Started 10/3 2013 or somewhere around there.

**HughesLiuZimmermann1981**

- Material domain in motion??.
- Try to understand examples better.

**BraessWriggers2000**

- Free surface flow as an example of the ALE method permitting a good mesh (no distortion) and an accurate description of boundaries at the same time (due to Eulerian description being used horizontally and Lagrangian description vertically).
- I should emphasize that the convective velocity isn't physical!
- Aspect ratio of elements as r_out/r_in.
- (p.103) In case of Lagrangian description: mesh distortion a problem, but can be solved using remeshing. Computationally costly though, and a source of discretization errors.
- Remeshing is used.

**FrancaFarhat1995**

- Stabilization operator
  $$P(v) = -\sigma v + a\nabla v + \kappa\Delta v$$
  instead of
  $$P(v) = \sigma v + a\nabla v + \kappa\Delta v$$

**BankWelfert1990**

-

**Stromberg2011**

- Uses decoupled thermomechanical equations.
- Doesn't seem to model heat exchange with environment.
- Coded in Fortran? Not clear.
- Global sliding assumed!
- S-U-stabilization implemented.
- Why call the kinematical model "Eulerian"? Due to no translation of wheel? (Then, no linear acceleration effects?)
- No problems with energy dissipation?
- Only wheel–brake block contact. No translation.

**PaukYevtushenko1997**

- Sliding using an Eulerian approach.
- The effect of rail chill shown (more prominent for higher Pe).

**WauerSchweizer2010**

- Many good thermoelasticity-references in the beginning.
- Eulerian disc-brakeblock-model (as in Stromberg2011) *using cylindrical coordinates.*

- No mention of numerical instability or measures to mitigate it, probably because there is none, due to the use of cyl. coords.
- Insight: If *any* convection, then not Lagrangian description. If *any* movement of nodes, then not Eulerian description. Therefore ALE.

**SchweizerWauer2001**

- Very nice introduction, discussing thermomechanical coupling.
- Metals – polychrystalline. Rubber – polymeric.

**Galantucci1999**

- Lagrangian formulation of hot rolling (>800 deg used). Rolling $2\pi/4$ and $2\pi$.

**Belytschko1980**

- FSI application with ALE (called quasi-Eulerian here).

**Hughes1995**

- Stabilized methods originate from a particular class of subgrid scale methods.
- $\tau$ can be defined exactly: it can be derived from first principles.
- Check Hughes publications following this paper.
- Bubbles identified as approximations of element Green's functions.
- Bubbles in the subgrid + static condensation ≈ subgrid scale model + assumption of subgrid contribution vanishing on element boundaries.
- Expression for $\tau$ derived, involves $g(x, y)$.

**AmsdenHirt1973book**

- Mostly describes the code YAQUI (which uses finite differences).
- Other techniques L/E/ALE mentioned, e.g. ICE (?).
- I should check out reference 2 (HirtAmsdenCook1997), it describes the ALE technique in detail.

**HirtAmsdenCook1997**

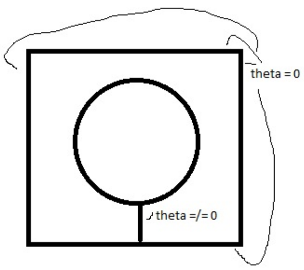
- A very simple ALE description seems to be used. It seems to be described in terms of moving mesh points rather than some formulation preceding discretization based on continuum mechanics.
- Each time step involves 3 phases, the last of which is optional re-zoning: moving vertices to keep a good mesh quality and thereby causing convection. An upwind averaging scheme is here used to avoid numerical instability (p. 209).
- The method seems so closely tied to the mesh and the practical computational algorithm. They don't start from first principles (continuum mechanics) as for instance Donea & Huerta, Nackenhorst etc.

**FrancaFarhat1994**

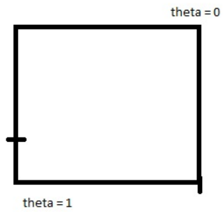
- In a pure diffusion prolem, the node values at the nodes included in the linear mesh are the same whether linear or quadratic interpolation is used.
- I'm wondering what the 1D example has to do with bubbles. It seems that linear–quadratic is what's compared, rather than bubbles on/off.
- But int he 2D example, they use an approximation consisting of a linear interpolation + a higher order bubble (for which they need to introduce an extra node).

**FrancaFrey1992**

- Compares a stabilization method by Douglas–Wang with the GLS method for Stokes flow.
- Hints that the pure least-squares method gives too dissipative results for the advection–diffusion equation.




- Example:




- Example:
  Here, quad is considerably better than lin for equal number of dofs.
- Some development in the choice of $\tau$.

**BurmanHansbo2004**

- Term added to the weak form penalizing gradient jumps across element boundaries.
- It is said that the source of the added stability is the fact that the edge of operator controls the projection error of the term $\boldsymbol{\beta} \cdot \nabla v$ ($\boldsymbol{a} \cdot \nabla \theta$) in the case of convection–diffusion.
- Some error estimates are given and proven.
- Something called "shock capturing" is mentioned, which was used in a previous paper. The numerical examples both do and don't use shock capturing.

**GaoHuang2006**

- Insightful introduction about:
  - Disc brakes, pros and failure modes, including thermomechanical effects.
  - How fatigue cracks initiate and spread.
  - Comment on the assumption of a constant sliding velocity in braking simulations.
- Disc braking model with variable speed (down to 0).
- The model used uses simplifying geometrical and thermal assumptions that make it very specific to the given application.
- FEM using ANSYS.
- On hot spots and how their related temperature spikes can be avoided.
- Results show fluctuating temperatures and stresses. Up due to frictional heat generation and down due to dissipation/convection. (I don't really understand) when studied point is outside of the contact region (the part in parenthesis above was strikethrough:ed, I don't know what I meant by that). One cycle per revolution.

- Unclear whether Eulerian or Lagrangian description.
- Contact model unclear.

**HuertaLiu1988**

- History of ALE (just as in DoneaHuerta2003book).
- Spurious oscillations when Galerkin method is used for convection–dominated problems is because the FE equations are not self-adjoint.
- "The [ALE] reference frame is fixed, but its movement [...] is arbitrary...". If this isn't a contradiction, then what does "fixed" mean in this context?
- Tsunami, dam-break and sloshing problem considered. Large free-surface motion.
- The use "a SUPG method for the mesh updating equation". What does this mean?
- Kinematical description different for each problem, but in general Eulerian in the horizontal direction and Lagrangian in the vertical direction.

**Baiocchi1993**

- Intro mentions bubbles + static condensation (SC) viewed as the addition of a stabilizing term to the FE problem.
- Addition of bubbles + SC formulated in abstract form. The resulting stabilization term added to the RHS is derived for the general case...
- ...and then for two specific examples, one of which is the advection–diffusion problem.
- If bubble functions + SC is equivalent to the SUPG among others, why does it work better in the rotating disc case? Because the space used for the bubble functions is of a particularly high quality?
- Paper establishes "a rather general link between two major families of stabilizing techniques". For the first time I guess? Nice.

**BrezziBristeau1992**

- Similar to Baiocchi1993 (Brezzi & Franca authors on both). The equivalence between Galerkin + bubbles and stabilized FE methods is established. The limit $\kappa \to 0$ is investigated.
- The stability of different methods are investigated quantitatively.
- Relationship between bubbles and SUPG was first noted in Rog's thesis (so, I should cite that if I write about it).

**XuJiang2001**

- Introduction:
  - On rolling contact, where it happens and associated problems (damage phenomena).
  - Stress analysis, of its importance to understand fatigue/wear and how it's done: semianalyt or FEM. *many* references on each method. I should check them out to understand: a) why "semi-"? b) How to do rolling contact with FEM without ALE?
  - Claims to be first paper to do elastic-plastic stress analysis with stick/slip.
  - History of stick/slip, experimental/theoretical.
  - Partial slip more damaging than full?
- This paper: Stresses/deformations due to stick/slip rolling. Uses analytical load on meshed rail (2D plane strain).
- Uses finite elements!

- ABAQUS used
- 2-4 hours per rolling cycle!
- Results similar to those of JiangSehitoglu1999's semi-analytical theory (next reference in this list)

**JiangSehitoglu1999**

- Introduction:
  - Distinguishes between engineering/research models for life prediction of rolling contact (references Tallian).
  - Problems with FE models: computationally expensive, convergence issues.
  - Current paper uses a semi-analytical model, shown to agree with FE results.
- Talks about critical plane approaches to fatigue.
- Stress distribution obtained from Hertzian contact theory. Full slip assumed.
- Contact pressure distribution assumed to be unaffected by stick/slip, geometry changes, etc.

**Special section** Summary of my quick look at references 3–13 and 14–24 in **XuJiang2001** (these groups of references refer to semi-analytical vs. FE-based analysis of rolling contact)
<u>3–13</u>

- Semi-analytical: Hertzian contact theory gives stress distribution/deformation in contacting bodies. Numerical methods modelling e.g. inelastic material response or wear or fatigue are used in conjunction with these solutions.
- Significant gain in computational cost compared to FE methods (see eg. number 9). No convergence problems, especially in 3D.
- Isn't no. 11 a FE method?

<u>14–24</u>

- Papers using elliptic pressure distribution on meshed rail: all of them.

**Kulkarni1991**

- Introduction:
  - Important to account for thermal effects.
  - Present paper: Transiently elasto-plastic, thermomechanical FE model of 2D frictional rolling contact (of type: analytical Hertz pressure distr. on FE mesh).
- Lets a purely thermal load pass before the thermomechanical one. Why?
- Fig 9 is nice. Looks just like you'd expect.
- Observes residual tensile stresses in the rail (an explanation is given).

**Special remark** Now, let's look at papers on rolling contact including <u>two</u> bodies, preferably with AND without ALE. (How is two-body contact without ALE done?)
**Ibrahimbegovic2003**

- In short: non-linear dynamics of flexible systems with constraints (large deformations)
- Rolling contact example exemplifying non-holonomic constraints. But very simple: rigid bodies, no slip.
- Lagrangian kinematics all the way, it seems.

**Amirouche1993**

- About dynamic analysis of rolling contact in flexible multibody systems. Large deformations.
- Intro mentions other rolling contact references and seems to hint at ALE implementations.
- Node–node contact model used.
- Eqs. of motion "presented in matrix form making use of Kane's equations and FEM". Chapter 2 contains a derivation. I don't understand a thing.
- According to Lesser1992, Kane's eqs. are "equations of motion for systems of constrained particles and rigid bodies". So how does it apply to deformable bodies?
- **Very** coarse mesh used in the wheel–rail rolling contact numerical example.

**Arnold1984**

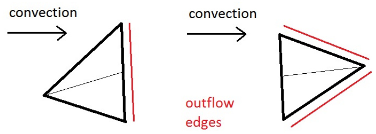
- Proof that the MINI element (linear shape functions + bubble function) satisfies the inf-sup condition for the Stokes problem (coupled velocity-pressure).

**BrezziMarini1998**

- Residual-free bubbles were "recently introduced" in 1997.
- Great description of "layers" (boundary and internal) in which the solution changes abruptly due to convection dominated flow. Also on how a Galerkin approach with a mesh that is too coarse to resolve these layers leads to numerical oscillations spreading all over the domain. WHY?? (see DoneaHuerta2003book?)
- On the relation between SUPG and bubble functions: the problem of choosing $\tau$ can be transferred to the problem of choosing a suitable bubble space, since $\tau$ can be expressed exactly as soon as the latter has been done.

Q If SUPG and bubble function method are equivalent, why do bubbles (but not SUPG) work in my case (especially since I choose the extra node $P$ in a fairly ad hoc way)? Hmm, bubbles *do* work better than SUPG in this paper, see next Q.

- Paper is dedicated to approximationg $b_k$, the function involved in the exact expression for $\tau$.
- Strategy: Look for an approximation to $b_k$ in the shape of a pyramid (i.e. look for a solution to the differential equation (35), which governs $b_k$, in the shape of a pyramid). Unknowns: position of $P$ and height of pyramid.
- Method used: Choose $P$ along the thin line in the triangles in the figure (the thin line can be chosen parallel to the convective velocity field as well). Note that in the left figure, there is one outflow edge, while in the other there is two. The thin line goes from either the corner opposite the only outflow edge or from the corner in between the two outflow edges, and to the middle of the opposite side.




Q Examples show how SUPG is over-diffusive compared to bubbles and refined Galerkin. But I wonder how coarse Galerkin would look. Over-diffusive (damped) or just oscillatory?

- They don't seem to mention how $\tau$ is chosen in the SUPG method they use in the examples. Is there a standard choice?

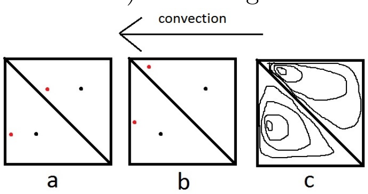
Q Can it be that the pure over-diffusion that happens in my case (i.e. no oscillation) is due to the absence of boundary layers (since the convection streamlines are all closed: no outflow boundary).

- One more thing: this paper doesn't assume that convection dominates (in that case, finding a suitable bubble space is allegedly easy), but offers a method for the general case.

**BrezziHauke2003**

- Introduction:
  - First paper on RFB (one I can't get) is Brezzi, Russo: "Choosing bubbles for advection–diffusion problems".
  
  Q Should I talk about "advection" rather than "convection"?
  - Their point was to find a suitable value for $\tau$.
  - Stabilization (or removing the need for it, rather) by proper choice of grid!
  - Current paper merges the above and bubbles.
- RFB: The bubble space is $H_0^1(K)$, i.e. all functions with a continuous first derivative, with the element $K$ as support and zero at $\partial K$.
- Origin of the name Residual Free!!!
- Elaborated: In the RFB approach, the approximation is enriched by a bubble function in each element, taken from the space $H_0^1(K)$. This enriched basis function ($V_L + V_B$) is capable of exactly satisfying the governing equation (the strong form) in each element ($\mathcal{L}u = f$, say), hence "Residual free".
- Interesting observation that if bubbles are the same type of function as the approximation (e.g. p.w. linear), then the process of adding extra nodes locally in each element and condensing is really just the same thing as "smart" mesh refinement (since condensation – being optional – doesn't change the mathematical nature of the procedure).

Q Maybe I don't get good effects using bubbles in the rail because the nodes there are incorrectly placed? How would the exact bubbles look? In the figure below. a) shows how I do it in Paper 2–4: placement of the extra node (red dot) along a line that is parallel with the convection stream line, and that intersects the centroid (black dot) of the triangle. b) How I perhaps should place it instead, which is based on c) which I guess is some kind of solution to the convective heat equation across the triangle.




**MouradDolbow2007**

- Not particularly relevant. Also, I don't really understand what is meant by "embedded interfaces".
- They admit that their choice of bubble space is naive. I identified with that.

**Zeid1981**

- Quite sophisticated FEM-based contact formulation from a LONG time ago: friction, rigid rail, "Galilean" ($\Leftrightarrow$ Eulerian I guess) transform, unsure of if the contact formulation is more like PM or LMM, purely mechanical case.
- I don't understand what the Galilean transform does.

**Padovan1987**

- Part 1/3 of this series of papers.
- Very sophisticated wheel–rail contact model: 3D, simulation of bumps/holes, viscoelasticity + all of what was in Zeid1981. Still only mechanical though.

R Search for keywords "thermomechanical rolling contact" in the next search for papers.

- Coriolis acc. included in formulation. Do I have it in mine? Probably, but which term is it?
- This paper contains field equations, material model and FEM formulation.

**Kennedy1987**

- Part 2/3 of this series of papers.
- Contains 3D formulation, shell formulation, contact formulation (in 3D), and numerical examples.
- Purely mechanical, so high speeds not surprising. Rigid ground when included.
- Same Galilean transform here as in Zeid1981?
- Pantographing mentioned: it is the antenna-like assembly of thin metal "sticks" that connects e.g. trams to the power lines above them.

**Nakajima1987**

- Part 3/3 of this series of papers.
- When they talk about "slip–stick" situation is these papers, do they mean total slip/total stick or mixed, as in the examples in my last two papers?

R Having a rigid rail would enable me to implement a non-smooth contact surface that wouldn't be as "cheating" as the one in paper 1 for instance.

- Contains contact impact methodology and numerical examples, involving rolling over bumps (triangular protrusions) and holes (triangular depressions)

**ZavariseWriggers1992_2**

- Introduction:
  - Constant penalty parameters (for mechanical/thermal contact) not enough to accurately model contact behaviour.
  - Micro-geometry important to take into account, especially (?) for macroscopically smooth surfaces. Current paper will do this.
  - Clean surfaces, no friction assumed (no rolling contact of course).
- Thermal contact resistance is modelled as three resistors in parallel: these correspond to thermal resistances due to spots, gas and radiation.
- Relations for $c_i = 1/R_i$ for spots and gas derived. They depend on microgeometrical considerations such as real contact area (dependent on $p$), cavity size (dependent on $p$) and statistical parameters.
- Comparison with experiments gives unknown parameters. They note: this is no ordinary curve fitting that only gives a model applicable to a narrow set of situations, but a calibration of a sophisticated theoretical model.
- FEM residuals are consistently linearized so that Newton's method can be used.
- Model is concluded to give high precision and a detailed understanding of mechanisms in the contact zone.
- No elaboration on mechanical contact formulation, they probably just use a constant penalty parameter, as in ZavariseWriggers1992.

**ZavariseWriggers1992**

- Introduction:
  - They make a point about how primitive the constant contact conductivity models are in comparison with theirs.

  - Another paper about roughly the same thing as in ZavariseWriggers1992_2: Development of a thermomechanical contact element based on material laws representing "true" contact mechanisms.

- Frictionless contact, clean (oxide-free) surfaces assumed (allegedly complicated to treat non-clean surfaces). Frictional formulation under construction (said in conclusions).
- Uses same 5-node contact element that I do.
- Constant normal stiffness used! They almost apologize for it, see p. 771 "we have focussed [sic] our interest more on the thermomechanical coupling".

**SchreflerZavarise1993**

- More of the same (microgeometrical thermomechanical contact formulation).
- Nice passage in the introduction about required accuracy of the contact formulation depending on implementation.
- Much more detailed/sophisticated mechanical contact formulation here than in the 2 previous papers.

**WriggersZavarise1993_2**

- Pure micromechanics here (no thermal contact).
- No friction, small displacements, nodal contact formulation.
- Augmented Lagrangian technique used.

**WriggersZavarise1993**

- Node-to-node formulation. No friction.
- Microscopic statistical interface laws both for mechanical & thermal problem.
- Remark 1 on p. 50 discusses element area-dependent normal mechanical penalty parameter.
- Interfacial thermal conductivity depends on pressure.

**WriggersMiehe1994**

- The paper presents a microgeometrical contact interface model for thermomechanical frictional contact.
- Contact geometry described in detail.
- Sophisticated interface laws for normal contact stress and contact heat flux given. Seems like no mention of frictional heat generation? Slip is split into an elastic part and a plastic part.
- "The resistance agains heat transfer is mainly due to the low percentage of physical surface area which is really in contact."
- Thermoelasticity constitutive response assumed.
- Gough–Joule coupling effect included (no mention about neglecting it).
- A staggered scheme (problem split into "a purely mechanical subproblem (M) at frozen temperature" (called a "predictor") and "a purely thermal subproblem (T) at frozen configuration (called a "corrector")). The algorithm is said to be identical to the one suggested in Argyris and Doltsinis [16].
- Numerical examples provided. Most of them feature a thermoelastic body pressed against a rigid but heat-conducting body. Idealized thermal isolation or fixed temperatures are assumed at surfaces.

**Nackenhorst2004**

- General summary: Lays out the mathematical foundations for elastic, transient rolling contact based on an ALE kinematical description. The kin. descr., balance laws, weak form, rolling contact kinematics are discussed + FE approach of stationary rolling including linearizations.
- Introduction:
  - ALE-like methods used before (goes back to Padovan, Oden&Lin). Inelastic material behaviour has even been addressed in the ALE context (engineering approaches relying on structured meshes I think (references [10], [29])) or the Streamline Upwind-based method of LeTallec and Rahier. However, Nackenhorst says, previous works do not handle the contact problem stringently, and don't provide a sound mathematical basis for inelastic or rate dependent material behaviour.
- He uses 3 domains. All use ground-fixed coordinate systems! His no. 1 = my no. 1; his no. 3 = my no. 4; his no. 2 is my no. 2 but with a ground-fixed coord. syst; his no. 3 is my no. 4 but with a ground-fixed coord. syst! I.e. his $\boldsymbol{w}$ (my $\bar{\boldsymbol{v}}$) contains a translational part! Despite the above, his <u>displacements</u> are the same as mine.
- His $\boldsymbol{w}$ is called the "guiding velocity". *His* "convective velocity" is Grad$\boldsymbol{\phi} \cdot \boldsymbol{w}$ which I'll have to agree is the true convective velocity).

Q How do we handle mass balance? Just by $\hat{\rho} = \rho J$? Aha, we assume homogeneity.

- He performs a symmetrization by partial integration (p. 4306).
- I actually think he forgets a term in eq. (28).
- He computes the term involving $\hat{\boldsymbol{n}} \cdot \boldsymbol{w}$! For perfectly round boundaries, this term vanishes. I therefore neglect it. Nackenhorst mentions that it is best to include it, however, since it has a real influence in discretized geometries. Neglecting it introduces an error.
- In discussion of FE formulation, he restricts the discussion to stationary rolling contact and a rigid rolling surface.
- Unsure of whether he uses a staggered solution procedure or if he just computes successively refined start guesses, like I do (probably the former, based on later papers).

**ZiefleNackenhorst2007**

- Introduction:
  - So Oden and Lin and Padovan (and Zeid I guess) are the first examples of FEM being used for rolling contact. They used a relative kinematics approach or whatever it's called. (I guess it is impossible to model rolling contact using FEM using a purely Lagrangian description.) ALE is more general though.
  - "[...] the history of particles which initially got into contact has to be traced to evaluate the friction law". Why? Aren't slip velocities enough? "A common procedure [regarding treatment of frictional contact within an ALE context] is the penalization of the slip velocities which are computed directy within the relative kinematic framwork". Is that what I'm doing?. "A further complication arises from the fact that by this approach the tangential contact traction is not computed directly from a constitutive law". I don't understand this, I thought that was what I was doing.
- The slip distances can be computed from the slip velocities by temporal integration, which can be exchanged for spatial integration due to the fact that time derivatives are exchanged for spatial derivatives in the convective description (is this true for the transient case as well?)
- Slip distances introduced as extra variables.
- "Once [the slip distribution has been obtained], the standard concepts for the treatment of frictional contact problems can be used, see eg. [18] [a paper by Wriggers].". Why is the slip distance (undifferentiated) needed in a friction law? How is "slip distance = 0" a valid stick condition?

**ZiefleNackenhorst2007_2**

- Introduction:
  - "First approaches on the numerical treatment of rolling contact by finite element methods based on rather simple relative kinematic formulations...". Reference given to Oden and Lin.
  - Check out reference 6 (Faria et. al.). I supposed to mention ALE (but I don't find any explicit reference to it, not by that name anyway).
  - Claims that ABAQUS can handle treatment of inelastic material parameters in an ALE-context. I didn't find anything on this though. Perhaps they call it something other than ALE.
  - "No sound mathematical basis for the treatment of inelastic material properties within the ALE-framework is available so far, which for example enables for error estimation based on computed results. This contribution is aimed to close these gaps".
- Insight: I should know the physical interpretation of all my terms.
- An additional update procedure for the internal variables has to be provided. This requires tracing the material particles as they move through the fixed FE mesh. This problem is well known in fluid mechanics.
- Insight from the above: Any advection-related problem of the ALE approach can probably be found (as well as its solution) in fluid mechanics.
- In the ALE context, time derivatives in the advection equation are replaced by spatial derivatives (at least in the stationary case).
- Perhaps "convection": the mathematical concept and "advection": specifically referring to the transport of physical quantities.
- The "advection equation" is solved to update the internal variables.
- Different numerical schemes and associated stabilization methods are compared. This is said: "The upstream method is stable, but suffers strongly under diffusion effects". I'm wondering if these are the same as those we observe in paper 2. Perhaps not, since this is 1D.
- Overshoot-effects result from differences between inflow and the outflow within one element.
- A fractional-step method (a staggered approach) is used to incorporate the solution of the advection equation in the FE-problem: 1. Solve nonlinear ALE rolling contact problem. 2. Smooth out internal variables for a $C^0$-smooth representation. 3. Solution of the advection problem by TDG-schema.
- Question regarding part 2 above: Isn't information lost as the internal variable data is smoothed out in coarser regions of the mesh?
- As material particles are convected through the contact region, the contact pressure distribution becomes asymmetric for medium convective velocities: for high conv. vel.: no relaxation, for low conv. vel.: total relaxation.
- One disadvantage of the ALE approach is additional computational effort when inelastic material properties have to be taken into account.

**ZiefleNackenhorst2008**

- Seems like a combination of the last two papers.
- Introduction:
  - ALE methods have been developed rapidly in other fields of engineering application, i.e. FSI and metal forming processes (see review paper [13]).

- Traditional engineering approaches to treatment of inelastic material properties involves integrating the history along predefined rings of integration points (e.g. Faria, Oden/Lin). However, this poses a problem with unstructured meshes and isn't built upon a sound mathematical basis. Le Tallec and Rahier [22] represents a first step towards something more mathematically sound.

- Insight: I should include the expression for $\boldsymbol{v}$ as well and discuss terms such as "guiding velocity" and actual convective velocity.

- "the accurate and efficient numerical solution of advection equations is still part of the current research in the field of computational fluid dynamics". Implying that ALE approaches can benefit from this research as well.

- "The TDG approach uses a coupled space-time-discretization instead of the common concept of semi-discretization" apparently.

- The smoothing out of the internal variables is said to be performed using a "super-convergent patch recovery technique".

- Regarding Figure 7 of the rolling kinematics. Isn't this a very idealized picture? Is it actually used?

- "The asymmetric of the contact pressure distribution in the mid-frequency range results in a torque which contributes to the rolling resistance."

- He uses the slip

$$S = \frac{V_T - V_C}{VT}$$

while I use

$$S = 2\frac{V_T - V_C}{VT + V_C}$$

- "If inelastic material properties are applied to the model, the computation time increases significantly". I'm wondering: How big is the increase in computational effort in the *Lagrangian* case when inelastic material props are applied?

- "An explicit scheme has been suggested for the computation of rolling contact problems of inelastic bodies, known as *Fractional-Step*-method from other established ALE-applications, because a fully implicit algorithm seems to be not computable for real life problems yet." Because of excessive computational effort, or what? So the use of the fractional-step method is by necessity rather than by choice?

- "concerning the frictional rolling contact problem a novel and fully implicit approach for the treatment of tractive rolling contact within the ALE framework for steady-state rolling has been presented". What does "implicit" mean? Requiring iterations?

- Ref [10] seems to be another good summary paper on ALE methods (by Donea et. al.).

**BrinkmeierNackenhorst2009**

- Paper focuses on "the transient dynamics response with respect to rolling noise prediction". A modal analysis will be used. The transient response due to excitation from different road surface textures will be investigated next.

- Quite applied, a little bit outside my area of interest.

**Suwannachit2013**

- Paper is about thermomechanical analysis of stationary rolling tires using the ALE approach. Large deformations are implemented along with a thermoviscoelastic constitutive model including internal dissipation.

- Introduction:
  - Internal dissipation is well known to be the main source of temperature rise in tires.

- – "The coupled momentum and energy balance equations are solved with an isentropic operator-split scheme".
  – The transport of inelastic variables during rolling is solved using the TDG method as described in ref. [26].

- Constitutive parameters are temperature-dependent.

- Viscous hysteresis decreases with increasing environmental temperature and even vanishes eventually. This is captured by their model.

- They don't want to solve the balance eqs. simultaneously, because the tangent operator is "large" and "nonsymmetrical" in that case. They use a staggered (fractional-step) approach. They here choose the isentropic operator-split method over the isothermal method because the former is unconditionally stable.

- Frictionless contact considered (and frictional heating is as a consequence not considered) "in order to focus on the heat generation caused by inelastic constitutive behaviour".

- On the TDG approach: "The major advantage of this approach is its control over numerical inaccuracies, like diffusion and oscillation".

- "although the global representation of balance equations is time-independent for stationary rolling, integration in time is still needed for the evolution of inelastic strains." A staggered computational scheme is used: i) mechanical subproblem. ii) nonlinear heat conduction, iii) transportation of internal variables.

- The Gough–Joule effect is neglected (both the thermoelastic and the thermoviscoelastic part of it).

- For a clear observation of the hysteretic heating, the heat transfer with environmental air is neglected. (perhaps I should have motivated my neglection of this effect better as well).

- Cool simulation: first inflation (statically), then setting the wheel into rotation, then bringing it into contact with the surface. I guess all this is done "statically" and not transiently.

- Consistent linearization is employed and quadratic convergence rate of both mech. and therm. subproblems are achieved (using Newton-Raphson iterations).

- "In our numerical studies, the stationary rolling state was obtained within only few revolutions". I don't get this, isn't everything stationary?