Demostrar $\mathcal{E}\{x\} = \mu$ cuando

$\mathcal{E}\{x\} = \int x \, N(x|\mu,\sigma^2)\,dx$

$$\mathcal{E}\{x\} = \int_{x \in X} \frac{x}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(\frac{-|x-\mu|^2}{2\sigma^2}\right) dx$$

$z = x - \mu \qquad dz = dx \qquad x = z + \mu$

$$= \int_{-\infty}^{\infty} (z+\mu) \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(\frac{-|z|^2}{2\sigma^2}\right) dz$$

$\rightarrow 0 \qquad \rightarrow$ Simetría (Impar · Par = Impar)

$$= \cancel{\int_{-\infty}^{\infty} z \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(\frac{-|z|^2}{2\sigma^2}\right) dz} + \int \frac{\mu}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(\frac{-|z|^2}{2\sigma^2}\right) dz$$

$\rightarrow 1 \rightarrow p(N(x|\mu,\sigma^2)) = 1$

$$= \mu \cancel{\int \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(\frac{-|z|^2}{2\sigma^2}\right) dz}$$

$\mathcal{E}\{x\} = \mu$ cuando $x \sim N(x|\mu,\sigma^2)$

Demostrar $\mathcal{E}\{x^2\} = \int_{-\infty}^{\infty} N(x|\mu,\sigma^2)\, x^2\, dx = \mu^2 + \sigma^2$

$$\int_{-\infty}^{\infty} x^2 \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(\frac{-|x-\mu|^2}{2\sigma^2}\right) dx$$

$z = x - \mu$

$dz = dx$

$x = z + \mu$

$$\int \frac{(z+\mu)^2}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(\frac{-|z|^2}{2\sigma^2}\right) dz$$

$$= \int \left(z^2 + 2zu + u^2\right) \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(\frac{-|z|^2}{2\sigma^2}\right) dz$$

$$= E\{z^2\} + 2\mu \cancel{E\{z\}}^{\to 0} + \mu^2 \cdot (1)$$

$$= E\{z^2\} + \mu^2 \implies E\{x^2 - 2x\mu + \mu^2\} + \mu^2$$

$$= \underline{E\{x^2\}} - \underbrace{2\mu E\{x\}}_{\mu^2 = E^2\{x\}} + \underbrace{E\{\mu^2\}}_{E^2\{x\}} + \mu^2$$

$$= E\{x^2\} - 2E^2\{x\} + E^2\{x\} + \mu^2$$

$$= \underbrace{E\{x^2\} - E^2\{x\}}_{\sigma^2} + \mu^2 \implies \boxed{\sigma^2 + \mu^2}$$