Lab05 - Rocket fuel optymalization, Pawel Drapiewski 11.03.2018 r.

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In this problem we will be searching for the most optimal rocket trace, where there will be the smallest fuel amount burned and the rocket reach all points.

```
clear all;
close all;

% import data for the excersice from outter file
run thrusters_data.m
```

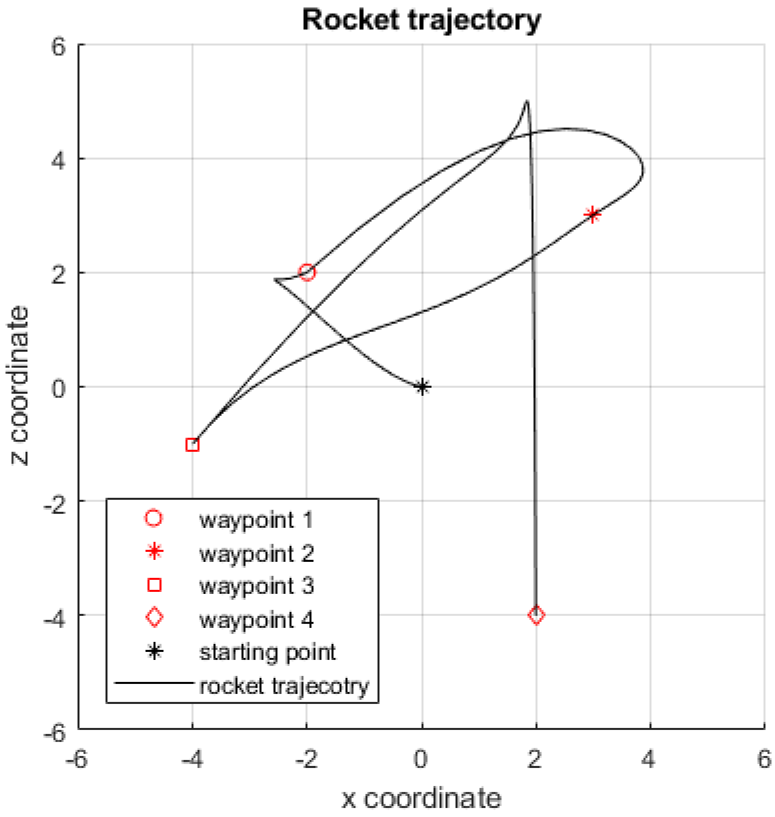
Part 1: No trust constrains

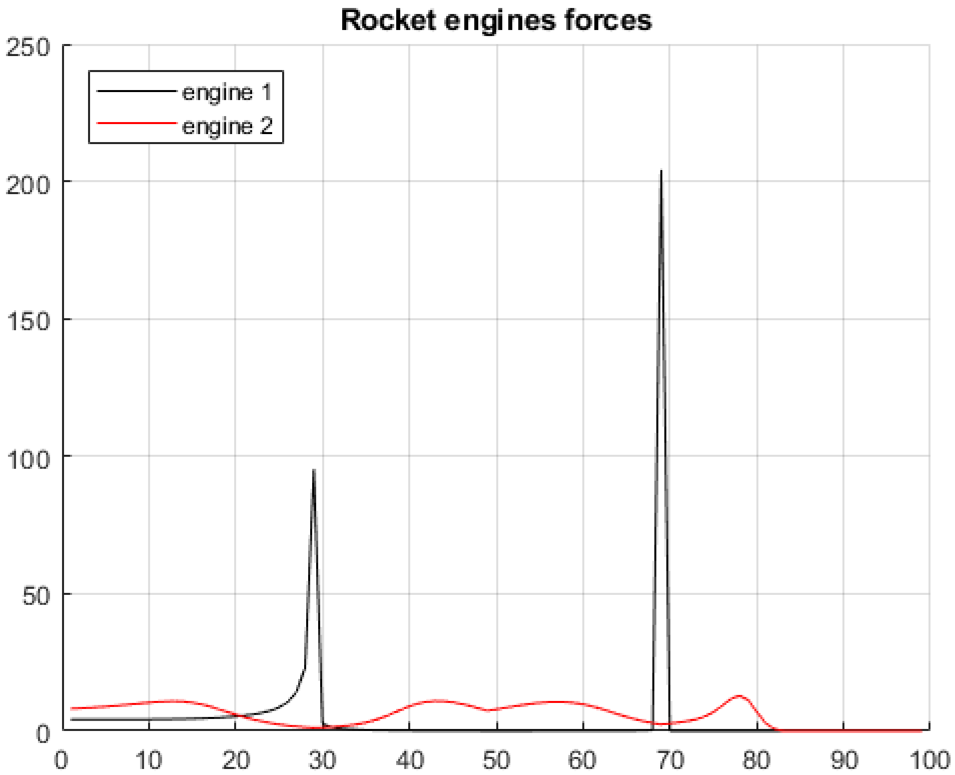
```
cvx_solver('sdpt3');
cvx_begin
variables engine1_power(K-1) engine2_power(K-1) force_vector_x(K) force_vector_y(K
minimize (sum(engine1_power)+sum(engine2_power))
subject to
    % where rocket starts
    points_x(1) == 0;
    points_y(1) == 0;
    % rocket wasn't moving on the start
    velocity_x(1) == 0;
    velocity_y(1) == 0;

    points_x(k1) == w1(1);
    points_y(k1) == w1(2);

    points_x(k2) == w2(1);
    points_y(k2) == w2(2);

    points_x(k3) == w3(1);
    points_y(k3) == w3(2);

    points_x(k4) == w4(1);
    points_y(k4) == w4(2);

    % engine power must be grather than 0!
    engine1_power >= 0; engine2_power >= 0;
    % bounds of the trajecory
    abs(points_x) <= pmax; abs(points_y) <= pmax;

    for k=1:K-1
     % Rocket model
     force_vector_x(k)== cos(theta1)*engine1_power(k) +
 cos(theta2)*engine2_power(k) + 0;
     force_vector_y(k)== sin(theta1)*engine1_power(k) +
 sin(theta2)*engine2_power(k) - m*g;

     velocity_x(k+1)==(1-alpha)*velocity_x(k) + (h/
m)*force_vector_x(k);
     velocity_y(k+1)==(1-alpha)*velocity_y(k) + (h/
m)*force_vector_y(k);

     points_x(k+1) == points_x(k) + h*velocity_x(k);
     points_y(k+1) == points_y(k) + h*velocity_y(k);
    end
cvx_end

total_fuel_consumption = (sum(engine1_power)+sum(engine2_power));
disp('Found minimal fuel consumption: ')
disp(total_fuel_consumption)


% draw the waypoints and rocket trajectory
figure(1);
hold on;
axis equal;
grid on;
title("Rocket trajectory");
xlim([-pmax-1 pmax+1]);
ylim([-pmax-1 pmax+1]);
xlabel("x coordinate");
ylabel("z coordinate");

point1 = plot(w1(1), w1(2) ,'ro');
point2 = plot(w2(1), w2(2) ,'r*');
point3 = plot(w3(1), w3(2) ,'rs');
point4 = plot(w4(1), w4(2) ,'rd');
point_start = plot(points_x(1), points_y(1), 'black*');
rocket_trajectory = plot(points_x, points_y, '-black');
legend([point1 point2 point3 point4 point_start
 rocket_trajectory],'waypoint 1','waypoint 2','waypoint 3','waypoint
 4', 'starting point', 'rocket trajecotry','Location','southwest');
hold off;

% draw plot of enigines trust over the time
figure(2);
hold on; grid on;
title('Rocket engines forces');
engine1 = plot(engine1_power, 'black-');
engine2 = plot(engine2_power, 'red-');
legend([engine1 engine2],'engine 1','engine
 2','Location','Northwest');
```

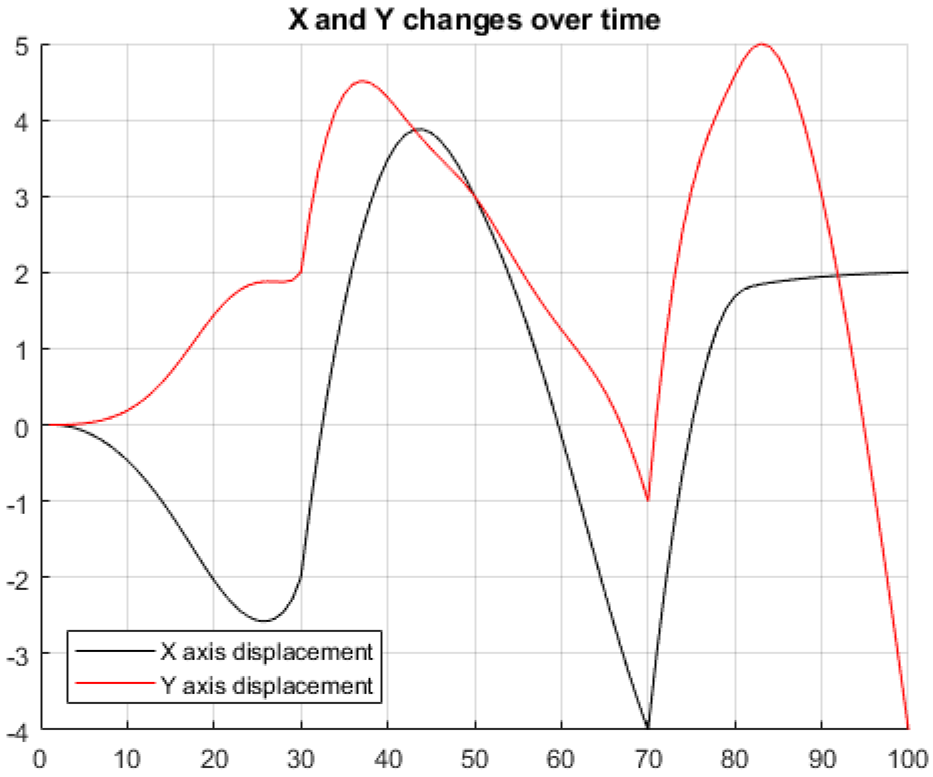
```
hold off

% Draw plot of  the X and Y position
figure(3);
hold on; grid on;
title('X and Y changes over time');
points_x = plot (points_x, 'black-');
points_y = plot(points_y, 'red-');
legend([points_x points_y],'X axis displacement','Y axis
 displacement','Location','Southwest');
hold off
```

```
Calling SDPT3 4.0: 800 variables, 408 equality constraints
------------------------------------------------------------

 num. of constraints = 408
 dim. of socp   var  = 400,   num. of socp blk  = 200
 dim. of linear var  = 398
 dim. of free   var  =  2 *** convert ublk to lblk
*******************************************************************
   SDPT3: Infeasible path-following algorithms
*******************************************************************
 version  predcorr  gam  expon  scale_data
    NT      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj
 cputime
-------------------------------------------------------------------
 0|0.000|0.000|9.4e+01|3.1e+01|5.9e+06| 1.459619e+05  0.000000e+00|
 0:0:00| chol  1  1
 1|0.588|0.122|3.9e+01|2.8e+01|2.6e+06| 6.248028e+04 -7.286643e+03|
 0:0:00| chol  1  1
 2|0.886|1.000|4.4e+00|2.5e-01|3.5e+05| 1.038611e+04 -4.445138e+04|
 0:0:00| chol  1  1
 3|0.877|1.000|5.4e-01|7.5e-02|6.3e+04| 2.349045e+03 -3.169547e+04|
 0:0:00| chol  1  1
 4|0.750|0.794|1.3e-01|2.1e-02|2.1e+04| 1.484675e+03 -1.495913e+04|
 0:0:00| chol  1  1
 5|0.758|0.904|3.2e-02|2.7e-03|8.1e+03| 1.239459e+03 -6.014713e+03|
 0:0:01| chol  1  1
 6|0.788|0.972|6.9e-03|6.6e-03|2.9e+03| 1.148724e+03 -1.661217e+03|
 0:0:01| chol  1  1
 7|0.997|0.880|2.1e-05|2.2e-03|4.5e+02| 1.090408e+03  6.412857e+02|
 0:0:01| chol  1  2
 8|0.961|0.944|8.6e-07|1.3e-04|1.8e+02| 1.065187e+03  8.824841e+02|
 0:0:01| chol  2  2
 9|0.941|0.936|2.8e-07|8.6e-06|3.0e+01| 1.057253e+03  1.027106e+03|
 0:0:01| chol  1  1
10|1.000|0.741|1.0e-07|2.3e-06|1.2e+01| 1.056206e+03  1.044692e+03|
 0:0:01| chol  2  2
11|1.000|0.950|2.5e-08|1.4e-07|4.4e+00| 1.055632e+03  1.051257e+03|
 0:0:01| chol  1  1
12|0.951|0.872|1.2e-08|2.5e-08|1.0e+00| 1.055405e+03  1.054366e+03|
 0:0:01| chol  2  1
```

```
13|0.980|0.982|4.6e-09|3.5e-09|1.9e-02| 1.055345e+03  1.055326e+03|
 0:0:01| chol  1  1
14|0.989|0.989|9.2e-10|1.8e-08|2.7e-04| 1.055344e+03  1.055344e+03|
 0:0:01| chol  2  2
15|0.991|0.989|1.3e-11|1.3e-09|5.4e-06| 1.055344e+03  1.055344e+03|
 0:0:01|
  stop: max(relative gap, infeasibilities) < 1.49e-08
-------------------------------------------------------------------
 number of iterations   = 15
 primal objective value =  1.05534376e+03
 dual   objective value =  1.05534375e+03
 gap := trace(XZ)       = 5.42e-06
 relative gap           = 2.57e-09
 actual relative gap    = 2.12e-09
 rel. primal infeas (scaled problem)   = 1.31e-11
 rel. dual     "        "       "      = 1.26e-09
 rel. primal infeas (unscaled problem) = 0.00e+00
 rel. dual     "        "       "      = 0.00e+00
 norm(X), norm(y), norm(Z) = 2.5e+02, 1.2e+02, 4.9e+00
 norm(A), norm(b), norm(C) = 1.8e+02, 1.1e+02, 1.5e+01
 Total CPU time (secs)  = 0.61
 CPU time per iteration = 0.04
 termination code       =  0
 DIMACS: 3.6e-11  0.0e+00  9.5e-09  0.0e+00  2.1e-09  2.6e-09
-------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +1055.34

Found minimal fuel consumption:
   1.0553e+03
```


Rocket trajectory
z coordinate
x coordinate
6
4
2
0
-2
-4
-6
-6
-4
-2
0
2
4
6
waypoint 1
waypoint 2
waypoint 3
waypoint 4
starting point
rocket trajecotry


Rocket engines forces
250
200
150
100
50
0
0
10
20
30
40
50
60
70
80
90
100
engine 1
engine 2



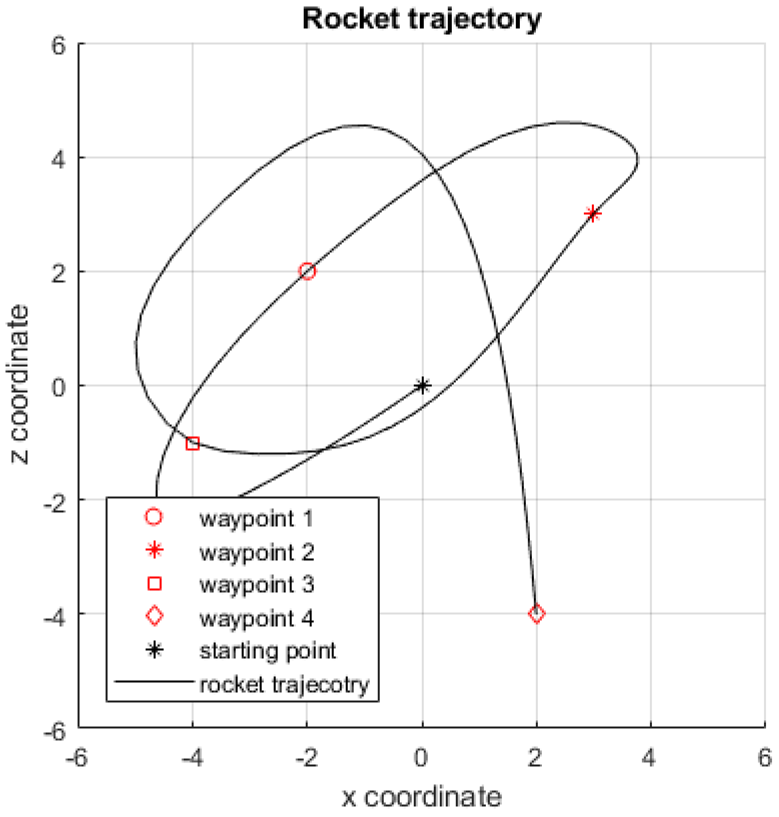
Part 2: Trust less than 20 for each engine

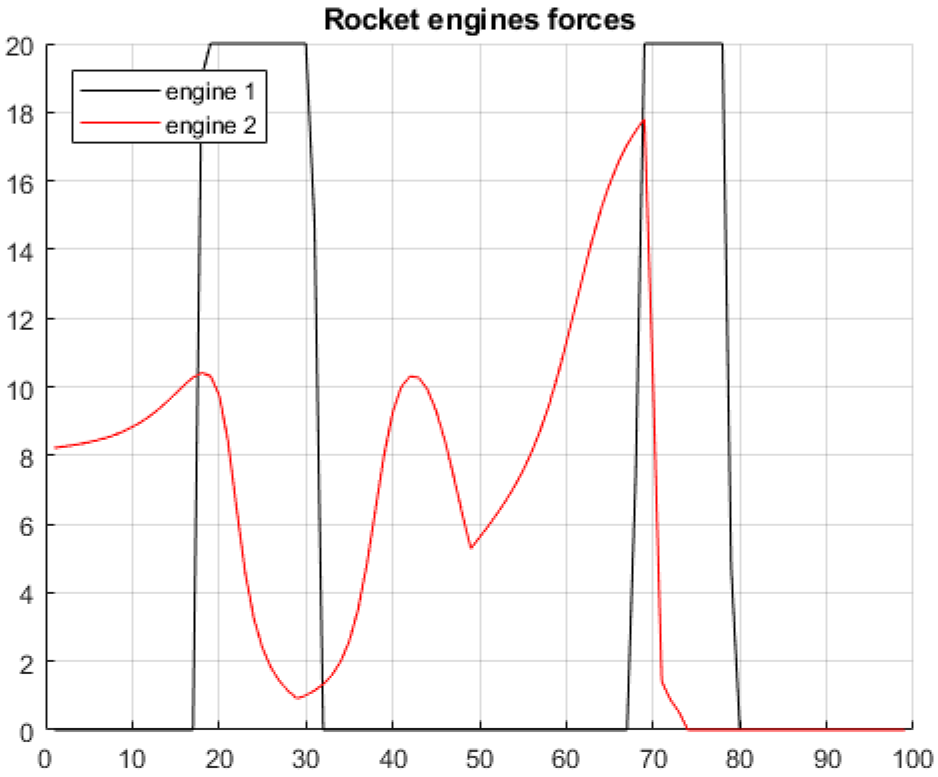
```
cvx_solver('sdpt3');
cvx_begin
variables engine1_power(K-1) engine2_power(K-1) force_vector_x(K) force_vector_y(K
minimize (sum(engine1_power)+sum(engine2_power))
subject to
    % where rocket starts
    points_x(1) == 0;
    points_y(1) == 0;
    % rocket wasn't moving on the start
    velocity_x(1) == 0;
    velocity_y(1) == 0;

    points_x(k1) == w1(1);
    points_y(k1) == w1(2);

    points_x(k2) == w2(1);
    points_y(k2) == w2(2);

    points_x(k3) == w3(1);
    points_y(k3) == w3(2);

    points_x(k4) == w4(1);
    points_y(k4) == w4(2);
    % engine power must be grather than 0!
    engine1_power >= 0; engine2_power >= 0;
    engine1_power < 20; engine2_power < 20;
    % bounds of the trajecory
    abs(points_x) <= pmax; abs(points_y) <= pmax;

    for k=1:K-1
     % Rocket model
     force_vector_x(k)== cos(theta1)*engine1_power(k) +
 cos(theta2)*engine2_power(k) + 0;
     force_vector_y(k)== sin(theta1)*engine1_power(k) +
 sin(theta2)*engine2_power(k) - m*g;

     velocity_x(k+1)==(1-alpha)*velocity_x(k) + (h/
m)*force_vector_x(k);
     velocity_y(k+1)==(1-alpha)*velocity_y(k) + (h/
m)*force_vector_y(k);

     points_x(k+1) == points_x(k) + h*velocity_x(k);
     points_y(k+1) == points_y(k) + h*velocity_y(k);
    end
cvx_end

total_fuel_consumption = (sum(engine1_power)+sum(engine2_power));
disp('Found minimal fuel consumption: ')
disp(total_fuel_consumption)


% draw the waypoints and rocket trajectory
figure(4);
hold on;
axis equal;
grid on;
title("Rocket trajectory");
xlim([-pmax-1 pmax+1]);
ylim([-pmax-1 pmax+1]);
xlabel("x coordinate");
ylabel("z coordinate");

point1 = plot(w1(1), w1(2) ,'ro');
point2 = plot(w2(1), w2(2) ,'r*');
point3 = plot(w3(1), w3(2) ,'rs');
point4 = plot(w4(1), w4(2) ,'rd');
point_start = plot(points_x(1), points_y(1), 'black*');
rocket_trajectory = plot(points_x, points_y, '-black');
legend([point1 point2 point3 point4 point_start
 rocket_trajectory],'waypoint 1','waypoint 2','waypoint 3','waypoint
 4', 'starting point', 'rocket trajecotry','Location','southwest');
hold off;

% draw plot of enigines trust over the time
figure(5);
hold on; grid on;
```

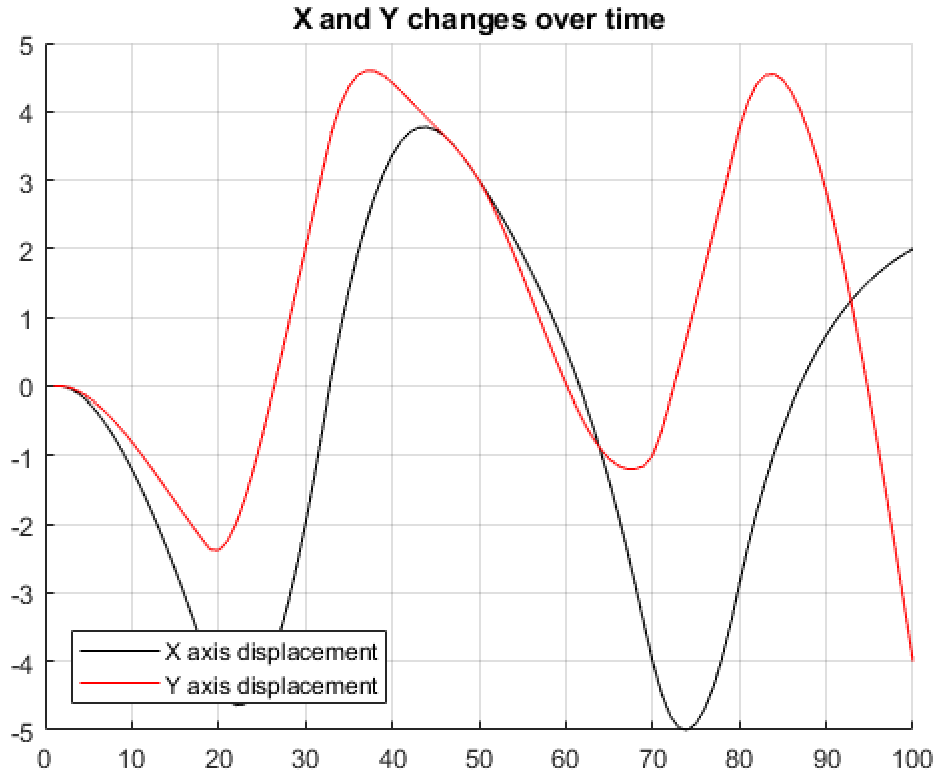
```
title('Rocket engines forces');
engine1 = plot(engine1_power, 'black-');
engine2 = plot(engine2_power, 'red-');
legend([engine1 engine2],'engine 1','engine
 2','Location','Northwest');
hold off

% Draw plot of  the X and Y position
figure(6);
hold on; grid on;
title('X and Y changes over time');
points_x = plot (points_x, 'black-');
points_y = plot(points_y, 'red-');
legend([points_x points_y],'X axis displacement','Y axis
 displacement','Location','Southwest');
hold off
```

```
Warning: The use of strict inequalities in CVX is strongly
 discouraged,
    because solvers treat them as non-strict inequalities. Please
    consider using "<=" instead.

Warning: The use of strict inequalities in CVX is strongly
 discouraged,
    because solvers treat them as non-strict inequalities. Please
    consider using "<=" instead.


Calling SDPT3 4.0: 998 variables, 606 equality constraints
------------------------------------------------------------

 num. of constraints = 606
 dim. of socp   var  = 400,   num. of socp blk  = 200
 dim. of linear var  = 596
 dim. of free   var  =  2 *** convert ublk to lblk
*******************************************************************
   SDPT3: Infeasible path-following algorithms
*******************************************************************
 version  predcorr  gam  expon  scale_data
    NT      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj
 cputime
-------------------------------------------------------------------
 0|0.000|0.000|9.3e+01|4.3e+01|1.3e+07| 1.786164e+05  0.000000e+00|
 0:0:00| spchol  1  1
 1|0.872|1.000|1.2e+01|5.0e-01|2.0e+06| 2.376211e+04 -1.281282e+05|
 0:0:00| spchol  1  1
 2|0.938|1.000|7.3e-01|1.5e-01|2.1e+05| 2.561084e+03 -1.072436e+05|
 0:0:00| spchol  1  1
 3|0.718|0.998|2.1e-01|1.5e-02|6.1e+04| 1.555192e+03 -4.199545e+04|
 0:0:00| spchol  1  1
 4|0.687|0.778|6.5e-02|4.5e-03|2.0e+04| 1.274367e+03 -1.547453e+04|
 0:0:00| spchol  1  1
 5|0.489|0.736|3.3e-02|1.4e-02|1.1e+04| 1.204352e+03 -7.994095e+03|
 0:0:00| spchol  1  1 
 6|0.756|0.527|8.1e-03|1.4e-02|6.9e+03| 1.138706e+03 -5.355988e+03|
 0:0:00| spchol  1  1 
 7|0.761|0.457|1.9e-03|9.0e-03|4.6e+03| 1.113686e+03 -3.392635e+03|
 0:0:00| spchol  1  1 
 8|1.000|0.957|2.8e-07|7.7e-04|5.9e+02| 1.092380e+03  5.070567e+02|
 0:0:00| spchol  2  2 
 9|1.000|0.950|8.5e-08|3.9e-05|2.2e+02| 1.068731e+03  8.494662e+02|
 0:0:00| spchol  2  2 
10|1.000|0.940|1.1e-07|2.3e-06|4.2e+01| 1.061178e+03  1.018998e+03|
 0:0:00| spchol  2  2 
11|0.718|0.941|5.1e-08|1.6e-07|2.0e+01| 1.059604e+03  1.039687e+03|
 0:0:00| spchol  2  2 
12|0.906|0.847|1.4e-08|3.7e-08|8.7e+00| 1.058483e+03  1.049811e+03|
 0:0:00| spchol  2  2 
13|0.171|0.434|9.6e-09|2.4e-08|8.0e+00| 1.058120e+03  1.050078e+03|
 0:0:00| spchol  2  2 
14|0.886|0.604|2.7e-09|1.2e-08|4.4e+00| 1.057683e+03  1.053313e+03|
 0:0:00| spchol  2  2 
15|1.000|0.952|4.5e-10|1.4e-09|2.2e+00| 1.057451e+03  1.055234e+03|
 0:0:00| spchol  1  1 
16|1.000|0.901|1.5e-11|3.4e-10|5.7e-01| 1.057237e+03  1.056665e+03|
 0:0:00| spchol  1  1 
17|0.927|0.983|8.1e-12|7.1e-11|1.4e-02| 1.057174e+03  1.057160e+03|
 0:0:00| spchol  1  1 
18|0.983|0.982|1.6e-11|2.1e-10|2.9e-04| 1.057170e+03  1.057169e+03|
 0:0:00| spchol  1  1 
19|0.989|0.989|8.0e-12|2.1e-11|6.1e-06| 1.057170e+03  1.057170e+03|
 0:0:00|
  stop: max(relative gap, infeasibilities) < 1.49e-08
-------------------------------------------------------------------
 number of iterations   = 19
 primal objective value =  1.05716958e+03
 dual   objective value =  1.05716957e+03
 gap := trace(XZ)       = 6.14e-06
 relative gap           = 2.90e-09
 actual relative gap    = 2.62e-09
 rel. primal infeas (scaled problem)   = 8.03e-12
 rel. dual     "        "       "      = 2.09e-11
 rel. primal infeas (unscaled problem) = 0.00e+00
 rel. dual     "        "       "      = 0.00e+00
 norm(X), norm(y), norm(Z) = 2.7e+02, 1.2e+02, 5.2e+00
 norm(A), norm(b), norm(C) = 1.8e+02, 3.0e+02, 1.5e+01
 Total CPU time (secs)  = 0.25  
 CPU time per iteration = 0.01  
 termination code       =  0
 DIMACS: 6.0e-11  0.0e+00  1.6e-10  0.0e+00  2.6e-09  2.9e-09
-------------------------------------------------------------------
 
------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +1057.17
```

```
Found minimal fuel consumption:
   1.0572e+03
```




Rocket engines forces
engine 1
engine 2
20
18
16
14
12
10
8
6
4
2
0
0
10
20
30
40
50
60
70
80
90
100


X and Y changes over time
X axis displacement
Y axis displacement
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
10
20
30
40
50
60
70
80
90
100

Part 3: Trust less than 15

```
cvx_solver('sdpt3');
cvx_begin
variables engine1_power(K-1) engine2_power(K-1) force_vector_x(K) force_vector_y(K
minimize (sum(engine1_power)+sum(engine2_power))
subject to
    % where rocket starts
    points_x(1) == 0;
    points_y(1) == 0;
    % rocket wasn't moving on the start
    velocity_x(1) == 0;
    velocity_y(1) == 0;

    points_x(k1) == w1(1);
    points_y(k1) == w1(2);

    points_x(k2) == w2(1);
    points_y(k2) == w2(2);

    points_x(k3) == w3(1);
    points_y(k3) == w3(2);

    points_x(k4) == w4(1);
    points_y(k4) == w4(2);

    % engine power must be grather than 0!
    engine1_power >= 0; engine2_power >= 0;
    engine1_power < 15; engine2_power < 15;
    % bounds of the trajecory
    abs(points_x) <= pmax; abs(points_y) <= pmax;

    for k=1:K-1
     % Rocket model
     force_vector_x(k)== cos(theta1)*engine1_power(k) +
 cos(theta2)*engine2_power(k) + 0;
     force_vector_y(k)== sin(theta1)*engine1_power(k) +
 sin(theta2)*engine2_power(k) - m*g;

     velocity_x(k+1)==(1-alpha)*velocity_x(k) + (h/
m)*force_vector_x(k);
     velocity_y(k+1)==(1-alpha)*velocity_y(k) + (h/
m)*force_vector_y(k);

     points_x(k+1) == points_x(k) + h*velocity_x(k);
     points_y(k+1) == points_y(k) + h*velocity_y(k);
    end
cvx_end

total_fuel_consumption = (sum(engine1_power)+sum(engine2_power));
disp('Found minimal fuel consumption: ')
disp(total_fuel_consumption)
```

```
% In that case, the result should be false because we can't find it
 for
% this constrains (less than 15)

Warning: The use of strict inequalities in CVX is strongly
 discouraged,
    because solvers treat them as non-strict inequalities. Please
    consider using "<=" instead.

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Calling SDPT3 4.0: 998 variables, 606 equality constraints
------------------------------------------------------------

 num. of constraints = 606
 dim. of socp   var  = 400,   num. of socp blk  = 200
 dim. of linear var  = 596
 dim. of free   var  =  2 *** convert ublk to lblk
*******************************************************************
   SDPT3: Infeasible path-following algorithms
*******************************************************************
 version  predcorr  gam  expon  scale_data
    NT      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj
 cputime
-------------------------------------------------------------------
 0|0.000|0.000|1.2e+02|4.3e+01|1.3e+07| 1.786164e+05  0.000000e+00|
 0:0:00| spchol  1  1
 1|0.849|1.000|1.8e+01|5.0e-01|2.4e+06| 2.785466e+04 -1.058592e+05|
 0:0:00| spchol  1  1
 2|0.946|1.000|9.7e-01|1.5e-01|2.1e+05| 2.606972e+03 -9.452570e+04|
 0:0:00| spchol  1  1
 3|0.709|1.000|2.8e-01|1.5e-02|7.0e+04| 1.573383e+03 -4.636439e+04|
 0:0:00| spchol  1  1
 4|0.591|0.761|1.2e-01|4.7e-03|2.9e+04| 1.317431e+03 -2.107729e+04|
 0:0:00| spchol  1  1
 5|0.376|0.706|7.2e-02|1.7e-03|1.8e+04| 1.248321e+03 -1.290063e+04|
 0:0:00| spchol  1  1
 6|0.530|0.475|3.4e-02|1.5e-02|1.3e+04| 1.183609e+03 -9.722901e+03|
 0:0:00| spchol  1  1
 7|0.342|0.833|2.2e-02|9.4e-03|8.3e+03| 1.162200e+03 -4.797431e+03|
 0:0:00| spchol  2  2
 8|0.236|0.501|1.7e-02|9.2e-03|7.4e+03| 1.150760e+03 -3.088114e+03|
 0:0:00| spchol  2  2
 9|0.355|0.580|1.1e-02|7.3e-03|5.6e+03| 1.136231e+03  1.326401e+03|
 0:0:00| spchol  2  2
10|0.170|0.052|9.1e-03|9.1e-03|4.8e+03| 1.131223e+03  6.309853e+03|
 0:0:00| spchol  2  2
11|0.037|0.905|8.8e-03|2.7e-03|6.5e+04| 1.130272e+03  2.423033e+06|
 0:0:00| spchol  2  3
```

```
12|0.381|0.382|5.5e-03|3.4e-03|7.8e+08| 1.196691e+03  1.175159e+09|
 0:0:00| spchol  3  3
13|0.001|0.933|5.4e-03|1.3e-03|4.6e+09| 1.111708e+03  6.798526e+09|
 0:0:00| spchol  2  2
14|0.232|0.932|4.2e-03|1.2e-03|1.2e+10| 1.187193e+03  3.965130e+10|
 0:0:00| spchol  2  2
15|0.208|0.918|3.3e-03|1.9e-03|2.0e+10| 1.151780e+03  7.457735e+11|
 0:0:00| spchol  3  2
16|0.782|0.902|8.6e-04|1.6e+00|1.5e+15| 1.105180e+03  9.219602e+14|
 0:0:00| spchol  3  3
17|0.001|0.905|8.6e-04|3.8e+00|3.7e+15| 1.181947e+03  2.227928e+15|
 0:0:00| spchol  2  2
18|0.201|0.905|6.9e-04|1.2e+01|6.4e+15| 1.103024e+03  5.157799e+15|
 0:0:00| spchol  2  2
19|0.389|0.835|4.2e-04|4.6e+01|5.7e+15| 1.138442e+03  1.721931e+16|
 0:0:00| spchol  2  2
20|0.176|0.918|3.5e-04|9.2e+02|2.0e+16| 1.138697e+03  3.374393e+17|
 0:0:00|
  sqlp stop: primal problem is suspected of being infeasible
-------------------------------------------------------------------
 number of iterations   = 20
 residual of primal infeasibility
 certificate (y,Z)      = 4.13e-14
 reldist to infeas.    <= 6.31e-18
 Total CPU time (secs)  = 0.22
 CPU time per iteration = 0.01
 termination code       =  1
 DIMACS: 2.0e-03  0.0e+00  7.0e+03  0.0e+00  -1.0e+00  6.0e-02
-------------------------------------------------------------------

------------------------------------------------------------
Status: Infeasible
Optimal value (cvx_optval): +Inf

Found minimal fuel consumption:
   NaN
```

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