Demostrar

$$\mathcal{E}_x\{x\} = \int x \, p(x)\, dx \qquad x \in \mathbb{R}$$

$$\int_{-\infty}^{\infty} N(x \mid \mu, \sigma^2)\, dx = 1$$

TIPS: → Por partes
→ función generadora de momentos

1. $\mathcal{E}\{x\} = \int_{-\infty}^{\infty} N(x \mid \mu, \sigma^2)\, x\, dx = \mu$

TIPS: $var(x) = \sigma_x^2 = \mathcal{E}\{(x-\mu)^2\}$

2. $\mathcal{E}\{x^2\} = \int_{-\infty}^{\infty} N(x \mid \mu, \sigma^2)\, x^2 dx = \mu^2 + \sigma^2$

$var\{x\} = \sigma^2$

Demostración ⇒

1. $\mathcal{E}\{x\} = \int x\, p(x)\, dx = \mu$

$p(x) \sim N(x \mid \mu, \sigma^2)$

$$= \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{|x-\mu|^2}{2\sigma^2}\right) dx \;, \quad z = x-\mu$$

$dz = dx$

$x = z + \mu$

$$= \int_{-\infty}^{\infty} (z+\mu) \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-|z|^2}{2\sigma^2}\right) dz$$

$$= \cancel{\int_{-\infty}^{\infty} \frac{z}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-|z|^2}{2\sigma^2}\right) dz} \xrightarrow{} 0 + \int \frac{\mu}{2\pi\sigma^2} \exp\left(\frac{-|z|^2}{2\sigma^2}\right) dz$$

impar

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$$= \mu \int_{-\infty}^{\infty} \frac{1}{2\pi\sigma^2} \exp\left(\frac{-|z|^2}{2\sigma^2}\right) dz \rightarrow 1$$

$$\int_{-\infty}^{\infty} p(z) = 1$$

$$\boxed{\mathcal{E}\{x\} = \mu}$$

2. $\mathcal{E}\{x^2\} = \int_{-\infty}^{\infty} N(x|\mu, \sigma^2)\, x^2 dx = \mu^2 + \sigma^2$

$\text{var}\{x\} = \sigma^2$

Tomar en cuenta = $\text{var}\{x\} = \mathcal{E}\{(x-\mu)^2\} = \mathcal{E}\{x^2\} - \mathcal{E}^2\{x\}$

$$\mathcal{E}\{x^2\} = \mu^2 + \sigma^2 \qquad \mu^2 = \mathcal{E}^2\{x\}$$

$$\sigma^2 = \text{var}\{x\} = \mathcal{E}\{x^2\} - \mathcal{E}^2\{x\}$$

$$\mathcal{E}\{x^2\} = \cancel{\mathcal{E}^2\{x\}} + \mathcal{E}\{x^2\} - \cancel{\mathcal{E}^2\{x\}}$$

$$\boxed{\mathcal{E}\{x^2\} = \mathcal{E}\{x^2\}}$$

Demostrado por igualdades