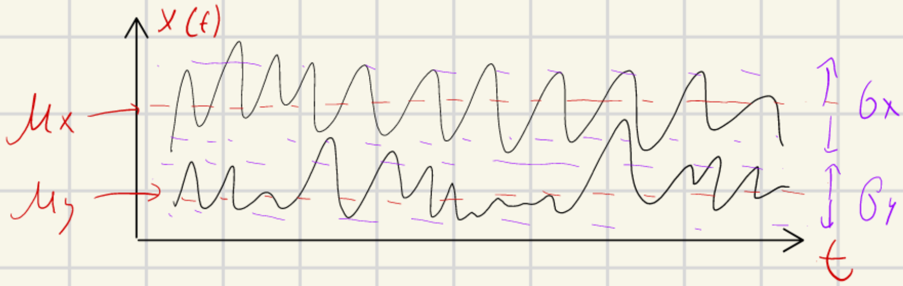

$$\operatorname{cov}\{x,y\} = \mathcal{E}_{x,y}\{(x-\mu_x)(y-\mu_y)\}$$

$$\operatorname{cov}\{x,y\} = \int_{x,y} (x-\mu_x)(y-\mu_y)\, p(x,y)\, dxy$$

Demostrar

$$\operatorname{var}\{x\} = \mathcal{E}\{(x-\mu_x)^2\} = \mathcal{E}\{x^2\} - \mathcal{E}^2\{x\}$$

$$\mathcal{E}\{(x-\mu_x)^2\} = \mathcal{E}\{(x-\mu_x)(x-\mu_x)\} \Rightarrow$$

$$\mathcal{E}\{x-\mu_x\}\,\mathcal{E}\{x-\mu_x\} = (\mathcal{E}\{x\} - \mathcal{E}\{\mu_x\})(\mathcal{E}\{x\} - \mathcal{E}\{\mu_x\})$$

$$\Rightarrow \mathcal{E}\{x\}^2 - \mathcal{E}\{x\}\mathcal{E}\{\mu_x\} - \mathcal{E}\{x\}\mathcal{E}\{\mu_x\} + \mathcal{E}^2\{\mu_x\}$$

$$\mathcal{E}^2\{x\} - 2\mu_x\,\mathcal{E}\{x\} - \mu_x\,\mathcal{E}\{x\} + \mathcal{E}^2\{\mu_x\}$$

$$\Rightarrow \mathcal{E}\{x^2\} - 2\mu_x \underbrace{\mathcal{E}\{x\}}_{\leftarrow \mu_x} + \mu_x^2 \Rightarrow \mathcal{E}\{x^2\} - 2\mu_x^2 + \mu_x^2$$

$$= \mathcal{E}\{x^2\} - \mu_x^2 = \mathcal{E}\{x^2\} - E^2\{x\}$$

$$\operatorname{cov}\{x,y\} = E_{x,y}\{(x-\mu_x)(y-\mu_y)\} = E_{x,y}\{xy\} - E\{x\}E\{y\}$$

$$= E\{(x-E\{x\})(y-E\{y\})\} = E\{xy - xE\{y\} - yE\{x\} + E\{x\}E\{y\}\}$$

$$= E\{xy\} - E\{xE\{y\}\} - E\{E\{x\}y\} + E\{E\{x\}E\{y\}\}$$

$$= E\{xy\} - \cancel{E\{x\}E\{y\}} - E\{x\}E\{y\} + \cancel{E\{x\}E\{y\}}$$

$$\boxed{= E\{xy\} - E\{x\}E\{y\}}$$

$$\operatorname{cov}(X,Y) = E\{XY^T\} - E\{X\}E\{Y\} \qquad x,y \in \mathbb{R}^{D\times 1}$$

$$\operatorname{cov}(x,y) = E\{(x-E\{x\})(y-E\{y\})^T\}$$

$$= E\{xy^T - xE\{y\}^T - E\{x\}y^T + E\{x\}E\{y\}^T\}$$

$$= E\{xy^T\} - E\{xE\{y\}^T\} - E\{E\{x\}y^T\} + E\{E\{x\}E\{y\}^T\}$$

$$= E\{xy^T\} - E\{x\}E\{y\} - \cancel{E\{x\}E\{y\}^T} + \cancel{E\{x\}E\{y\}^T}$$

$$\boxed{= E\{xy^T\} - E\{x\}E\{y\}}$$