

3D Scanning & Motion Capture

Exercise - 3

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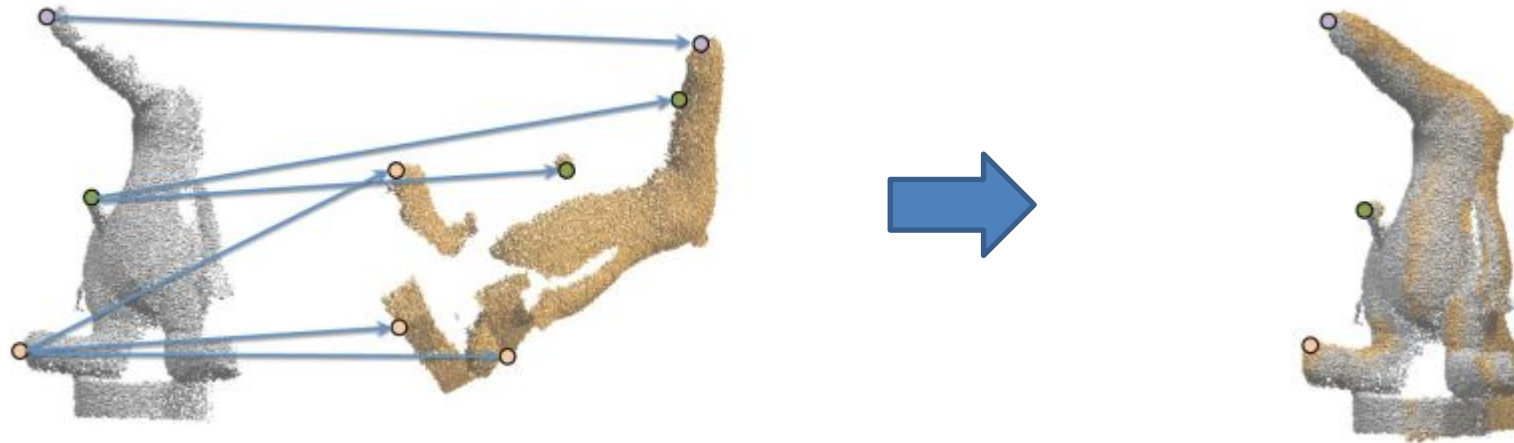


Exercises – Overview

1. Exercise ☐ Camera Intrinsics, Back-projection, Meshes
2. Exercise ☐ Surface Representations
3. **Exercise ☐ Coarse Alignment (Procrustes)**
4. Exercise ☐ Optimization
5. Exercise ☐ Object Alignment, ICP

Procrustes

- Problem: Align two objects using known correspondences
 - scaling, translation, rotation

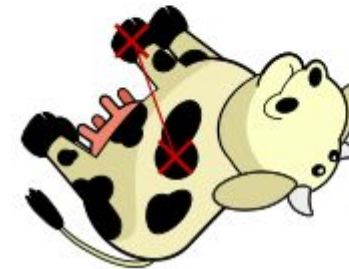


Procrustes

- Problem: Align two objects using known correspondences

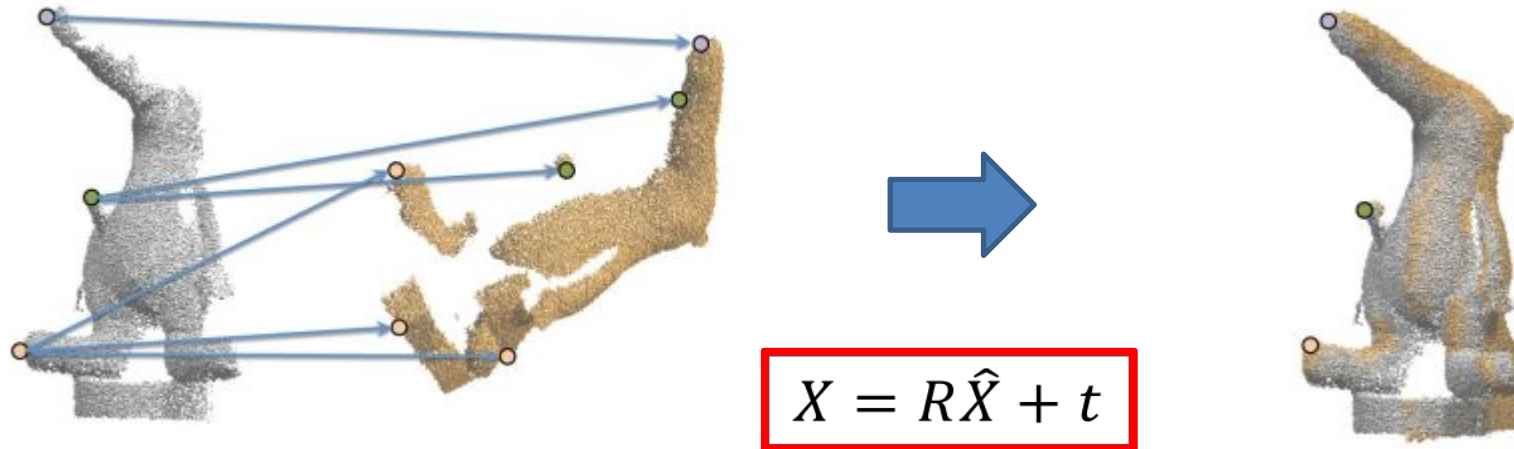
□ **scaling**, translation, rotation

- Compute center of gravity of both objects
- Scale one object to match the avg. distance from all vertices to the center of gravity



Procrustes

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 - scaling, translation, rotation



Procrustes

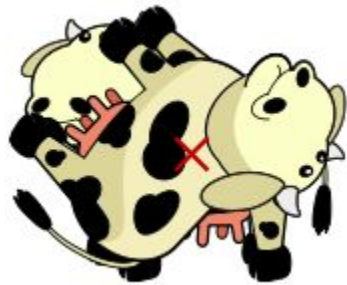
- Problem: Align two objects using known correspondences
 - scaling, **translation**, rotation
 - The translation vector will depend on the rotation, so we will analyse it later

Procrustes

- Problem: Align two objects using known correspondences

→ scaling, translation, **rotation**

- Assume objects that are zero-centered
 - Target object: $\{x_0, \dots, x_{n-1}\}$
 - Moving object: $\{\hat{x}_0, \dots, \hat{x}_{n-1}\}$



$$\sum_i \|x_i - R \cdot \hat{x}_i\|_2^2 \rightarrow \min$$

$$\|X - \hat{X}R^T\|_F^2 \rightarrow \min$$

Procrustes

- Problem: Align two objects using known correspondences
 - scaling, translation, **rotation**

$$\|X - \hat{X}R^T\|_F^2 \rightarrow \min$$

$$\|X - \hat{X}R^T\|_F^2 = \text{trace}(X^T X - X^T \hat{X}R^T - (\hat{X}R^T)^T X + (\hat{X}R^T)^T (\hat{X}R^T)) \rightarrow \min$$

$$\text{trace}(-X^T \hat{X}R^T - (\hat{X}R^T)^T X + (\hat{X}R^T)^T (\hat{X}R^T)) \rightarrow \min$$

$$-2 \cdot \text{trace}(X^T \hat{X}R^T) \rightarrow \min$$

$$\text{trace}(X^T \hat{X}R^T) \rightarrow \max$$

$$\text{trace}(USV^T R^T) \rightarrow \max$$

$$\text{trace}(SV^T R^T U) \rightarrow \max$$

Singular values

□ positive

Product of orthogonal matrices

□ max if equal to Identity

$$\|A\|_F^2 = \text{trace}(A^T A)$$

Cyclic invariance of trace:

$$\text{trace}(ABC) = \text{trace}(CAB)$$

$$\text{SVD: } X^T \hat{X} = USV^T$$

Procrustes

- Problem: Align two objects using known correspondences
 - scaling, translation, **rotation**

$$\|X - \hat{X}R^T\|_F^2 \rightarrow \min$$

- Compute SVD of the Cross-Covariance Matrix

$$X^T \hat{X} = USV^T$$

- Compute the rotation

$$R = UV^T$$

Procrustes

- Problem: Align two objects using known correspondences

→ scaling, translation, **rotation**

- The computed rotation might be a mirroring!
- The determinant of a rotation matrix must be 1
- If $\det(UV^T) = -1$, compute the rotation as:

$$R = U \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} V^T$$

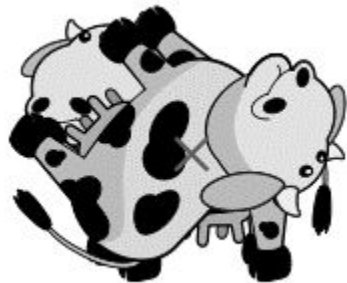
Procrustes

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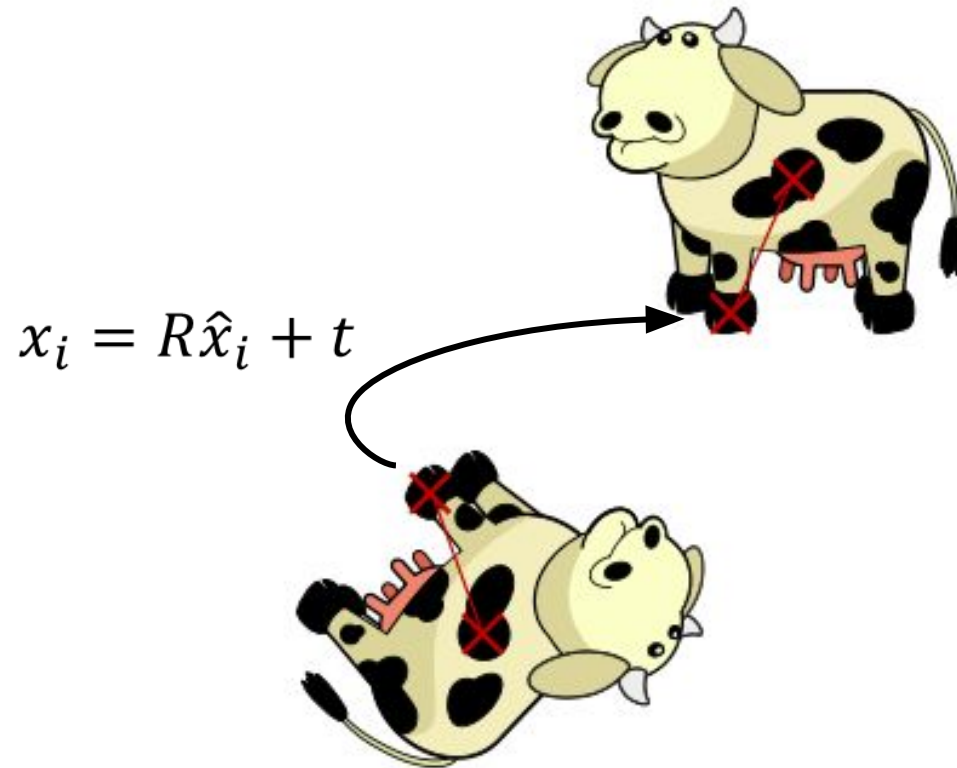


$$\sum_i \|x_i - R \cdot \hat{x}_i\|_2^2 \rightarrow \min$$

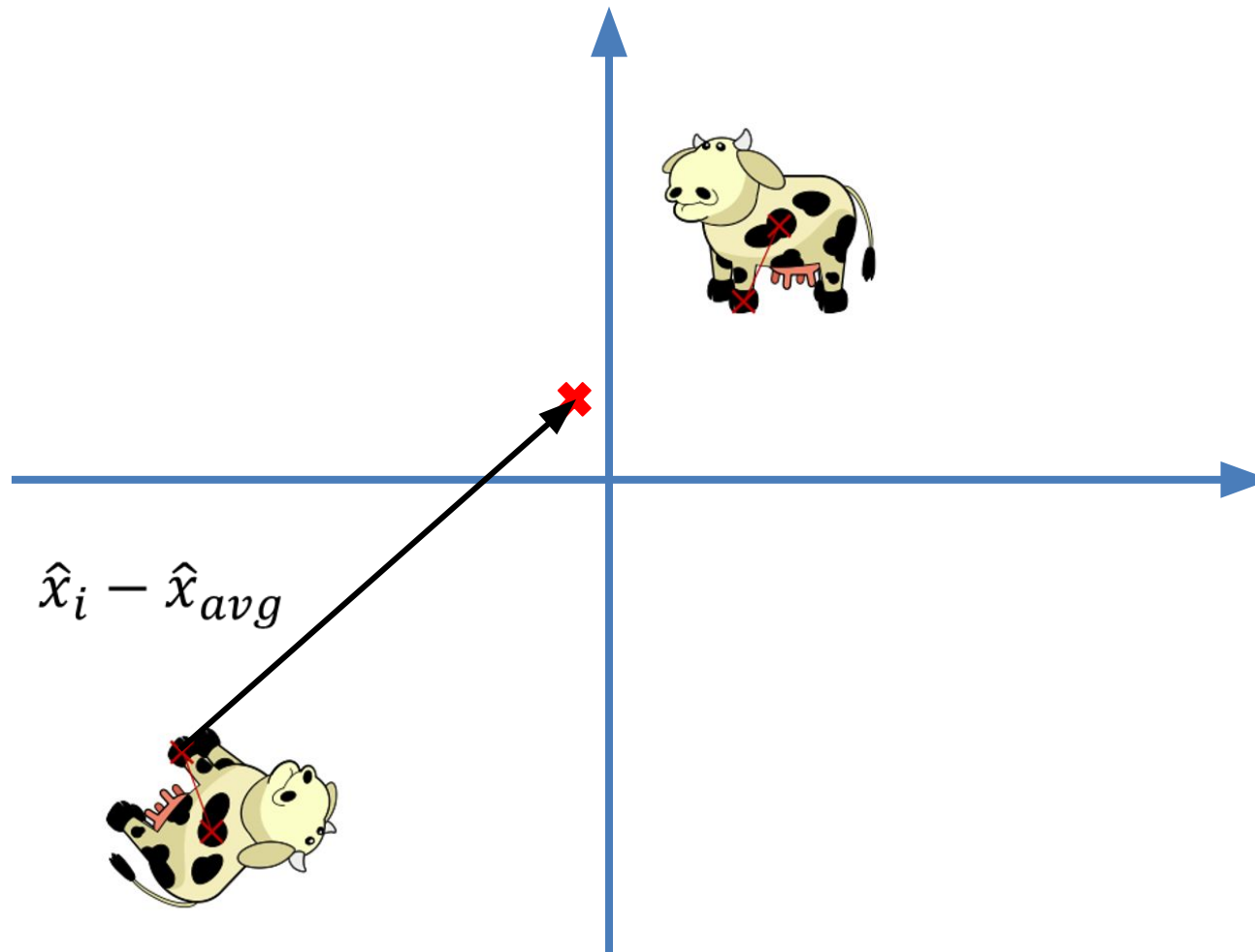
$$\|X - \hat{X}R^T\|_F^2 \rightarrow \min$$

Procrustes

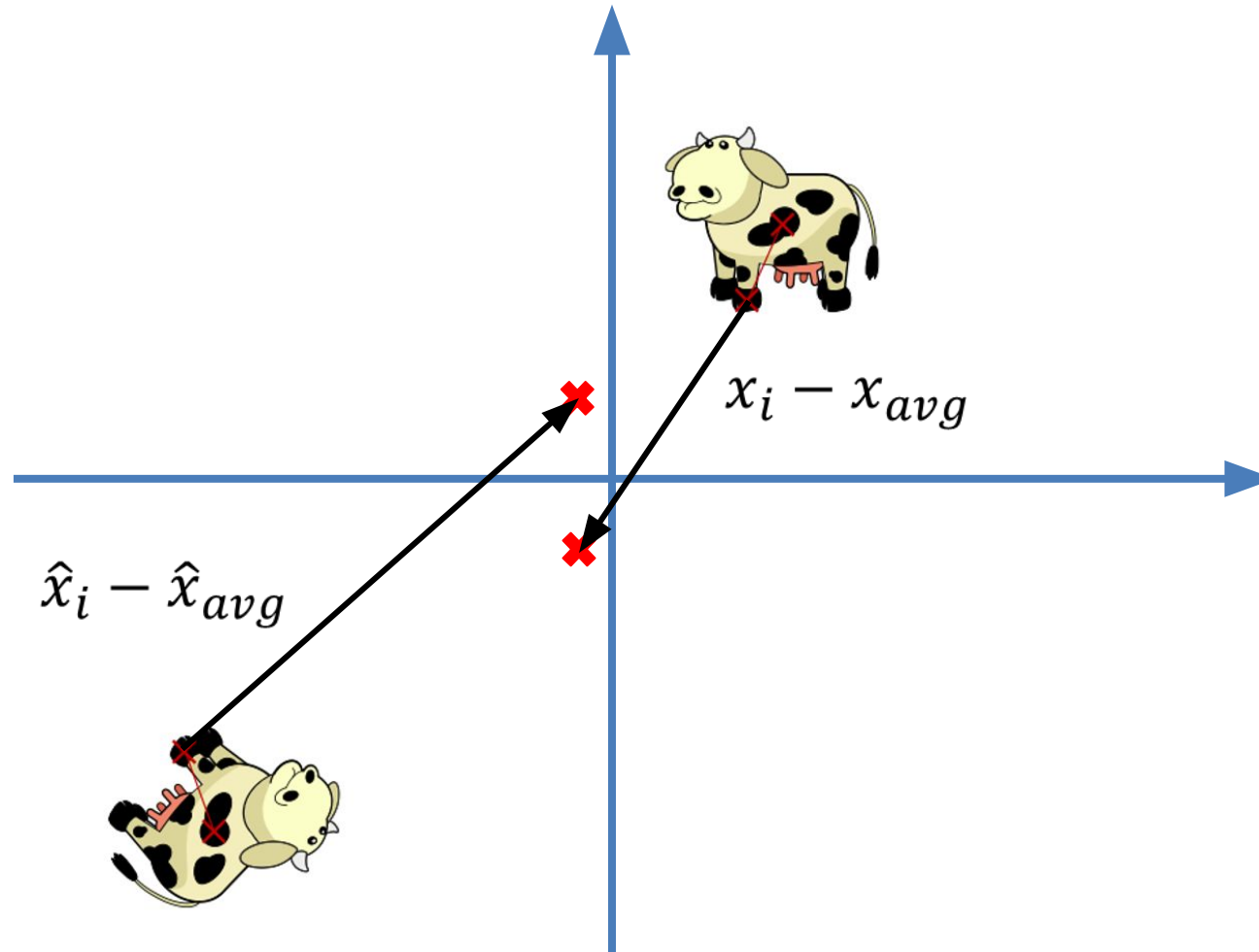
- Problem: Align two objects using known correspondences



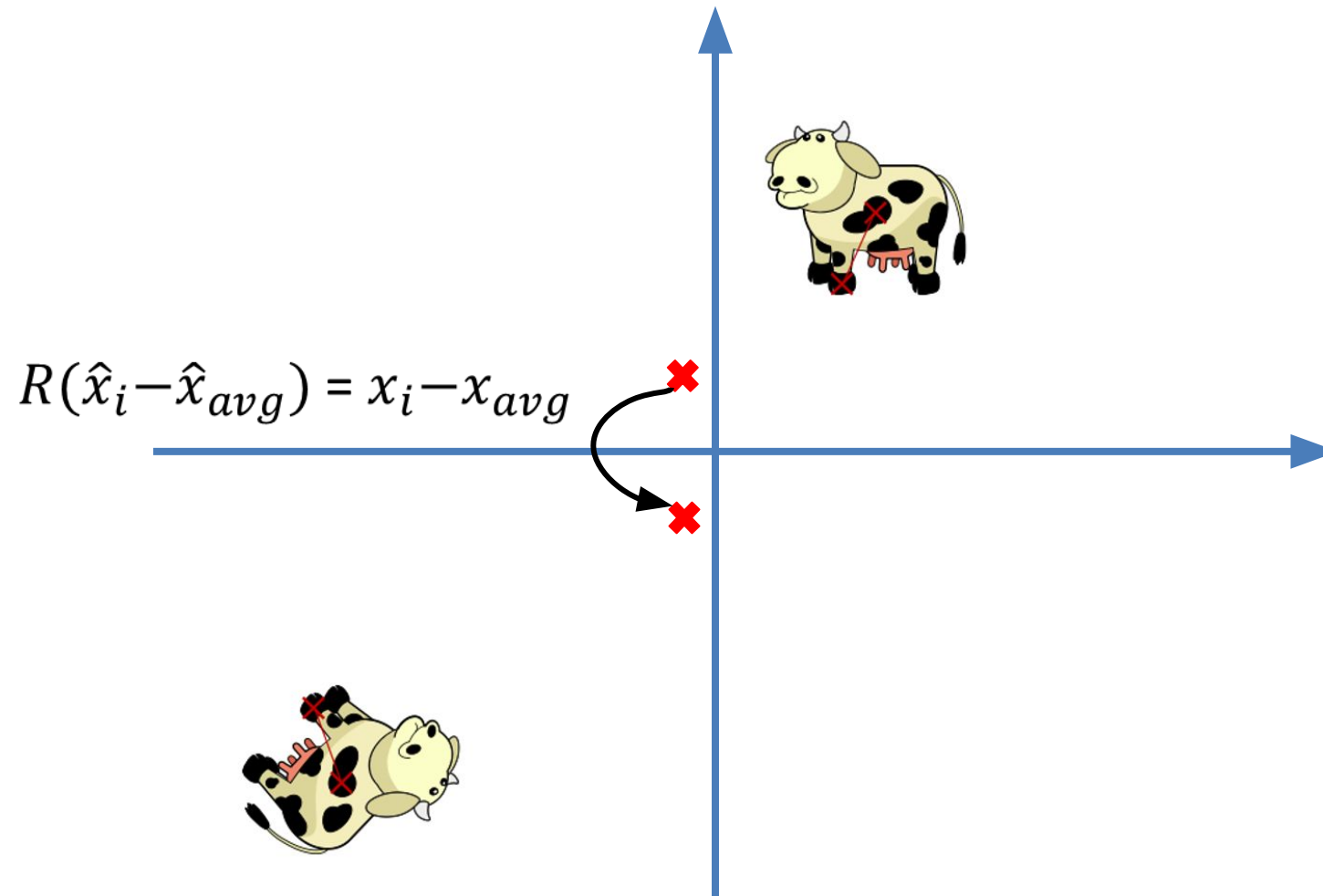
Procrustes



Procrustes



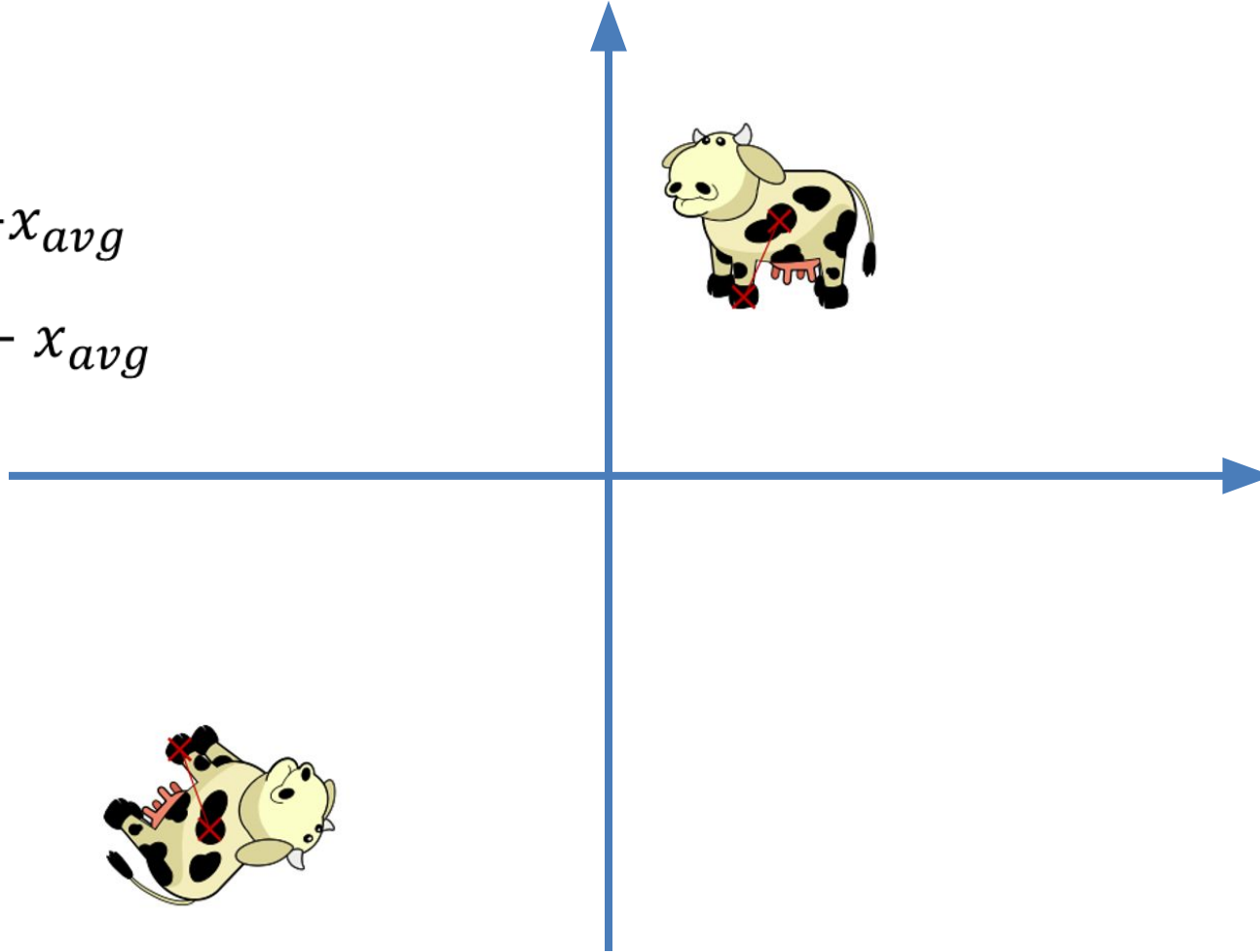
Procrustes



Procrustes

$$R(\hat{x}_i - \hat{x}_{avg}) = x_i - x_{avg}$$

$$x_i = R(\hat{x}_i - \hat{x}_{avg}) + x_{avg}$$

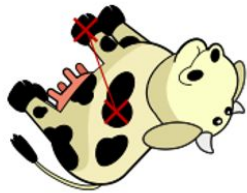


Procrustes

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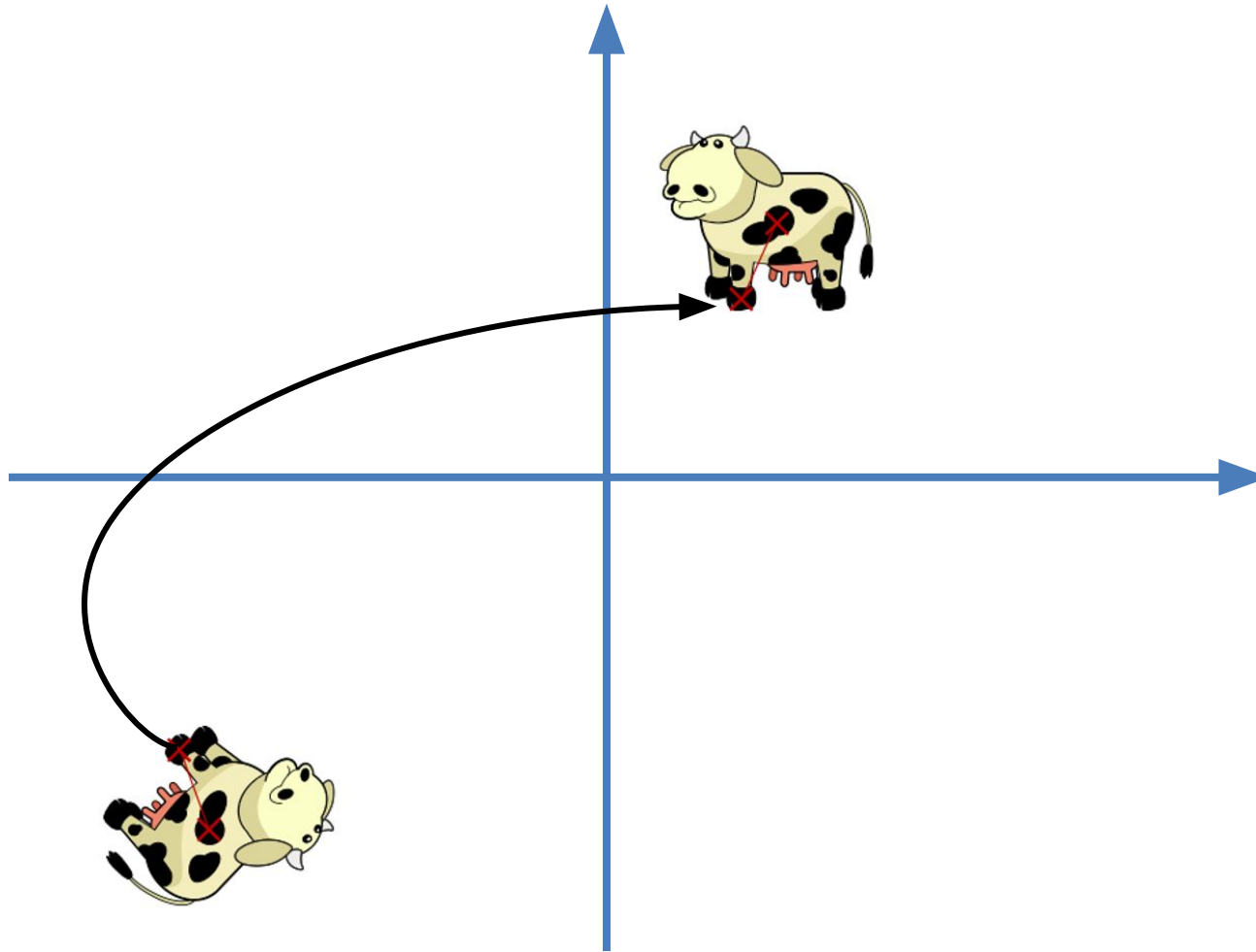


$$x_i = R\hat{x}_i - R\hat{x}_{avg} + x_{avg}$$



Procrustes

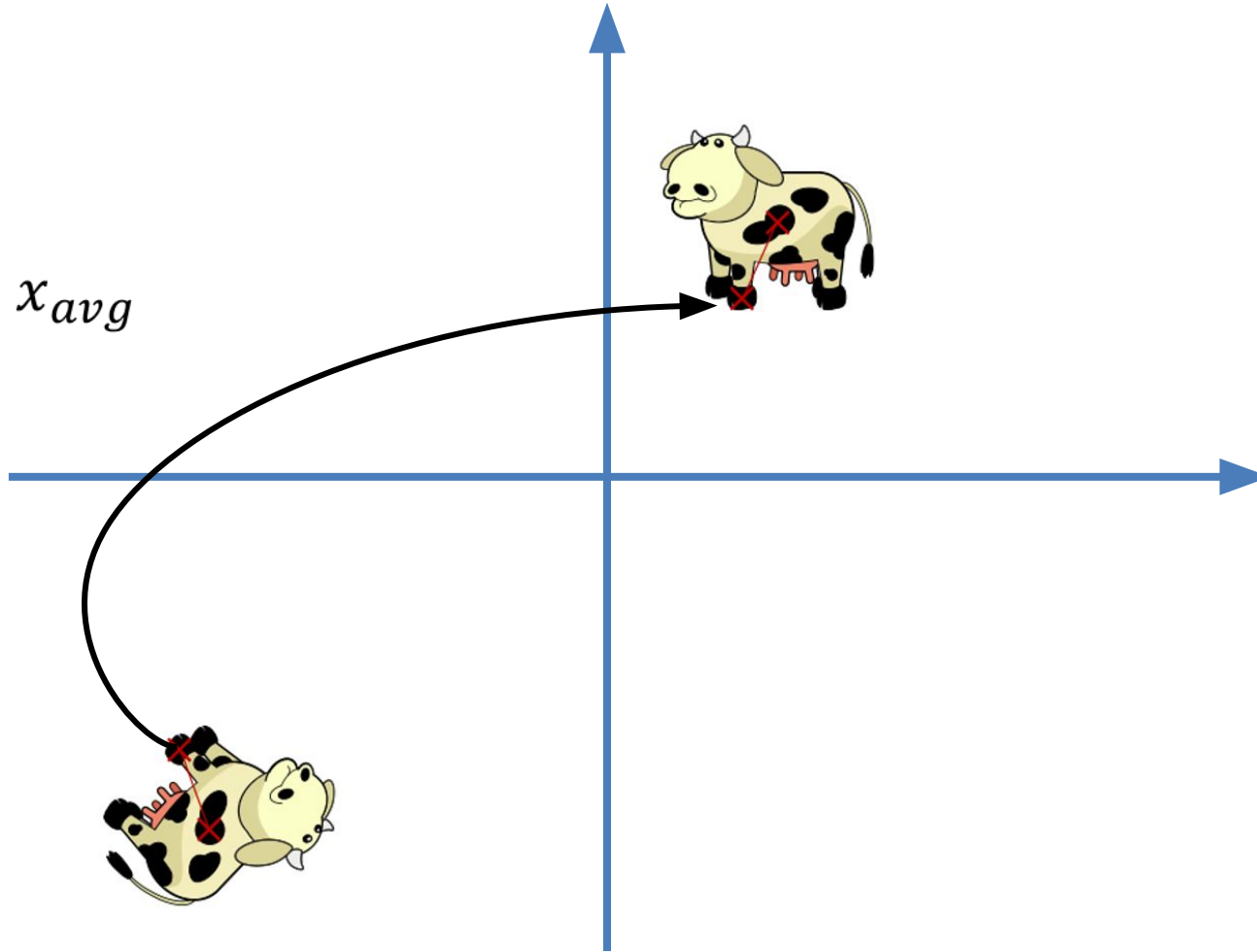
$$x_i = R\hat{x}_i + t$$



Procrustes

$$x_i = R\hat{x}_i + t$$

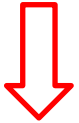
$$x_i = R\hat{x}_i - R\hat{x}_{avg} + x_{avg}$$



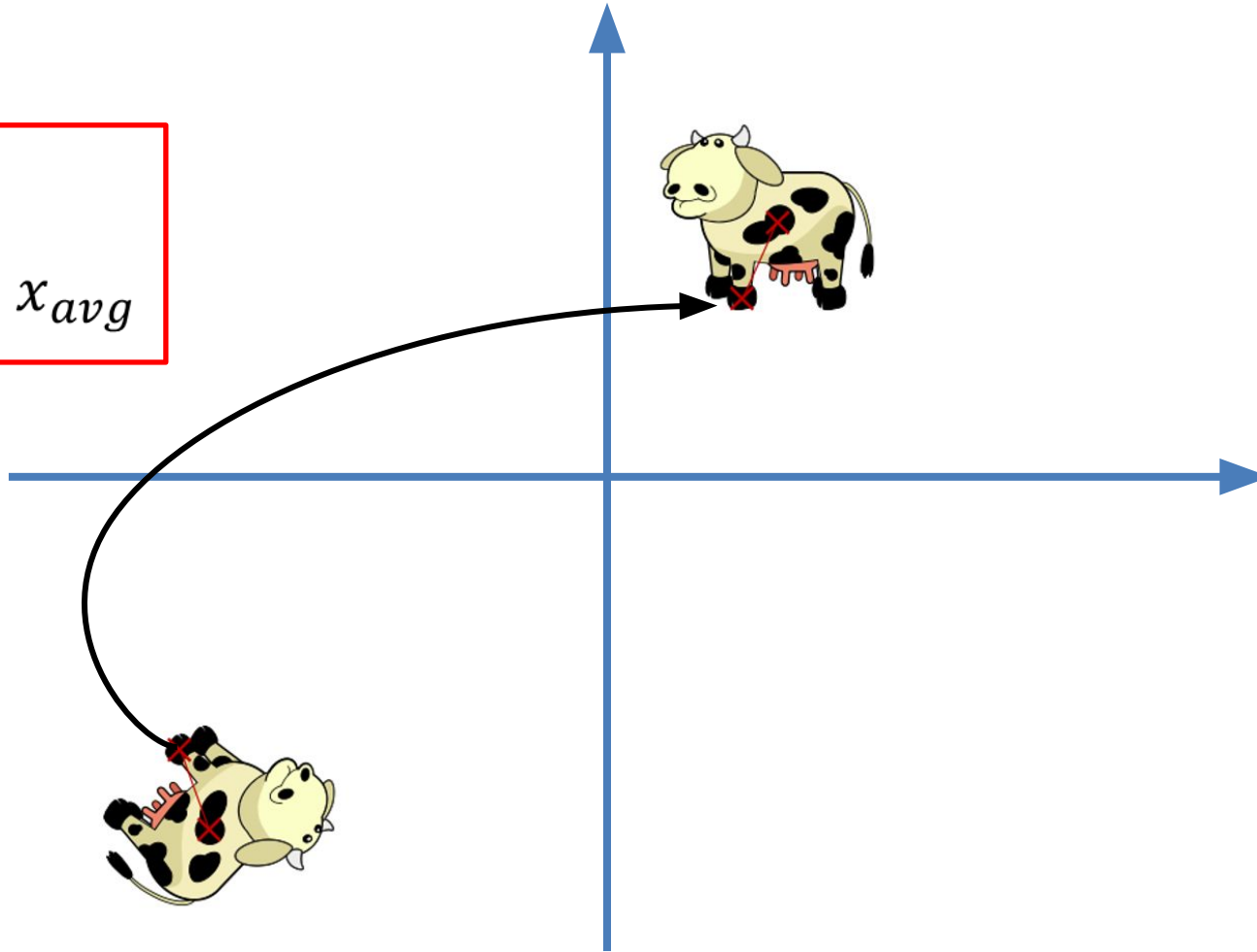
Procrustes

$$x_i = R\hat{x}_i + t$$

$$x_i = R\hat{x}_i - R\hat{x}_{avg} + x_{avg}$$

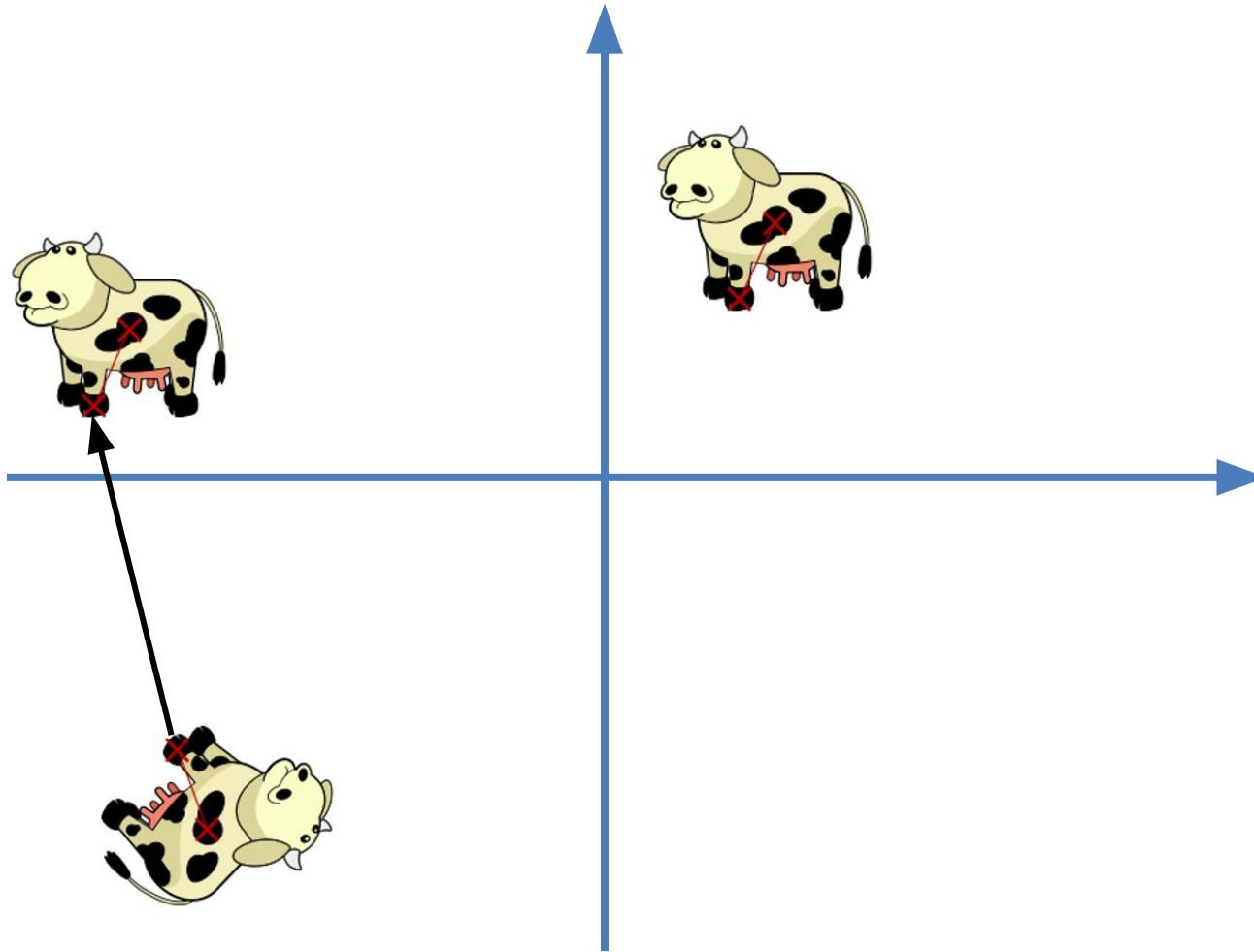


$$t = -R\hat{x}_{avg} + x_{avg}$$



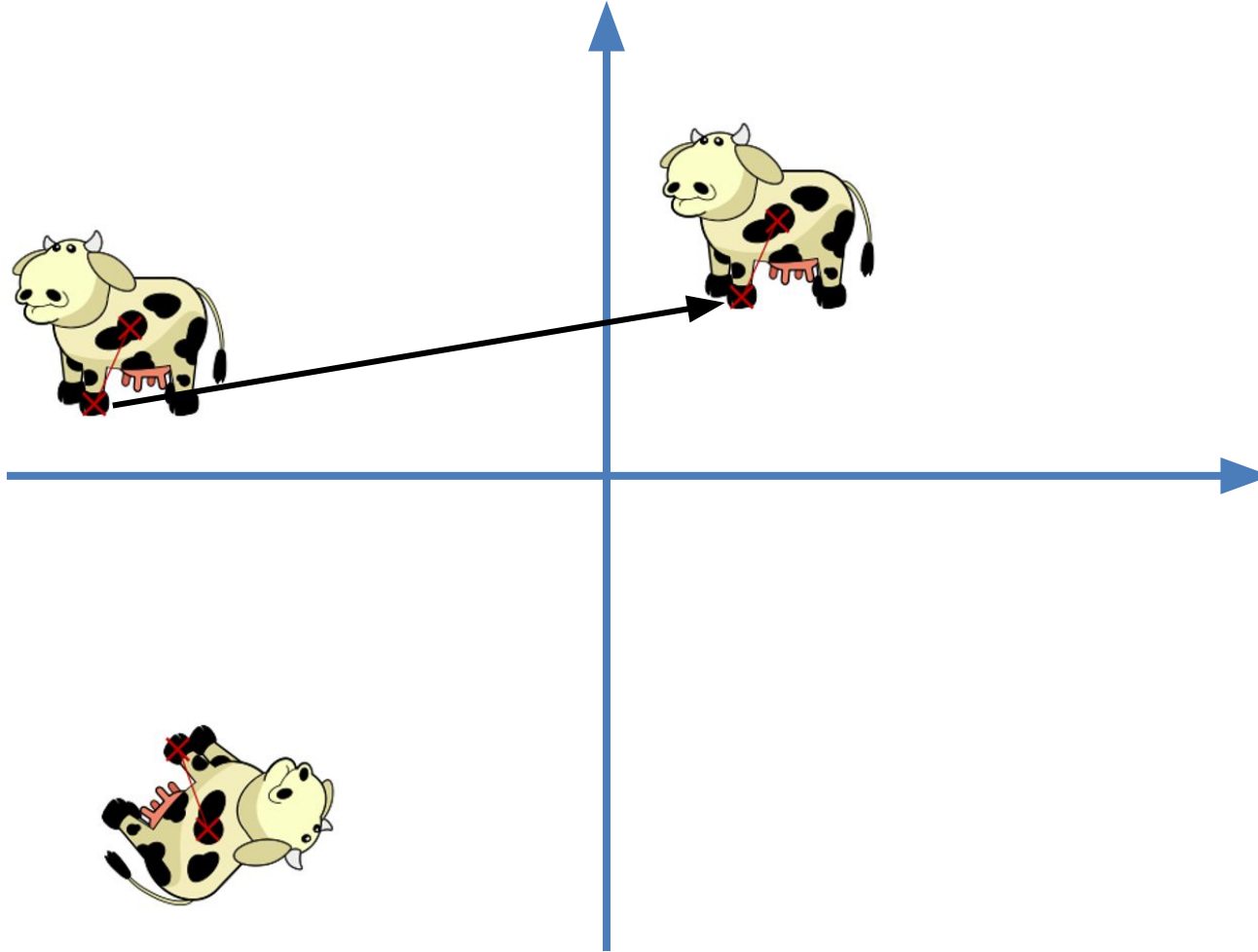
Procrustes

$$x_i = R\hat{x}_i + t$$

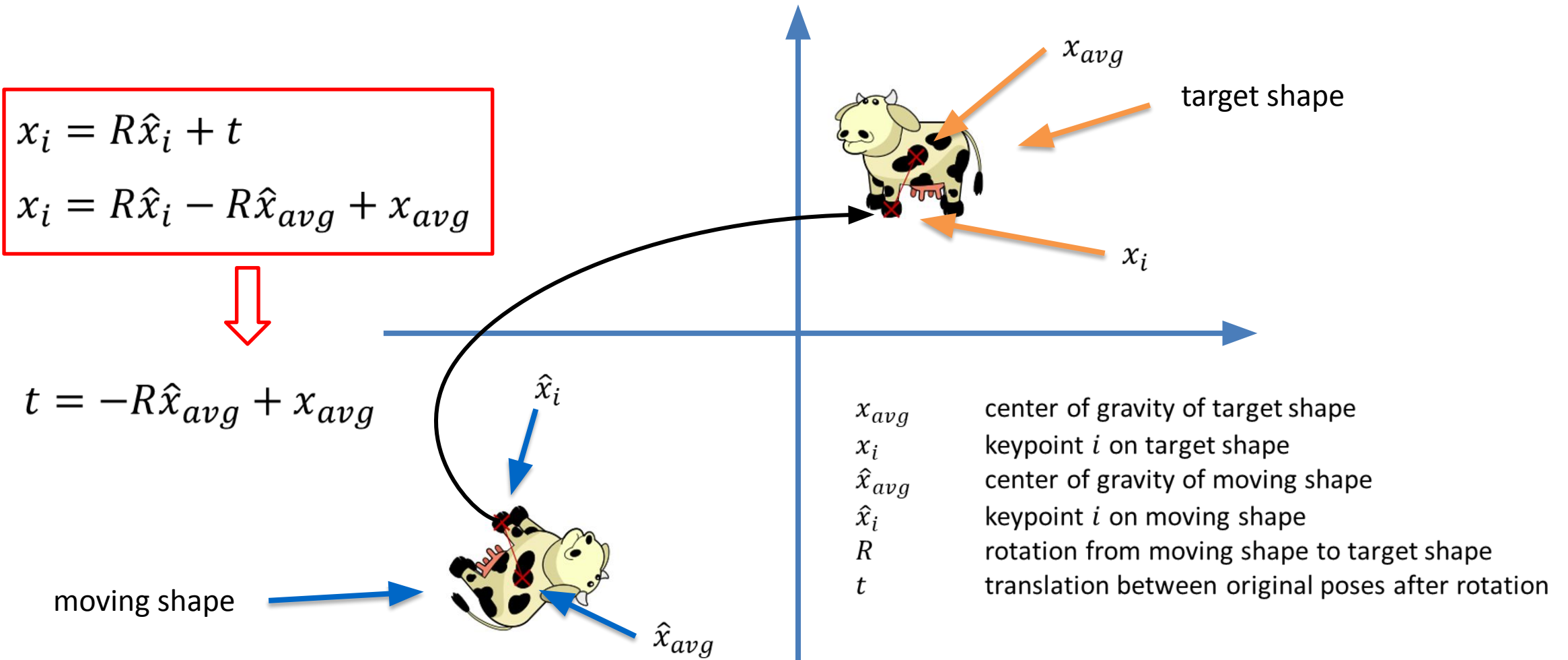


Procrustes

$$x_i = R\hat{x}_i + t$$



Procrustes



See you next time!

