Group Equivariant Convolutional Networks

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Agenda

- Introduction and Motivation
- Related Works
- Background: Groups, Group Actions, and Equivariance
- Methods
- Results
- Demonstration of the Code
- Conclusion

Introduction

- Purpose of the research/study
- Importance and implications
- Brief overview of the approach

Related Works

- Overview of existing research in the area
- Comparative analysis with previous studies

Groups

- A *group* is a set G with a binary operation $\cdot$ such that:
  1. G is closed under $\cdot$
  2. $\cdot$ is associative
  3. There exists an identity element $e \in G$ such that $e \cdot g = g \cdot e = g$ for all $g \in G$
  4. For each $g \in G$, there exists an inverse $g^{-1} \in G$ such that $g \cdot g^{-1} = g^{-1} \cdot g = e$
- In this paper, the authors focus on groups of rigid transformations of the plane (e.g. subgroups of $SE(2)$, the 2-dimensional special Euclidean group)

$p4$ and $p4m$

- $p4$ – all 2-dimensional integer translations and rotations by multiples of $\frac{\pi}{2}$
  - The underlying set can be described as a set of matrices where $r \in \{0,1,2,3\}$ and $u, v \in \mathbb{Z}$

$$g(r,u,v) = \begin{bmatrix} \cos(\frac{\pi}{2}r) & -\sin(\frac{\pi}{2}r) & u \\ \sin(\frac{\pi}{2}r) & \cos(\frac{\pi}{2}r) & v \\ 0 & 0 & 1 \end{bmatrix}$$

- $p4m$ – all 2-dimensional integer translations, rotations by multiples of $\frac{\pi}{2}$, and mirror reflections
  - For $r \in \{0,1,2,3\}$, $u, v \in \mathbb{Z}$, and $m \in \{0,1\}$

$$g(m,r,u,v) = \begin{bmatrix} (-1)^m \cos(\frac{\pi}{2}r) & (-1)^{m+1} \sin(\frac{\pi}{2}r) & u \\ \sin(\frac{\pi}{2}r) & \cos(\frac{\pi}{2}r) & v \\ 0 & 0 & 1 \end{bmatrix}$$

- In both cases, the binary operation is matrix multiplication

Group Actions

- Critical to this paper, the authors use the fact that these groups act on the set of images.
- A group G is said to act on a set X if there exists a function $\gamma : G \times X \to X$ such that

$$\gamma(e, x) = x \text{ (e is the identity element of G)}$$

$$\gamma(g_1, \gamma(g_2, x)) = \gamma(g_1 g_2, x)$$

- $p4$ and $p4m$ act on $\mathbb{Z}^2$ (specifically $\mathbb{Z}^2 \times \{1\} \subset \mathbb{R}^3$) by matrix-vector multiplication:
  - Ex: For $A \in p4$ and $[u, v, 1]^T \in \mathbb{Z}^2 \times \{1\}$,

$$A[u, v, 1]^T = [u', v', 1]^T \in \mathbb{Z}^2 \times \{1\}$$

Acting on the set of images

- The authors describe the set of images as the collection of functions $f : \mathbb{Z}^2 \to \mathbb{R}^K$ (where f has compact (rectangular) support)
- $p4$ and $p4m$ act on the the set of images $\{f\}$ with the function L:

$$L_g(f)(x) = f(g^{-1}x)$$

  where $g^{-1}x$ denotes matrix-vector multiplication

Equivariance

- A function $f : X \to Y$ is said to be equivariant with respect to the action of G on X and Y if

$$f(\gamma(g,x)) = \gamma(g, f(x))$$

- In this paper, the authors are interested in functions $f : \{f\} \to \{f\}$ that are equivariant with respect to the action of $p4$ and $p4m$ on $\{f\}$
- The authors define a convolutional layer to be equivariant if the function f is equivariant with respect to the action of $p4$ and $p4m$ on $\{f\}$

Methods

- Detailed description of the methodology used
- Explain the algorithms, models, and tools

Results

- Presentation of the key findings and outcomes
- Statistical analysis and data interpretation
- Discuss the figures and tables from the paper that showcase the results

Demonstration of the Code

- Walkthrough of the main segments of the code
- Execution of the code and presentation of results in real-time (if possible)

Conclusion

- Summarize the main points from the presentation
- Implications of the results
- Possible directions for future work

References