Bayesian Network

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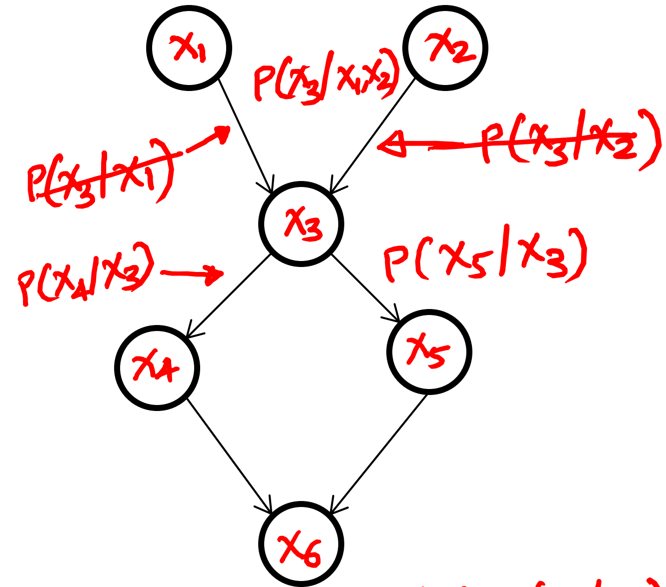
Outline

- Introduction
- Dependency graph
- Conditional Probability Table (CPT)
- Inference
- Exact Inference
- Approximate inference (using sampling)

Introduction

- *A Bayesian belief network describes the joint probability distribution for a set of variables.*
- Bayesian networks are a type of probabilistic graphical model,
  - nodes : Random Variables
  - directed edges : Dependence
- Captures both conditionally dependent and conditionally independent relationships between random variables.
- Models can be prepared by experts or learned from data, then used for inference to estimate the probabilities for causal or subsequent events.



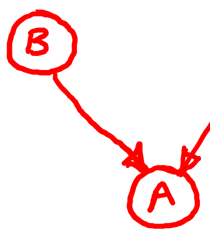
$$P(x_1, x_2 \dots x_5) = P(x_6|x_4,x_5)\cdot P(x_4|x_3)\cdot P(x_5|x_3) \cdot P(x_3|x_1,x_2)$$

Formally, a Bayesian network is a directed graph G=(V,E)~~G=(V,E)~~ together with

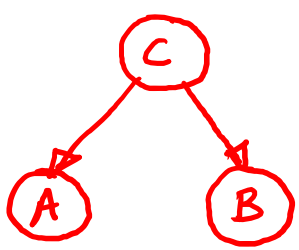
- A random variable Xi for each node ∈V.
- One conditional probability distribution (CPD) p(Xi|Xpi) per node, specifying the probability of xi conditioned on its parents' (Xpi) values.

Draw Bayesian Network

P(A, B, C) = P(A/B, C)* P(B) * P(C)



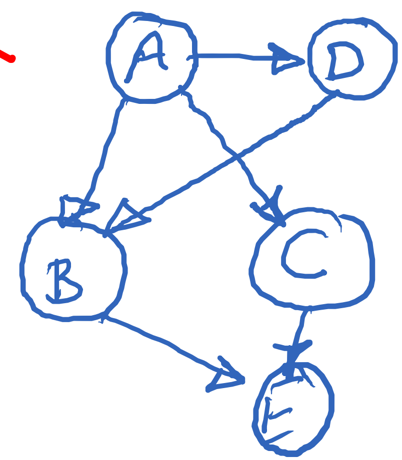
P(A, B, C) = P(A/C)* P(B/C) * P(C)



P(A, B, C, D, E) =
P(B/A,D)*P(C/A)*P(D/A)*P(B/D)*P(E/B,C) · P(A)

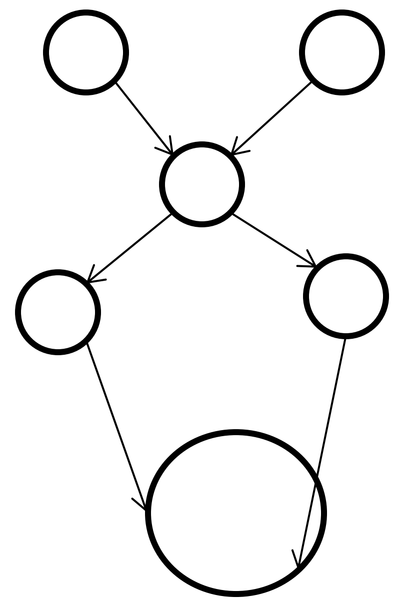
P(B/A, D)   P (B| D)

" A is conditionally independent of B, given D "



D-Separation

- "Each variable is conditionally independent of its non-descendants, given its parents."
- Procedure to check conditional independence
  - Draw ancestral graph
  - Moralize
  - Disorient
  - Delete Given
  - If the variables are disconnected in this graph, they are guaranteed to be independent.
  - If the variables are connected in this graph, they are not guaranteed to be independent.
  - If one or both of the variables are missing (because they were givens, and were therefore deleted), they are independent.



x_1 x_2 x_3 x_4 x_5 x_6

...onal Independence

$\perp x_2 \mid x_3$? No

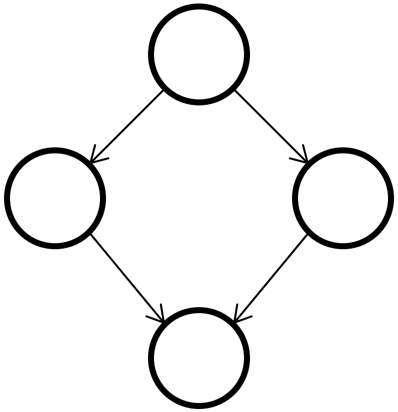
...l Independence

$\perp x_3$? 2

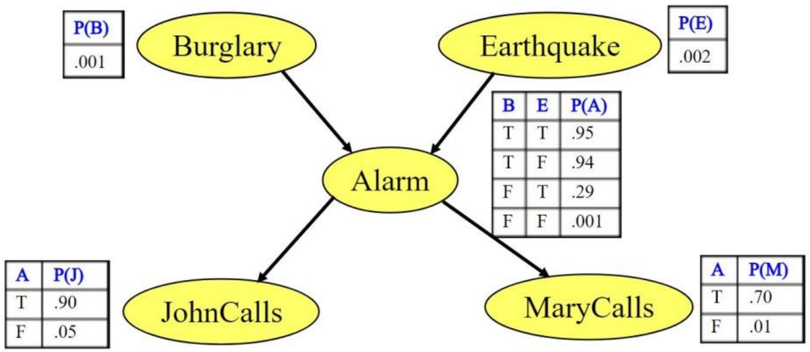
Yes

Learn CPT (Conditional Probability Tables)

X1	X2	X3	X4
1	0	1	1
0	0	1	0
1	1	0	0
0	0	0	0
0	1	0	1
1	0	0	1
1	1	0	1
0	0	1	1
1	1	1	1
1	0	1	0



Inference



What is the probability that MaryCalls if Alarm rings?

What is the probability that MaryCalls if Burglary happens?

What is the probability of false alarm and both John and Mary calling?

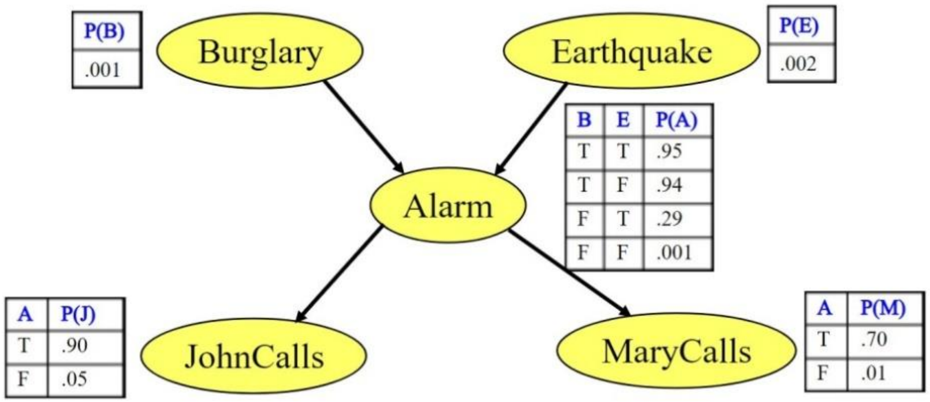
P(~B, ~E, A, J, M)

Approximate Inference through sampling

- Exact Inference over Bayesian Network requires O(a^n) computations.
- There have been some efforts to propose better algorithms, e.g. Variable Elimination Method
- Exact inference over Bayesian Network is an NP-Hard problem.

- Bayesian Network is a generative model.
- Generate samples using sampling algorithms.
- Use sample to perform inference.
- Approximate any desired conditional or marginal probability by empirical frequencies

Random Sampling

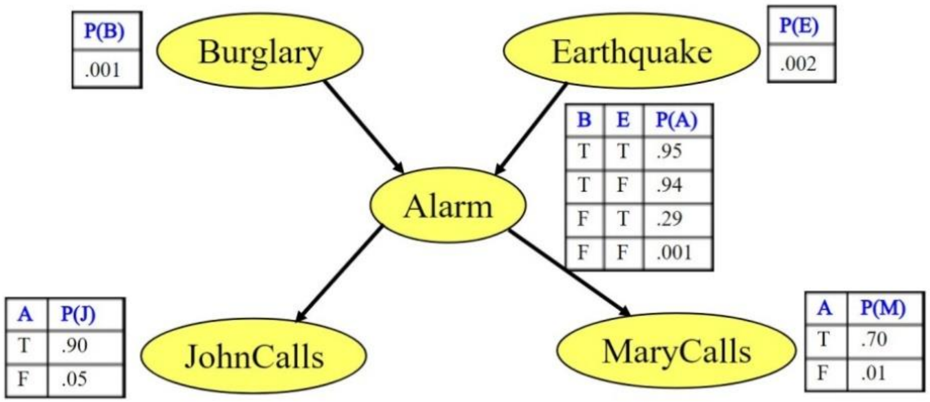


X :Generate 10 Random Values (1-10000)

Decide values of Random Variable using CPT and X.

B	E	A	J	M

Random Sampling



X :Generate 10 Random Values (1-10000)

Decide values of Random Variable using CPT and X.

B	E	A	J	M

Rejection Sampling