System identification of power grids

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I. MODEL

The power-grid dynamics can be modeled as a structure-preserving network of coupled first- and second-order Kuramoto oscillators. In this model, the generators dynamics are governed by the so-called swing equation,

$$\begin{aligned} \dot{\phi}_i &= \omega_i, \\ \frac{2H_i}{\omega_R}\dot{\omega}_i &= A_i - \frac{D_i}{\omega_R}\omega_i - \sum_{j=1, j\neq i} K_{ij}\sin(\phi_j - \phi_i - \gamma_{ij}) \end{aligned} \tag{1}$$

for $i = 1, \ldots, n_\mathrm{g}$, and the dynamics of load buses and generator terminals are described as first-order phase oscillators,

$$\frac{D_i}{\omega_\mathrm{R}}\dot{\phi}_i = P_i + \sum_{j=1, j\neq i}^{N} K_{ij}\sin(\phi_j - \phi_i), \tag{2}$$

for $i = n_\mathrm{g}+1, \ldots, N$, where n_g is the number of generators, n_l is the number of load buses, $N = 2n_\mathrm{g} + n_\mathrm{l}$ is the number of oscillators (nodes or buses), and $n = N + n_\mathrm{g}$ is the state-space dimension. To explain in more detail: note that each generator node is directly connected to a generator terminal (another node), which then connects to other nodes according to some interconnection matrix (the admittance matrix Y). Therefore, the power grid consists of n_g generators and its n_g terminals, as well as n_l. The dynamics of the generators are governed by 2nd-order equation (1) and the dynamics of the loads and generator terminals are governed by the 1st-order equation (2). See Fig. 1 for an example.

Here, $\phi_i(t)$ is the phase angle of oscillator i at time t relative to the frame rotating at reference frequency $\omega_\mathrm{R} = 2\pi f$ (where typically $f = 50$ or 60 Hz). The parameters H_i and D_i are the inertia and damping constants, respectively.

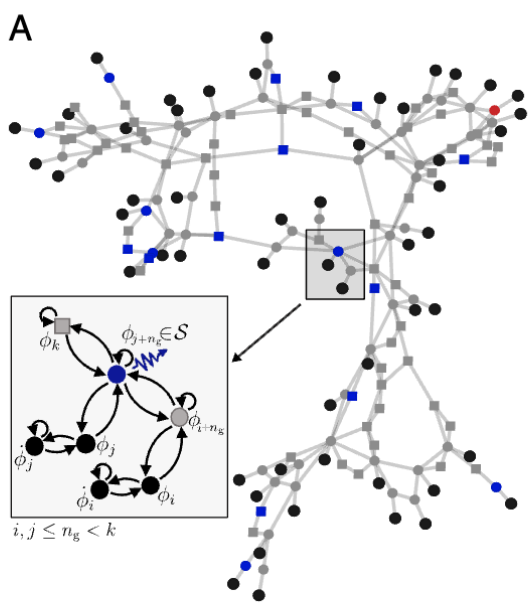

FIG. 1. Network structure of a power grid. Generator buses are represented by black circles, generator terminals by gray circles, and loads by gray squares. Ignore the inset and the colored nodes, they served other purpose in another publication. Note how each generator bus is directly connected to *one* generator terminal, which then interconnects to the rest of the grid. The structure of this system is encompassed by the (sparse) admittance matrix Y or the coupling matrix K.

In addition, K_{ij} is related to the admittance matrix Y and thus determines the interconnection between nodes (for instance, if there is no line connecting buses i and j, $K_{ij} = 0$). The power injection A_i is related to the power generation ($A_i > 0$) and power consumption ($A_i < 0$). The phase shift γ_{ij} is related to the admittance of the line (i, j).[1]

Together with the equations above, the power grids follow the conservation law (power flow) determined by the algebraic equations:

$$\begin{aligned} P_i &= \sum_{j=1}^{N} |V_i V_j Y_{ij}| \sin(\phi_i - \phi_j - \gamma_{ij}), \\ Q_i &= -\sum_{j=1}^{N} |V_i V_j Y_{ij}| \cos(\phi_i - \phi_j - \gamma_{ij}), \end{aligned} \tag{3}$$

for $i = 1, \ldots, N$, where P_i is the active power, Q_i is the reactive power, and V_i is the voltage at bus i (and is assumed to be constant). The matrix entry Y_{ij} is the admittance of the transmission line interconnecting buses (i, j). At steady-state, $\sum_i P_i = 0$.

II. DATASET

The MATPOWER case 3 consists of 6 buses: 2 generators (and its 2 generator terminals) plus 2 loads. The dataset contains the true values of matrices K, Y and γ, which are assumed to be *unknown*. The data file 'timeseries.csv' contains temporal data of the measured variables in a power grid: first column is itme, next columns are all phases ϕ_i, then the generator's frequencies ω_i, and finally the active powers P_i. These quantities are plotted in Fig. 2.

The vector $\phi_(t)$ is sorted in the same order: first the generator buses, then the generator terminals, and finally the loads. Matrices K and Y (also provided in the data files) follow the same order. The data file 'staticparams' the static parameters of the model: V_i, A_i, D_i, H_i and ω_R.

The numerical setup is very simple: every 2s, before the system reachers complete steady state, a perturbation to the system state is inputted, and the system undergoes a transient. This is not the most realistic case, but I guess it is a good start to test the identification method. If it works, I'll invest more time in a more realistic scenario.

[1] *I did not set γ to zero.* Setting it to zero has some additional complications that prevent the use of real datasets (we would have to fabricate some dataset that works), so I'd suggest not setting it to zero; however values are typically very low, less than 10 degrees.

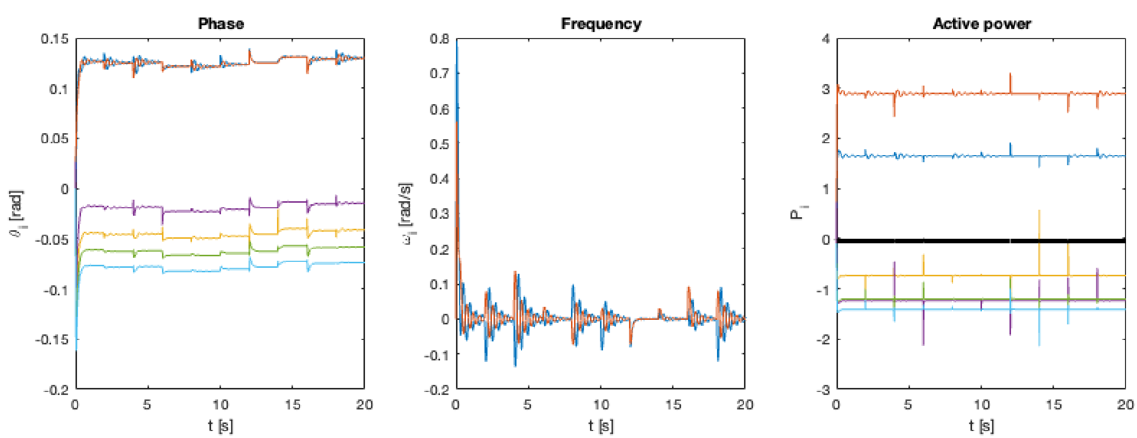

FIG. 2. Phase, frequency, and power as a function of time. The black solid line represents $\sum_i P_i(t)$ (which approaches 0 close to steady state). I'll try to get around this problem meanwhile.