# Source reconstruction

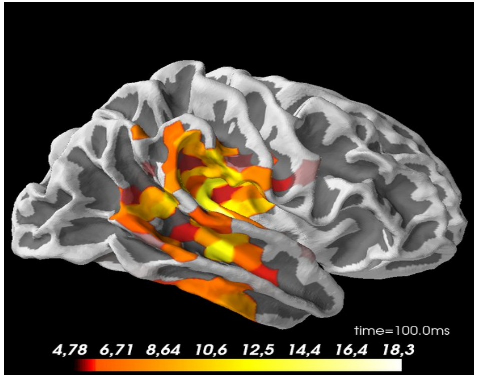
Advanced cognitive neuroscience
October 3 and October 5
2023

# Learning goals

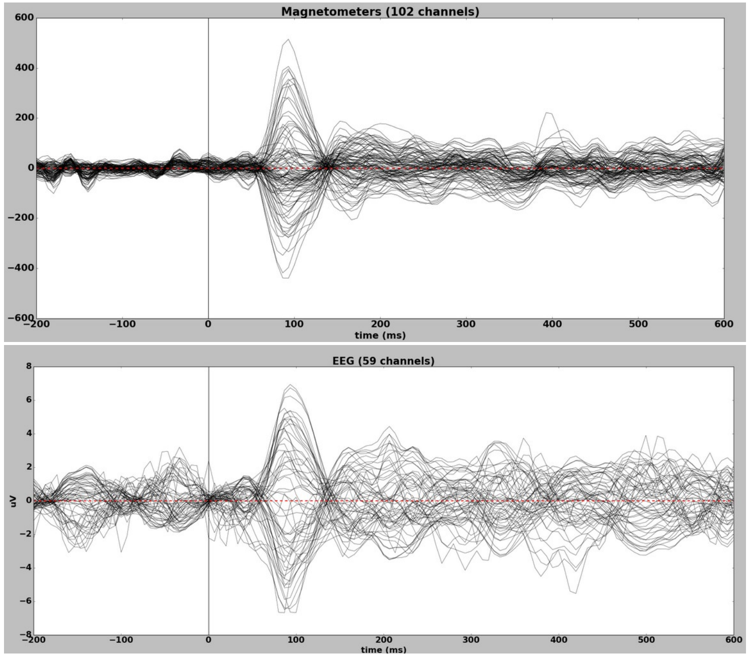
- Learning
  - what the ingredients of a forward model are
  - how the forward model links sources of the brain to sensors, electrodes or magnetic sensors
  - why the volume conductors needed for MEG and EEG respectively differ
  - what co-registration of MRI and MEG amounts to

# We want to go here:

Brain activation

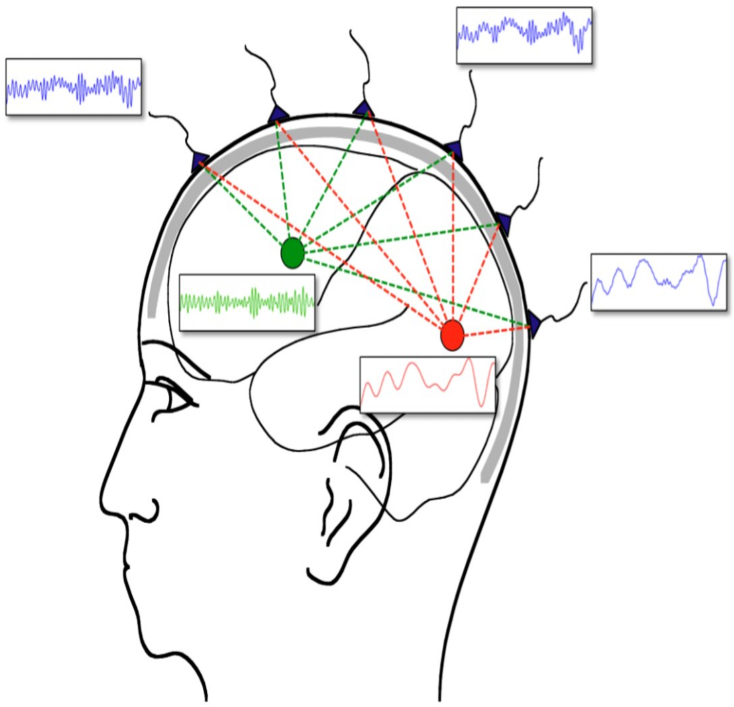



But we only have magnetic fields and scalp potentials outside the brain...



# Problem

Superposition of source activity

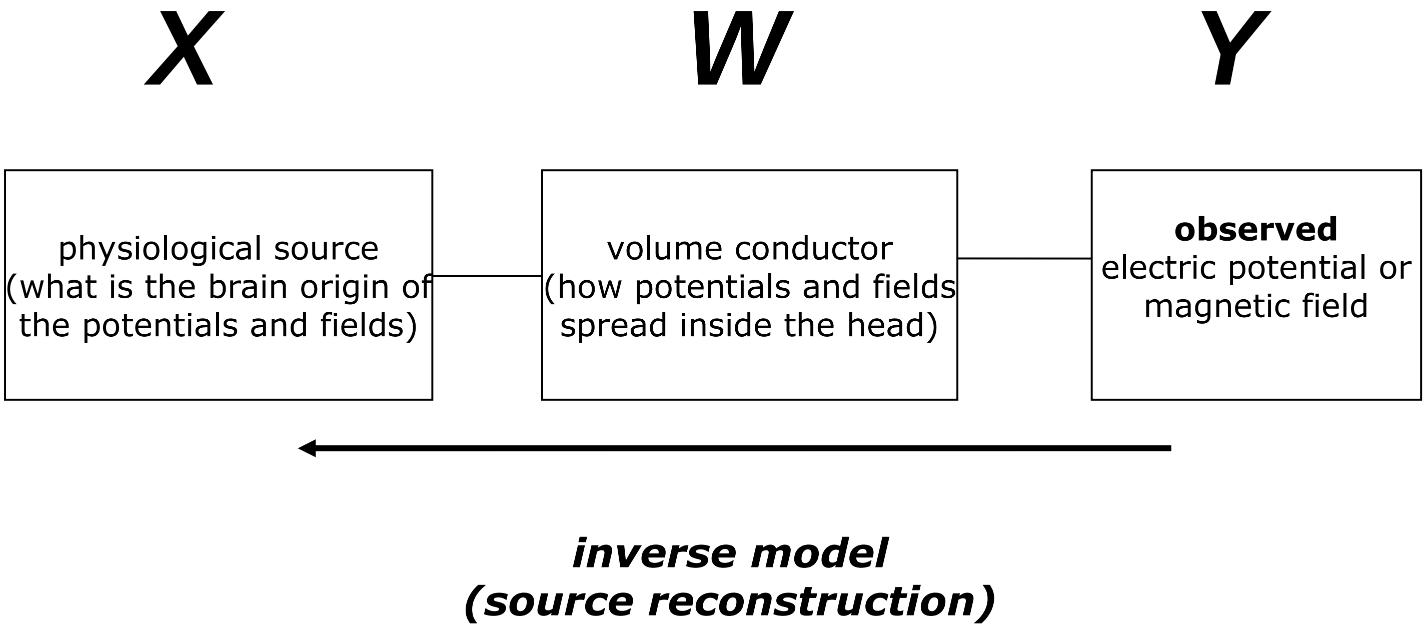


# The problem in a nutshell

$$\boldsymbol{Y} = \boldsymbol{W}\boldsymbol{X}$$

***Y***: the measured signal (magnetic field)
***X***: the neural generators
***W***: a weighting matrix (the leadfield)

# Inverse modelling



# Forward modelling

***forward model***

$$X \xrightarrow{\quad W \quad} Y$$

physiological source
(what is the brain origin of
the potentials and fields)

volume conductor
(how potentials and fields
spread inside the head)

***expected***
electric potential or
magnetic field

IMPORTANT

Without a plausible forward model restricting the solution space, an infinite number of inverse models could explain the data

# Ingredients for a forward model

- A source model *src*
  - Telling us *the origin* of brain activity
- A volume conductor *bem*
  - Telling us how the volume currents *spread* on their way to the sensors *bem solution*
- Sensor positions *info*
  - Telling us *where* the sensors are relative to the sources

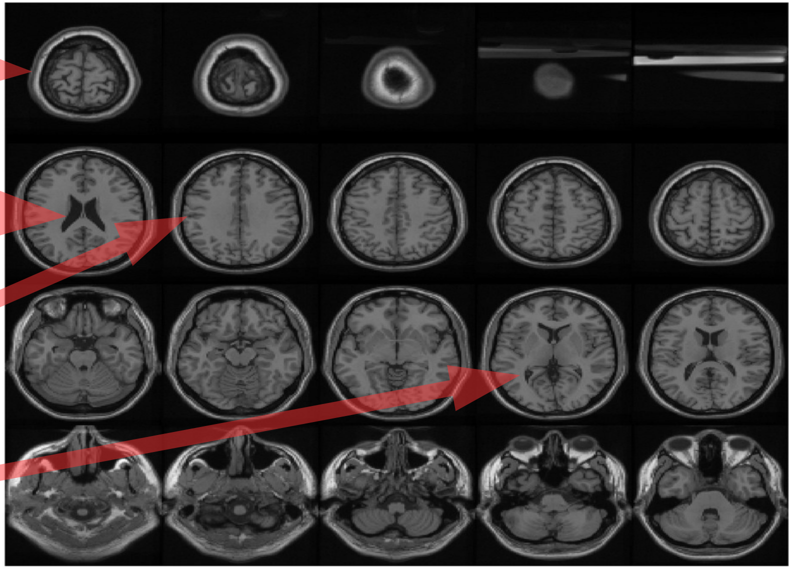
# What do we see?

Skin and skull

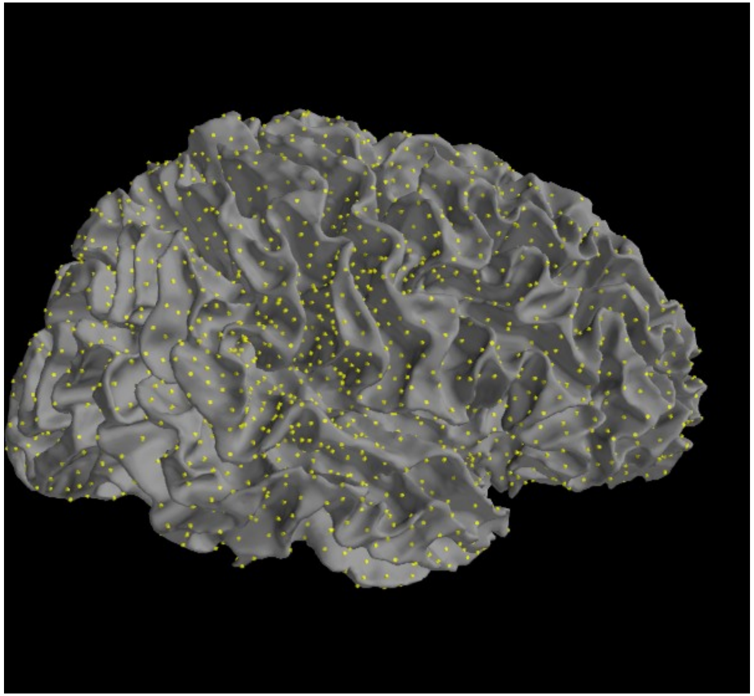
Cerebrospinal fluid

Grey matter

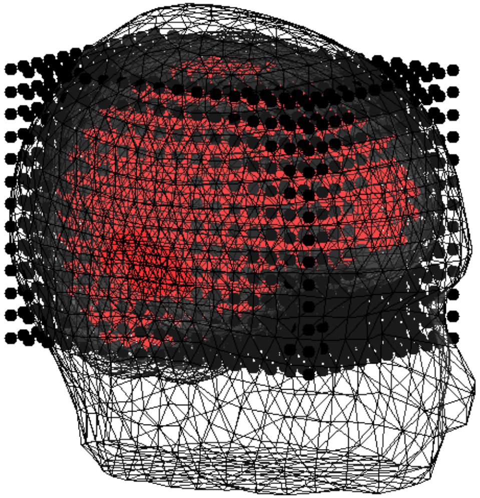
White matter



# Source model examples



Restricted to the cortical surface

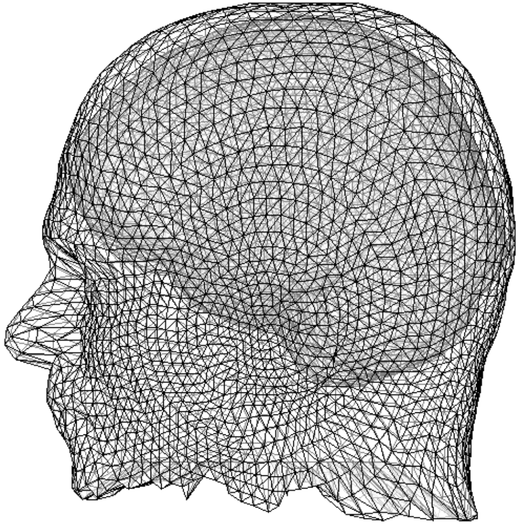


Volumetric grid

# Volume conductor (head model)

An anatomical model that models the conductivities of different tissues

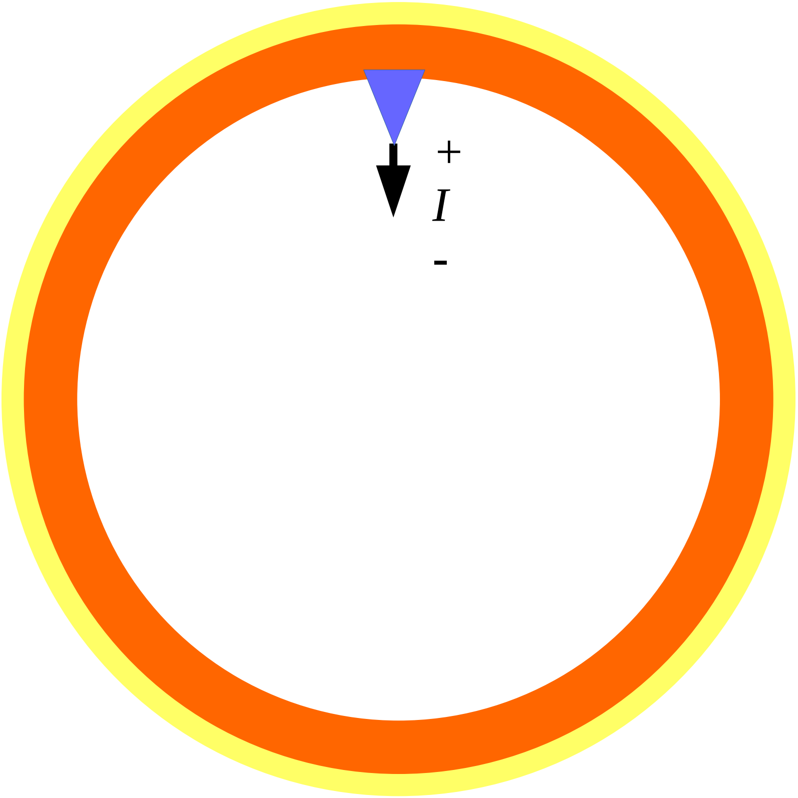
*bem*



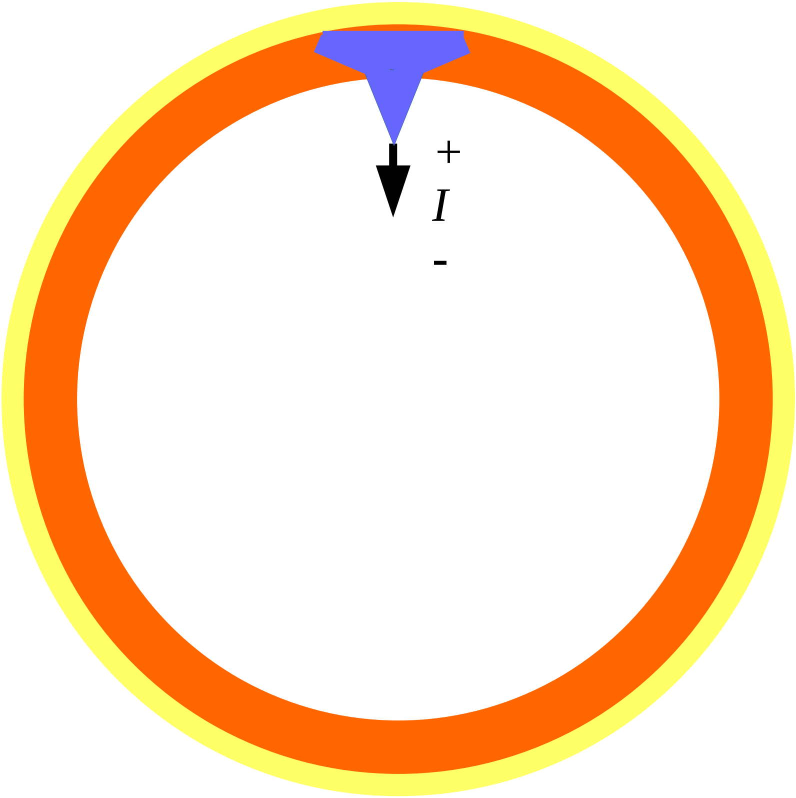
# Electric conductivity

- How “allowing” a given material is in allowing an electrical potential of passing through
  - Think of an analogy
    - Throwing a ball under water versus throwing a ball in the air

# Electrical Conductivity

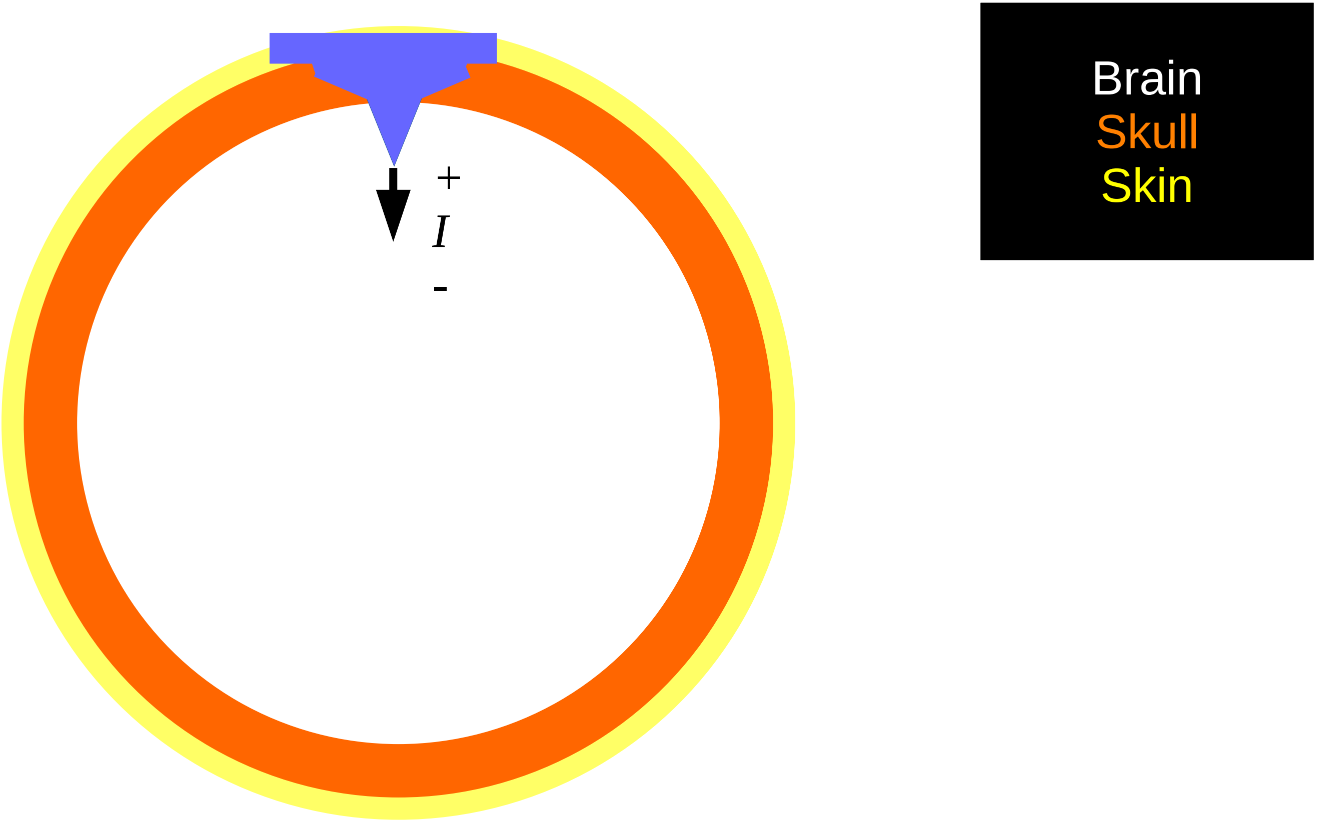



# Electrical Conductivity

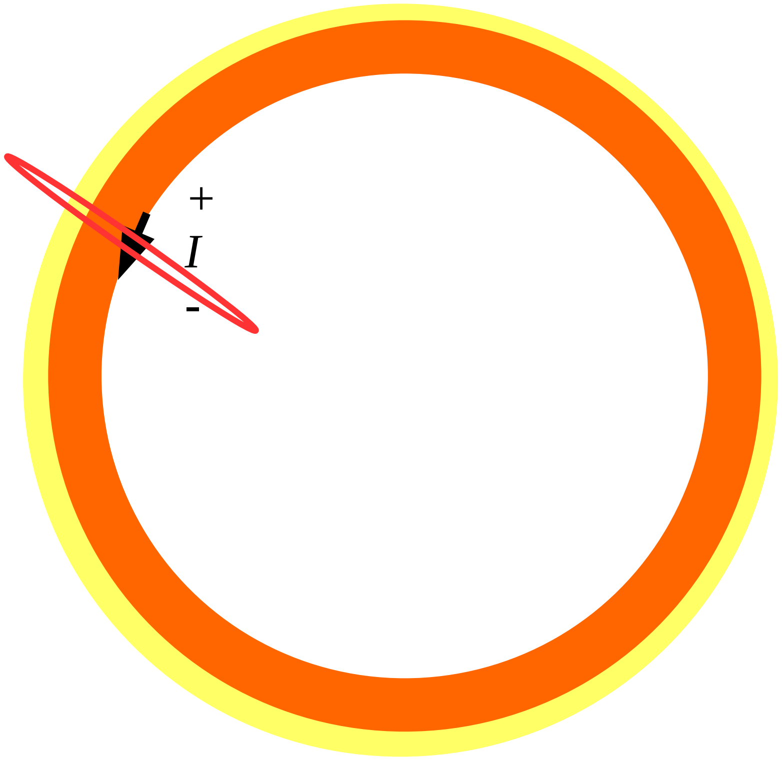



Brain
Skull
Skin

# Electrical Conductivity



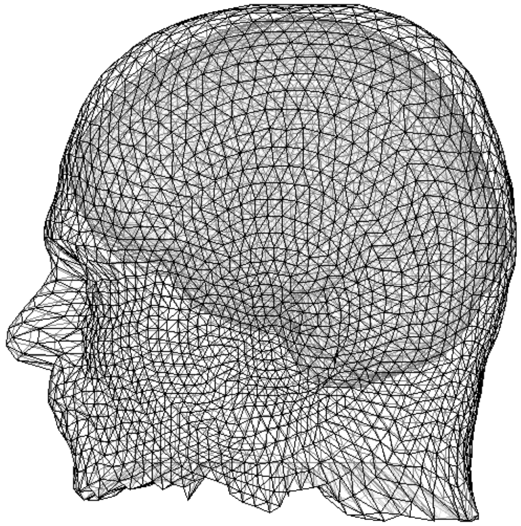
# Primary current: Magnetic field – does not depend on the medium (inside a spherical (!) conductor)




Brain
Skull
Skin

# Volume conductor (head model)

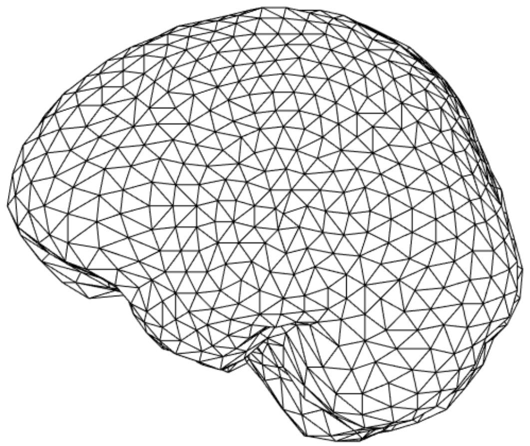
For *EEG*, we need to model the brain, skull and scalp with different conductivities



# Volume conductor (head model)

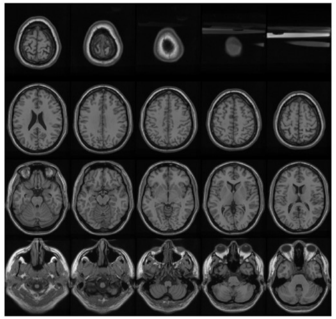
For MEG, the model can be much simpler since the magnetic field spreads homogeneously

(This is why MEG has better spatial resolution than EEG)



# Sensor positions

- The MR-images and the helmet/electrode data will be represented in different coordinate systems
  - Thus, they have to be co-registered



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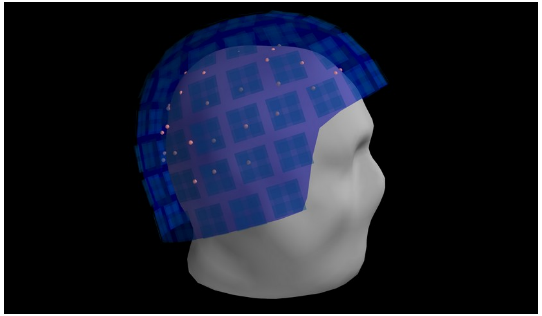
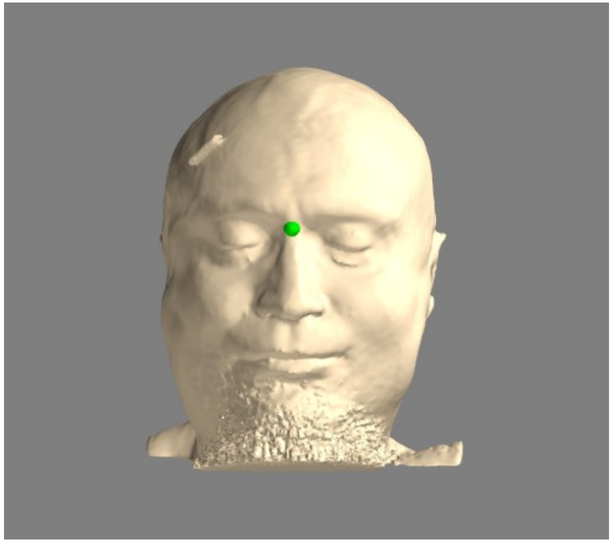


# Co-registration

Construct a head model based on the MRI

# Co-registration

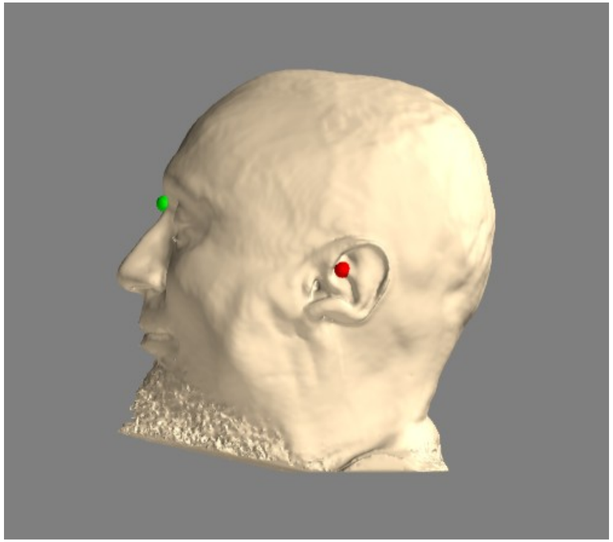
1. Nasion



Plot in fiducial points (these are acquired before recording)

# Co-registration

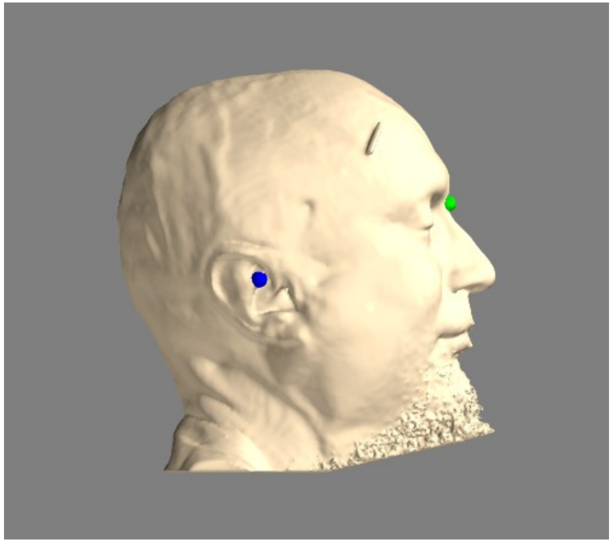
1. Nasion
2. Left pre-auricular point



Plot in fiducial points (these are acquired before recording)

# Co-registration

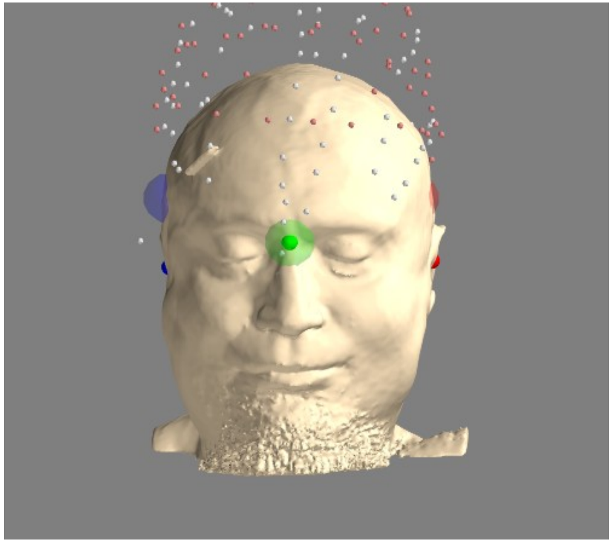
1. Nasion
2. Left pre-auricular point
3. Right pre-auricular point



Plot in fiducial points (these are acquired before recording)

# Co-registration

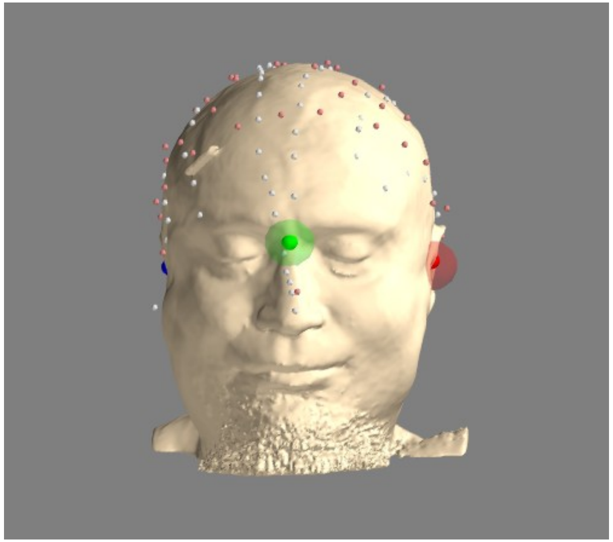
1. Nasion
2. Left pre-auricular point
3. Right pre-auricular point



Plot extra head points (as seen on lab tour)

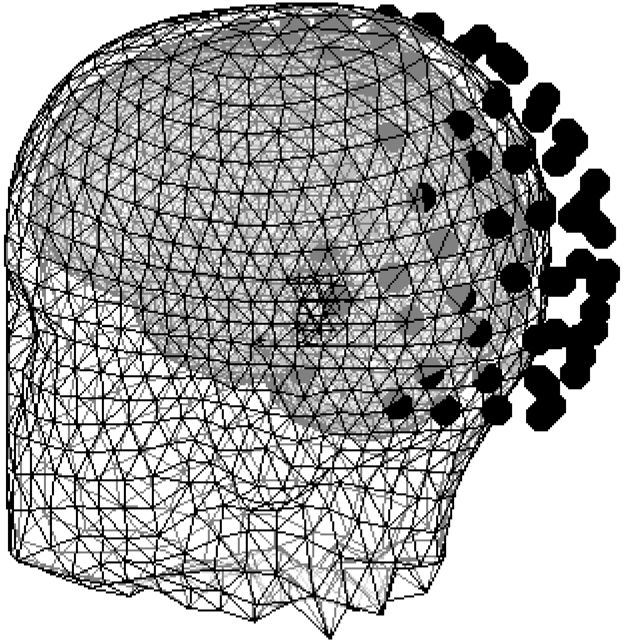
# Co-registration

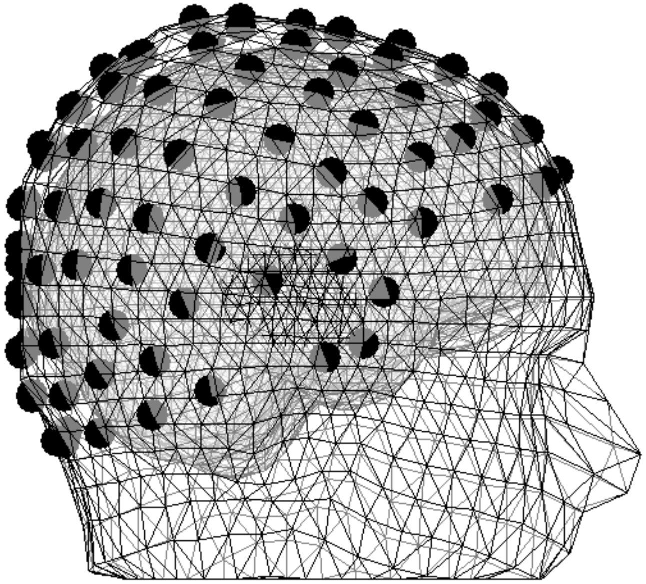
1. Nasion
2. Left pre-auricular point
3. Right pre-auricular point



Use an algorithm to minimize the distance between points and head shape. The MR and the MEG are now co-registered

**Without co-registration** **With co-registration**





# Now we can create the forward model (leadfield)

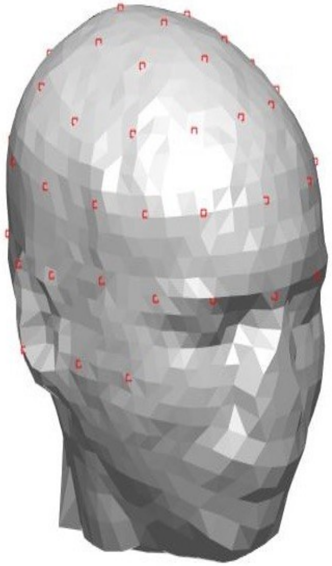
The forward model models how each source in the source space *would* be seen by the sensors in the helmet, *given* that that source is active

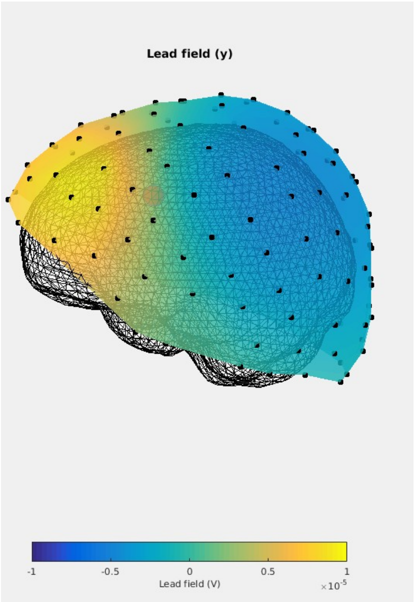
***forward model***

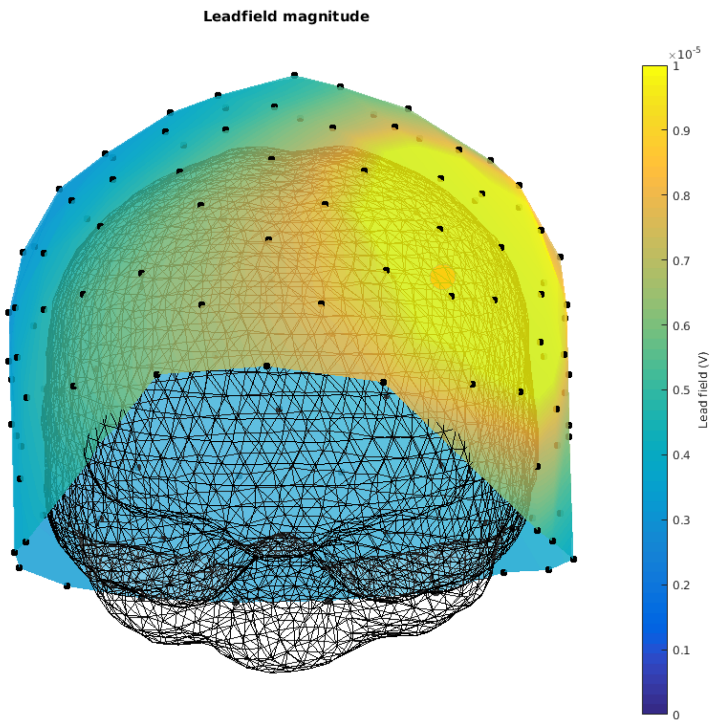
physiological source
(source model)

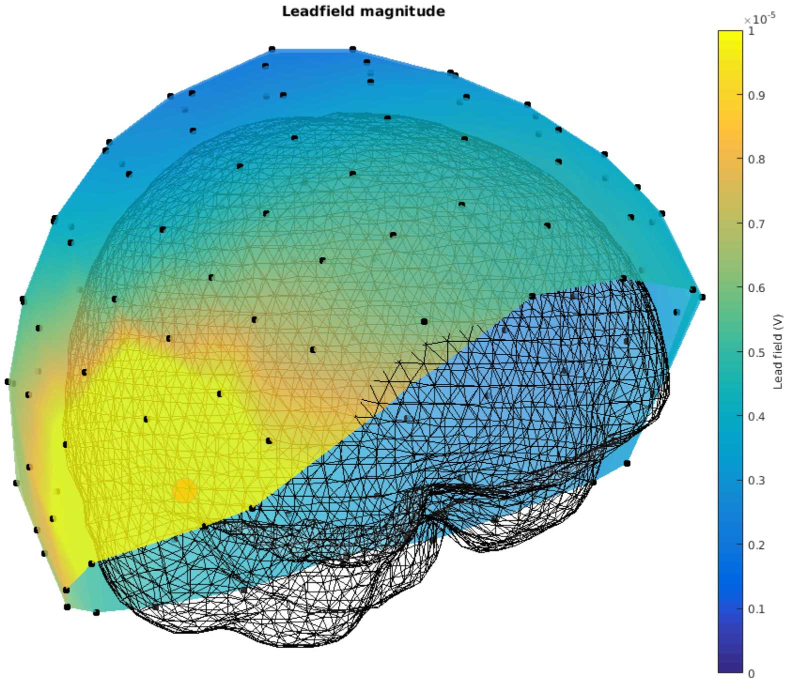
volume conductor
(head model)

**expected**
potential or field
(lead field)







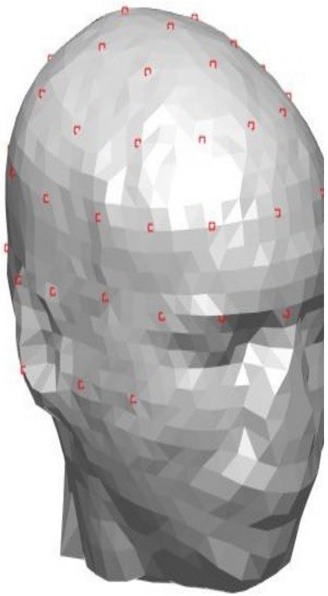

and so on for each source in the source model ...

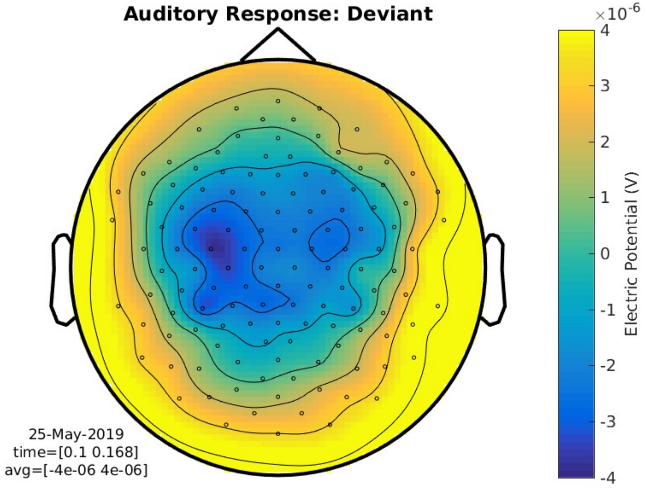
***inverse model***

physiological source
(source model)

volume conductor
(head model)

**observed**
potential or field
**(actual data)**





# Ill-posed problems and how we solve them

## The minimum norm estimate

based on

Sekihara, K., Nagarajan, S.S., 2008. Adaptive Spatial Filters for Electromagnetic Brain Imaging. Springer Science & Business Media.

# Equations

$$\boldsymbol{b}(t)=\sum_{n=1}^{N}\boldsymbol{L}(\boldsymbol{r}_{\boldsymbol{n}})\boldsymbol{s}(\boldsymbol{r}_{\boldsymbol{n}},t)$$

$\boldsymbol{b}(t)$: summed magnetic field measured over a time course $t$

$\boldsymbol{L}(r_n)$: leadfield (forward model) for each of $N$ sources, $r$

$\boldsymbol{s}(r_n, t)$: source time courses for each of $N$ sources, $r$ as they develop over time $t$

# Equations

$$\boldsymbol{b}(t) = \sum_{n=1}^{N} \boldsymbol{L}(\boldsymbol{r}_n)\boldsymbol{s}(\boldsymbol{r}_n, t)$$

$$\boldsymbol{v}_{\boldsymbol{vox}}(t) = \begin{bmatrix} \boldsymbol{s}(\boldsymbol{r}_1, t) \\ \boldsymbol{s}(\boldsymbol{r}_2, t) \\ \vdots \\ \boldsymbol{s}(\boldsymbol{r}_n, t) \end{bmatrix}$$

$$\boldsymbol{L}_{\boldsymbol{V}} = [\boldsymbol{L}(\boldsymbol{r}_1), \boldsymbol{L}(\boldsymbol{r}_2), \ldots, \boldsymbol{L}(\boldsymbol{r}_n)]$$

$$\boldsymbol{b}(t) = \boldsymbol{L}_{\boldsymbol{V}}\boldsymbol{v}_{\boldsymbol{vox}}(t)$$

# Equations

$$\boldsymbol{b}(t) = \boldsymbol{L_V}\boldsymbol{v_{vox}}(t)$$

if these were scalar variables,
how would you isolate $\boldsymbol{v}_{vox}(t)$ ?

$$(\boldsymbol{Y} = \boldsymbol{WX})$$

We cannot divide with matrices, but multiplying by the inverse is possible, e.g.

$$\boldsymbol{Y} = \boldsymbol{X}\,\boldsymbol{Z}$$

$$\boldsymbol{X}^{-1}\boldsymbol{Y} = \boldsymbol{X}^{-1}\boldsymbol{X}\,\boldsymbol{Z}$$

$$\boldsymbol{X}^{-1}\boldsymbol{Y} = \boldsymbol{I}\,\boldsymbol{Z}$$

$$\boldsymbol{X}^{-1}\boldsymbol{Y} = \boldsymbol{Z}$$

$$\boldsymbol{Z} = \boldsymbol{X}^{-1}\boldsymbol{Y}$$

but $\boldsymbol{L}_v$ is an $M \times 3N$ matrix and $M < 3N$

$(M = \text{number of sensors}, N = \text{number of sources})$

Is the inverse matrix of $\boldsymbol{L}_v$ then defined?

# No, but the generalised inverse is $\boldsymbol{L}_{\boldsymbol{V}}^{+}$

$$\boldsymbol{L}_{\boldsymbol{V}}^{+}=\boldsymbol{L}_{\boldsymbol{V}}^{\boldsymbol{T}}[\boldsymbol{L}_{\boldsymbol{V}}\boldsymbol{L}_{\boldsymbol{V}}^{\boldsymbol{T}}]^{(-1)}$$

thus we can isolate $\boldsymbol{v}_{vox}(t)$ by multiplying both sides with the generalised inverse

$$\boldsymbol{b}(t)=\boldsymbol{L}_{\boldsymbol{V}}\boldsymbol{v}_{\boldsymbol{vox}}(t)$$

and our estimate for $\boldsymbol{v}_{vox}(t)$ becomes:

$$\hat{\boldsymbol{v}}_{\boldsymbol{vox}}(t)=\boldsymbol{L}_{\boldsymbol{V}}^{\boldsymbol{T}}[\boldsymbol{L}_{\boldsymbol{V}}\boldsymbol{L}_{\boldsymbol{V}}^{\boldsymbol{T}}]^{(-1)}\boldsymbol{b}(t)$$

$$\hat{\boldsymbol{v}}_{\boldsymbol{vox}}(t) = \boldsymbol{L}_{\boldsymbol{V}}^{\boldsymbol{T}} [\boldsymbol{L}_{\boldsymbol{V}} \boldsymbol{L}_{\boldsymbol{V}}^{\boldsymbol{T}}]^{(-1)} \boldsymbol{b}(t)$$

$$\boldsymbol{G} = \boldsymbol{L}_{\boldsymbol{V}} \boldsymbol{L}_{\boldsymbol{V}}^{\boldsymbol{T}}$$

$$\hat{\boldsymbol{v}}_{\boldsymbol{vox}}(t) = \boldsymbol{L}_{\boldsymbol{V}}^{\boldsymbol{T}} \boldsymbol{G}^{(-1)} \boldsymbol{b}(t)$$

***G*** is square (306 x 306) so in principle invertible, but it is rank-deficient, because sensors see the same things

How can we make it invertible?

# Regularisation

$$\hat{\boldsymbol{v}}_{\boldsymbol{vox}}(t) = \boldsymbol{L}_{\boldsymbol{V}}^{\boldsymbol{T}}(\boldsymbol{G} + \epsilon \boldsymbol{I})^{(-1)} \boldsymbol{b}(t)$$

$\epsilon$ **:** a scalar

$\boldsymbol{I}$ **:** the identity matrix

and minimising the cost function is done by *F*:

$$F = \|\boldsymbol{b}(t) - \boldsymbol{L}_{\boldsymbol{V}} \hat{\boldsymbol{v}}_{vox}(t)\|^2 \quad + \quad \epsilon \|\hat{\boldsymbol{v}}_{vox}(t)\|^2$$

# Voila – the minimum norm estimate – optimising two constraints

$$F = \left\| \boldsymbol{b}(t) - \boldsymbol{L}_{\boldsymbol{v}} \hat{\boldsymbol{v}}_{vox}(t) \right\|^2 \quad + \quad \epsilon \left\| \hat{\boldsymbol{v}}_{vox}(t) \right\|^2$$

Minimising the unexplained variance of the solution

Minimising the norm of the solution (less current is better than more current)