https://iclr.cc/Conferences/2025/PosterInstructions

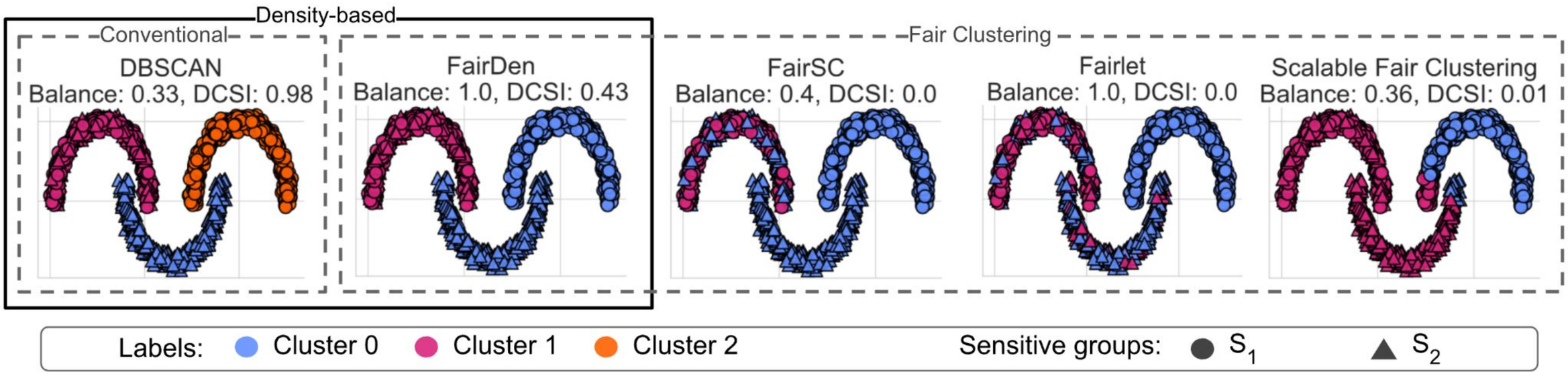
FairDen: Fair Density-Based Clustering

Lena Krieger*[1], Anna Beer*[2], Pernille Matthews[3], Anneka Myrup Thiesson[3], Ira Assent[1,3]

[1] IAS-8, Forschungszentrum Jülich [2] Faculty of Computer Science, University of Vienna [3] Department of Computer Science, Aarhus University

*Equal Contribution.

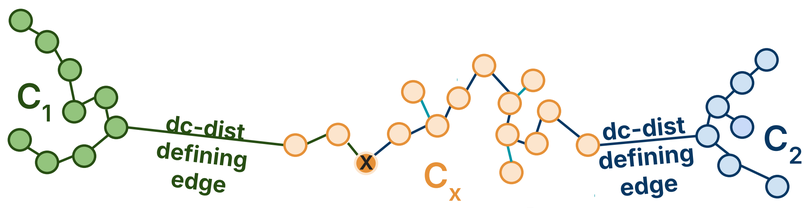
Fairness in Density-based Clustering



For a binary sensitive attribute indicated by the shape, a **balanced** density-based cluster has the **same ratio** of sensitive groups (circles and triangles) as in the overall dataset. The three equally sized moons are density-based clusters that have different ratios: the first moon has 50% triangles and 50% circles, the second moon has only triangles, the third moon has only circles. We show the balance and DCSI values (higher is better), for five clustering methods: FairDen achieves optimal balance and a high density-connectivity of clusters.

Method

Capture Density-connectivity structure:



The *density-connectivity distance* [1] is based on the mutual reachability distance *dm* that takes the density of points into account by using the points' core distances d_{core} :

Ensure balance of sensitive attributes:

$$f_p^{S_x} - (|S_x|/n) \cdot \mathbf{1}_n$$

a. Binary encoding for each *sensitive group*

	△	◯	◯	△
△	1	0	0	1
◯	0	1	1	0

b. Columns of fairness matrix

$$\triangle(1\ 0\ 0\ 1) - \frac{|\triangle|}{n} \cdot \mathbf{1}_n$$

Combine density-connectivity and fairness:

$$\min_{\mathcal{H} \in \mathbb{R}^{n \times k}} Tr(\mathcal{H}^\top \mathcal{L} \mathcal{H})$$

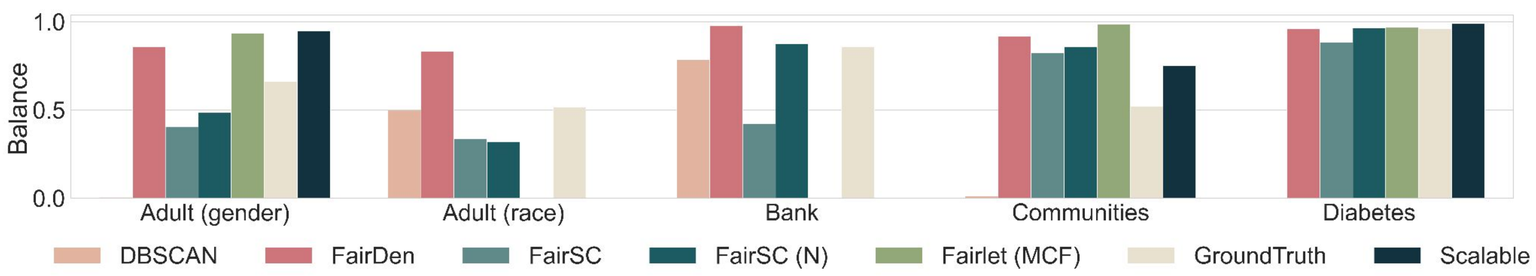
$$\text{subject to } \mathcal{H}^\top \mathcal{D} \mathcal{H} = \mathcal{I}_k \text{ and } \mathcal{F}^\top \mathcal{H} = 0$$

Imposing the fairness constraint ($\mathcal{F}^\top\mathcal{H}$) to a hierarchy of density-connected clusters transforms the problem into a ***graph-cut problem*** solvable with ***spectral clustering*** **[2]**.

Results

FairDen finds more balanced clusterings w.r.t. sensitive attributes than other density-based methods and detects density-based clusters better than other fair methods.

Experiments regarding fairness: Balance values for all competitors on benchmark datasets. Fairlet (MCF) and Scalable Fair Clustering are not always included as they cannot handle non-binary sensitive groups.



Experiments regarding clustering quality: Number of clusters ***k***, **Balance** (assessing group-level fairness), **DCSI** (assessing density-connectivity of clusters), and **ARI** (assessing similarity to ground truth clustering) for real-world benchmark data.

	k	Algorithm	Balance	DCSI	ARI
Adult (gender)	2	DBSCAN	0.01	**0.97**	0.00
	2	FairDen	0.86	0.04	0.05
	2	FairSC	0.40	0.00	0.23
	2	FairSC (N)	0.49	0.00	0.27
	2	Fairlet (MCF)	0.94	0.00	0.00
	2	GroundTruth	0.66	0.00	1.00
	2	Scalable	**0.95**	0.01	-0.01
Adult (race)	2	DBSCAN	0.50	**0.99**	0.02
	2	FairDen	**0.83**	0.09	0.05
	2	FairSC	0.34	0.00	-0.03
	2	FairSC (N)	0.32	0.00	0.16
	2	Fairlet (MCF)	-	-	-
	2	Scalable	-	-	-
	2	GroundTruth	0.52	0.00	1.00
Bank	2	DBSCAN	0.79	**0.99**	0.01
	2	FairDen	**0.98**	0.14	0.21
	2	FairSC	0.42	0.00	-0.06
	2	FairSC (N)	0.88	0.00	-0.04
	2	Fairlet (MCF)	-	-	-
	2	Scalable	-	-	-
	2	GroundTruth	0.86	0.00	1.00

	k	Algorithm	Balance	DCSI	ARI
Communities	2	DBSCAN	0.01	**0.65**	-0.03
	2	FairDen	0.92	0.15	0.09
	2	FairSC	0.82	0.13	0.03
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	2	Scalable	0.75	0.08	-0.03
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Diabetes	2	DBSCAN	-	-	-
	2	FairDen	0.96	**0.08**	0.01
	2	FairSC	0.89	0.00	-0.01
	2	FairSC (N)	0.96	0.00	0.01
	2	Fairlet (MCF)	0.97	0.00	0.00
	2	Scalable	**0.99**	0.01	0.00
	2	GroundTruth	0.96	0.00	1.00
	4	DBSCAN	0.01	**0.88**	-
	4	FairDen	0.95	0.24	-
	4	FairSC	0.23	0.01	-
	4	FairSC (N)	0.61	0.19	-
	4	Fairlet (MCF)	0.95	0.00	-
	4	Scalable	**0.96**	0.07	-
	4	GroundTruth	-	-	-

References

[1] Anna Beer, Andrew Draganov, Ellen Hohma, Philipp Jahn, Christian MM Frey, and Ira Assent. "Connecting the Dots--Density-Connectivity Distance unifies DBSCAN, k-Center and Spectral Clustering." In *Proceedings of the 29th ACM SIGKDD Conference on Knowledge Discovery and Data Mining*, pp. 80-92. 2023.

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Code

Paper

ECML PKDD 2025
Originally published at ICLR

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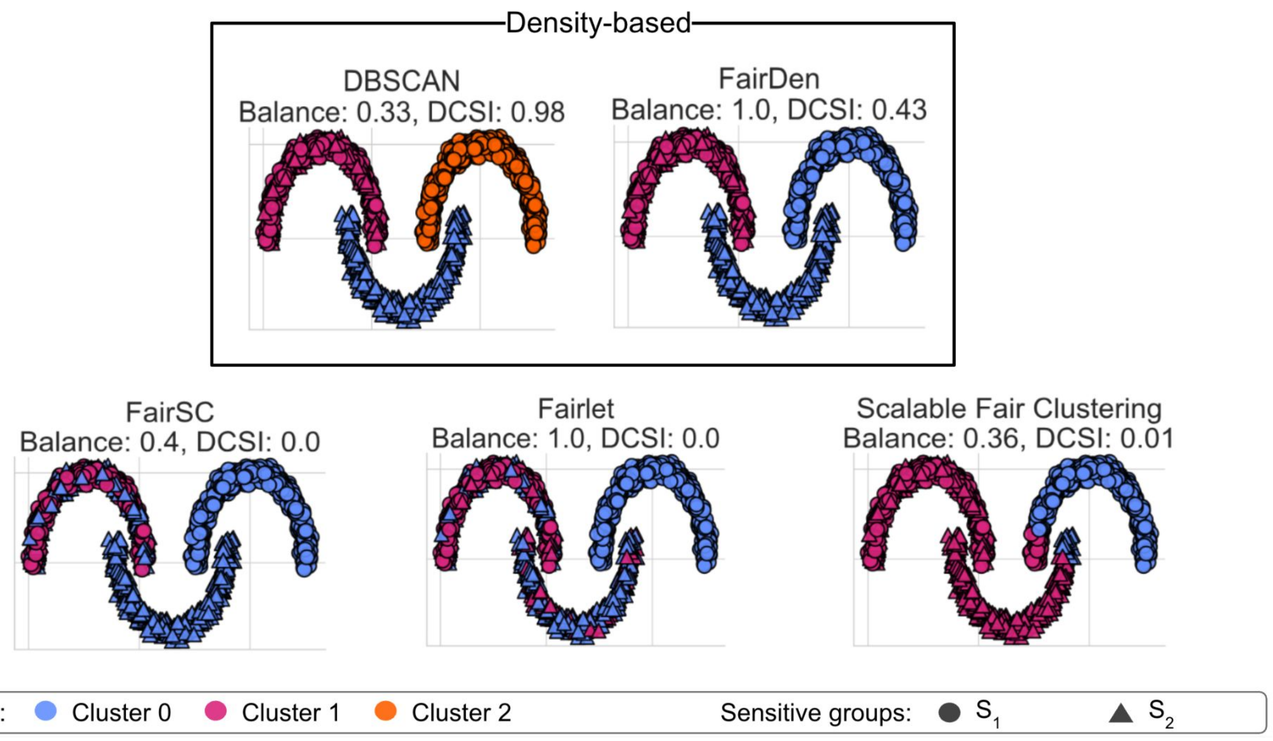


For a binary sensitive attribute indicated by the shape, a **balanced** density-based cluster has the **same ratio** of sensitive groups (circles and triangles) as in the overall dataset. The three equally sized moons are density-based clusters that have different ratios: the first moon has 50% triangles and 50% circles, the second moon has only triangles, the third moon has only circles. We show the balance and DCSI values (higher is better), for five clustering methods: FairDen achieves optimal balance and a high density-connectivity of clusters.

Summary

FairDen is the first **density-based group-level fair** clustering algorithm.
It can even cluster data with:

- **multiple sensitive attributes**,
- **multiple sensitive groups**, and
- **categorical features.**

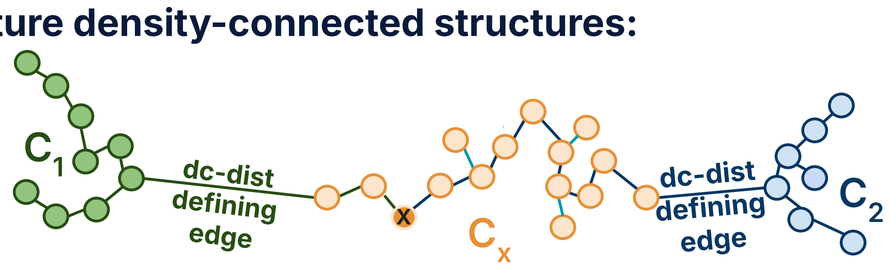
Related Work

Most other state-of-the-art group-level fair clustering methods cannot handle multiple sensitive attributes, multiple sensitive groups, or categorical values in the non-sensitive attributes.

Algorithm	Fairlet [3]	Scalable Fair Clustering [4]	Fair SC [2]	FairDen (Ours)
Density-based	✗	✗	✗	✓
Multiple sensitive attributes	✗	✗	✗	✓
Multiple (>2) sensitive groups	✗	✗	✓	✓
Categorical Features	✗	✗	✗	✓

Method

Capture density-connected structures:



The *density-connectivity distance* [1] is based on the mutual reachability distance d_m that takes the density of points into account by using the points' core distances d_{core}:

$$d_m(x,y) = \max\left(d_{core}(x), d_{core}(y), d_{eucl}(x,y)\right)$$

The longest edge e along the min-max path on the graph given by d_m indicates the density-connectivity between two points x and y

$$d_{dc}(x,y) = \min_{P\in\mathcal{P}} \max_{e\in p(x,y)} |e|$$

Ensure balance of sensitive attributes:

$$f_p^{S_x} - (|S_x|/n)\cdot \mathbf{1}_n$$

a. Binary encoding for each *sensitive group* S_x

	△	○	○	△
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b. Columns of fairness matrix:

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Combine density-connectivity and fairness:

$$\min_{\mathcal{H}\in\mathbb{R}^{n\times k}} Tr(\mathcal{H}^\top \mathcal{L}\mathcal{H}) \text{ subject to}$$

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Imposing the fairness constraint (F^TH) to a hierarchy of density-connected clusters transforms the problem into a ***graph-cut problem*** solvable with ***spectral clustering*** **[2]**.

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FairDen finds more balanced clusterings with respect to sensitive attributes than other density-based methods and detects density-based clusters better than other fair methods.

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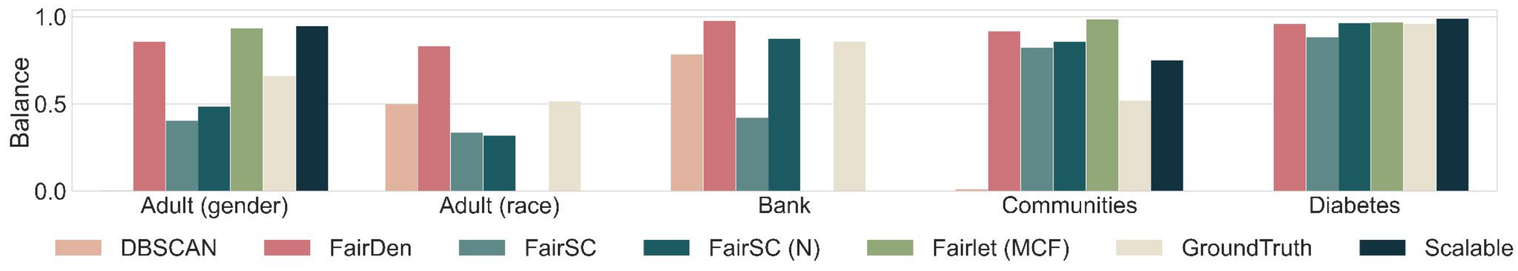

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Code

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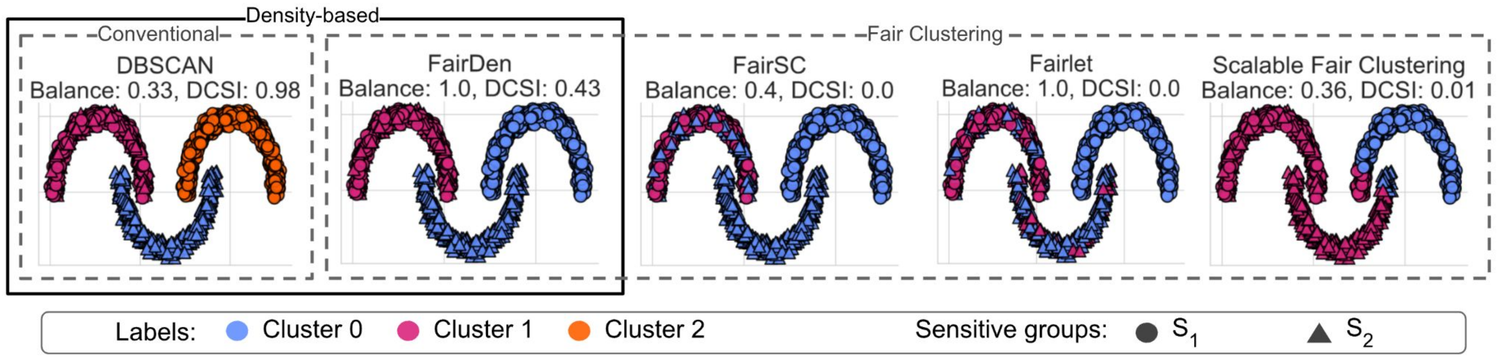

Figure 1: Top row: method, balance, and DCSI values (higher is better), respectively. A balanced density-based clustering has the same ratios of circles and triangles as in the overall dataset. Shapes and edge color indicate membership to one of two sensitive groups: the first moon has 50% triangles and 50% circles, the second moon has only triangles, the third moon has only circles, and all moons have the same number of data points.

Group-level Fair Clustering Approaches

Algorithm	Fairlets	Scalable Fair Clustering	Fair SC	FairDen (Ours)
Density-based	×	×	×	✓
Multiple sensitive attributes	×	×	×	✓
Multiple (>2) sensitive groups	×	×	✓	✓
Categorical Features	×	×	×	✓

Table 1. Properties of our group-level fair competitors. ***Sensitive attributes*** denote the name of the feature, e.g., gender, while ***sensitive groups*** encapsulate the values of the feature per group, e.g., female, male.

Method

Capture Density-connectivity structure:

$$d_m(x,y) = \max\left(d_{core}(x), d_{core}(y), d_{eucl}(x,y)\right)$$

$$d_{dc}(x,y) = \min_{P \in \mathcal{P}} \max_{e \in p(x,y)} |e|$$

a. Binary encoding for each *sensitive group*

△ ○ ○ △
△ (1 0 0 1)
○ (0 1 1 0)

b. Columns of fairness matrix

$$\triangle(1\ 0\ 0\ 1) - \frac{|\triangle|}{n} \cdot \mathbf{1}_n$$

$$\min_{\mathcal{H} \in \mathbb{R}^{n \times k}} Tr(\mathcal{H}^\top \mathcal{L} \mathcal{H})$$

$$\text{subject to } \mathcal{H}^\top \mathcal{D} \mathcal{H} = \mathcal{I}_k \text{ and } \mathcal{F}^\top \mathcal{H} = 0$$

Results

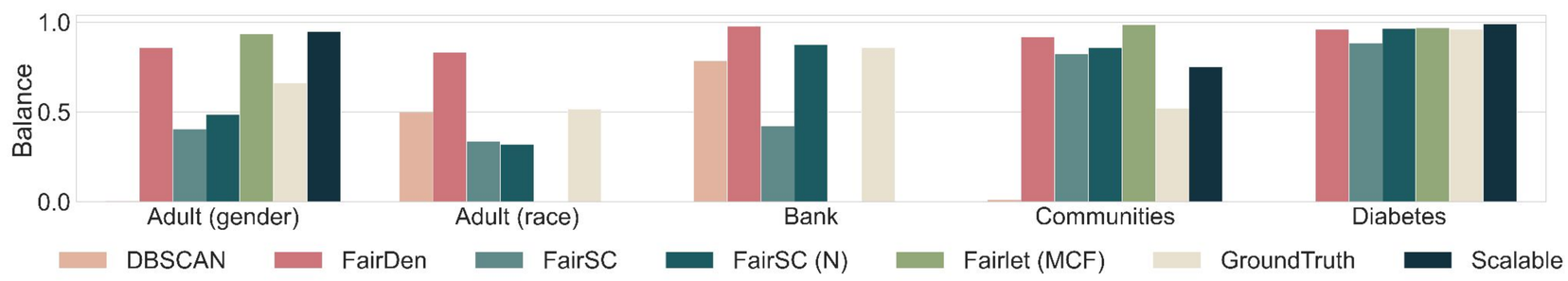

Figure 3: Balances for all competitors and benchmark datasets. Fairlet (MCF) and Scalable Fair Clustering are not included for settings including non-binary sensitive groups.

FairDen determines more balanced clusterings w.r.t. sensitive attributes than other density-based methods and detects density-based clusters better than other fair methods.

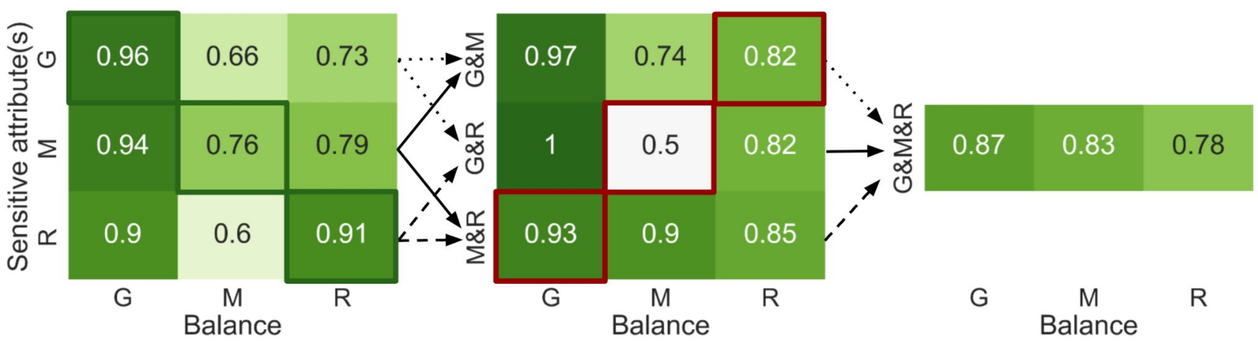

Figure 4: Columns show balance with respect to sensitive attributes *gender* (G), *marital status* (M) and *race* (R) in FairDen clusterings of the Adult dataset. **Left**: results for clustering when including only one of the sensitive attributes, *highest* column-wise values/balance for the attribute labeled as sensitive is framed in green. **Middle**: results for clusterings when including two of the sensitive attributes as indicated by the row description. *Lowest* column-wise values/ balance of attribute that is *not* labeled as sensitive is framed in red. **Right**: results for including all three sensitive attributes.

In contrast to state-of-the-art competitors, FairDen inherently handles categorical attributes, noise, and data with several sensitive attributes or groups.

	k	Algorithm	Balance	DCSI	ARI
Adult (gender)	2	DBSCAN	0.01	**0.97**	0.00
	2	FairDen	0.86	0.04	0.05
	2	FairSC	0.40	0.00	0.23
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	4	Fairlet (MCF)	0.95	0.00	-
	4	Scalable	**0.96**	0.07	-
	4	GroundTruth	-	-	-

Table 2: Number of clusters k, Balance, DCSI, ARI for real-world benchmark data. Diabetes dataset for k=2 (Ground truth) and k=4 (DBSCAN clusters).

References

[1] Beer, Anna, et al. "Connecting the Dots--Density-Connectivity Distance unifies DBSCAN, k-Center and Spectral Clustering." Proceedings of the 29th ACM SIGKDD Conference on Knowledge Discovery and Data Mining. 2023.

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[2]

Code **Paper**

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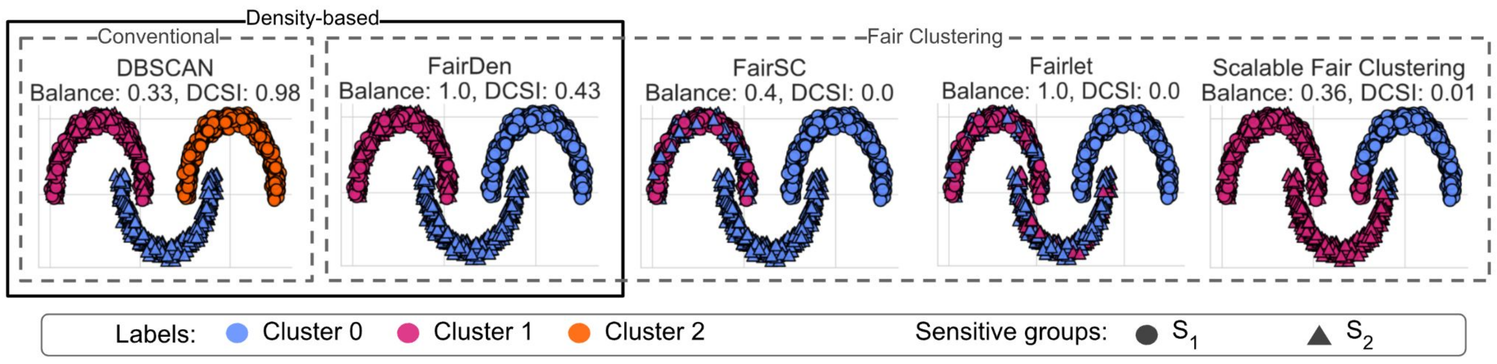

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Multiple (>2) sensitive groups	×	×	✓	✓
Categorical Features	×	×	×	✓

Table 1. Properties of our group-level fair competitors. ***Sensitive attributes*** denote the name of the feature, e.g., gender, while ***sensitive groups*** encapsulate the values of the feature per group, e.g., female, male.

Method

1) Capture the density-connectivity structure

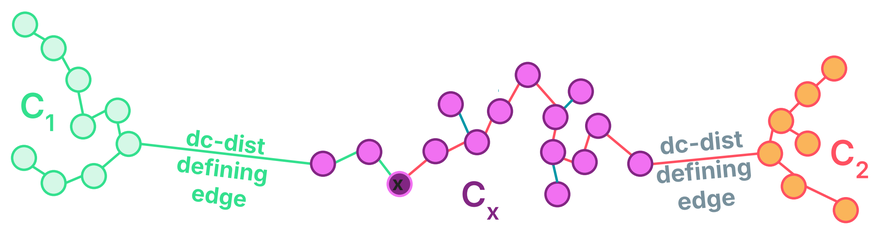

Figure 2: Regarding the *Density-connectivity distance* **[1]** C_2 is closer to x than C_1.

2) Ensure balance with fairness matrix

a. Binary encoding for each *sensitive group*

△ (1 0 0 1)
● (0 1 1 0)

b. Columns of fairness matrix

$$\triangle\,(1\ 0\ 0\ 1) - \frac{|\triangle|}{n} \cdot \mathbf{1}_n$$

3) Combine both aspects

Imposing the fairness constraint to a hierarchy of density-connected clusters transforms the problem into a ***graph-cut problem*** solvable with ***spectral clustering*** **[2]**.

Results

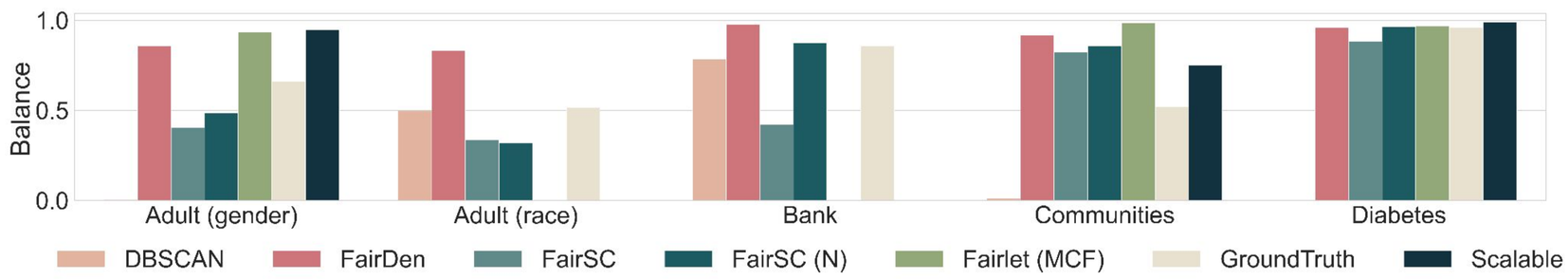

Figure 3: Balances for all competitors and benchmark datasets. Fairlet (MCF) and Scalable Fair Clustering are not included for settings including non-binary sensitive groups.

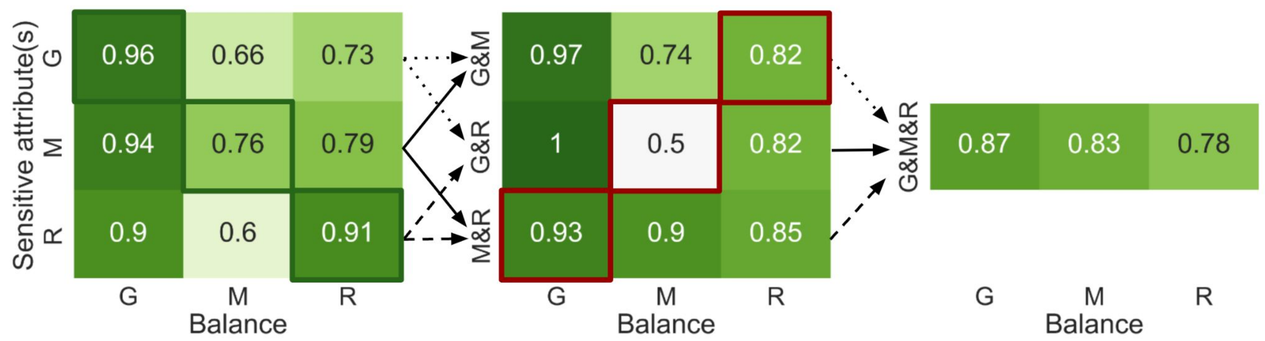

Figure 4: Columns show balance with respect to sensitive attributes *gender* (G), *marital status* (M) and *race* (R) in FairDen clusterings of the Adult dataset. **Left**: results for clustering when including only one of the sensitive attributes, *highest* column-wise values/balance for the attribute labeled as sensitive is framed in green. **Middle**: results for clusterings when including two of the sensitive attributes as indicated by the row description. *Lowest* column-wise values/ balance of attribute that is *not* labeled as sensitive is framed in red. **Right**: results for including all three sensitive attributes.

FairDen determines more balanced clusterings w.r.t. sensitive attributes than other density-based methods and detects density-based clusters better than other fair methods.

In contrast to state-of-the-art competitors, FairDen inherently handles categorical attributes, noise, and data with several sensitive attributes or groups.

	k	Algorithm	Balance	DCSI	ARI
Adult (gender)	2	DBSCAN	0.01	**0.97**	0.00
	2	FairDen	0.86	0.04	0.05
	2	FairSC	0.40	0.00	0.23
	2	FairSC (N)	0.49	0.00	0.27
	2	Fairlet (MCF)	0.94	0.00	0.00
	2	GroundTruth	0.66	0.00	1.00
	2	Scalable	**0.95**	0.01	-0.01
Adult (race)	2	DBSCAN	0.50	**0.99**	0.02
	2	FairDen	**0.83**	0.09	0.05
	2	FairSC	0.34	0.00	-0.03
	2	FairSC (N)	0.32	0.00	0.16
	2	Fairlet (MCF)	-	-	-
	2	Scalable	-	-	-
	2	GroundTruth	0.52	0.00	1.00
Bank	2	DBSCAN	0.79	**0.99**	0.01
	2	FairDen	**0.98**	0.14	0.21
	2	FairSC	0.42	0.00	-0.06
	2	FairSC (N)	0.88	0.00	-0.04
	2	Fairlet (MCF)	-	-	-
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	2	GroundTruth	0.86	0.00	1.00
Communities	2	DBSCAN	0.01	**0.65**	-0.03
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	2	FairSC (N)	0.86	0.13	0.03
	2	Fairlet (MCF)	**0.99**	0.05	0.16
	2	Scalable	0.75	0.08	-0.03
	2	GroundTruth	0.52	0.07	1.00
Diabetes	2	DBSCAN	-	-	-
	2	FairDen	0.96	**0.08**	0.01
	2	FairSC	0.89	0.00	-0.01
	2	FairSC (N)	0.96	0.00	0.01
	2	Fairlet (MCF)	0.97	0.00	0.00
	2	Scalable	**0.99**	0.01	0.00
	2	GroundTruth	0.96	0.00	1.00
	4	DBSCAN	0.01	**0.88**	-
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	4	GroundTruth	-	-	-

Table 2: Number of clusters k, Balance, DCSI, ARI for real-world benchmark data. Diabetes dataset for k=2 (Ground truth) and k=4 (DBSCAN clusters).

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Code

Paper

FairDen: Fair Density-Based Clustering

Lena Krieger*[1], Anna Beer*[2], Pernille Matthews[3], Anneka Myrup Thiesson[3], Ira Assent[1,3]

[1]Forschungszentrum Jülich [2]University of Vienna [3]Aarhus University

Fairness in Density-based Clustering

Density-based Clustering (DBSCAN)

1. Determine core points
2. Select random (core) point to start a new cluster C_i
3. Merge all points with distance < ε from C_i to C_i
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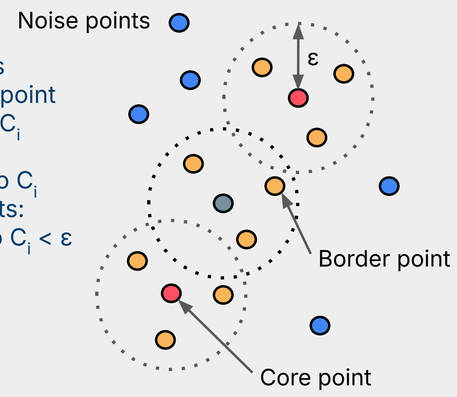


Figure 1: Illustration of DBSCAN workflow when *minPts=4*.

Group-level Fair / Balanced Clustering

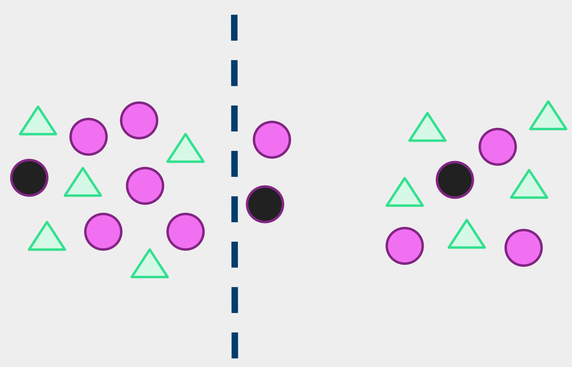


Figure 2: Line illustrates cut for group-level fair clustering. ***Sensitive groups*** are denoted in green and pink. This dataset includes only one ***sensitive attribute*** with two ***sensitive groups***.

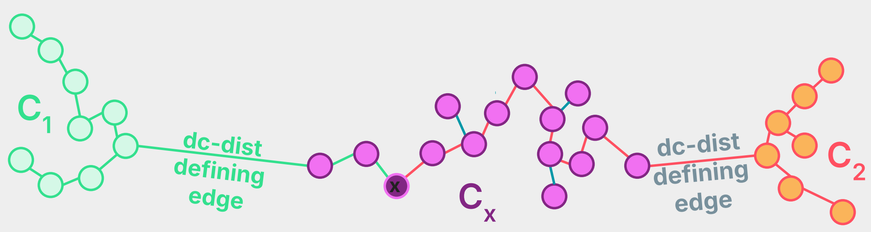
Group-level Fair Clustering Approaches

Algorithm	Fairlets	Scalable Fair Clustering	Fair SC	FairDEN (Ours)
Density-based	×	×	×	✓
Multiple sensitive attributes	×	×	×	✓
Multiple (>2) sensitive groups	×	×	✓	✓
Categorical Features	×	×	×	✓

Table 1. Properties of our group-level fair competitors. ***Sensitive attributes*** denote the name of the feature, e.g., gender, while ***sensitive groups*** encapsulate the values of the feature per group, e.g., female, male.

Method

1) Capture the density-connectivity structure



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2) Ensure balance with fairness matrix

a. Binary encoding for each *sensitive group*

△ (1 0 0 1)
● (0 1 1 0)

b. Columns of fairness matrix

$$\triangle\ (1\ 0\ 0\ 1) - \frac{|\triangle|}{n} \cdot \mathbf{1}_n$$

3) Combine both aspects

Imposing the fairness constraint to a hierarchy of density-connected clusters transforms the problem into a ***graph-cut problem*** solvable with ***spectral clustering*** **[2]**.

Results

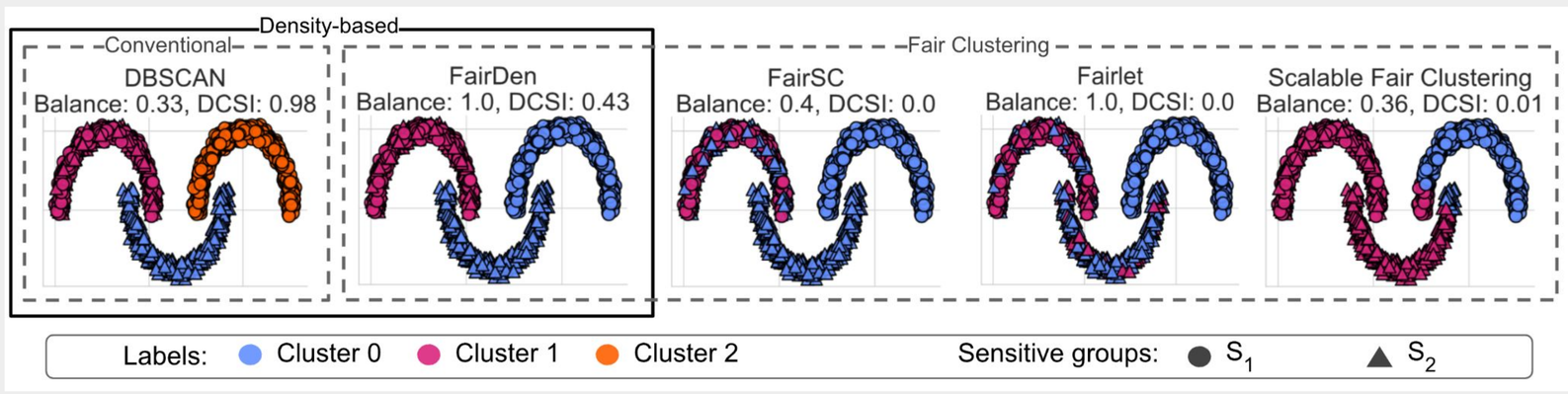

Figure 4: Top row: method, balance, and DCSI values (higher is better), respectively. A balanced density-based clustering has the same ratios of circles and triangles as in the overall dataset. Shapes and edge color indicate membership to one of two sensitive groups: the first moon has 50% triangles and 50% circles, the second moon has only triangles, the third moon has only circles, and all moons have the same number of data points.

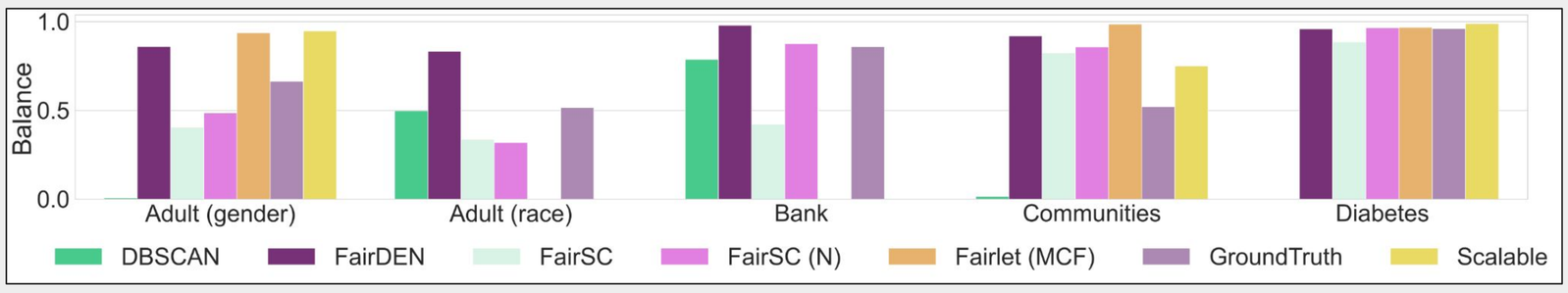

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Table 2: Number of clusters k, Balance, DCSI, ARI for real-world benchmark data. Diabetes dataset for k=2 (Ground truth) and k=4 (DBSCAN clusters).

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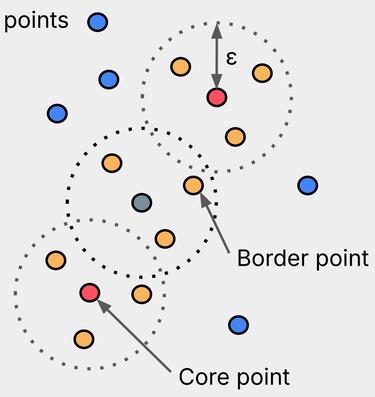
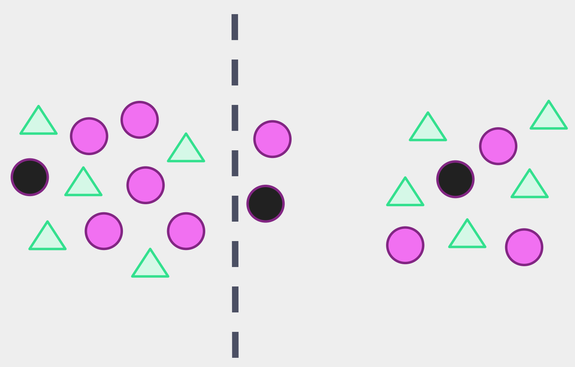


Figure 1: Illustration of DBSCAN workflow when *minPts=4*.

Group-level Fair / Balanced Clustering



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Group-level Fair Clustering Approaches

Algorithm	Fairlets	Scalable Fair Clustering	Fair SC	FairDEN (Ours)
Density-based	✗	✗	✗	✓
Multiple sensitive attributes	✗	✗	✗	✓
Multiple (>2) sensitive groups	✗	✗	✓	✓
Categorical Features	✗	✗	✗	✓

Table 1. Properties of our group-level fair competitors. ***Sensitive attributes*** denote the name of the feature, e.g., gender, while ***sensitive groups*** encapsulate the values of the feature per group, e.g., female, male.

Method

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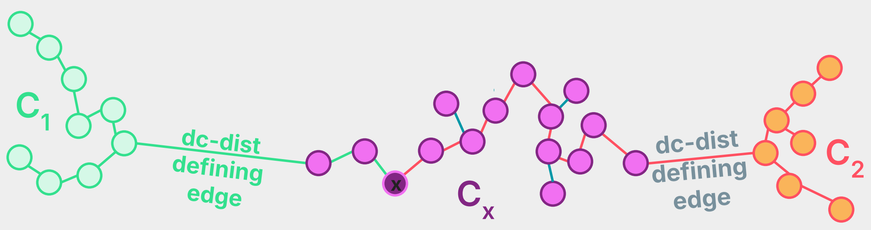

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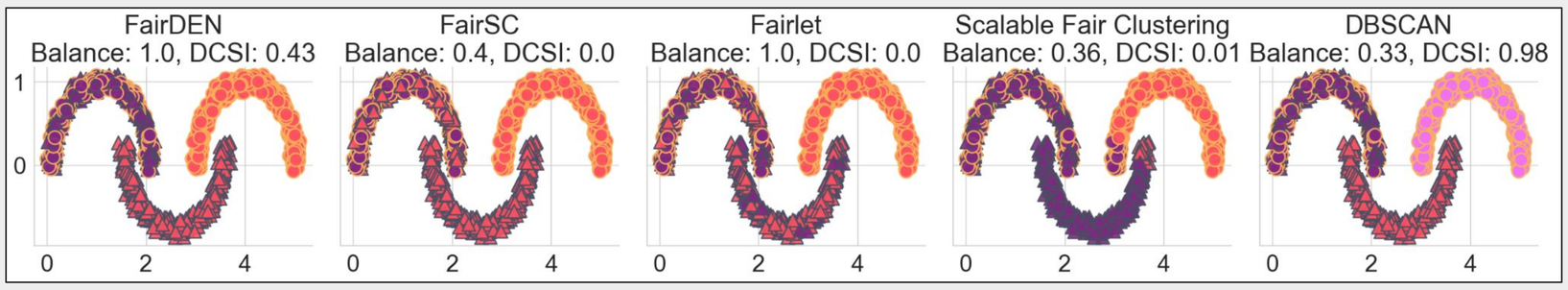

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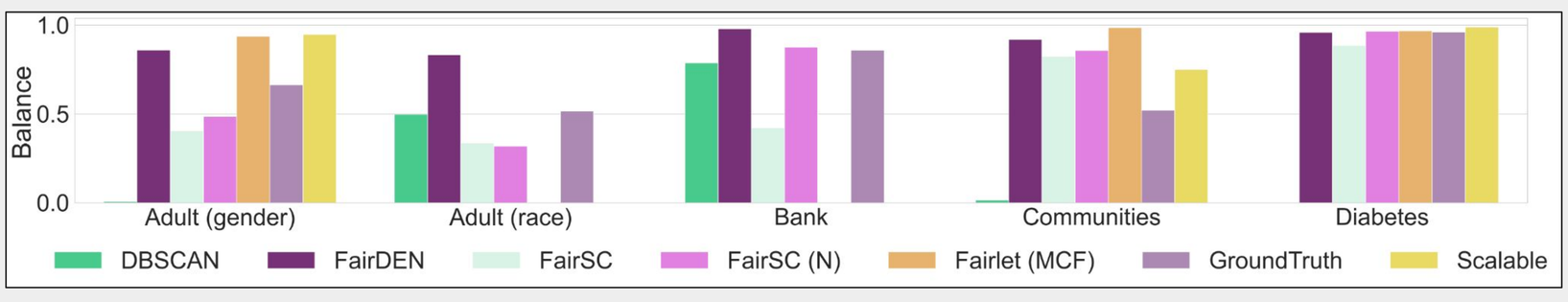

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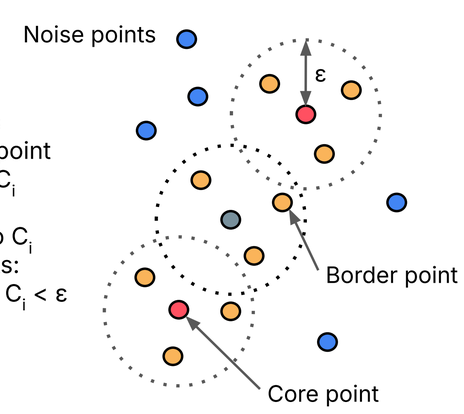

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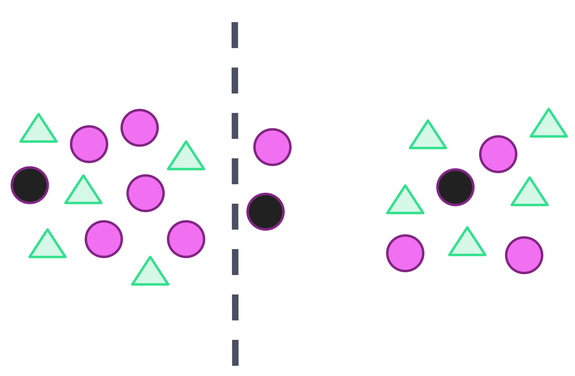

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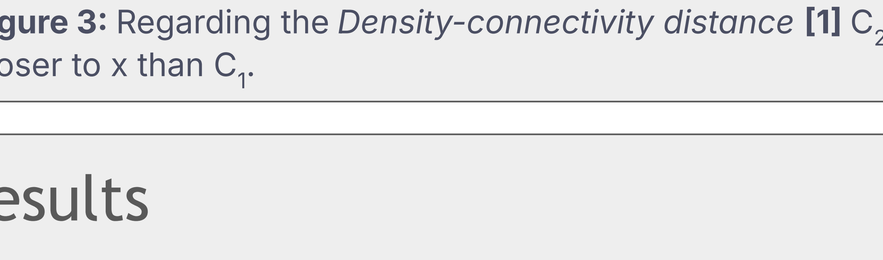

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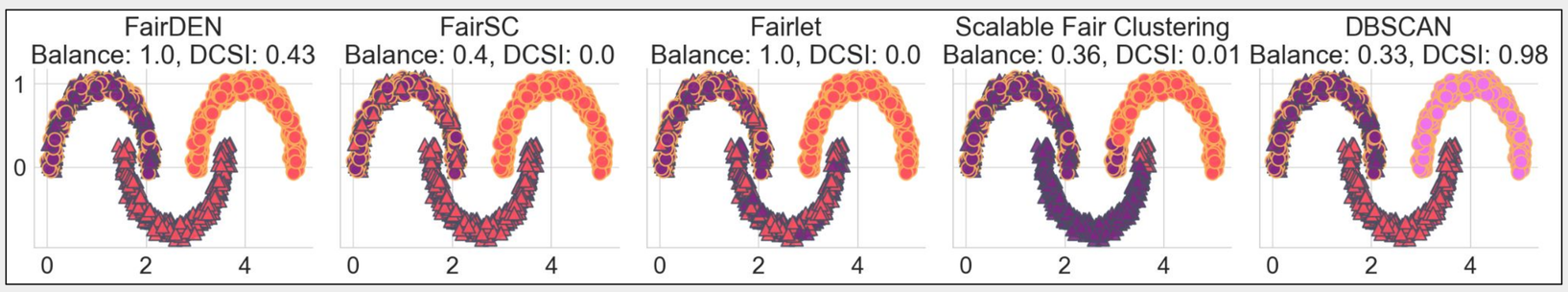

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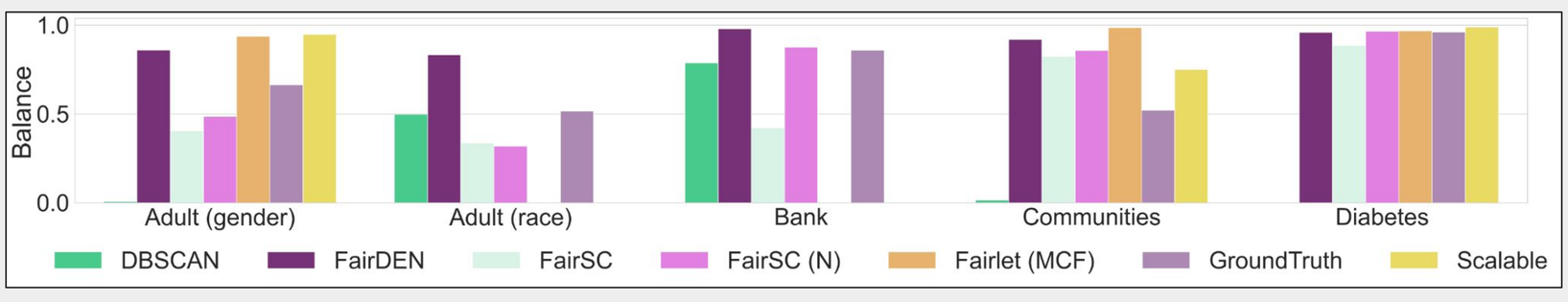

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	2	FairDen	**0.83**	0.09	0.05
	2	FairSC	0.34	0.00	-0.03
	2	FairSC (N)	0.32	0.00	0.16
	2	Fairlet (MCF)	-	-	-
	2	Scalable	-	-	-
	2	GroundTruth	0.52	0.00	1.00
Bank	2	DBSCAN	0.79	**0.99**	0.01
	2	FairDen	**0.98**	0.14	0.21
	2	FairSC	0.42	0.00	-0.06
	2	FairSC (N)	0.88	0.00	-0.04
	2	Fairlet (MCF)	-	-	-
	2	Scalable	-	-	-
	2	GroundTruth	0.86	0.00	1.00
Communities	2	DBSCAN	0.01	**0.65**	-0.03
	2	FairDen	0.92	0.15	0.09
	2	FairSC	0.82	0.13	0.03
	2	FairSC (N)	0.86	0.13	0.03
	2	Fairlet (MCF)	**0.99**	0.05	0.16
	2	Scalable	0.75	0.08	-0.03
	2	GroundTruth	0.52	0.07	1.00
Diabetes	2	DBSCAN	-	-	-
	2	FairDen	0.96	**0.08**	0.01
	2	FairSC	0.89	0.00	-0.01
	2	FairSC (N)	0.96	0.00	0.01
	2	Fairlet (MCF)	0.97	0.00	0.00
	2	Scalable	**0.99**	0.01	0.00
	2	GroundTruth	0.96	0.00	1.00
	4	DBSCAN	0.01	**0.88**	-
	4	FairDen	0.95	0.24	-
	4	FairSC	0.23	0.01	-
	4	FairSC (N)	0.61	0.19	-
	4	Fairlet (MCF)	0.95	0.00	-
	4	Scalable	**0.96**	0.07	-
	4	GroundTruth	-	-	-

Table 2: Number of clusters k, Balance, DCSI, ARI for real-world benchmark data. Diabetes dataset for k=2 (Ground truth) and k=4 (DBSCAN clusters).

References

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