Compiler Construction

Lexical Analysis

Chapter -3

Convert Finite Automata To Regular Expression

- **Arden's Theorem**
- **State Elimination Method**

Arden’s Theorem

It states that-

Let P and Q be two regular expressions over ∑.

If **P does not contain a null string**
then
R = Q + RP has a unique solution i.e. R = QP*

Theorem is applicable to only Deterministic Finite Automata

Arden’s Theorem (Proof)

Theorem:
Let P and Q be two regular expressions over ∑.

If **P does not contain a null string**
then
. **R = Q + RP has a unique solution i.e. R = QP***

Proof –

R = Q + (Q + RP)P [After putting the value R = Q + RP]

= Q + QP + RPP

When we put the value of **R** recursively again and again, we get the following equation –

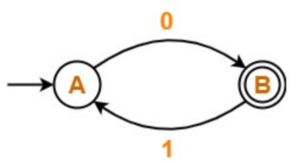
$R = Q + QP + QP^2 + QP^3 \ldots\ldots$

$R = Q (\varepsilon + P + P^2 + P^3 + \ldots\ldots)$

R = QP* [As P* represents (ε + P + P2 + P3 + ….)]

Hence, proved.

Arden's Theorem



For State A:

A= ε + B1

For State B:

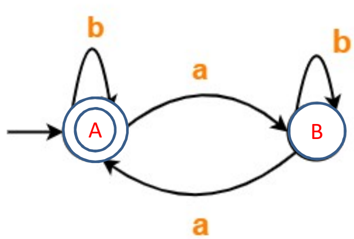
B = A0

Bring accepting state in the form R=Q+RP,

B= (ε + B1)0
B= ε0 + B10
B= 0+B10

Here, B=R, Q=0 and P=10
As per R=QP*, **solution will be : 0(10)***

Arden's Theorem



For State A:

A= ε + Ab + Ba

For State B:

B = Aa + Bb

Applying Arden's Theorem to State B,
Here, R=B, Q=Aa and P=b

B= Aab*

Bring accepting state in the form R=Q+RP,

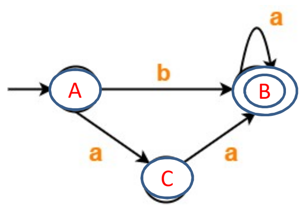
A= ε + Ab + Aab*a

A= ε + A(b + ab*a)

Here, A=R, Q= ε and P=b + ab*a

As per R=QP*, **solution will be : (b + ab*a)***

Arden’s Theorem



For State A:

A= ε

For State B:

B =Ab+ Ba + Ca

For State C:

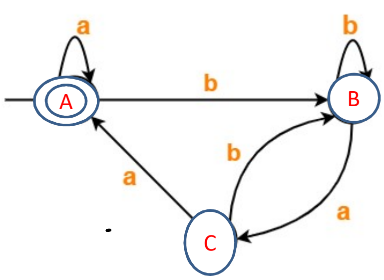
C =Aa

Bring accepting state in the form R=Q+RP,

B= Ab + Ba + Aaa
B= εb + Ba + εaa
B= b + Ba + aa
B= b+ aa + Ba

Here, B=R, Q=b+aa and P=a
As per R=QP*, **solution will be : (b+aa)a***

Arden’s Theorem



For State A:
A= ε + Aa + Ca
For State B:
B =Ab+ Bb + Cb
For State C:
C =Ba

B= Ab+Bb+Bab

Applying Arden’s Theorem
B=Ab(b+ab)*

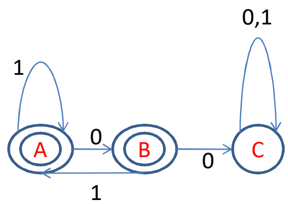
C=Ab(b+ab)*a

A= ε + Aa + Ab(b+ab)*aa
A= ε + A(a+b(b+ab)*aa)

Bring accepting state in the form R=Q+RP,

As per R=QP*, **solution will be :** (a + b(b+ab)*aa)*

Arden's Theorem



For State A:

A= ε +A1 + B1

For State B:

B =A0   B= (1+01)*0

Bring accepting state in the form R=Q+RP,

A= ε +A1 + A01

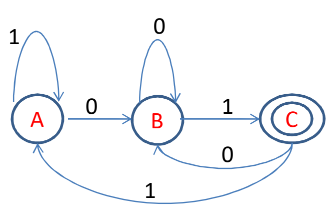
A= ε +A (1+01)

Here, A=R, Q= ε and P=(1+01)
As per R=QP*, **A= (1+01)***
B= (1+01)*0

Final Solution is : (1+01)* + (1+01)*0

Eliminate dead state directly.
Also remove any incoming arrow to the dead state

Arden's Theorem

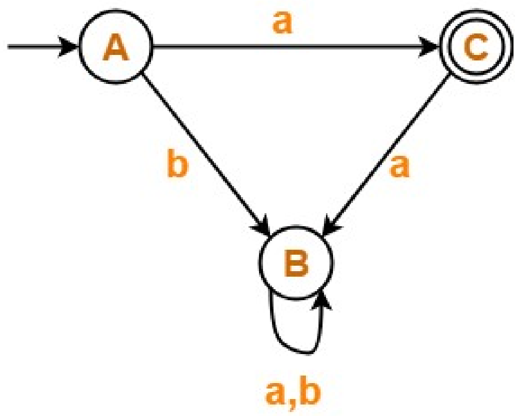


For State A:
A= ε + A1 + C1
For State B:
B =A0+ B0 + C0
For State C:
C =B1
Applying Arden's Theorem to state A
A=(ε+C1)1*
A= 1*+C11*

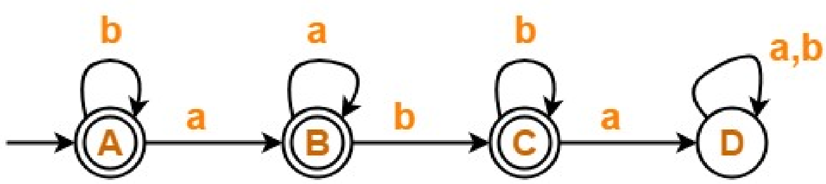
B= (1*+C11*)0+B0+C0
B= 1*0+C11*0+C0+B0
Applying Arden's Theorem to state B
B= (1*0+C11*0+C0)0*

Bring accepting state in the form R=Q+RP,
C=(1*0+C11*0+C0)0*1
C= 1*00*1+C11*00*1+C00*1
C=1*00*1+C(11*00*1+00*1)
As per R=QP*,
solution will be : 1*00*1(11*00*1+00*1)*

Try Yourself



Try Yourself



Try Yourself