Demostrar $\sigma^2_{\mu_L} = \frac{1}{N} \sum_{n=1}^{N} (x_n - \mu_{\mu_L})^2$

dado $\Rightarrow \mu_{\mu_L} = \frac{1}{N} \sum_{n=1}^{N} x_n$

$$\sigma^2_{\mu_L} = \frac{1}{N} \sum_{n=1}^{N} \left(x_n^2 - 2 x_n \mu_{\mu_L} + \mu^2_{\mu_L} \right)$$

$$\sigma^2_{\mu_L} = \frac{1}{N} \left(\sum_{n=1}^{N} x_n^2 - 2\mu_{\mu_L} \underbrace{\sum_{n=1}^{N} x_n}_{\mu_{\mu_L} \cdot N} + \underbrace{\sum_{n=1}^{N} \mu^2_{\mu_L}}_{N \cdot \mu^2_{\mu_L}} \right)$$

$$\sigma^2 = \frac{1}{N} \sum_{n=1}^{N} x_n^2 - 2\mu^2_{\mu_L} + \mu^2_{\mu_L}$$

$$\sigma^2_{\mu_L} = \frac{1}{N} \sum_{n=1}^{N} x_n^2 - \underbrace{\mu^2_{\mu_L}}_{\left(\frac{1}{N} \sum_{n=1}^{N} x_n\right)^2} \rightarrow \sigma^2_{\mu_L} = \frac{1}{N} \sum_{n=1}^{N} (x_n - \mu_{\mu_L})^2$$