Lab23 - optimalization labolatory, Pawel Drapiewski 29.05.2018 r.

Solve matura excercise of maximal tapeze area describe under y=2-1/2*x^2 function

Solution made using KKT

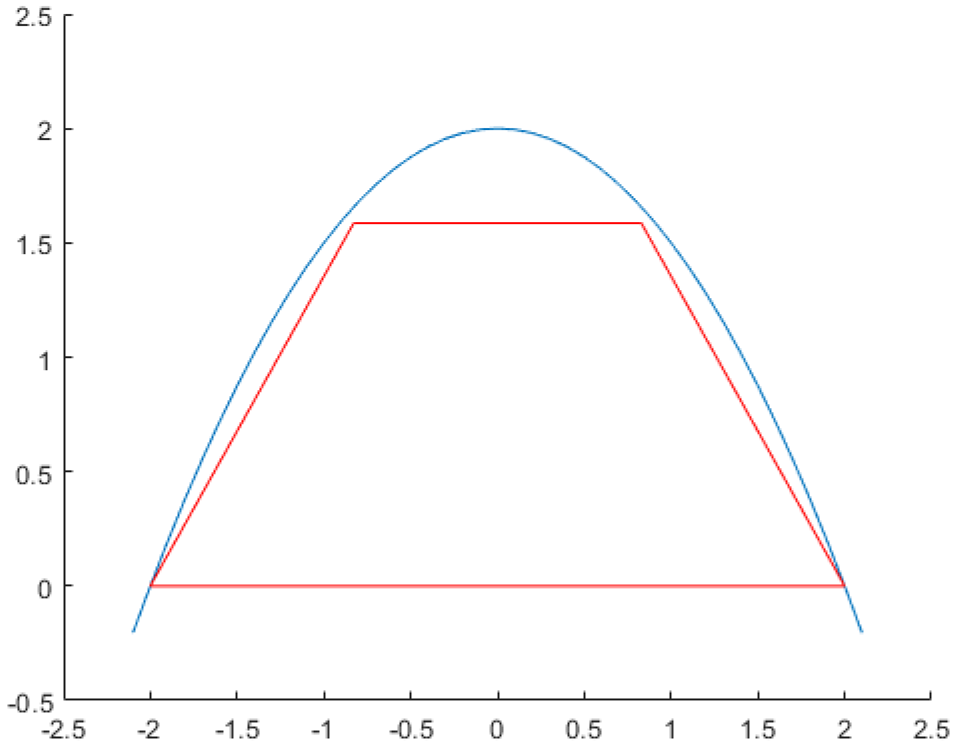
```
clear all, close all;

% Solution is written on the paper. Attatched below.

B = [0.82842712 1.5857865];
x = -2.1:0.1:2.1;
y = 2 - 1/2 * x.^2;

figure (1)
hold on;
ylim([-0.5 2.5])
plot(x, y);
% draw trapeze bounds lines
plot([-B(1) B(1)], [B(2) B(2)], 'r')
plot([-2 -B(1)], [0 B(2)], 'r')
plot([2 B(1)], [0 B(2)], 'r')
plot([-2 2], [0 0], 'r')
%plot()
hold off;
fprintf("Coords of found B points are: x=%d y=%d\n", B(1), B(2));
```

```
Coords of found B points are: x=8.284271e-01 y=1.585786e+00
```



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Podejście 3

$$\max \; xy + 2y$$

s.t.:

$$y = 2 - \frac{1}{2}x^2$$

$$\cancel{y \geq 0}$$

$$-y < 0$$

$$L(x, y, \lambda_1, \lambda_2) = xy + 2y + \lambda_2(0 + y) + \lambda_1\left(2 - \frac{1}{2}x^2 - y\right) =$$

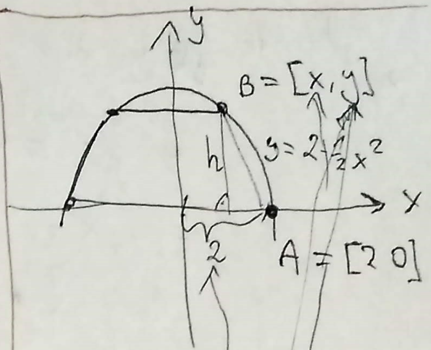
$$= xy + 2y + \lambda_1\left(-\frac{1}{2}x^2 - y + 2\right) + \lambda_2 y$$

c1:

$$\frac{\partial L}{\partial x} = y - \lambda_1 x = 0$$

c2:

$$\frac{\partial L}{\partial y} = x + 2 - \lambda_1 + \lambda_2 = 0$$



$$\max(P_\square)$$

$$P_\square = \frac{(a+b)}{2} \cdot h$$

$$a = 2 \cdot 2 = 4$$

$$b = 2 \cdot x$$

$$h = y$$

$$P_\square = (2 + x)y$$

complexity constraint 1:

$$\lambda_1\left(-\frac{1}{2}x^2 - y + 2\right) = 0$$

complexity constraint 2:

$$\lambda_2 y = 0$$

<u>scenario 1</u>: both constrains active

$$\left[\begin{array}{l} -\frac{1}{2}x^2 - y + 2 = 0 \\ \lambda_1 > 0 \end{array}\right. \wedge \quad \begin{array}{l} y = 0 \\ \lambda_2 > 0 \end{array}$$

$x = 2$ and $x = -2$

$$y = 2 - \frac{1}{2}x^2$$

$$0 = 2 - \frac{1}{2}x^2$$

$$\frac{1}{2}x^2 = 2$$

$$x^2 = 4$$

$$|x| = 2$$

$$x = 2 \vee x = -2$$

cond 1:

$$\lambda_1\left(-\frac{1}{2} \cdot 4 - 0 + 2\right) = 0$$

$$\lambda_1(-2 + 2) = 0$$

$\lambda_1 = 0$ can't be, because of

<u>scenario 2</u>: first slack, second active

$$\left[\begin{array}{l} -\frac{1}{2}x^2 - y + 2 > 0 \\ \lambda_1 = 0 \end{array}\right. \wedge \quad \begin{array}{l} y = 0 \\ \lambda_2 > 0 \end{array}$$

comp cond 1:

$$0 \cdot \ldots = 0$$

comp cond 2:

$$\lambda_2 \cdot y = 0$$

$$0 \cdot y = 0$$

$$y < -\frac{1}{2}x^2 + 2$$

$$0 < -\frac{1}{2}x^2 + 2$$

$$-2 < -\frac{1}{2}x^2 \; / \cdot (-2)$$

$$4 > x^2 \; / \sqrt{}$$

$$|x| < 2 \qquad x < 2 \wedge x > -2$$

c1:

$$0 - 0 \cdot x = 0$$

c2:

$$x + 2 - 0 + \lambda_2 = 0$$

$$x + \underset{\geq 0}{\lambda_2} = -2$$

$$\underset{>0}{\lambda_2} = \underbrace{-2 - x}_{>0}$$

$$-2 - x > 0$$

$$x < -2$$

$$x \in \emptyset$$

scenario 3: first active, second slack

$$\boxed{\begin{array}{ll} -\frac{1}{2}x^2 - y + 2 = 0 & \wedge \quad y > 0 \\ \lambda_1 > 0 & \lambda_2 = 0 \end{array}}$$

$$y = 2 - \frac{1}{2}x^2$$

$\Downarrow$ C1: $y - \lambda_1 x = 0$

$$2 - \frac{1}{2}x^2 - \lambda_1 x = 0$$

$$-\lambda_1 x = -2 + \frac{1}{2}x^2 \quad / : -x$$

$$\lambda_1 = \frac{2}{x} - \frac{x}{2}$$

C2: $x + 2 - \lambda_1 + \lambda_2 = 0$

$$x + 2 - \frac{2}{x} - \frac{x}{2} + 0 = 0$$

$$\frac{1}{2}x = \frac{2}{x} - 2 \quad / \cdot x$$

$$\frac{1}{2}x^2 + 2x - 2 = 0$$

$$\Downarrow$$

$$x = 2(\sqrt{2}-1) \text{ or } x = -2(1+\sqrt{2})$$

$$x = 0.82842712$$

$$y = 2 - \frac{1}{2}(0.828427)^2 = 1.5857865$$

$\wedge$ var 1 ✓

scenario 4: both slack

$$\boxed{\begin{array}{ll} -\frac{1}{2}x^2 - y + 2 > 0 & \wedge \quad y > 0 \\ \lambda_1 = 0 & \lambda_2 = 0 \end{array}}$$

$$y < 2 - \frac{1}{2}x^2$$

C1: $y - \underbrace{\lambda_1}_{=0} x = 0$

$$y = 0$$

$$0 < 2 - \frac{1}{2}x^2$$

(solution on previous page)

$$x < 2 \ \wedge \ x < -2$$

C2: $x + 2 - \underbrace{\lambda_1}_{=0} + \underbrace{\lambda_2}_{=0} = 0$

$$x + 2 = 0$$

$$x = -2$$

$$\Rightarrow x = \emptyset$$

Summary

Only one scenario had result. So the solution is $x = 0.82842712$

$y = 1.5857865$