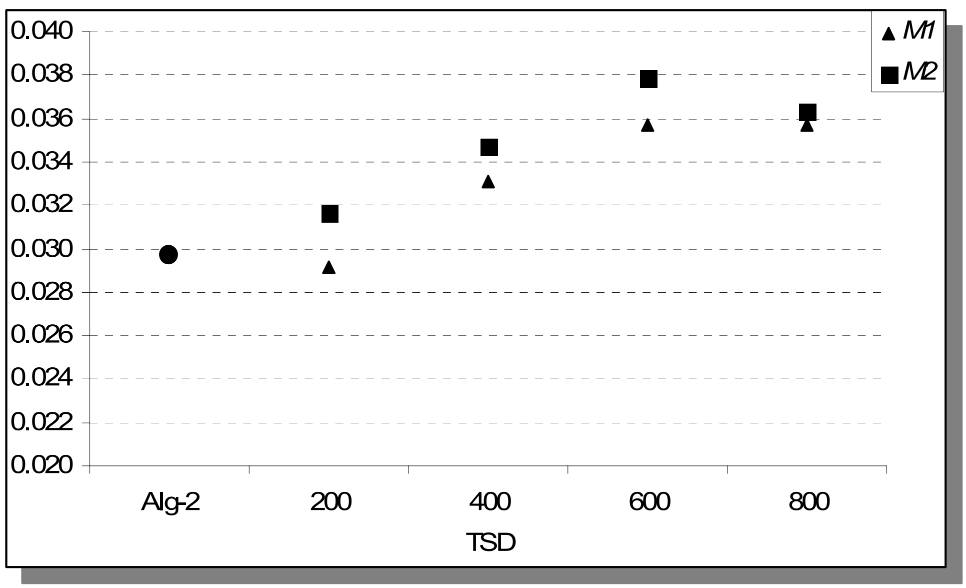

Figure 81. Average Sharpe Ratios obtained with both *M1* and *M2*.

The distributions of the Sharpe Ratios in both Figures 79 and 80 imply an improvement led by the implementation of the proposed approach when compared to the base strategy, Alg-2, where the positive effect is clearly identified through the average values in Figure 81. One can make three observations from these aggregate results:

1. Almost all problem settings with the new approach (except *M1* with *TSD*=200) outperforms the base strategy of the original Alg-2. This can be attributed to the potential benefit of employing the proposed regression-based approach itself.

2. Usage of *M2* leads to superior results when compared to M1 regardless the value of *TSD*. This can be attributed to $(r_{i,t-1}^2 + r_{i,t-2}^2 + r_{i,t-3}^2)$ being a better proxy for instantaneous volatility than $\left|r_{i,t-1} r_{i,t-2} r_{i,t-3}\right|$. This is not unexpected considering the wide usage of squared returns as a proxy for volatility. The difference between two measures become more apparent when at least one of the return values is close to zero, in which case $\left|r_{i,t-1} r_{i,t-2} r_{i,t-3}\right|$ attains small values regardless the magnitude of the other returns.

3. The maximum performance is achieved when *TSD* = 600 regardless *M1* or *M2* is used as the proxy for volatility. This outcome can be attributed to the significance of regression coefficients, which are usually measured via *p*-values. As commonly interpreted, lower *p*-values support the rejection of the null-hypothesis that independent variables do not provide information about the dependent variable. Due to the high number of estimations carried over (i.e., 24,000), we provide here only the average *p*-values for each time window in Table 86. The last row in Table 86 shows that *p*-values get the lowest magnitude when *TSD* is

set to 600, which could be a valid explanation for the performance being maximized in this setting. In fact, the ranking of the other average p-values is in accordance with the average Sharpe Ratios as given in Table 87. In other words, the order of average p-values is the reverse of the order of average Sharpe Ratios for both *M1* and *M2*.

Table 86. Average p-values for each time window.

	TSD			
Time Window	200	400	600	800
TW1	0.216966	0.065138	0.000096	0.000148
TW2	0.334505	0.468634	0.000068	0.000012
TW3	0.277518	0.154268	0.021265	0.022551
TW4	0.238465	0.334029	0.005683	0.012273
TW5	0.215589	0.142874	0.004237	0.000112
Overall Average	**0.256609**	**0.232988**	**0.006270**	**0.007019**

Table 87. Average p-values vs. average Sharpe Ratios.

	TSD			
Statistics	200	400	600	800
Avg. p-value	0.256609	0.232988	0.006270	0.007019
Avg. Sharpe Ratio (*M1*)	0.029172	0.033090	0.035714	0.035709
Avg. Sharpe Ratio (*M2*)	0.031684	0.034698	0.037837	0.036320

In this chapter we proposed a general approach that is based on updating the investment decisions led by a given trading strategy. We implemented this approach considering Alg-2 as the underlying strategy and logistic regression as the learning model. We calibrated the main parameter *TSD* to 600, which provides the best return/risk profile among the tested values.

We should note that the magnitude of improvement on the performance measures and the corresponding parameter values can vary when different trading strategies, assets, and learning models are considered. However, the experimentation carried over in this section provides significant empirical support for the proposed approach.

5. CONCLUSION

In this study we presented a multi-period financial portfolio optimization framework based on stochastic programming (SP). We proposed two new scenario generation algorithms each capturing the uncertainty in random variables through creating discrete probability distributions on a scenario tree. We also presented a general multi-stage SP model to obtain the investment decisions to maximize the expected wealth given the generated scenario tree and limits on risk exposure at each decision node.

We implemented the proposed methodology considering different parameter and data sets in order to evaluate the performance in different settings and to obtain insight about the sensitivity of the results. We next summarize our methodological approach and computational results.

NOTE 1: In the following section we use the term *dominate* in order to compare two different strategies. We use the phrase "Strategy A dominates Strategy B" to mean than Strategy A yields a more efficient strategy such that it provides the investor with higher average return (less risk) than Strategy B given a fixed level of risk exposure (average return).

5.1. Multi-Stage Linear SP model

We presented a multi-stage linear SP model with the objective of maximizing expected final wealth where the risk exposure is controlled by linear constraints on CVaR. For the implementation of the proposed scenario generation algorithms, we considered an asset allocation problem, where the investor has an initial amount of wealth and wants to allocate this amount among a risk-free asset and risky assets. Note that the multi-stage linear SP model presented in Chapter 2 is a comprehensive ALM model since it is also capable of handling cash inflows and outflows. In addition, risk measures different than the CVaR are included in the general model.

One aspect that is unique to our model to the best of our knowledge is the state dependent risk control. Instead of measuring risk merely at the horizon, we include linear constraints that limit the realized CVaR for each node of the scenario tree except the leaf nodes. In our computational analysis, we set all CVaR limits to the same value in each experiment. However, our general SP model is capable of setting different parameters for risk control at different decision epochs. Given a complete scenario tree, the decision maker is able to set a specific limit on CVaR depending on the state at a particular node or a particular time period.

The size of the deterministic version of the SP model presented and implemented in this study is determined mainly by the topology of the scenario tree. The experiments run for Alg-2 suggest the usage of larger scenario trees for improved performance; however, the problems become significantly larger as the number of scenarios is increased (see Table 88).

Table 88. Problem size for different tree topologies

Tree Topology	**[8 4]**	**[30 15]**	**[80 40]**
Number of Scenarios	32	450	3200
Number of Constraints	1,604	41,952	617,852
Number of Variables	876	11,810	83,460

5.2. Alg-1A and Alg-1B

We present our first approach in the form of two separate, but very similar, scenario generation algorithms, Alg-1A and Alg-1B, to create scenario trees with the objective of capturing the true uncertainty associated with the random variables in question. Both scenario generation algorithms are based on the same idea of reducing the historical data set into a smaller but more relevant set such that the similarity between the current situation and the historical data set is increased. This reduction is achieved using similarity scores, which are computed via distance measures. There are two differences between Alg-1A and Alg-1B. First one is the type of the distance measure used to assign similarity scores. Alg-1A is based on *weighted* Euclidean distance whereas Alg-1B is built on the new *UD* distance measure proposed in Chapter 2. Another difference is that Alg-1B is implemented on price series rather than return series. The proposed approach can be implemented also via other distance measures such as *Mahalanobis* distance, which is equal to $\sqrt{(X-Y)^T \Sigma^{-1}(X-Y)}$ for random vectors X and Y. Note that this particular distance measure assumes a covariance structure given by Σ and has the Euclidean distance as a special case.

We implemented the resulting framework (i.e., first scenario generation approach and the proposed SP model) assuming that the investor has two alternatives for investing, one risky asset and one risk-free asset. As a proxy for the risky asset, we considered the weekly S&P 500 index.

Our main interest has been on a multi-period investment problem where the investor continues investing the portfolio resulting from the previous periods. However, we also carried over several experiments to see the performance of Alg-1A in a single period investing scheme. We considered both in-sample and out-of-sample testing and compared the proposed approach with the static strategy. The results show that Alg-1A dominates the static strategy. The level of dominance is much higher when average shortfall is assumed to be the risk measure.

The remaining computations presented in this study are implemented assuming a multi-period investment setting. For the analysis in a multi-period investing scheme, we ran the scenario generation and optimization phases repetitively for consecutive periods such that the portfolio is revalued after realizing returns and reinvested using the results of the next run. We implemented both Alg-1A and Alg-1B using this methodology. For benchmarking purposes, we compared the results first with the B&H strategy as the alternative portfolio management strategy. Next we obtained the results for AR(2) and GBM as the alternative scenario generation methods using the same proposed SP model. We observe from the results that the proposed approach, regardless whether we employ Alg-1A or Alg-1B, outperforms the strategies led by B&H, AR(2), and GBM. The dominance is stronger especially when risk averseness is low (i.e., in the high risk region).

We performed further computations to observe the sensitivity against variation in input parameters and data. For instance, changing the weights used in weighted Euclidean distance revealed outcomes that suggest putting more weight to lower lags (i.e., more recent periods), which is in accordance with our original consideration and initial vector of weights. The sensitivity with respect to time unit has been also investigated by implementing the proposed approach over the *daily* S&P 500 index data (i.e., the time window denoted by TW1) instead of *weekly* data. We also implemented the aforementioned alternative strategies (i.e., B&H as the alternative portfolio management strategy and AR(2) and GBM as the alternative scenario generation tools) using the same set of data. The results suggest that the proposed approach dominates B&H, AR(2), and GBM regardless of the risk measure taken and regardless whether Alg-1A or Alg-1B is used for scenario generation.

Following the experiments considering a different time unit, we investigated the sensitivity with respect to time window. We considered four additional time windows, TW2-TW5, each having exclusive *test periods* of 100 days, which resulted in a high number of outcomes representing return/risk profiles. We presented the individual and average Sharpe Ratios obtained from each experiment in order to have a concise benchmarking process. The results we obtained from these additional experiments confirmed the conclusions suggested by the previous experiments. Considering the aggregate results from all time windows, the results suggest that the proposed approach dominates B&H, AR(2), and GBM. Among the proposed algorithms, Alg-1B outperformed the Alg-1A given the five time windows of daily S&P 500 index data. This is quite promising for the new *UD* distance measure proposed in this study.

5.3. Alg-2

Separate than the first approach, we proposed another approach, Alg-2, which aims to find a discrete probability distribution for the scenarios such that the first four moments of the generated scenarios are closely matched with target moments through a linear program. This approach incorporates state dependency of variance via the well-known EGARCH model. The first two target moments are set dynamically by EGARCH and CCC-GARCH over the scenario tree (i.e., the innovations on each node are used to estimate the second moments in the following node) whereas the third and fourth moments are set by historical estimation. Once the target moments are set, a linear programming model is run for the entire scenario tree to obtain the scenario probabilities.

During the design stage of this algorithm, we also investigated an issue led by using moment matching technique for a multi period model. During the moment matching process for a particular source node, a number of nodes (i.e., scenarios) following the source node could be assigned probabilities that are either equal or close to zero. Considering the multi-period structure of the scenario tree, we discussed the likely event of having zero probabilities for many scenarios at the horizon since the final probabilities are computed by multiplying all probabilities on the underlying path. We presented a test case to illustrate this issue and argued that it might decrease the performance of a multi-period SP model. Therefore, we proposed setting lower-bounds on probabilities, which would be calibrated in accordance with the data used.

For our computations regarding Alg-2, we assumed that the investor appropriates a multi-period investing scheme and has three alternatives for investing, two risky assets and one risk-free asset. As proxies for the risky assets, we considered the weekly S&P 500 (large-cap) and Russell 2000 (small-cap) stock indices.

We first investigated the effects of setting lower bounds on scenario probabilities during the moment matching. The results supported our initial discussion since setting a lower bound led to an improved return/risk profile. The lower bound was calibrated through multiple experiments which show that exposing high lower-bounds worsen the performance. This was expected since setting high lower-bounds would lead to large deviations from target moments.

The next set of experiments was carried over in order to reveal the effect of incorporating EGARCH into the scenario generation methodology. For this purpose, we implemented the same framework without EGARCH (i.e., state-independent variance) and lower-bounds on probabilities. The results strongly favored Alg-2 since removing EGARCH resulted in lowered performance.

For benchmarking purposes, we compared the results with B&H strategy and Markowitz's MV model as the alternative portfolio management strategies over the same set of data. Regardless of the risk measure used, the results suggested that Alg-2 dominates both B&H and MV strategies. The dominance was found to be more significant especially in the high risk region. Next we obtained the results for VAR(2) as the alternative scenario generation method using the same proposed SP model. We observed that the trading strategy with Alg-2 dominates the one with VAR(2) for all risk regions when variance is taken as the risk measure whereas the dominance is clearly observed only in the high risk region when average shortfall is taken as the risk measure. We also observed that VAR(2) led to a trading strategy with high turnover, which leads to higher transaction costs. In a new set of computations, we implemented both strategies with a higher unit transaction cost and Alg-2 outperformed VAR(2) in this experiment regardless of the risk region and risk measure taken.

For sensitivity analysis, we first considered the weights used in the objective function of the moment matching model. Considering the base tree topology of [30 15], the results were not too sensitive to different weight vectors; however, we observed an increase in sensitivity when we made experiments with a different tree topology, [8 4]. This was expected since the decrease in the number of scenarios leads to fewer variables (i.e., probabilities) to fit the all four moments. From this same set of experiments, we had the opportunity to evaluate the sensitivity with respect to the tree topology, or sensitivity against the number of scenarios. The change from a topology of [8 4] to [30 15] provides a significant and positive change in the output. The comparison of Sharpe Ratios reveals that allowing more scenarios in the tree pays off by an increase in performance, which is in accordance with the well known trade-off for SP models.

Following the same procedure mentioned in Section 5.2, the next set of observations are pulled out from experiments involving *daily* S&P 500 and Russell 2000 data in order to analyze the sensitivity with respect to both time unit and time windows. For instance similar to the experiments on weekly data we carried of several experiments on the first daily time window TW1, which was used for Alg-1A and Alg-1B, to calibrate the lower bound parameter for the daily investment setting. The resulting parameter was found to be smaller than the one corresponding to the weekly data. Setting the calibrated lower bound parameter, we considered the four time windows TW2-TW5, which were used for Alg-1A and Alg-1B, for the rest of the computations.

Another set of computations were achieved to investigate the pure contribution of EGARCH modeling in scenario generation as described for the weekly experiments. The results obtained from five time windows TW1-TW5 indicate that usage of EGARCH improves the return/risk profile of the trading strategy. This observation is in agreement with the one obtained from the weekly data.

Sensitivity with respect to weights used in the objective function of the moment matching model has been reinvestigated considering the daily data. Even though there were no significant discrepancies among the different weight vectors, the Sharpe Ratios imply that the performance over daily data is improved when maximum weight is used for the fourth moment. This can be *partially* explained by the fat-tail characteristic of the high frequency financial data.

Using the weekly data we had obtained results in alignment with the tradeoff between the problem size and the performance of the SP based approaches since the strategy based on a [30 15] scenario tree dominated the one based on the [8 4] tree. We extended this sensitivity analysis over the daily time windows TW1-TW5. Plus, we implemented the proposed approach with [80 40] scenario tree topology in addition to [30 15] and [8 4]. The switch from [30 15] to [80 40] changed the performance; however, the sensitivity was much higher when compared to the transition from [8 4] to [30 15]. Nevertheless, the results suggest that increase in problem size is compensated by an increase in performance. Therefore, computations regarding the scenario tree suggest that the performance is sensitive to the scenario tree topology; however, the possible gains from an increased number of scenarios might be diminishing depending on the starting tree topology.

The final set of computations aimed at benchmarking the proposed approach with the alternative strategies considering a daily investment environment. Similar to the weekly setting, we implemented B&H and Markowitz's MV strategy as the alternative portfolio management strategies over the time windows TW1-TW5. The aggregate results presented by the Sharpe Ratios show that the proposed strategy outperforms both B&H and MV strategies when implemented by a scenario tree topology of [80 40]. When the [30 15] scenario tree is implemented, it is only slightly outperformed by MV whereas it still outperforms the B&H strategy. Next we obtained the results for VAR(2) as the alternative scenario generation method using the same proposed SP model over the daily time windows TW1-TW5. The combination of Alg-2 and the proposed SP model generated a higher performance than VAR(2) combined with the same SP model regardless we consider a scenario tree with the topology of [30 15] or [80 40].

5.4. A General Approach for Loss Reduction

In Chapter 4, we presented a general approach aiming to modify investment decisions led by a given trading strategy in accordance with the state of the market in terms of volatility. The proposed methodology is based on utilizing all information realized up to time tick t-1 within a learning model to infer probability of a significant loss at time tick t. In particular, we considered logistic regression to make predictions and take corrective actions on the original investment decisions with the intention of preventing significant losses and enhancing the return/risk profile.

We considered two simple proxies for volatility at a given time to be used as the independent variables in logistic regression, which are functions of the past three return values. We implemented the combined approach over time windows TW1-TW5 having Alg-2 and the SP model presented in Chapter 2 as the underlying trading strategy. The implementation process involves setting a threshold value, which identifies a trading decision as a significant loss in the following period.

We tested four different threshold values with two different proxies adding up to eight different trading strategies, all of which improved performance except one. The second proxy (i.e., sum of squared returns) outperformed the first measure (i.e., absolute value of the product of returns) at all threshold values, whereas the maximum performance is observed when the threshold is set to 600 for both proxies. This observation is in alignment with the average p-values obtained from regression estimations.

NOTE 2: It can be argued how sensitive the results presented in Chapters 3 and 4 are with respect to the composition of S&P 500 company list as this index is considered as one of the two representative risky assets in this study, the other one being Russell 2000 index. We conjecture that the sensitivity will be extremely low because of two reasons:

1. The turnover in the company list is low when we consider the additions/deletions provided in Table 89. Moreover, the weekly and daily time windows considered in this study (i.e., 2001-2007 and 2003-2006) have average number of additions/deletions to be 21.5 and 26.6, which are lower than the average number of additions/deletions over 2000-2008, which is 32.2.

2. Standard & Poor's requires certain criteria for the companies to be eligible for S&P 500 index such as market capitalization, liquidity, financial viability, etc. Therefore, the removal of the companies that fail the criteria and inclusion of those that meet the criteria is expected to keep the index to be based on similar underlying components and less sensitive to additions/deletions over time.

Table 89. Changes in S&P 500 company listing

Year	Number of Additions/Deletions
2009*	17
2008	44
2007	43
2006	34
2005	21
2004	22
2003	9
2002	25
2001	32
2000	60

*As of 8/18/09

NOTE 3: The portfolio management strategies considered in this study have different levels of *portfolio turnover*. The lowest turnover is observed for the static B&H strategy, in which case there is no trade after the initial asset allocation. Another strategy implemented in this study with a low portfolio turnover is the MV approach, which is based on solving a quadratic optimization problem with the inputs of covariance matrix and expected return vector. These input parameters were estimated for each test period; however, they are not expected to change significantly from one period to the next period considering the entire historical data set. Therefore, we obtained very low turnover for MV approach. The highest portfolio turnover was observed when scenarios are generated via VAR(2). This issue is further discussed in Section 3.2.5 based on the experiments carried over with different transaction costs. Different than the alternative approaches, the proposed approaches led to moderate levels of portfolio turnover at various experiments.

NOTE 4: Throughout the implementation process presented in this study, we paid special attention on the experimental setup for alternative strategies. Experiments with different input parameters and data provided a significant number of performance measures and therefore enabled us to achieve an extensive sensitivity analysis. We increased the precision of the estimates for the performance measures led by alternative strategies by keeping the number of experiments specifically high. For instance, *average return* is the major driver of the performance measures considered in this study and the standard error of this measure is inversely proportional to the square root of the number of observations. Considering the high number of runs at different experiments (e.g., 2,500-3,000), we can conclude that the estimates for the performance measures are satisfactorily precise from the statistical significance point of view.

5.5. Research Contributions

We identify the contributions of this study briefly as follow:

- We developed a new scenario generation approach based on reducing historical data and assigning probabilities via distance measures. The algorithm does not require estimation or optimization procedures.
    - We proposed a new distance measure to capture the similarities between data patterns. It is used as an alternative to the weighted Euclidean distance to implement the proposed scenario generation approach.
- We developed another new scenario generation approach that combines the moment matching technique with the ARCH-type modeling approach and provides input for an SP model.
    - In particular, we incorporated state dependent second moments into the scenario generation phase via EGARCH and CCC-GARCH.
    - We addressed the issue of having extremely low scenario probabilities and proposed imposing lower-bounds that are calibrated to navigate the trade-off between fitting the moments and the overall performance of the SP model.
- We proposed a comprehensive multi-stage linear SP formulation that is capable of addressing a diverse set of parameters, constraints, and objectives.

  - We presented constraints that enable the decision maker to control risk with different risk tolerances at each decision epoch. Each node in the scenario tree except the leaf nodes has its own set of constraints to control the realized CVaR.
- We proposed a general methodology that can be appended to any trading strategy to increase the original performance from a return/risk point of view.
  - The implementation, which is presented in Chapter 4, yielded empirical results that strongly support the proposed approach when used in conjunction with Alg-2 and the SP model presented Chapter 2.

5.6. Directions for Future Research

Stochastic programming is a broad area of operations research providing flexible models that are able to solve complex real life problems faced in various domains. The success of a SP based approach depends strictly on its capability to capture the underlying uncertainty associated with the random parameters in question. This makes the scenario generation phase especially critical. Therefore, developing new methodologies to generate representative scenario trees is still an open research area. We conjecture that domain-specific algorithms would perform better than generic algorithms.

The scenario generation algorithms proposed in this study can be combined with actuarial methodologies to generate representative liability scenarios so that a complete ALM system is obtained. The combined framework would be applicable to additional problems where positive and negative cash flows are involved.

An extension of this study that could be quite rewarding is the further development of the general decision rule presented in Chapter 4 through further computations and modeling effort. A learning model that is able to capture possible states of the market and relate this to the performance of a trading strategy can produce significant improvement as shown in this study. In fact the general approach described in Section 4.2 can also be implemented using a model other than logistic regression and combined with a scenario generation algorithm or a SP model in order to obtain a portfolio management framework that can adapt to changing market conditions and provide enhanced investment decisions.

We conjecture that external factors such as interest rates (i.e., risk-free return) could affect the relative and absolute performances of portfolio optimization models. In our analyses we assumed that the risk-free rates are fixed throughout the horizon, which is a sound assumption since the planning horizon is short (i.e., several days/weeks). Another possible way of addressing the portfolio management problem is to consider longer planning horizons (i.e., several months/years), which could benefit from incorporating a stochastic interest rate model (e.g., Hull and White (1994a-b) into the scenario generation process.

APPENDIX A

Table 2. Probability values for a test case of 450 scenarios

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0574	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0032	0.1967	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0319
0.0017	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0198
0.0156	0.0000	0.0000	0.0000	0.0000	0.0732	0.0000	0.0000	0.0000	0.0000
0.0053	0.0819	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0272	0.0178	0.0000	0.0000	0.0000	0.0478	0.0000	0.0000	0.0000	0.0000
0.0418	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0117	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0169	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0159
0.0000	0.0000	0.0000	0.0000	0.0000	0.0421	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0033
0.0266	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0290	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0279	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0864	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0287	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0903	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

APPENDIX B (Alg-1)

Table 3. Decisions table for Alg-1A, LCVAR=10 (Single Period)

t	CASH	SP500	W	t	CASH	SP500	W
201	631.73	367.91	1004.81	251	635.43	364.21	996.09
202	1000.00	0.00	1000.40	252	729.70	270.03	995.01
203	781.08	218.70	1000.62	253	1000.00	0.00	1000.40
204	1000.00	0.00	1000.40	254	1000.00	0.00	1000.40
205	1000.00	0.00	1000.40	255	1000.00	0.00	1000.40
206	640.12	359.52	994.39	256	639.63	360.01	1001.02
207	1000.00	0.00	1000.40	257	1000.00	0.00	1000.40
208	677.10	322.58	1000.17	258	1000.00	0.00	1000.40
209	652.93	346.72	1025.93	259	1000.00	0.00	1000.40
210	683.47	316.22	1007.90	260	514.98	484.54	1003.22
211	1000.00	0.00	1000.40	261	599.98	399.62	996.37
212	1000.00	0.00	1000.40	262	649.23	350.42	990.17
213	1000.00	0.00	1000.40	263	1000.00	0.00	1000.40
214	1000.00	0.00	1000.40	264	603.17	396.44	968.17
215	1000.00	0.00	1000.40	265	1000.00	0.00	1000.40
216	633.58	366.05	1000.15	266	1000.00	0.00	1000.40
217	728.91	270.82	1004.36	267	1000.00	0.00	1000.40
218	1000.00	0.00	1000.40	268	1000.00	0.00	1000.40
219	1000.00	0.00	1000.40	269	690.79	308.90	995.10
220	1000.00	0.00	1000.40	270	1000.00	0.00	1000.40
221	1000.00	0.00	1000.40	271	674.94	324.74	991.85
222	1000.00	0.00	1000.40	272	1000.00	0.00	1000.40
223	1000.00	0.00	1000.40	273	518.93	480.59	1003.99
224	1000.00	0.00	1000.40	274	668.21	331.46	997.65
225	515.30	484.22	1006.79	275	1000.00	0.00	1000.40
226	1000.00	0.00	1000.40	276	608.37	391.24	1006.33
227	1000.00	0.00	1000.40	277	1000.00	0.00	1000.40
228	1000.00	0.00	1000.40	278	1000.00	0.00	1000.40
229	672.60	327.08	1001.12	279	1000.00	0.00	1000.40
230	1000.00	0.00	1000.40	280	621.09	378.53	1010.75
231	1000.00	0.00	1000.40	281	1000.00	0.00	1000.40
232	626.88	372.75	1002.22	282	691.46	308.23	1001.60
233	572.07	427.51	1004.90	283	1000.00	0.00	1000.40
234	659.61	340.05	1003.83	284	571.20	428.38	1000.84
235	1000.00	0.00	1000.40	285	609.25	390.36	1005.43
236	1000.00	0.00	1000.40	286	1000.00	0.00	1000.40
237	624.58	375.04	1000.02	287	616.63	382.99	986.74
238	606.00	393.60	1004.59	288	1000.00	0.00	1000.40
239	719.32	280.40	1003.96	289	708.53	291.18	1000.11
240	623.10	376.53	1011.35	290	643.85	355.79	1015.34
241	1000.00	0.00	1000.40	291	537.46	462.08	1000.85
242	1000.00	0.00	1000.40	292	1000.00	0.00	1000.40
243	610.67	388.94	1029.04	293	1000.00	0.00	1000.40
244	707.87	291.84	1003.06	294	1000.00	0.00	1000.40
245	1000.00	0.00	1000.40	295	1000.00	0.00	1000.40
246	662.01	337.65	1004.99	296	1000.00	0.00	1000.40
247	1000.00	0.00	1000.40	297	674.92	324.75	992.09
248	582.58	417.00	1009.31	298	603.25	396.35	1002.30
249	1000.00	0.00	1000.40	299	556.55	443.01	1009.59
250	1000.00	0.00	1000.40	300	1000.00	0.00	1000.40

Table 4. Decisions table for Alg-1A, LCVAR=15 (Single Period)

t	CASH	SP500	W	t	CASH	SP500	W
201	454.67	544.79	1006.92	251	460.16	539.30	994.01
202	1000.00	0.00	1000.40	252	599.60	400.00	992.42
203	683.29	316.39	1000.72	253	1000.00	0.00	1000.40
204	1000.00	0.00	1000.40	254	1000.00	0.00	1000.40
205	1000.00	0.00	1000.40	255	1000.00	0.00	1000.40
206	467.10	532.37	991.50	256	466.37	533.09	1001.31
207	1000.00	0.00	1000.40	257	1000.00	0.00	1000.40
208	521.85	477.67	1000.06	258	1000.00	0.00	1000.40
209	486.07	513.41	1038.20	259	1000.00	0.00	1000.40
210	531.29	468.25	1011.50	260	281.80	717.49	1004.57
211	1000.00	0.00	1000.40	261	407.66	591.74	994.42
212	1000.00	0.00	1000.40	262	480.59	518.89	985.25
213	1000.00	0.00	1000.40	263	1000.00	0.00	1000.40
214	1000.00	0.00	1000.40	264	412.38	587.03	952.66
215	1000.00	0.00	1000.40	265	1000.00	0.00	1000.40
216	457.42	542.04	1000.04	266	1000.00	0.00	1000.40
217	501.33	498.17	1007.68	267	1000.00	0.00	1000.40
218	1000.00	0.00	1000.40	268	1000.00	0.00	1000.40
219	1000.00	0.00	1000.40	269	542.13	457.41	992.55
220	1000.00	0.00	1000.40	270	1000.00	0.00	1000.40
221	1000.00	0.00	1000.40	271	518.66	480.86	987.74
222	1000.00	0.00	1000.40	272	1000.00	0.00	1000.40
223	1000.00	0.00	1000.40	273	287.64	711.65	1005.72
224	1000.00	0.00	1000.40	274	508.69	490.82	996.33
225	282.27	717.01	1009.85	275	1000.00	0.00	1000.40
226	1000.00	0.00	1000.40	276	420.09	579.34	1009.19
227	1000.00	0.00	1000.40	277	1000.00	0.00	1000.40
228	1000.00	0.00	1000.40	278	1000.00	0.00	1000.40
229	515.19	484.33	1001.46	279	1000.00	0.00	1000.40
230	1000.00	0.00	1000.40	280	438.72	560.72	1015.73
231	1000.00	0.00	1000.40	281	1000.00	0.00	1000.40
232	447.50	551.95	1003.09	282	543.13	456.42	1002.18
233	366.33	633.04	1007.06	283	1000.00	0.00	1000.40
234	495.96	503.53	1005.47	284	365.04	634.33	1001.05
235	1000.00	0.00	1000.40	285	421.39	578.03	1007.85
236	1000.00	0.00	1000.40	286	1000.00	0.00	1000.40
237	444.09	555.35	999.84	287	432.32	567.12	980.18
238	416.58	582.84	1006.62	288	1000.00	0.00	1000.40
239	584.38	415.21	1005.67	289	568.39	431.18	999.97
240	441.89	557.55	1016.60	290	472.62	526.85	1022.52
241	1000.00	0.00	1000.40	291	315.08	684.23	1001.06
242	1000.00	0.00	1000.40	292	1000.00	0.00	1000.40
243	423.49	575.94	1042.81	293	1000.00	0.00	1000.40
244	567.42	432.15	1004.34	294	1000.00	0.00	1000.40
245	1000.00	0.00	1000.40	295	1000.00	0.00	1000.40
246	499.51	499.99	1007.20	296	1000.00	0.00	1000.40
247	1000.00	0.00	1000.40	297	518.63	480.89	988.10
248	381.90	617.49	1013.61	298	412.51	586.90	1003.21
249	1000.00	0.00	1000.40	299	343.35	655.99	1014.00
250	1000.00	0.00	1000.40	300	1000.00	0.00	1000.40

Table 5. Decisions table for Alg-1A, LCVAR=20 (Single Period)

t	CASH	SP500	W	t	CASH	SP500	W
201	277.61	721.66	1009.03	251	284.88	714.40	991.94
202	1000.00	0.00	1000.40	252	469.50	529.97	989.82
203	580.39	419.19	1000.82	253	1000.00	0.00	1000.40
204	1000.00	0.00	1000.40	254	1000.00	0.00	1000.40
205	1000.00	0.00	1000.40	255	1000.00	0.00	1000.40
206	294.08	705.21	988.60	256	293.12	706.17	1001.61
207	1000.00	0.00	1000.40	257	1000.00	0.00	1000.40
208	366.61	632.76	999.95	258	1000.00	0.00	1000.40
209	319.21	680.11	1050.48	259	1000.00	0.00	1000.40
210	379.11	620.27	1015.09	260	48.61	950.43	1005.91
211	1000.00	0.00	1000.40	261	215.35	783.87	992.49
212	1000.00	0.00	1000.40	262	311.96	687.36	980.33
213	1000.00	0.00	1000.40	263	1000.00	0.00	1000.40
214	1000.00	0.00	1000.40	264	221.59	777.63	937.16
215	1000.00	0.00	1000.40	265	1000.00	0.00	1000.40
216	281.26	718.02	999.91	266	1000.00	0.00	1000.40
217	339.30	660.04	1010.05	267	1000.00	0.00	1000.40
218	1000.00	0.00	1000.40	268	1000.00	0.00	1000.40
219	1000.00	0.00	1000.40	269	393.47	605.92	990.00
220	1000.00	0.00	1000.40	270	1000.00	0.00	1000.40
221	1000.00	0.00	1000.40	271	362.38	636.99	983.63
222	1000.00	0.00	1000.40	272	1000.00	0.00	1000.40
223	1000.00	0.00	1000.40	273	56.36	942.70	1007.45
224	1000.00	0.00	1000.40	274	349.17	650.18	995.00
225	49.24	949.81	1012.92	275	1000.00	0.00	1000.40
226	1000.00	0.00	1000.40	276	231.80	767.43	1012.03
227	1000.00	0.00	1000.40	277	1000.00	0.00	1000.40
228	1000.00	0.00	1000.40	278	1000.00	0.00	1000.40
229	357.78	641.57	1001.79	279	1000.00	0.00	1000.40
230	1000.00	0.00	1000.40	280	256.34	742.91	1020.71
231	1000.00	0.00	1000.40	281	1000.00	0.00	1000.40
232	268.11	731.16	1003.96	282	394.79	604.60	1002.75
233	160.59	838.57	1009.21	283	1000.00	0.00	1000.40
234	332.32	667.02	1007.14	284	158.88	840.28	1001.25
235	1000.00	0.00	1000.40	285	233.53	765.70	1010.26
236	1000.00	0.00	1000.40	286	1000.00	0.00	1000.40
237	263.60	735.66	999.66	287	248.00	751.24	973.60
238	227.16	772.07	1008.64	288	1000.00	0.00	1000.40
239	449.44	550.01	1007.38	289	428.26	571.17	999.83
240	260.69	738.57	1021.86	290	301.40	697.90	1029.71
241	1000.00	0.00	1000.40	291	92.71	906.38	1001.28
242	1000.00	0.00	1000.40	292	1000.00	0.00	1000.40
243	236.31	762.93	1056.58	293	1000.00	0.00	1000.40
244	426.97	572.46	1005.62	294	1000.00	0.00	1000.40
245	1000.00	0.00	1000.40	295	1000.00	0.00	1000.40
246	337.02	662.32	1009.41	296	1000.00	0.00	1000.40
247	1000.00	0.00	1000.40	297	362.34	637.02	984.10
248	181.21	817.97	1017.88	298	221.77	777.46	1004.13
249	1000.00	0.00	1000.40	299	130.16	868.98	1018.43
250	1000.00	0.00	1000.40	300	1000.00	0.00	1000.40

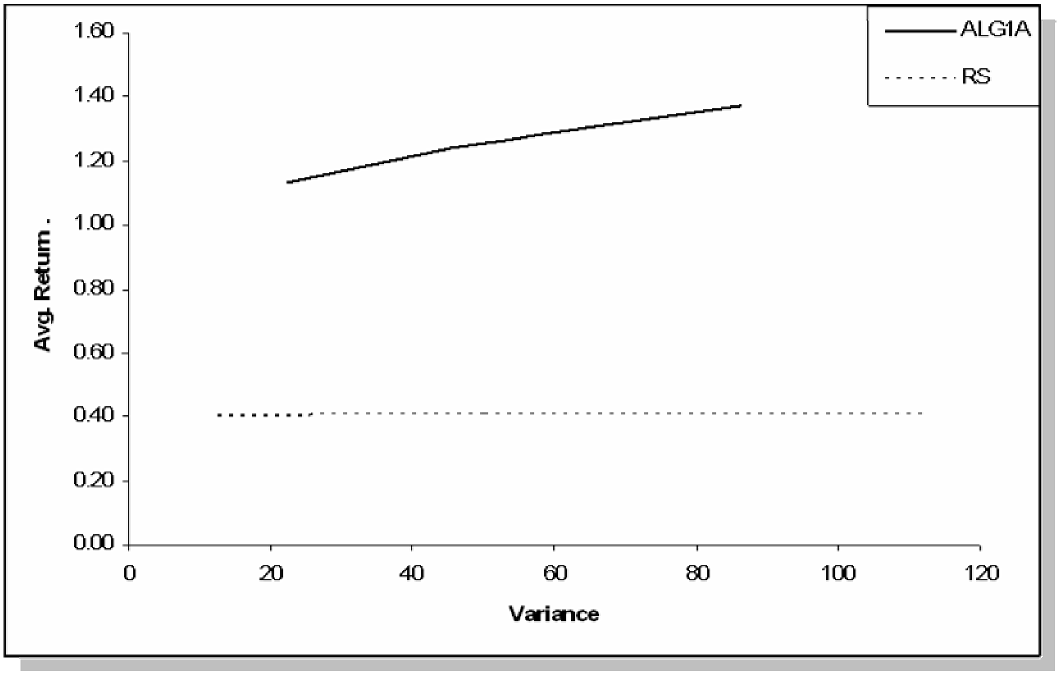

Figure 17. Comparison of Alg-1A and RS over Sample-2 (Variance)

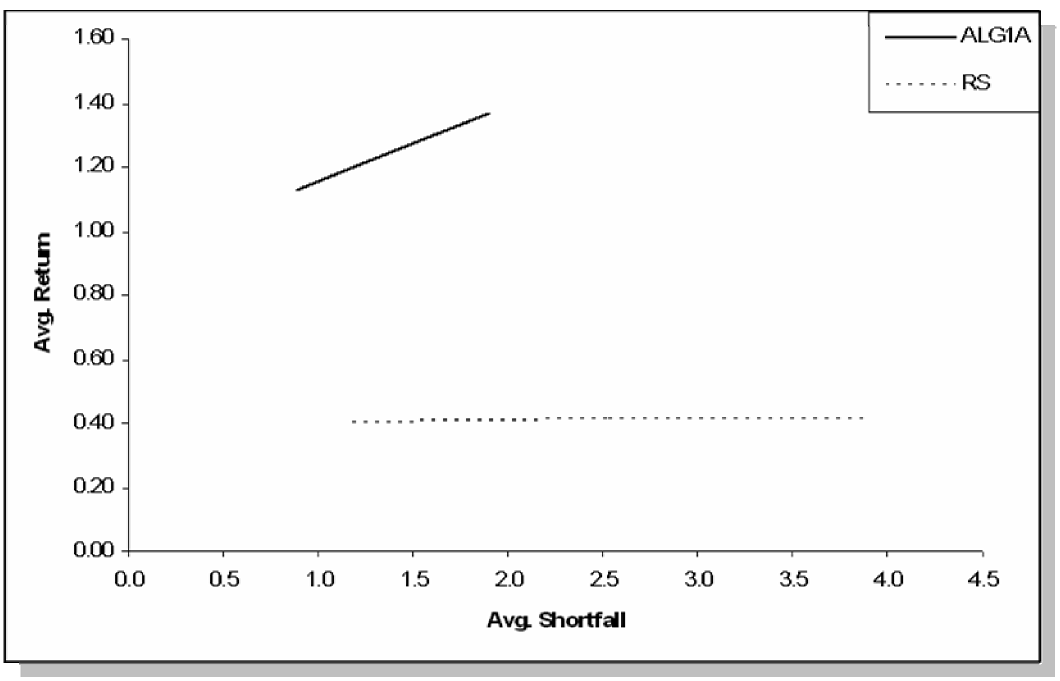

Figure 18. Comparison of Alg-1A and RS over Sample-2 (Avg. Shortfall)

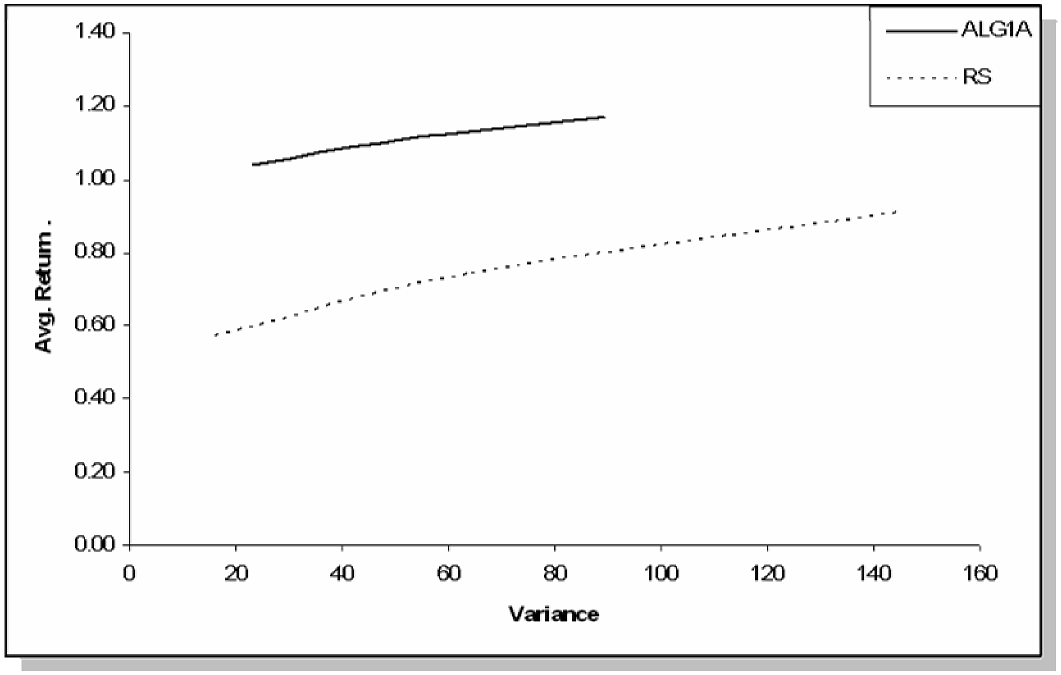

Figure 19. Comparison of Alg-1A and RS over Sample-3 (Variance)

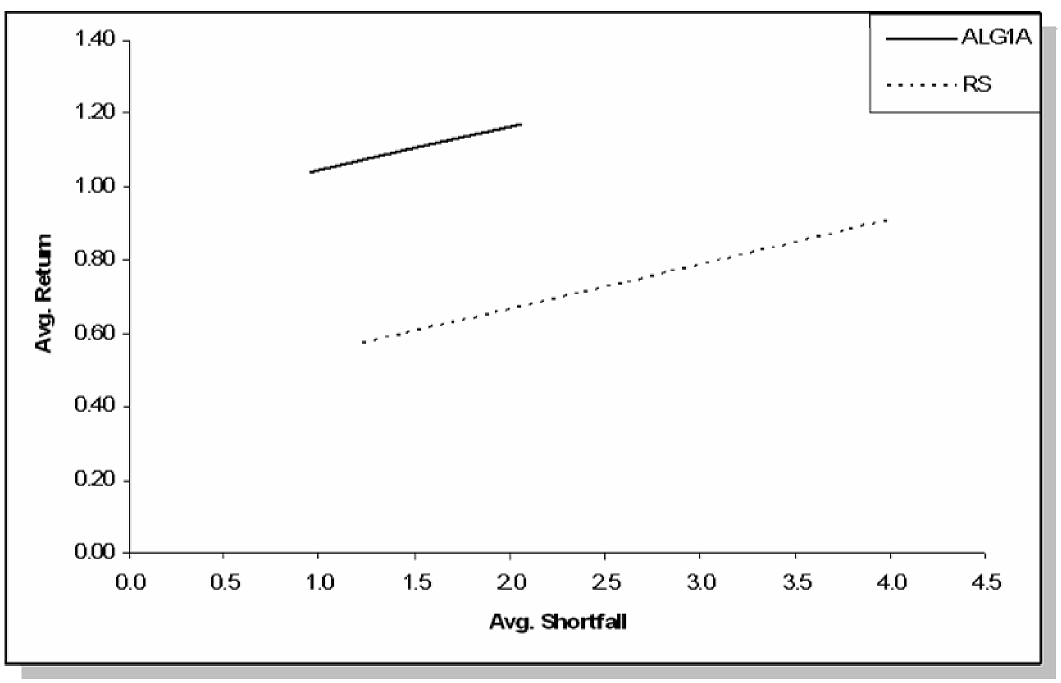

Figure 20. Comparison of Alg-1A and RS over Sample-3 (Avg. Shortfall)

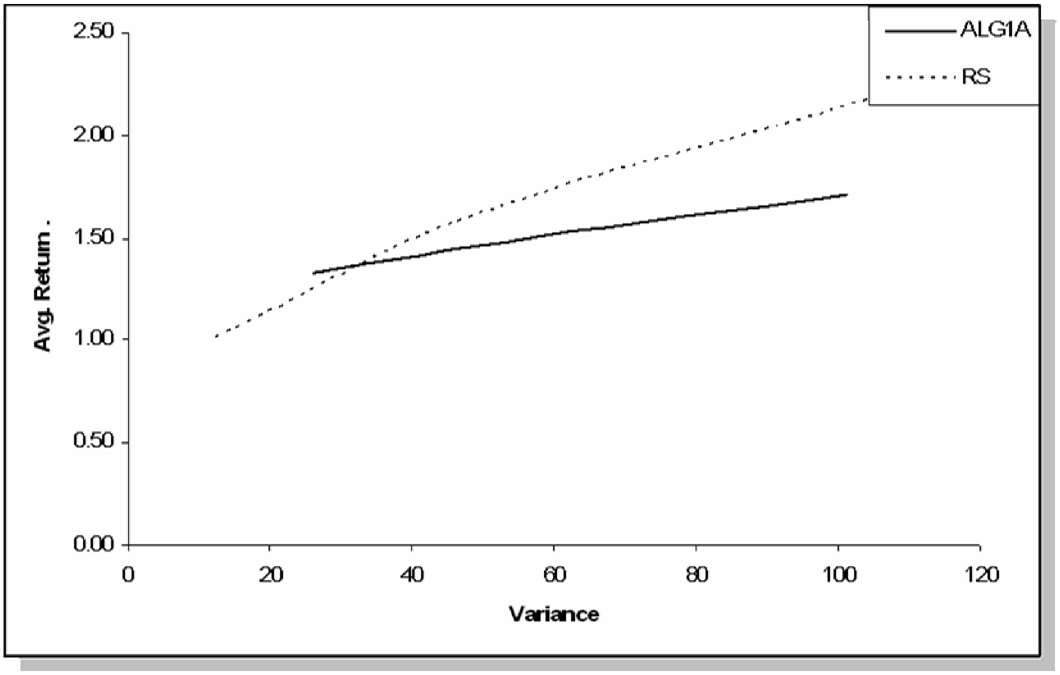

Figure 21. Comparison of Alg-1A and RS over Sample-4 (Variance)

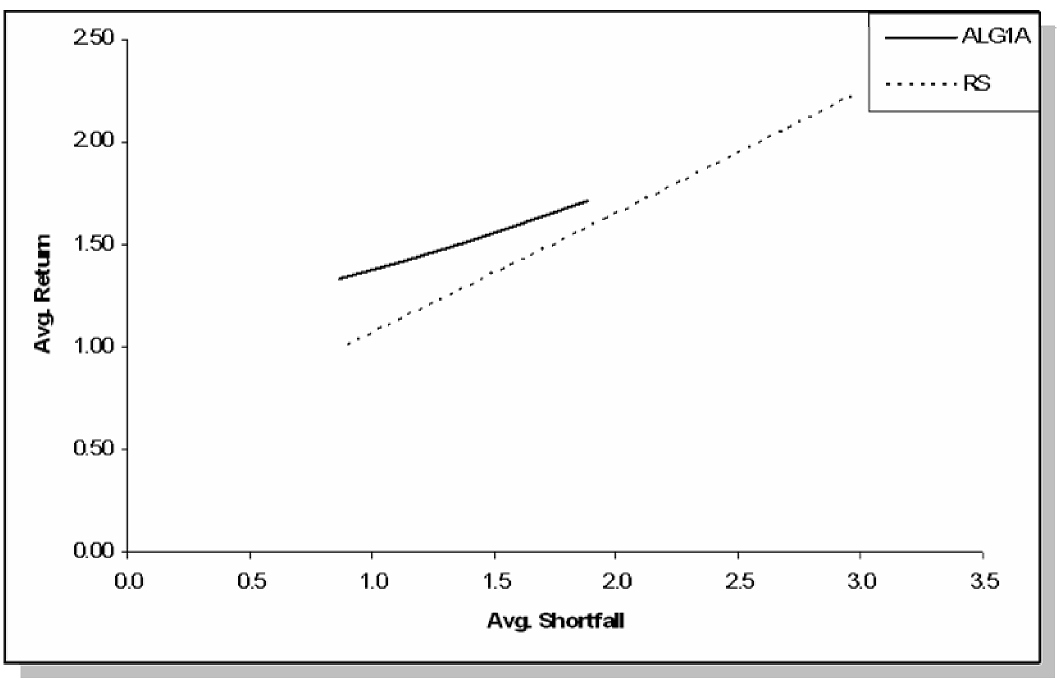

Figure 22. Comparison of Alg-1A and RS over Sample-4 (Avg. Shortfall)

Table 11. Decisions table for Alg-1A, LCVAR=10 (Single Period - Out of Sample)

t	CASH	SP500	W	t	CASH	SP500	W
201	631.98	363.50	995.49	251	635.68	360.62	996.30
202	1000.40	0.00	1000.40	252	729.99	277.61	1007.60
203	781.39	222.91	1004.31	253	1000.40	0.00	1000.40
204	1000.40	0.00	1000.40	254	1000.40	0.00	1000.40
205	1000.40	0.00	1000.40	255	1000.40	0.00	1000.40
206	640.38	363.52	1003.90	256	639.89	365.76	1005.64
207	1000.40	0.00	1000.40	257	1000.40	0.00	1000.40
208	677.37	320.06	997.43	258	1000.40	0.00	1000.40
209	653.19	344.68	997.87	259	1000.40	0.00	1000.40
210	683.74	321.27	1005.01	260	515.19	490.30	1005.48
211	1000.40	0.00	1000.40	261	600.22	400.49	1000.71
212	1000.40	0.00	1000.40	262	649.49	352.66	1002.15
213	1000.40	0.00	1000.40	263	1000.40	0.00	1000.40
214	1000.40	0.00	1000.40	264	603.41	401.26	1004.68
215	1000.40	0.00	1000.40	265	1000.40	0.00	1000.40
216	633.83	364.40	998.23	266	1000.40	0.00	1000.40
217	729.20	272.53	1001.73	267	1000.40	0.00	1000.40
218	1000.40	0.00	1000.40	268	1000.40	0.00	1000.40
219	1000.40	0.00	1000.40	269	691.07	312.68	1003.75
220	1000.40	0.00	1000.40	270	1000.40	0.00	1000.40
221	1000.40	0.00	1000.40	271	675.21	326.48	1001.69
222	1000.40	0.00	1000.40	272	1000.40	0.00	1000.40
223	1000.40	0.00	1000.40	273	519.14	487.76	1006.89
224	1000.40	0.00	1000.40	274	668.48	331.41	999.88
225	515.51	485.35	1000.86	275	1000.40	0.00	1000.40
226	1000.40	0.00	1000.40	276	608.61	398.45	1007.06
227	1000.40	0.00	1000.40	277	1000.40	0.00	1000.40
228	1000.40	0.00	1000.40	278	1000.40	0.00	1000.40
229	672.87	325.60	998.47	279	1000.40	0.00	1000.40
230	1000.40	0.00	1000.40	280	621.34	361.83	983.17
231	1000.40	0.00	1000.40	281	1000.40	0.00	1000.40
232	627.13	370.44	997.57	282	691.74	304.74	996.48
233	572.30	427.72	1000.02	283	1000.40	0.00	1000.40
234	659.87	338.38	998.25	284	571.43	423.83	995.26
235	1000.40	0.00	1000.40	285	609.49	396.65	1006.15
236	1000.40	0.00	1000.40	286	1000.40	0.00	1000.40
237	624.83	379.38	1004.21	287	616.88	391.29	1008.17
238	606.24	383.35	989.59	288	1000.40	0.00	1000.40
239	719.61	275.14	994.75	289	708.81	293.43	1002.24
240	623.35	380.43	1003.78	290	644.11	355.84	999.95
241	1000.40	0.00	1000.40	291	537.68	467.27	1004.94
242	1000.40	0.00	1000.40	292	1000.40	0.00	1000.40
243	610.91	388.70	999.62	293	1000.40	0.00	1000.40
244	708.15	290.20	998.35	294	1000.40	0.00	1000.40
245	1000.40	0.00	1000.40	295	1000.40	0.00	1000.40
246	662.28	336.40	998.67	296	1000.40	0.00	1000.40
247	1000.40	0.00	1000.40	297	675.19	324.92	1000.11
248	582.81	418.38	1001.19	298	603.49	403.49	1006.98
249	1000.40	0.00	1000.40	299	556.77	449.40	1006.17
250	1000.40	0.00	1000.40	300	1000.40	0.00	1000.40

Table 12. Decisions table for Alg-1A, LCVAR=15 (Single Period - Out of Sample)

t	CASH	SP500	W	t	CASH	SP500	W
201	454.85	538.26	993.12	251	460.34	533.98	994.32
202	1000.40	0.00	1000.40	252	599.84	411.23	1011.07
203	683.56	322.48	1006.05	253	1000.40	0.00	1000.40
204	1000.40	0.00	1000.40	254	1000.40	0.00	1000.40
205	1000.40	0.00	1000.40	255	1000.40	0.00	1000.40
206	467.29	538.29	1005.58	256	466.56	541.60	1008.16
207	1000.40	0.00	1000.40	257	1000.40	0.00	1000.40
208	522.06	473.94	996.00	258	1000.40	0.00	1000.40
209	486.26	510.39	996.65	259	1000.40	0.00	1000.40
210	531.50	475.72	1007.22	260	281.91	726.01	1007.93
211	1000.40	0.00	1000.40	261	407.82	593.03	1000.85
212	1000.40	0.00	1000.40	262	480.78	522.20	1002.99
213	1000.40	0.00	1000.40	263	1000.40	0.00	1000.40
214	1000.40	0.00	1000.40	264	412.55	594.17	1006.72
215	1000.40	0.00	1000.40	265	1000.40	0.00	1000.40
216	457.60	539.59	997.20	266	1000.40	0.00	1000.40
217	501.53	501.32	1002.85	267	1000.40	0.00	1000.40
218	1000.40	0.00	1000.40	268	1000.40	0.00	1000.40
219	1000.40	0.00	1000.40	269	542.35	463.01	1005.35
220	1000.40	0.00	1000.40	270	1000.40	0.00	1000.40
221	1000.40	0.00	1000.40	271	518.87	483.43	1002.30
222	1000.40	0.00	1000.40	272	1000.40	0.00	1000.40
223	1000.40	0.00	1000.40	273	287.76	722.26	1010.02
224	1000.40	0.00	1000.40	274	508.89	490.74	999.64
225	282.38	718.69	1001.07	275	1000.40	0.00	1000.40
226	1000.40	0.00	1000.40	276	420.26	590.02	1010.28
227	1000.40	0.00	1000.40	277	1000.40	0.00	1000.40
228	1000.40	0.00	1000.40	278	1000.40	0.00	1000.40
229	515.40	482.14	997.54	279	1000.40	0.00	1000.40
230	1000.40	0.00	1000.40	280	438.90	535.98	974.88
231	1000.40	0.00	1000.40	281	1000.40	0.00	1000.40
232	447.68	548.53	996.21	282	543.35	451.25	994.60
233	366.48	633.35	999.83	283	1000.40	0.00	1000.40
234	496.16	501.05	997.21	284	365.19	627.59	992.78
235	1000.40	0.00	1000.40	285	421.56	587.35	1008.91
236	1000.40	0.00	1000.40	286	1000.40	0.00	1000.40
237	444.27	561.77	1006.04	287	432.49	579.42	1011.91
238	416.75	567.66	984.41	288	1000.40	0.00	1000.40
239	584.61	407.43	992.04	289	568.62	434.51	1003.13
240	442.07	563.33	1005.40	290	472.81	526.93	999.74
241	1000.40	0.00	1000.40	291	315.21	691.91	1007.12
242	1000.40	0.00	1000.40	292	1000.40	0.00	1000.40
243	423.66	575.59	999.25	293	1000.40	0.00	1000.40
244	567.65	429.72	997.37	294	1000.40	0.00	1000.40
245	1000.40	0.00	1000.40	295	1000.40	0.00	1000.40
246	499.71	498.13	997.84	296	1000.40	0.00	1000.40
247	1000.40	0.00	1000.40	297	518.84	481.14	999.98
248	382.05	619.53	1001.59	298	412.68	597.48	1010.15
249	1000.40	0.00	1000.40	299	343.49	665.45	1008.93
250	1000.40	0.00	1000.40	300	1000.40	0.00	1000.40

Table 13. Decisions table for Alg-1A, LCVAR=20 (Single Period - Out of Sample)

t	CASH	SP500	W	t	CASH	SP500	W
201	277.72	713.02	990.74	251	284.99	707.35	992.35
202	1000.40	0.00	1000.40	252	469.69	544.85	1014.54
203	580.62	427.26	1007.89	253	1000.40	0.00	1000.40
204	1000.40	0.00	1000.40	254	1000.40	0.00	1000.40
205	1000.40	0.00	1000.40	255	1000.40	0.00	1000.40
206	294.20	713.06	1007.25	256	293.24	717.45	1010.68
207	1000.40	0.00	1000.40	257	1000.40	0.00	1000.40
208	366.76	627.82	994.58	258	1000.40	0.00	1000.40
209	319.34	676.11	995.45	259	1000.40	0.00	1000.40
210	379.26	630.17	1009.43	260	48.63	961.72	1010.35
211	1000.40	0.00	1000.40	261	215.44	785.58	1001.02
212	1000.40	0.00	1000.40	262	312.09	691.75	1003.83
213	1000.40	0.00	1000.40	263	1000.40	0.00	1000.40
214	1000.40	0.00	1000.40	264	221.68	787.09	1008.77
215	1000.40	0.00	1000.40	265	1000.40	0.00	1000.40
216	281.37	714.78	996.15	266	1000.40	0.00	1000.40
217	339.44	664.21	1003.64	267	1000.40	0.00	1000.40
218	1000.40	0.00	1000.40	268	1000.40	0.00	1000.40
219	1000.40	0.00	1000.40	269	393.63	613.33	1006.96
220	1000.40	0.00	1000.40	270	1000.40	0.00	1000.40
221	1000.40	0.00	1000.40	271	362.53	640.39	1002.92
222	1000.40	0.00	1000.40	272	1000.40	0.00	1000.40
223	1000.40	0.00	1000.40	273	56.38	956.76	1013.14
224	1000.40	0.00	1000.40	274	349.31	650.08	999.39
225	49.26	952.03	1001.29	275	1000.40	0.00	1000.40
226	1000.40	0.00	1000.40	276	231.89	781.57	1013.47
227	1000.40	0.00	1000.40	277	1000.40	0.00	1000.40
228	1000.40	0.00	1000.40	278	1000.40	0.00	1000.40
229	357.92	638.67	996.60	279	1000.40	0.00	1000.40
230	1000.40	0.00	1000.40	280	256.44	710.14	966.58
231	1000.40	0.00	1000.40	281	1000.40	0.00	1000.40
232	268.22	726.63	994.84	282	394.95	597.75	992.70
233	160.65	838.98	999.63	283	1000.40	0.00	1000.40
234	332.45	663.74	996.19	284	158.94	831.36	990.30
235	1000.40	0.00	1000.40	285	233.62	778.04	1011.66
236	1000.40	0.00	1000.40	286	1000.40	0.00	1000.40
237	263.71	744.16	1007.87	287	248.10	767.53	1015.63
238	227.25	751.97	979.22	288	1000.40	0.00	1000.40
239	449.62	539.70	989.32	289	428.43	575.59	1004.02
240	260.79	746.22	1007.02	290	301.52	698.01	999.53
241	1000.40	0.00	1000.40	291	92.75	916.55	1009.30
242	1000.40	0.00	1000.40	292	1000.40	0.00	1000.40
243	236.41	762.47	998.87	293	1000.40	0.00	1000.40
244	427.14	569.24	996.38	294	1000.40	0.00	1000.40
245	1000.40	0.00	1000.40	295	1000.40	0.00	1000.40
246	337.16	659.86	997.01	296	1000.40	0.00	1000.40
247	1000.40	0.00	1000.40	297	362.49	637.36	999.84
248	181.28	820.68	1001.96	298	221.86	791.47	1013.33
249	1000.40	0.00	1000.40	299	130.21	881.51	1011.72
250	1000.40	0.00	1000.40	300	1000.40	0.00	1000.40

Table 16. Decisions table for Alg-1A, LCVAR=35 (Multi-Period)

t	Cash	SP500	W	t	Cash	SP500	W
201	0.00	999.00	999.00	251	1065.36	0.00	1065.36
202	0.00	987.03	987.03	252	0.00	1064.72	1064.72
203	0.00	997.62	997.62	253	0.00	1094.61	1094.61
204	1015.81	0.00	1015.81	254	1087.46	0.00	1087.46
205	241.03	774.41	1015.45	255	1087.90	0.00	1087.90
206	0.00	1001.15	1001.15	256	0.00	1087.25	1087.25
207	0.00	1012.29	1012.29	257	0.00	1104.61	1104.61
208	0.00	985.18	985.18	258	0.00	1100.52	1100.52
209	0.00	977.49	977.49	259	1117.04	0.00	1117.04
210	0.00	971.74	971.74	260	1117.49	0.00	1117.49
211	312.38	674.56	986.93	261	0.00	1116.82	1116.82
212	312.50	686.79	999.29	262	0.00	1119.25	1119.25
213	50.78	956.58	1007.36	263	1125.28	0.00	1125.28
214	50.80	967.08	1017.88	264	0.00	1124.60	1124.60
215	0.00	1033.33	1033.33	265	1137.15	0.00	1137.15
216	0.00	1030.74	1030.74	266	1137.60	0.00	1137.60
217	0.00	1026.09	1026.09	267	1138.06	0.00	1138.06
218	0.00	1032.57	1032.57	268	0.00	1137.37	1137.37
219	80.58	953.00	1033.58	269	0.00	1148.07	1148.07
220	1017.37	0.00	1017.37	270	0.00	1162.11	1162.11
221	1017.78	0.00	1017.78	271	0.00	1148.82	1148.82
222	1018.19	0.00	1018.19	272	1153.80	0.00	1153.80
223	0.00	1017.58	1017.58	273	0.00	1153.11	1153.11
224	0.00	1035.51	1035.51	274	0.00	1170.30	1170.30
225	0.00	1019.63	1019.63	275	0.00	1170.11	1170.11
226	0.00	1022.01	1022.01	276	0.00	1163.31	1163.31
227	0.00	1038.35	1038.35	277	1183.56	0.00	1183.56
228	1039.07	0.00	1039.07	278	1184.04	0.00	1184.04
229	0.00	1038.45	1038.45	279	1184.51	0.00	1184.51
230	0.00	1033.76	1033.76	280	0.00	1183.80	1183.80
231	1053.55	0.00	1053.55	281	0.00	1131.58	1131.58
232	1053.97	0.00	1053.97	282	0.00	1144.36	1144.36
233	0.00	1053.34	1053.34	283	0.00	1131.40	1131.40
234	0.00	1053.85	1053.85	284	0.00	1171.50	1171.50
235	1047.61	0.00	1047.61	285	0.00	1159.06	1159.06
236	1048.03	0.00	1048.03	286	1176.56	0.00	1176.56
237	1048.45	0.00	1048.45	287	0.00	1175.86	1175.86
238	1048.87	0.00	1048.87	288	244.86	956.25	1201.11
239	0.00	1048.24	1048.24	289	244.95	962.51	1207.47
240	0.00	1028.59	1028.59	290	245.05	969.95	1215.00
241	0.00	1039.25	1039.25	291	0.00	1215.01	1215.01
242	1044.74	0.00	1044.74	292	0.00	1228.64	1228.64
243	0.00	1044.12	1044.12	293	1221.75	0.00	1221.75
244	0.00	1043.48	1043.48	294	1222.24	0.00	1222.24
245	0.00	1037.61	1037.61	295	0.00	1221.51	1221.51
246	0.00	1059.04	1059.04	296	89.20	1152.67	1241.87
247	0.00	1055.11	1055.11	297	0.00	1219.00	1219.00
248	0.00	1030.69	1030.69	298	0.00	1219.64	1219.64
249	0.00	1034.10	1034.10	299	566.07	674.98	1241.05
250	1064.94	0.00	1064.94	300	566.29	684.71	1251.00

Table 17. Decisions table for Alg-1B, LCVAR=35 (Multi-Period)

t	Cash	SP500	W	t	Cash	SP500	W
200	270.78	728.50	999.27	250	0.00	1085.94	1085.94
201	0.00	992.79	992.79	251	0.00	1086.62	1086.62
202	113.68	867.10	980.78	252	0.00	1075.91	1075.91
203	58.48	931.59	990.07	253	135.14	970.83	1105.97
204	332.31	675.45	1007.76	254	0.00	1100.52	1100.52
205	350.41	655.52	1005.93	255	0.00	1114.05	1114.05
206	0.00	993.74	993.74	256	0.00	1103.77	1103.77
207	0.00	1004.80	1004.80	257	314.85	806.23	1121.08
208	0.00	977.89	977.89	258	119.85	998.18	1118.03
209	0.00	970.26	970.26	259	347.23	786.62	1133.85
210	0.00	964.55	964.55	260	513.45	628.46	1141.91
211	0.00	979.94	979.94	261	654.02	495.42	1149.44
212	293.67	703.75	997.42	262	478.15	672.46	1150.61
213	217.05	788.81	1005.87	263	586.58	568.40	1154.98
214	0.00	1014.39	1014.39	264	586.82	563.02	1149.84
215	450.48	579.70	1030.18	265	500.31	656.53	1156.84
216	240.91	787.79	1028.70	266	1166.02	0.00	1166.02
217	363.63	661.49	1025.12	267	1166.49	0.00	1166.49
218	443.68	585.68	1029.36	268	1166.95	0.00	1166.95
219	443.86	586.30	1030.16	269	1167.42	0.00	1167.42
220	0.00	1020.48	1020.48	270	1167.89	0.00	1167.89
221	325.02	725.51	1050.53	271	1168.36	0.00	1168.36
222	1051.15	0.00	1051.15	272	1168.82	0.00	1168.82
223	0.00	1050.52	1050.52	273	1169.29	0.00	1169.29
224	431.39	637.21	1068.60	274	1169.76	0.00	1169.76
225	244.66	814.15	1058.81	275	1170.23	0.00	1170.23
226	0.00	1060.57	1060.57	276	1170.69	0.00	1170.69
227	466.98	610.08	1077.06	277	1171.16	0.00	1171.16
228	520.82	557.40	1078.23	278	1171.63	0.00	1171.63
229	166.93	910.20	1077.13	279	1172.10	0.00	1172.10
230	375.16	697.72	1072.88	280	1172.57	0.00	1172.57
231	217.24	869.69	1086.94	281	0.00	1171.87	1171.87
232	402.30	681.68	1083.98	282	619.90	564.58	1184.48
233	0.00	1079.51	1079.51	283	405.45	772.67	1178.12
234	182.20	897.65	1079.85	284	1204.87	0.00	1204.87
235	464.96	610.27	1075.22	285	647.57	557.23	1204.79
236	279.47	806.24	1085.71	286	1213.47	0.00	1213.47
237	353.33	732.01	1085.34	287	1213.95	0.00	1213.95
238	1093.20	0.00	1093.20	288	1214.44	0.00	1214.44
239	0.00	1092.55	1092.55	289	1214.93	0.00	1214.93
240	0.00	1072.06	1072.06	290	1215.41	0.00	1215.41
241	0.00	1083.17	1083.17	291	1215.90	0.00	1215.90
242	153.65	936.19	1089.84	292	1216.38	0.00	1216.38
243	0.00	1063.64	1063.64	293	1216.87	0.00	1216.87
244	0.00	1063.00	1063.00	294	1217.36	0.00	1217.36
245	0.00	1057.02	1057.02	295	1217.84	0.00	1217.84
246	0.00	1078.84	1078.84	296	1218.33	0.00	1218.33
247	0.00	1074.84	1074.84	297	1218.82	0.00	1218.82
248	0.00	1049.97	1049.97	298	1219.31	0.00	1219.31
249	0.00	1053.44	1053.44	299	1219.79	0.00	1219.79

Table 23. Decisions table for AR(2), LCVAR=60 (Multi-Period)

t	Cash	SP500	W	t	Cash	SP500	W
200	1000.00	0.00	1000.00	250	1024.68	0.00	1024.68
201	0.00	999.40	999.40	251	1025.09	0.00	1025.09
202	0.00	1010.12	1010.12	252	1025.50	0.00	1025.50
203	0.00	1029.57	1029.57	253	0.00	1024.89	1024.89
204	0.00	1026.61	1026.61	254	0.00	1037.49	1037.49
205	1006.84	0.00	1006.84	255	0.00	1027.92	1027.92
206	0.00	1006.24	1006.24	256	1043.29	0.00	1043.29
207	0.00	979.29	979.29	257	1043.71	0.00	1043.71
208	0.00	971.65	971.65	258	0.00	1043.08	1043.08
209	0.00	965.93	965.93	259	0.00	1053.81	1053.81
210	0.00	981.35	981.35	260	0.00	1066.33	1066.33
211	0.00	999.14	999.14	261	1067.58	0.00	1067.58
212	1010.07	0.00	1010.07	262	0.00	1066.94	1066.94
213	0.00	1009.46	1009.46	263	0.00	1056.84	1056.84
214	0.00	1025.62	1025.62	264	0.00	1069.70	1069.70
215	0.00	1023.06	1023.06	265	0.00	1085.43	1085.43
216	0.00	1018.44	1018.44	266	0.00	1085.23	1085.23
217	1023.84	0.00	1023.84	267	0.00	1081.95	1081.95
218	1024.25	0.00	1024.25	268	0.00	1092.12	1092.12
219	1024.66	0.00	1024.66	269	0.00	1105.48	1105.48
220	1025.07	0.00	1025.07	270	0.00	1092.83	1092.83
221	1025.48	0.00	1025.48	271	0.00	1098.67	1098.67
222	1025.89	0.00	1025.89	272	1090.93	0.00	1090.93
223	0.00	1025.28	1025.28	273	1091.36	0.00	1091.36
224	0.00	1009.55	1009.55	274	0.00	1090.71	1090.71
225	0.00	1011.92	1011.92	275	0.00	1084.37	1084.37
226	0.00	1028.09	1028.09	276	1103.25	0.00	1103.25
227	1028.81	0.00	1028.81	277	0.00	1102.58	1102.58
228	1029.22	0.00	1029.22	278	0.00	1115.99	1115.99
229	1029.63	0.00	1029.63	279	1111.54	0.00	1111.54
230	0.00	1029.01	1029.01	280	1111.98	0.00	1111.98
231	0.00	1025.63	1025.63	281	1112.43	0.00	1112.43
232	0.00	1019.27	1019.27	282	1112.87	0.00	1112.87
233	0.00	1019.77	1019.77	283	0.00	1112.21	1112.21
234	1013.73	0.00	1013.73	284	0.00	1100.40	1100.40
235	1014.13	0.00	1014.13	285	0.00	1118.13	1118.13
236	0.00	1013.53	1013.53	286	0.00	1125.17	1125.17
237	0.00	1025.24	1025.24	287	0.00	1149.57	1149.57
238	997.55	0.00	997.55	288	0.00	1157.09	1157.09
239	0.00	996.95	996.95	289	0.00	1166.04	1166.04
240	0.00	1007.28	1007.28	290	0.00	1166.22	1166.22
241	1012.61	0.00	1012.61	291	0.00	1179.30	1179.30
242	1013.02	0.00	1013.02	292	1172.69	0.00	1172.69
243	1013.42	0.00	1013.42	293	1173.16	0.00	1173.16
244	198.77	814.24	1013.01	294	1173.63	0.00	1173.63
245	0.00	1029.71	1029.71	295	0.00	1172.93	1172.93
246	1024.85	0.00	1024.85	296	1148.56	0.00	1148.56
247	1025.26	0.00	1025.26	297	0.00	1147.87	1147.87
248	1025.67	0.00	1025.67	298	0.00	1168.55	1168.55
249	0.00	1025.06	1025.06	299	0.00	1185.40	1185.40

Table 24. Decisions table for GBM, LCVAR=60 (Multi-Period)

t	Cash	SP500	W	t	Cash	SP500	W
200	0.00	999.00	999.00	250	0.00	1047.65	1047.65
201	0.00	987.03	987.03	251	0.00	1037.32	1037.32
202	0.00	997.62	997.62	252	1065.37	0.00	1065.37
203	0.00	1016.83	1016.83	253	0.00	1064.73	1064.73
204	0.00	1013.91	1013.91	254	0.00	1077.82	1077.82
205	994.39	0.00	994.39	255	0.00	1067.88	1067.88
206	994.78	0.00	994.78	256	1083.85	0.00	1083.85
207	995.18	0.00	995.18	257	1084.28	0.00	1084.28
208	995.58	0.00	995.58	258	0.00	1083.63	1083.63
209	995.98	0.00	995.98	259	0.00	1094.78	1094.78
210	996.38	0.00	996.38	260	0.00	1107.78	1107.78
211	996.77	0.00	996.77	261	1109.09	0.00	1109.09
212	0.00	996.18	996.18	262	0.00	1108.42	1108.42
213	0.00	1007.11	1007.11	263	1096.83	0.00	1096.83
214	1022.21	0.00	1022.21	264	0.00	1096.17	1096.17
215	1022.61	0.00	1022.61	265	0.00	1112.29	1112.29
216	0.00	1022.00	1022.00	266	0.00	1112.09	1112.09
217	0.00	1028.45	1028.45	267	1107.62	0.00	1107.62
218	0.00	1029.54	1029.54	268	0.00	1106.95	1106.95
219	0.00	1013.01	1013.01	269	0.00	1120.50	1120.50
220	0.00	1043.17	1043.17	270	0.00	1107.67	1107.67
221	0.00	1044.92	1044.92	271	0.00	1113.59	1113.59
222	1022.70	0.00	1022.70	272	0.00	1106.85	1106.85
223	1023.11	0.00	1023.11	273	0.00	1123.35	1123.35
224	1023.52	0.00	1023.52	274	0.00	1123.17	1123.17
225	1023.93	0.00	1023.93	275	0.00	1116.64	1116.64
226	0.00	1023.31	1023.31	276	1136.08	0.00	1136.08
227	0.00	1025.05	1025.05	277	1136.54	0.00	1136.54
228	0.00	1023.30	1023.30	278	0.00	1135.86	1135.86
229	0.00	1018.69	1018.69	279	0.00	1132.46	1132.46
230	0.00	1039.22	1039.22	280	0.00	1082.50	1082.50
231	0.00	1035.80	1035.80	281	0.00	1094.73	1094.73
232	0.00	1029.38	1029.38	282	0.00	1082.33	1082.33
233	0.00	1029.88	1029.88	283	0.00	1120.69	1120.69
234	0.00	1024.81	1024.81	284	1107.68	0.00	1107.68
235	0.00	1042.42	1042.42	285	1108.13	0.00	1108.13
236	0.00	1041.89	1041.89	286	0.00	1107.46	1107.46
237	1052.88	0.00	1052.88	287	0.00	1131.47	1131.47
238	0.00	1052.25	1052.25	288	0.00	1138.88	1138.88
239	0.00	1032.52	1032.52	289	1146.54	0.00	1146.54
240	0.00	1043.22	1043.22	290	0.00	1145.85	1145.85
241	1048.74	0.00	1048.74	291	1157.55	0.00	1157.55
242	1049.16	0.00	1049.16	292	1158.02	0.00	1158.02
243	0.00	1048.53	1048.53	293	0.00	1157.32	1157.32
244	1041.59	0.00	1041.59	294	0.00	1135.73	1135.73
245	1042.01	0.00	1042.01	295	0.00	1154.74	1154.74
246	0.00	1041.38	1041.38	296	0.00	1131.88	1131.88
247	1016.27	0.00	1016.27	297	0.00	1132.47	1132.47
248	0.00	1015.66	1015.66	298	0.00	1152.88	1152.88
249	0.00	1046.99	1046.99	299	0.00	1169.50	1169.50

Table 28. Decisions table obtained by using $\mathbf{w}^2$, *LCVAR*=35.

t	Cash	SP500	W	t	Cash	SP500	W
200	1000.00	0.00	1000.00	250	1049.12	0.00	1049.12
201	0.00	999.40	999.40	251	1049.54	0.00	1049.54
202	0.00	1010.12	1010.12	252	1049.96	0.00	1049.96
203	1028.54	0.00	1028.54	253	1050.38	0.00	1050.38
204	0.00	1027.92	1027.92	254	0.00	1049.75	1049.75
205	0.00	1009.14	1009.14	255	0.00	1040.07	1040.07
206	0.00	1020.37	1020.37	256	0.00	1056.68	1056.68
207	175.24	817.63	992.87	257	0.00	1052.77	1052.77
208	0.00	986.38	986.38	258	0.00	1069.64	1069.64
209	0.00	980.58	980.58	259	197.00	883.45	1080.44
210	0.00	996.22	996.22	260	0.00	1090.82	1090.82
211	0.00	1014.29	1014.29	261	0.00	1093.20	1093.20
212	0.00	1026.41	1026.41	262	0.00	1100.18	1100.18
213	0.00	1037.67	1037.67	263	0.00	1089.77	1089.77
214	0.00	1054.28	1054.28	264	1101.92	0.00	1101.92
215	0.00	1051.65	1051.65	265	1102.36	0.00	1102.36
216	0.00	1046.90	1046.90	266	0.00	1101.70	1101.70
217	0.00	1053.51	1053.51	267	0.00	1098.37	1098.37
218	73.99	980.56	1054.55	268	0.00	1108.69	1108.69
219	1037.87	0.00	1037.87	269	0.00	1122.26	1122.26
220	1038.28	0.00	1038.28	270	0.00	1109.42	1109.42
221	1038.70	0.00	1038.70	271	1114.23	0.00	1114.23
222	93.23	944.94	1038.17	272	0.00	1113.56	1113.56
223	0.00	1054.77	1054.77	273	0.00	1130.17	1130.17
224	0.00	1038.59	1038.59	274	0.00	1129.99	1129.99
225	0.00	1041.02	1041.02	275	0.00	1123.41	1123.41
226	0.00	1057.66	1057.66	276	1142.97	0.00	1142.97
227	588.30	470.57	1058.87	277	1143.43	0.00	1143.43
228	0.00	1057.71	1057.71	278	0.00	1142.74	1142.74
229	0.00	1052.94	1052.94	279	0.00	1139.33	1139.33
230	0.00	1074.16	1074.16	280	0.00	1089.07	1089.07
231	313.50	756.82	1070.32	281	0.00	1101.37	1101.37
232	0.00	1065.43	1065.43	282	0.00	1088.89	1088.89
233	0.00	1065.95	1065.95	283	0.00	1127.49	1127.49
234	1059.64	0.00	1059.64	284	0.00	1115.52	1115.52
235	1060.07	0.00	1060.07	285	327.23	805.94	1133.17
236	1060.49	0.00	1060.49	286	0.00	1138.05	1138.05
237	0.00	1059.85	1059.85	287	383.21	779.12	1162.34
238	0.00	1032.26	1032.26	288	383.37	784.23	1167.59
239	0.00	1012.90	1012.90	289	383.52	790.29	1173.81
240	0.00	1023.40	1023.40	290	0.00	1173.70	1173.70
241	1028.81	0.00	1028.81	291	0.00	1186.87	1186.87
242	0.00	1028.20	1028.20	292	1180.22	0.00	1180.22
243	0.00	1027.57	1027.57	293	1180.69	0.00	1180.69
244	0.00	1021.79	1021.79	294	0.00	1179.98	1179.98
245	0.00	1042.89	1042.89	295	0.00	1199.74	1199.74
246	0.00	1039.02	1039.02	296	0.00	1175.98	1175.98
247	0.00	1014.98	1014.98	297	0.00	1176.60	1176.60
248	0.00	1018.34	1018.34	298	70.83	1126.91	1197.73
249	1048.70	0.00	1048.70	299	0.00	1213.93	1213.93

Table 34. Return/risk profiles of alternative approaches (TW2)

	Risk (LCVAR or SP/Cash)	Avg. Ret.	VAR	Avg. Shortfall
Alg-1A	2	-0.0042	1.3358	0.4563
	6	-0.1052	12.1137	1.4170
	10	-0.1579	29.7992	2.2185
	14	-0.1738	35.6242	2.4339
	18	-0.1703	36.0080	2.4440
Alg-1B	2	0.0599	0.0327	0.0191
	6	0.0801	0.3074	0.0684
	10	0.1001	0.8767	0.1193
	14	0.1215	1.7138	0.1687
	18	0.1404	2.8267	0.2187
B&H	0.20	0.0682	1.5380	0.4624
	0.35	0.0817	4.7102	0.8257
	0.50	0.0952	9.6127	1.1891
	0.65	0.1088	16.2455	1.5526
	0.80	0.1223	24.6086	1.9161
AR(2)	2	-0.0069	0.8120	0.3438
	6	-0.1232	7.4657	1.0856
	10	-0.2109	19.9377	1.7811
	14	-0.2636	29.3828	2.1838
	18	-0.2849	31.7478	2.2779
GBM	2	0.0478	0.5720	0.2451
	6	0.0397	5.2420	0.7877
	10	0.0440	14.4300	1.3157
	14	-0.0257	24.6616	1.7712
	18	-0.1131	27.3851	1.9195

Table 35. Return/risk profiles of alternative approaches (TW3)

	Risk (LCVAR or SP/Cash)	Avg. Ret.	VAR	Avg. Shortfall
Alg-1A	2	0.0238	1.8469	0.4683
	6	0.0006	16.2217	1.4087
	10	0.1978	33.1177	1.9677
	14	0.2937	37.9377	2.1316
	18	0.3361	38.5781	2.1351
Alg-1B	2	0.4839	5.9690	0.1481
	6	0.4439	12.1742	0.5881
	10	0.3769	15.9538	0.9273
	14	0.3014	20.7628	1.2176
	18	0.2408	25.5409	1.4646
B&H	0.20	0.0736	1.7407	0.4987
	0.35	0.0911	5.3307	0.8888
	0.50	0.1087	10.8791	1.2790
	0.65	0.1263	18.3856	1.6691
	0.80	0.1439	27.8504	2.0593
AR(2)	2	0.0800	0.7523	0.2692
	6	0.1404	6.7608	0.8543
	10	0.2154	18.5686	1.4243
	14	0.2818	31.3393	1.8595
	18	0.3364	34.2694	1.9550

Table 35. Continued

	Risk (LCVAR or SP/Cash)	Avg. Ret.	VAR	Avg. Shortfall
GBM	2	0.0024	0.5810	0.2680
	6	-0.0911	5.2544	0.8489
	10	-0.1889	14.5496	1.4289
	14	-0.1936	24.2012	1.8248
	18	-0.1937	26.8822	1.9287

Table 36. Return/risk profiles of alternative approaches (TW4)

	Risk (LCVAR or SP/Cash)	Avg. Ret.	VAR	Avg. Shortfall
Alg-1A	2	0.1194	1.4297	0.3333
	6	0.2630	12.9396	1.0428
	10	0.3738	34.2989	1.7272
	14	0.4324	39.7404	1.9404
	18	0.4737	40.8526	1.9767
Alg-1B	2	0.2847	12.5329	0.5340
	6	0.1797	15.5556	0.8541
	10	0.1667	19.4175	1.0671
	14	0.3120	27.1243	1.3851
	18	0.4145	31.8618	1.5031
B&H	0.20	0.1230	1.8041	0.4490
	0.35	0.1777	5.5252	0.8020
	0.50	0.2324	11.2759	1.1549
	0.65	0.2871	19.0563	1.5078
	0.80	0.3418	28.8663	1.8607
AR(2)	2	0.1289	0.7935	0.2306
	6	0.2926	7.1834	0.7320
	10	0.4541	19.4207	1.2174
	14	0.6226	28.8959	1.4902
	18	0.6454	31.6442	1.5899
GBM	2	0.0431	0.8292	0.2513
	6	0.0296	7.4919	0.7959
	10	0.0385	20.1365	1.3123
	14	0.0998	28.5612	1.5909
	18	0.0890	30.5057	1.6656

Table 37. Return/risk profiles of alternative approaches (TW5)

	Risk (LCVAR or SP/Cash)	Avg. Ret.	VAR	Avg. Shortfall
Alg-1A	2	0.0863	1.1826	0.3369
	6	0.1602	11.0522	1.0711
	10	0.2177	29.0240	1.7652
	14	0.1748	35.8977	2.0300
	18	0.1141	40.0757	2.2324
Alg-1B	2	0.0261	7.9006	0.3789
	6	0.0198	9.7421	0.5795
	10	0.0272	11.5352	0.7040
	14	0.0378	14.0366	0.8176
	18	0.0304	16.5656	0.9232

Table 37. Continued

	Risk (LCVAR or SP/Cash)	Avg. Ret.	VAR	Avg. Shortfall
B&H	0.20	0.0139	1.8703	0.5169
	0.35	-0.0132	5.7279	0.9228
	0.50	-0.0403	11.6897	1.3288
	0.65	-0.0675	19.7556	1.7348
	0.80	-0.0946	29.9257	2.1409
AR(2)	2	-0.0225	0.9382	0.3379
	6	-0.2018	8.8866	1.0870
	10	-0.3291	23.1721	1.7546
	14	-0.3218	31.6737	2.0318
	18	-0.3301	32.0511	2.0546
GBM	2	-0.0284	0.8119	0.2784
	6	-0.1935	7.6178	0.8911
	10	-0.3586	21.3360	1.5039
	14	-0.4694	30.7618	1.8624
	18	-0.4314	32.8911	1.9003

Table 38. Sharpe Ratios obtained from all time windows.

Time Win.	AR(2)	GBM	B&H	Alg-1A	Alg-1B
TW1	-0.0648	0.0005	0.0421	0.0637	0.0334
	-0.0608	0.0035	0.0420	0.0676	0.0593
	-0.0520	0.0044	0.0420	0.0586	0.0583
	-0.0290	0.0063	0.0420	0.0756	0.0467
	-0.0281	0.0114	0.0420	0.0855	0.0413
TW2	-0.0632	-0.0029	0.0146	-0.0469	0.0547
	-0.0634	-0.0045	0.0146	-0.0446	0.0543
	-0.0584	-0.0016	0.0146	-0.0381	0.0535
	-0.0578	-0.0152	0.0146	-0.0375	0.0546
	-0.0594	-0.0312	0.0146	-0.0367	0.0538
TW3	0.0346	-0.0624	0.0179	-0.0193	0.1776
	0.0348	-0.0616	0.0178	-0.0123	0.1129
	0.0384	-0.0626	0.0178	0.0257	0.0768
	0.0414	-0.0495	0.0178	0.0396	0.0552
	0.0489	-0.0470	0.0178	0.0461	0.0378
TW4	0.0885	-0.0076	0.0544	0.0580	0.0663
	0.0905	-0.0074	0.0543	0.0592	0.0329
	0.0917	-0.0026	0.0543	0.0553	0.0265
	0.1065	0.0093	0.0543	0.0607	0.0503
	0.1058	0.0071	0.0543	0.0663	0.0646
TW5	-0.0749	-0.0870	-0.0264	0.0334	-0.0085
	-0.0845	-0.0882	-0.0264	0.0331	-0.0097
	-0.0788	-0.0885	-0.0264	0.0311	-0.0067
	-0.0661	-0.0936	-0.0264	0.0208	-0.0033
	-0.0671	-0.0839	-0.0264	0.0101	-0.0048

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Table 52. Return / risk profiles obtained via different weights ([30 15], Weekly)

Weight Vector	LCVAR	Avg. Ret.	VAR	Avg. Shortfall
	10	0.9174	35.1402	1.8725
	20	1.4134	143.1724	3.9739
we^1	30	1.8827	324.0771	6.0899
	40	2.4760	520.3068	7.7436
	50	2.8195	672.6954	8.8001
	10	0.9177	35.1447	1.8721
	20	1.4141	143.1812	3.9730
we^2	30	1.8806	324.0618	6.0899
	40	2.4727	520.2894	7.7439
	50	2.8191	672.6513	8.8000
	10	0.9384	32.3565	1.7698
	20	1.4525	131.7087	3.7660
we^3	30	1.9725	298.3028	5.7620
	40	2.4445	508.3236	7.6494
	50	2.8516	677.3887	8.7921
	10	0.9541	33.1869	1.7899
	20	1.4881	134.7550	3.7987
we^4	30	2.0147	304.6229	5.8108
	40	2.5054	519.7018	7.6986
	50	2.8485	684.0514	8.8271

Table 53. Return / risk profiles obtained via different weights ([8 4], Weekly)

Weight Vector	LCVAR	Avg. Ret.	VAR	Avg. Shortfall
	10	0.5424	110.9310	3.3368
	20	0.8415	324.8729	6.0901
we^0	30	1.8990	456.9395	7.1939
	40	2.1604	534.2412	7.8904
	50	2.2805	579.4477	8.1722
	10	0.5430	110.8888	3.3361
	20	0.8416	324.8976	6.0904
we^1	30	1.8991	456.9117	7.1937
	40	2.1606	534.2624	7.8905
	50	2.2808	579.4304	8.1720
	10	0.6846	105.1095	3.1789
	20	1.0642	307.6657	5.8562
we^2	30	1.8572	469.3954	7.2968
	40	2.2127	573.4802	8.1291
	50	2.6013	652.2954	8.5627
	10	0.4587	112.2625	3.3790
	20	0.6845	329.0631	6.2061
we^3	30	1.6100	441.5174	7.2399
	40	2.1913	518.7228	7.7762
	50	2.4801	579.1909	8.0964

Table 53. Continued

Weight Vector	LCVAR	Avg. Ret.	VAR	Avg. Shortfall
we^4	10	0.4453	110.8810	3.3777
	20	0.7307	323.6759	6.1542
	30	1.7989	448.7608	7.1878
	40	2.1595	534.3835	7.8928
	50	2.2799	579.6558	8.1750

Table 54. Sharpe Ratios obtained via [8 4] and [30 15] tree topologies

Topology	we^0	we^1	we^2	we^3	we^4
[8 4]	0.013523	0.013583	0.027764	0.005542	0.004299
	0.024496	0.024498	0.037867	0.015685	0.018379
	0.070125	0.070132	0.067260	0.057584	0.066034
	0.076164	0.076168	0.075696	0.078652	0.076115
	0.078122	0.078133	0.086190	0.086430	0.078083
[30 15]	0.088178	0.087290	0.087333	0.094649	0.096182
	0.085574	0.084694	0.084753	0.091711	0.093731
	0.083252	0.082365	0.082249	0.091044	0.092512
	0.092320	0.091011	0.090868	0.090681	0.092353
	0.094575	0.093285	0.093275	0.094195	0.093615

Table 59. Return / risk profiles obtained via different lb^n values (TW2)

lb^n	LCVAR	Avg. Ret.	VAR	Avg. Shortfall
0.00	2	0.0331	1.4439	0.4642
	6	0.0108	13.2825	1.4509
	10	-0.0109	36.2741	2.4097
	14	-0.0703	59.9737	3.1268
	24	-0.1770	84.9352	3.7368
	32	-0.1904	85.2931	3.7480
0.05	2	0.0291	1.4460	0.4672
	6	-0.0125	13.2498	1.4613
	10	-0.0314	36.3465	2.4243
	14	-0.0496	59.1776	3.0964
	24	-0.1750	84.8842	3.7351
	32	-0.1904	85.2931	3.7480
0.10	2	0.0416	1.4562	0.4620
	6	0.0234	13.2452	1.4420
	10	0.0267	35.6657	2.3693
	14	-0.0624	59.4415	3.1121
	24	-0.1730	84.8355	3.7334
	32	-0.1904	85.2931	3.7480

Table 59. Continued

0.20	2	0.0441	1.4146	0.4501
	6	0.0168	12.6991	1.4073
	10	0.0057	35.1040	2.3568
	14	-0.0556	58.5297	3.0949
	24	-0.1692	84.7447	3.7302
	32	-0.1904	85.2931	3.7480

Table 60. Return / risk profile obtained via different lb^n values (TW3)

lb^n	LCVAR	Avg. Ret.	VAR	Avg. Shortfall
0.00	2	0.1191	1.5929	0.4446
	6	0.2616	14.2906	1.3751
	10	0.3979	39.9967	2.3237
	14	0.5506	73.0354	3.1360
	24	0.7035	111.1857	3.9024
	32	0.6935	112.4671	3.9299
0.05	2	0.1296	1.7930	0.4627
	6	0.2910	16.1696	1.4391
	10	0.4532	44.9905	2.4160
	14	0.5514	74.4104	3.1702
	24	0.7024	111.6120	3.9094
	32	0.6935	112.4671	3.9299
0.10	2	0.1227	1.7739	0.4648
	6	0.2666	15.9939	1.4470
	10	0.4116	44.4551	2.4296
	14	0.5099	74.1500	3.1971
	24	0.7053	111.0993	3.9002
	32	0.6935	112.4671	3.9299
0.20	2	0.1148	1.7096	0.4550
	6	0.2442	15.3412	1.4150
	10	0.3732	42.6288	2.3762
	14	0.4610	71.9999	3.1696
	24	0.6972	111.6246	3.9105
	32	0.6935	112.4671	3.9299

Table 61. Return / risk profile obtained via different lb^n values (TW4)

lb^n	LCVAR	Avg. Ret.	VAR	Avg. Shortfall
0.00	2	0.1415	1.4561	0.4106
	6	0.3315	13.2225	1.2762
	10	0.5219	36.7675	2.1411
	14	0.7150	70.8501	2.9676
	24	0.9736	108.5358	3.7062
	32	0.9720	108.5650	3.7082
0.05	2	0.1460	1.4703	0.4107
	6	0.3438	13.3466	1.2772
	10	0.5439	37.0898	2.1413
	14	0.7388	71.5043	2.9726
	24	0.9736	108.5358	3.7062
	32	0.9720	108.5650	3.7082
0.10	2	0.1457	1.3922	0.3976
	6	0.3436	12.6172	1.2366
	10	0.5415	35.1060	2.0760
	14	0.7444	68.1215	2.8946
	24	0.9736	108.5358	3.7062
	32	0.9720	108.5650	3.7082
0.20	2	0.1455	1.3883	0.3980
	6	0.3432	12.5873	1.2378
	10	0.5409	35.0185	2.0776
	14	0.7473	68.3541	2.9006
	24	0.9736	108.5358	3.7062
	32	0.9720	108.5650	3.7082

Table 62. Return / risk profile obtained via different lb^n values (TW5)

lb^n	LCVAR	Avg. Ret.	VAR	Avg. Shortfall
0.00	2	0.0226	1.5395	0.4471
	6	-0.0429	14.4013	1.4190
	10	-0.1068	40.2581	2.3891
	14	-0.2064	77.3709	3.3515
	24	-0.3487	121.7855	4.3002
	32	-0.3571	122.5404	4.3136
0.05	2	0.0245	1.5000	0.4404
	6	-0.0308	13.9571	1.3923
	10	-0.0907	38.9583	2.3437
	14	-0.1697	75.2614	3.2903
	24	-0.3479	121.7114	4.2988
	32	-0.3571	122.5404	4.3136

Table 62. Continued

0.10	2	0.0271	1.4662	0.4343
	6	-0.0264	13.6465	1.3755
	10	-0.0798	38.1708	2.3177
	14	-0.1558	73.8078	3.2503
	24	-0.3472	121.6561	4.2978
	32	-0.3571	122.5404	4.3136
0.20	2	0.0180	1.7214	0.4726
	6	-0.0540	15.9891	1.4914
	10	-0.1263	44.7239	2.5114
	14	-0.2381	84.7449	3.5084
	24	-0.3471	121.6412	4.2976
	32	-0.3571	122.5404	4.3136

Table 63. Sharpe Ratios obtained over different time windows and lb^n values.

Time Window	Alg-2 (0.00)	Alg-2 (0.05)	Alg-2 (0.10)	Alg-2 (0.20)
TW1	0.0360	0.0402	0.0290	0.0387
	0.0388	0.0415	0.0324	0.0411
	0.0397	0.0488	0.0413	0.0437
	0.0440	0.0521	0.0502	0.0486
	0.0614	0.0614	0.0614	0.0614
	0.0614	0.0614	0.0614	0.0614
TW2	-0.0140	-0.0174	-0.0069	-0.0050
	-0.0108	-0.0172	-0.0073	-0.0093
	-0.0101	-0.0135	-0.0039	-0.0075
	-0.0155	-0.0129	-0.0146	-0.0138
	-0.0246	-0.0244	-0.0242	-0.0238
	-0.0260	-0.0260	-0.0260	-0.0260
TW3	0.0548	0.0595	0.0546	0.0496
	0.0560	0.0599	0.0542	0.0496
	0.0550	0.0601	0.0542	0.0495
	0.0586	0.0581	0.0534	0.0484
	0.0620	0.0618	0.0622	0.0613
	0.0607	0.0607	0.0607	0.0607
TW4	0.0759	0.0792	0.0811	0.0811
	0.0774	0.0804	0.0826	0.0826
	0.0778	0.0811	0.0830	0.0830
	0.0790	0.0815	0.0841	0.0843
	0.0887	0.0887	0.0887	0.0887
	0.0885	0.0885	0.0885	0.0885
TW5	-0.0221	-0.0208	-0.0189	-0.0244
	-0.0245	-0.0216	-0.0207	-0.0260
	-0.0247	-0.0225	-0.0210	-0.0264
	-0.0292	-0.0253	-0.0240	-0.0313
	-0.0361	-0.0361	-0.0360	-0.0360
	-0.0368	-0.0368	-0.0368	-0.0368

Table 64. Return / risk profiles obtained via fixed second moment.

Time Window	LCVAR	Avg. Ret.	VAR	Avg. Shortfall
TW1	2	0.1272	2.3979	0.5368
	6	0.2881	21.7612	1.6663
	10	0.4525	58.9101	2.7623
	14	0.5929	88.9192	3.4932
	28	0.7103	115.5502	4.0620
TW2	2	0.0380	1.2830	0.4340
	6	0.0280	11.7669	1.3550
	10	-0.0639	32.3004	2.2867
	14	-0.1623	54.2613	3.0122
	28	-0.1892	85.2820	3.7468
TW3	2	0.1225	1.3681	0.4037
	6	0.2665	12.3879	1.2665
	10	0.3640	33.8973	2.1485
	14	0.4928	64.1600	2.9579
	28	0.7027	112.3258	3.9193
TW4	2	0.1276	1.5804	0.4296
	6	0.2880	14.3731	1.3352
	10	0.4524	39.6847	2.2339
	14	0.6132	69.3038	2.9901
	28	0.9586	108.1894	3.7082
TW5	2	0.0019	1.6674	0.4693
	6	-0.1027	15.5826	1.4847
	10	-0.2036	43.5497	2.4962
	14	-0.3688	77.9732	3.4542
	28	-0.3571	122.5404	4.3136

Table 65. Sharpe Ratios obtained via fixed second moments across five time windows

Time Window	Alg-2 Fixed VAR
TW1	0.0499
	0.0510
	0.0524
	0.0576
	0.0614
TW2	-0.0106
	-0.0064
	-0.0200
	-0.0288
	-0.0259
TW3	0.0620
	0.0615
	0.0539
	0.0553
	0.0616

Table 65. Cotinued

TW4	0.0617
	0.0628
	0.0639
	0.0676
	0.0874
TW5	-0.0373
	-0.0387
	-0.0384
	-0.0474
	-0.0368

Table 66. Return / risk profiles obtained via different weights ([30 15], TW1)

Weight Vector	LCVAR	Avg. Ret.	VAR	Avg. Shortfall
we^1	2	0.1086	2.5091	0.5675
	6	0.2379	22.7061	1.7509
	10	0.3797	56.3449	2.8007
	14	0.5029	87.7671	3.5372
	24	0.7103	115.5502	4.0620
	32	0.7103	115.5502	4.0620
we^2	2	0.1081	2.5196	0.5694
	6	0.2363	22.7970	1.7565
	10	0.3770	56.7650	2.8161
	14	0.5016	87.8518	3.5422
	24	0.7096	115.5341	4.0620
	32	0.7103	115.5502	4.0620
we^3	2	0.1040	2.3853	0.5513
	6	0.2246	21.5548	1.7022
	10	0.3671	53.9855	2.7394
	14	0.4868	87.7247	3.5469
	24	0.7103	115.5502	4.0620
	32	0.7103	115.5502	4.0620
we^4	2	0.1044	2.3458	0.5456
	6	0.2256	21.2046	1.6850
	10	0.3731	53.6462	2.7259
	14	0.5060	87.0972	3.5164
	24	0.7099	115.5403	4.0620
	32	0.7103	115.5502	4.0620

Table 67. Return / risk profile obtained via different weights ([30 15], TW2)

Weight Vector	LCVAR	Avg. Ret.	VAR	Avg. Shortfall
we^1	2	0.0428	1.4312	0.4535
	6	0.0110	12.8628	1.4229
	10	0.0058	34.8418	2.3472
	14	-0.0580	58.6124	3.0974
	24	-0.1692	84.7450	3.7302
	32	-0.1904	85.2931	3.7480
we^2	2	0.0420	1.4275	0.4531
	6	0.0116	12.7613	1.4157
	10	0.0010	34.9653	2.3542
	14	-0.0629	58.7879	3.1035
	24	-0.1692	84.7447	3.7302
	32	-0.1904	85.2931	3.7480
we^3	2	0.0348	1.3688	0.4470
	6	-0.0073	12.2845	1.3965
	10	-0.0443	33.6561	2.3248
	14	-0.0836	58.2558	3.0966
	24	-0.1747	84.8773	3.7348
	32	-0.1904	85.2931	3.7480
we^4	2	0.0437	1.3995	0.4488
	6	0.0290	12.6944	1.3996
	10	0.0279	35.1106	2.3386
	14	-0.0480	58.5263	3.0860
	24	-0.1692	84.7447	3.7302
	32	-0.1904	85.2931	3.7480

Table 68. Return / risk profile obtained via different weights ([30 15], TW3)

Weight Vector	LCVAR	Avg. Ret.	VAR	Avg. Shortfall
we^1	2	0.1161	1.6874	0.4506
	6	0.2484	15.1874	1.4027
	10	0.3814	42.2224	2.3557
	14	0.4665	71.5823	3.1569
	24	0.6972	111.6015	3.9104
	32	0.6935	112.4671	3.9299
we^2	2	0.1152	1.6887	0.4515
	6	0.2461	15.1948	1.4052
	10	0.3765	42.2378	2.3602
	14	0.4688	71.7230	3.1595
	24	0.6959	111.6568	3.9118
	32	0.6935	112.4671	3.9299
we^3	2	0.1225	1.6619	0.4440
	6	0.2662	14.9656	1.3840
	10	0.4110	41.5817	2.3237
	14	0.5040	70.9969	3.1261
	24	0.7030	111.8626	3.9126
	32	0.6935	112.4671	3.9299

Table 68. Continued

Weight Vector	LCVAR	Avg. Ret.	VAR	Avg. Shortfall
	2	0.1121	1.7173	0.4592
	6	0.2355	15.4528	1.4288
we4	10	0.3592	42.9298	2.3988
	14	0.4568	71.7437	3.1699
	24	0.6954	111.6511	3.9121
	32	0.6935	112.4671	3.9299

Table 69. Return / risk profile obtained via different weights ([30 15], TW4)

Weight Vector	LCVAR	Avg. Ret.	VAR	Avg. Shortfall
	2	0.1453	1.3945	0.3991
	6	0.3418	12.6726	1.2434
we1	10	0.5360	35.2287	2.0864
	14	0.7450	68.7702	2.9113
	24	0.9736	108.5358	3.7062
	32	0.9720	108.5650	3.7082
	2	0.1451	1.4232	0.4038
	6	0.3417	12.9273	1.2564
we2	10	0.5382	35.9904	2.1095
	14	0.7472	69.9830	2.9349
	24	0.9736	108.5358	3.7062
	32	0.9720	108.5650	3.7082
	2	0.1489	1.4309	0.4013
	6	0.3536	12.9559	1.2467
we3	10	0.5597	36.0216	2.0915
	14	0.7576	69.7391	2.9225
	24	0.9736	108.5358	3.7062
	32	0.9720	108.5650	3.7082
	2	0.1482	1.4084	0.3994
	6	0.3511	12.7576	1.2414
we4	10	0.5539	35.4882	2.0835
	14	0.7618	69.1343	2.9084
	24	0.9736	108.5358	3.7062
	32	0.9720	108.5650	3.7082

Table 70. Return / risk profile obtained via different weights ([30 15], TW5)

Weight Vector	LCVAR	Avg. Ret.	VAR	Avg. Shortfall
	2	0.0182	1.7083	0.4714
	6	-0.0545	15.8969	1.4895
we1	10	-0.1282	44.4763	2.5086
	14	-0.2402	84.7346	3.5114
	24	-0.3479	121.7191	4.2990
	32	-0.3571	122.5404	4.3136

Table 70. Continued

Weight Vector	LCVAR	Avg. Ret.	VAR	Avg. Shortfall
we^2	2	0.0182	1.7126	0.4721
	6	-0.0543	15.9302	1.4913
	10	-0.1264	44.5407	2.5103
	14	-0.2387	84.5943	3.5084
	24	-0.3469	121.6218	4.2972
	32	-0.3571	122.5404	4.3136
we^3	2	0.0143	1.7535	0.4777
	6	-0.0652	16.0725	1.4994
	10	-0.1424	44.8556	2.5214
	14	-0.2613	85.1136	3.5239
	24	-0.3468	121.6177	4.2972
	32	-0.3571	122.5404	4.3136
we^4	2	0.0182	1.7001	0.4701
	6	-0.0531	15.7912	1.4832
	10	-0.1257	44.1903	2.4985
	14	-0.2360	84.2306	3.4966
	24	-0.3561	122.5514	4.3127
	32	-0.3571	122.5404	4.3136

Table 71. Sharpe Ratios obtained via different weight vectors across five time windows

Time Window	we^0	we^1	we^2	we^3	we^4
TW1	0.0387	0.0370	0.0366	0.0350	0.0355
	0.0411	0.0394	0.0390	0.0376	0.0381
	0.0437	0.0439	0.0434	0.0432	0.0441
	0.0486	0.0483	0.0482	0.0466	0.0489
	0.0614	0.0614	0.0614	0.0614	0.0614
	0.0614	0.0614	0.0614	0.0614	0.0614
TW2	-0.0050	-0.0060	-0.0067	-0.0130	-0.0053
	-0.0093	-0.0109	-0.0107	-0.0163	-0.0059
	-0.0075	-0.0075	-0.0083	-0.0163	-0.0037
	-0.0138	-0.0141	-0.0147	-0.0175	-0.0128
	-0.0238	-0.0238	-0.0238	-0.0244	-0.0238
	-0.0260	-0.0260	-0.0260	-0.0260	-0.0260
TW3	0.0496	0.0509	0.0502	0.0563	0.0474
	0.0496	0.0509	0.0503	0.0559	0.0472
	0.0495	0.0510	0.0502	0.0560	0.0472
	0.0484	0.0492	0.0494	0.0539	0.0480
	0.0613	0.0613	0.0611	0.0617	0.0611
	0.0607	0.0607	0.0607	0.0607	0.0607
TW4	0.0811	0.0807	0.0797	0.0827	0.0828
	0.0826	0.0820	0.0811	0.0843	0.0843
	0.0830	0.0819	0.0814	0.0849	0.0846
	0.0843	0.0838	0.0833	0.0847	0.0856
	0.0887	0.0887	0.0887	0.0887	0.0887
	0.0885	0.0885	0.0885	0.0885	0.0885

Table 71. Continued

TW5	-0.0244	-0.0243	-0.0243	-0.0269	-0.0244
	-0.0260	-0.0262	-0.0261	-0.0287	-0.0260
	-0.0264	-0.0267	-0.0264	-0.0287	-0.0264
	-0.0313	-0.0315	-0.0314	-0.0337	-0.0312
	-0.0360	-0.0361	-0.0360	-0.0360	-0.0367
	-0.0368	-0.0368	-0.0368	-0.0368	-0.0368

Table 72. Return / risk profile obtained via [8 4] topology.

Time Window	LCVAR	Avg. Ret.	VAR	Avg. Shortfall
TW1	2	0.0622	12.6885	1.0448
	6	0.2778	40.4867	2.2653
	10	0.3002	67.7283	3.0969
	14	0.5159	94.8941	3.6513
	24	0.7117	115.5638	4.0502
	32	0.7178	115.6189	4.0498
TW2	2	-0.0827	11.4843	1.0842
	6	-0.2827	35.3282	2.3480
	10	-0.3627	56.3841	3.0907
	14	-0.3202	71.5978	3.4921
	24	-0.2140	85.0053	3.7612
	32	-0.2119	85.0900	3.7646
TW3	2	0.1627	5.6760	0.6611
	6	0.2647	28.8820	1.8517
	10	0.0229	58.5970	2.9683
	14	0.0867	80.2977	3.4133
	24	0.5365	108.1078	3.8456
	32	0.6548	110.6663	3.8574
TW4	2	0.1324	4.5743	0.6826
	6	0.4124	29.1108	1.7545
	10	0.6746	50.8027	2.3699
	14	0.7378	68.4311	2.7804
	24	1.0411	87.8960	3.1161
	32	1.1078	90.5975	3.1582
TW5	2	0.1166	4.1877	0.6142
	6	-0.0226	33.0614	2.0712
	10	-0.1954	64.5648	3.0231
	14	-0.3330	88.7699	3.6299
	24	-0.3934	119.9826	4.2907
	32	-0.3571	122.5404	4.3136

Table 73. Return / risk profile obtained via [80 40] topology.

Time Window	LCVAR	Avg. Ret.	VAR	Avg. Shortfall
TW1	2	0.1105	1.8090	0.4830
	6	0.2482	16.5443	1.5040
	10	0.3818	45.9829	2.5234
	14	0.5181	79.5616	3.3387
	24	0.7073	115.4955	4.0620
	32	0.7103	115.5502	4.0620
TW2	2	0.0292	1.1899	0.4204
	6	-0.0138	10.8876	1.3199
	10	-0.0616	30.4059	2.2254
	14	-0.0878	56.8003	3.0499
	24	-0.1904	85.2931	3.7480
	32	-0.1904	85.2931	3.7480
TW3	2	0.1235	1.1468	0.3638
	6	0.2720	10.3361	1.1394
	10	0.4201	28.7510	1.9170
	14	0.5686	56.3445	2.6930
	24	0.7014	111.2665	3.9025
	32	0.6935	112.4671	3.9299
TW4	2	0.1423	1.3880	0.3966
	6	0.3322	12.5643	1.2331
	10	0.5224	34.9377	2.0695
	14	0.7129	68.5809	2.9071
	24	0.9720	108.5650	3.7082
	32	0.9720	108.5650	3.7082
TW5	2	0.0219	1.3197	0.4117
	6	-0.0447	12.2569	1.3070
	10	-0.1078	34.4539	2.2060
	14	-0.1719	67.6833	3.1021
	24	-0.3571	122.5404	4.3136
	32	-0.3571	122.5404	4.3136

Table 74. Sharpe Ratios obtained via different tree topologies across five time windows

Time Window	[8 4]	[80 40]
TW1	0.0034	0.0450
	0.0358	0.0487
	0.0304	0.0489
	0.0478	0.0525
	0.0616	0.0612
	0.0621	0.0614
TW2	-0.0392	-0.0191
	-0.0560	-0.0193
	-0.0550	-0.0202
	-0.0438	-0.0183
	-0.0286	-0.0260
	-0.0284	-0.0260

Table 74. Continued

TW3	0.0473	0.0687
	0.0400	0.0691
	-0.0035	0.0690
	0.0041	0.0691
	0.0468	0.0618
	0.0575	0.0607
TW4	0.0385	0.0783
	0.0672	0.0796
	0.0876	0.0799
	0.0831	0.0801
	0.1057	0.0885
	0.1111	0.0885
TW5	0.0325	-0.0245
	-0.0126	-0.0270
	-0.0305	-0.0269
	-0.0407	-0.0270
	-0.0405	-0.0368
	-0.0368	-0.0368

Table 75. Return / risk profile obtained via MV strategy.

Time Window	Risk Level	Avg. Ret.	VAR	Avg. Shortfall
TW1	1	0.2214	7.6805	1.0250
	2	0.3887	30.1053	2.0557
	3	0.5518	66.3611	3.0682
	4	0.7103	115.5500	4.0620
TW2	1	-0.0102	5.2896	0.9163
	2	-0.0746	21.2199	1.8624
	3	-0.1430	47.8686	2.8125
	4	-0.1904	85.2929	3.7480
TW3	1	0.2115	6.8091	0.9448
	2	0.3694	27.4870	1.9258
	3	0.5236	62.4213	2.9208
	4	0.6935	112.4670	3.9299
TW4	1	0.2815	6.7207	0.9025
	2	0.5121	26.9739	1.8317
	3	0.7418	60.8826	2.7671
	4	0.9719	108.5644	3.7082
TW5	1	-0.0437	7.5391	1.0420
	2	-0.1430	30.3220	2.1216
	3	-0.2476	68.5832	3.2124
	4	-0.3571	122.5402	4.3137

Table 76. Return / risk profile obtained via B&H strategy.

Time Window	Cash %	Avg. Ret.	VAR	Avg. Shortfall
TW1	70%	0.1742	5.4963	0.8383
	35%	0.3497	29.2205	1.9734
	10%	0.5333	68.4204	3.0760
	5%	0.5848	81.5631	3.3742
	0%	0.6918	110.3983	3.9644
TW2	70%	0.0090	4.3101	0.8260
	35%	-0.0518	22.5144	1.9218
	10%	-0.1197	51.5996	2.9216
	5%	-0.1395	61.1610	3.1840
	0%	-0.1826	81.6862	3.6710
TW3	70%	0.1485	5.3860	0.8708
	35%	0.3026	28.6917	2.0289
	10%	0.4872	67.1073	3.0819
	5%	0.5430	79.9022	3.3516
	0%	0.6698	107.6026	3.8518
TW4	70%	0.2269	5.3803	0.7936
	35%	0.4748	28.3769	1.8595
	10%	0.7301	65.5748	2.8602
	5%	0.8012	77.8263	3.1238
	0%	0.9470	104.0132	3.6250
TW5	70%	-0.0268	5.8504	0.9141
	35%	-0.1354	31.2188	2.1451
	10%	-0.2485	73.1152	3.3091
	5%	-0.2802	87.0709	3.6196
	0%	-0.3458	117.2519	4.2163

Table 77. Return / risk profile obtained via VAR(2).

Time Window	LCVAR	Avg. Ret.	VAR	Avg. Shortfall
TW1	2	0.0271	2.6586	0.5899
	6	0.0167	21.9033	1.7440
	10	-0.1833	45.1787	2.6272
	14	-0.4270	60.3672	3.1542
	24	-0.6072	76.1459	3.6025
TW2	2	-0.0925	2.1300	0.5956
	6	-0.3131	17.3614	1.7426
	10	-0.4682	36.0433	2.5951
	14	-0.5983	50.0837	3.0532
	24	-0.5424	66.2408	3.4360
TW3	2	0.0083	3.5545	0.5826
	6	-0.3858	23.4916	1.9343
	10	-0.5735	43.8569	2.7363
	14	-0.8023	55.1573	3.1425
	24	-0.8807	62.0938	3.3723

Table 77. Continued

TW4	2	0.0282	2.8273	0.5872
	6	0.0553	22.6906	1.6763
	10	0.3582	42.3328	2.1885
	14	0.4605	59.7857	2.6125
	24	0.4448	83.5100	3.1902
TW5	2	0.0063	2.4316	0.5288
	6	-0.0984	21.2043	1.6018
	10	-0.1505	41.6755	2.2895
	14	-0.1885	54.7621	2.6711
	24	-0.2599	71.3851	3.1205

APPENDIX D

Table 78. Performance measures obtained considering *M1* (*TSD*=200)

Time Window	LCVAR	Avg. Return	VAR	Avg. Shortfall
TW1	2	0.0784	0.4209	0.2254
	6	0.1253	3.5285	0.6976
	10	0.1881	9.7430	1.1698
	14	0.2759	18.1540	1.5914
	24	0.4004	31.0326	2.0869
	32	0.4004	31.0326	2.0869
TW2	2	0.0356	0.3304	0.2113
	6	0.0046	2.8723	0.6681
	10	-0.0233	7.2847	1.0803
	14	-0.0200	12.9650	1.4321
	24	-0.0503	21.7582	1.8805
	32	-0.0597	21.9052	1.8891
TW3	2	0.0730	0.5379	0.2383
	6	0.1167	2.9843	0.6171
	10	0.1718	8.1685	1.0311
	14	0.2209	16.0083	1.4524
	24	0.3206	29.2627	1.9942
	32	0.3206	29.2627	1.9942
TW4	2	0.1138	0.3223	0.1724
	6	0.2159	2.6428	0.5446
	10	0.3286	7.3467	0.9208
	14	0.4409	14.4107	1.2975
	24	0.5301	27.6459	1.8473
	32	0.5301	27.6459	1.8473
TW5	2	0.0477	0.4718	0.2168
	6	-0.0025	3.3997	0.6751
	10	-0.0384	9.4996	1.1453
	14	-0.0791	18.6688	1.6176
	24	-0.1097	32.6215	2.1648
	32	-0.1107	32.6186	2.1657

Table 79. Performance measures obtained considering *M1* (*TSD*=400)

Time Window	LCVAR	Avg. Return	Variance	Avg. Shortfall
TW1	2	0.0812	0.4401	0.2305
	6	0.1500	3.9999	0.7391
	10	0.1929	10.0510	1.1869
	14	0.2702	18.6367	1.6186
	24	0.4044	31.8827	2.1190
	32	0.4044	31.8827	2.1190
TW2	2	0.0498	0.7582	0.3259
	6	0.0475	6.9199	1.0312
	10	0.0404	16.4095	1.6085
	14	-0.0083	20.5371	1.8262
	24	-0.0503	21.7581	1.8805
	32	-0.0597	21.9052	1.8891

Table 79. Continued

Time Window	LCVAR	Avg. Return	Variance	Avg. Shortfall
TW3	2	0.0761	0.6183	0.2530
	6	0.1279	5.5571	0.8036
	10	0.1664	8.8935	1.0700
	14	0.2129	17.4287	1.5074
	24	0.3020	31.0996	2.0540
	32	0.3020	31.0996	2.0540
TW4	2	0.1143	0.3250	0.1729
	6	0.2467	2.9544	0.5609
	10	0.3307	7.3942	0.9230
	14	0.4417	14.5261	1.3026
	24	0.5338	27.8353	1.8523
	32	0.5338	27.8353	1.8523
TW5	2	0.0483	0.4799	0.2180
	6	0.0421	4.5017	0.7147
	10	-0.0353	9.6160	1.1495
	14	-0.0745	18.8752	1.6227
	24	-0.1035	33.0105	2.1725
	32	-0.1045	33.0074	2.1734

Table 80. Performance measures obtained considering *M1* (*TSD*=600)

Time Window	LCVAR	Avg. Return	Variance	Avg. Shortfall
TW1	2	0.1173	1.4097	0.4015
	6	0.2631	12.7272	1.2524
	10	0.3706	30.3168	1.9387
	14	0.3995	36.8739	2.2331
	24	0.4251	45.9898	2.5499
	32	0.4251	45.9898	2.5499
TW2	2	0.0287	0.6858	0.3205
	6	-0.0122	6.3183	1.0191
	10	-0.0342	17.5743	1.7178
	14	-0.0209	24.3037	2.0128
	24	-0.0685	27.1527	2.1187
	32	-0.0833	27.4808	2.1316
TW3	2	0.1350	1.6760	0.4327
	6	0.3068	15.2191	1.3539
	10	0.3043	34.3943	2.0684
	14	0.2264	41.5346	2.3411
	24	0.1921	49.6274	2.5012
	32	0.1921	49.6274	2.5012
TW4	2	0.1614	0.8453	0.2775
	6	0.3894	7.6432	0.8737
	10	0.4808	19.0071	1.4151
	14	0.5907	28.2390	1.7499
	24	0.7403	49.2107	2.3531
	32	0.7402	49.2115	2.3532

Table 80. Continued

Time Window	LCVAR	Avg. Return	Variance	Avg. Shortfall
TW5	2	0.0520	0.5760	0.2374
	6	0.0532	5.4074	0.7741
	10	0.0676	14.5841	1.2884
	14	-0.0038	22.2744	1.7040
	24	-0.0100	39.6522	2.2984
	32	-0.0111	39.6482	2.2994

Table 81. Performance measures obtained considering *M1* (*TSD*=800)

Time Window	LCVAR	Avg. Return	Variance	Avg. Shortfall
TW1	2	0.1136	2.0094	0.5095
	6	0.2569	18.0445	1.5707
	10	0.4270	48.6361	2.5797
	14	0.3994	58.4036	2.8033
	24	0.4524	57.9996	2.8126
	32	0.4524	57.9996	2.8126
TW2	2	0.0393	1.4238	0.4602
	6	0.0176	13.0547	1.4406
	10	0.0135	35.9043	2.3955
	14	-0.0003	58.9748	3.0742
	24	-0.0059	37.2056	2.4348
	32	-0.0284	37.8817	2.4543
TW3	2	0.1397	1.7686	0.4534
	6	0.3202	15.9859	1.4133
	10	0.5007	44.4120	2.3723
	14	0.3162	54.7528	2.6766
	24	0.1924	49.6966	2.5026
	32	0.1924	49.6966	2.5026
TW4	2	0.1550	1.3786	0.3925
	6	0.3708	12.5248	1.2234
	10	0.5888	34.8422	2.0530
	14	0.6300	47.9324	2.3077
	24	0.7191	58.5327	2.5870
	32	0.7191	58.5327	2.5870
TW5	2	0.0332	1.4682	0.4309
	6	-0.0085	13.7540	1.3686
	10	-0.0492	38.3380	2.3017
	14	0.1543	54.9304	2.6351
	24	0.2303	56.8432	2.6209
	32	0.2258	56.9043	2.6253

Table 82. Performance measures obtained considering *M2* (*TSD*=200)

Time Window	LCVAR	Avg. Return	VAR	Avg. Shortfall
TW1	2	0.0857	0.5698	0.2606
	6	0.1247	3.7737	0.7201
	10	0.1887	10.3707	1.2045
	14	0.2742	19.2039	1.6353
	24	0.3759	32.1671	2.1339
	32	0.3759	32.1671	2.1339
TW2	2	0.0434	0.4189	0.2373
	6	0.0146	3.2611	0.7116
	10	-0.0058	8.0118	1.1233
	14	-0.0041	13.8319	1.4715
	24	-0.0467	21.2547	1.8520
	32	-0.0555	21.3860	1.8601
TW3	2	0.0890	0.5124	0.2377
	6	0.1333	3.0505	0.6228
	10	0.1999	8.3290	1.0388
	14	0.2597	16.3322	1.4639
	24	0.3711	30.5199	2.0313
	32	0.3711	30.5199	2.0313
TW4	2	0.1344	0.4490	0.1998
	6	0.2325	2.9520	0.5731
	10	0.3565	8.2052	0.9681
	14	0.4800	16.0945	1.3638
	24	0.5849	30.9042	1.9431
	32	0.5849	30.9042	1.9431
TW5	2	0.0213	0.7259	0.2822
	6	-0.0130	3.8219	0.7279
	10	-0.0566	10.6913	1.2345
	14	-0.1043	20.9965	1.7421
	24	-0.1482	37.0765	2.3446
	32	-0.1493	37.0727	2.3456

Table 83. Performance measures obtained considering *M2* (*TSD*=400)

Time Window	LCVAR	Avg. Return	VAR	Avg. Shortfall
TW1	32	0.0893	0.6573	0.2782
	36	0.1710	5.7535	0.8704
	40	0.1993	11.7460	1.2756
	44	0.2875	21.4286	1.7216
	54	0.3689	34.8876	2.2284
	62	0.3689	34.8876	2.2284
TW2	32	0.0574	0.7850	0.3256
	36	0.0705	7.1688	1.0305
	40	0.0473	16.2639	1.5940
	44	-0.0060	20.2240	1.8068
	54	-0.0467	21.2547	1.8520
	62	-0.0555	21.3860	1.8601

Table 83. Continued

Time Window	LCVAR	Avg. Return	VAR	Avg. Shortfall
TW3	32	0.0922	0.6076	0.2577
	36	0.1771	5.4882	0.8201
	40	0.2004	9.7889	1.1179
	44	0.2604	19.1912	1.5747
	54	0.3660	34.8580	2.1659
	62	0.3660	34.8580	2.1659
TW4	32	0.1368	0.4694	0.2040
	36	0.3140	4.2773	0.6558
	40	0.3675	8.6083	0.9881
	44	0.4954	16.8846	1.3916
	54	0.6071	32.4263	1.9840
	62	0.6071	32.4263	1.9840
TW5	32	0.0212	0.7407	0.2853
	36	-0.0424	6.9671	0.9224
	40	-0.0574	11.0959	1.2549
	44	-0.1036	21.6360	1.7655
	54	-0.1482	38.2652	2.3785
	62	-0.1493	38.2611	2.3795

Table 84. Performance measures obtained considering *M2* (*TSD*=600)

Time Window	LCVAR	Avg. Return	VAR	Avg. Shortfall
TW1	32	0.1104	1.5795	0.4403
	36	0.2403	14.1533	1.3642
	40	0.3210	32.4244	2.0722
	44	0.3255	47.2241	2.5491
	54	0.3259	59.7589	2.9175
	62	0.3259	59.7589	2.9175
TW2	32	0.0548	0.8651	0.3441
	36	0.0624	7.9124	1.0886
	40	0.0722	21.2670	1.8021
	44	0.0484	29.4831	2.1696
	54	-0.0004	34.6675	2.3435
	62	-0.0163	35.0777	2.3574
TW3	32	0.1385	1.6674	0.4334
	36	0.3159	15.1278	1.3564
	40	0.4743	40.7923	2.2401
	44	0.3797	51.8509	2.6087
	54	0.4229	66.9924	2.9404
	62	0.4229	66.9924	2.9404
TW4	32	0.1690	1.3016	0.3675
	36	0.4142	11.7606	1.1438
	40	0.6454	31.9485	1.8911
	44	0.7826	46.3486	2.2476
	54	0.9811	71.0139	2.8197
	62	0.9783	71.0392	2.8224

Table 84. Continued

Time Window	LCVAR	Avg. Return	VAR	Avg. Shortfall
TW5	32	0.0323	1.3502	0.3975
	36	-0.0158	13.6846	1.3093
	40	-0.0412	34.4640	2.1052
	44	-0.0453	41.1712	2.3697
	54	-0.1018	65.5114	3.0374
	62	-0.1036	65.5006	3.0391

Table 85. Performance measures obtained considering *M2* (*TSD*=800)

Time Window	LCVAR	Avg. Return	VAR	Avg. Shortfall
TW1	32	0.1137	2.0096	0.5095
	36	0.2571	18.0460	1.5707
	40	0.4273	48.6400	2.5797
	44	0.4365	68.2855	3.0485
	54	0.3147	68.0954	3.1035
	62	0.3147	68.0954	3.1035
TW2	32	0.0392	1.4242	0.4602
	36	0.0175	13.0579	1.4408
	40	0.0132	35.9137	2.3959
	44	-0.0010	58.9862	3.0749
	54	0.0363	48.8745	2.7628
	62	0.0182	49.3963	2.7782
TW3	32	0.1397	1.7686	0.4534
	36	0.3202	15.9859	1.4133
	40	0.5007	44.4120	2.3723
	44	0.5977	68.4152	2.9676
	54	0.4392	68.8318	2.9788
	62	0.4392	68.8318	2.9788
TW4	32	0.1569	1.4540	0.4035
	36	0.3771	13.1913	1.2551
	40	0.5984	36.6821	2.1058
	44	0.8641	66.0601	2.7528
	54	0.9712	73.9535	2.8919
	62	0.9686	73.9770	2.8944
TW5	32	0.0332	1.4682	0.4309
	36	-0.0085	13.7540	1.3686
	40	-0.0492	38.3381	2.3017
	44	0.0117	68.3824	3.0294
	54	-0.0684	75.1405	3.2429
	62	-0.0708	75.1322	3.2453

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