Lecture 7: Bayesian Belief Network

Background:

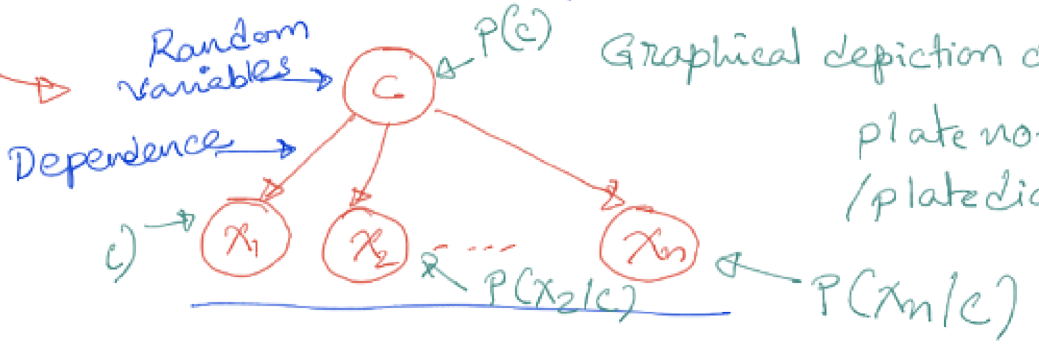
→ Model joint probability of features (x) and class (c)

$$P(x_1, x_2 \ldots x_n, c)$$

→ Naive assumption: $x_1, x_2 \ldots x_n$ are independent

$$P(x_1, x_2 \ldots x_n, c) = P(x_1/c) \cdot P(x_2/c) \ldots P(x_n/c) \cdot P(c) \quad (1)$$

→ Drop Naive Assumption and model dependence.

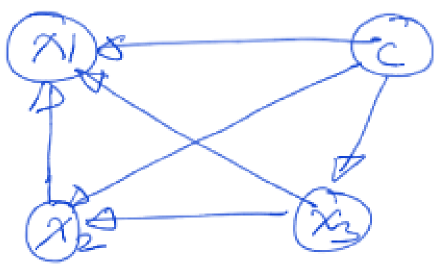


Graphical depiction of condi. dep.

plate notation /plate diagram

$$P(x_1, x_2, x_3, c)$$

– No independence assumption



$$P(x_1, x_2, x_3, c) = P(x_1/x_2, x_3, c) \cdot P(x_2/x_3, c) \cdot P(x_3/c) \cdot P(c)$$

Difficult to compute

Covid-19 Example

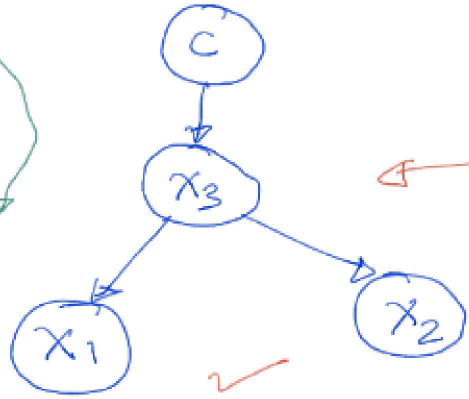
x_1 = Symptom $\quad x_2$ = Contact $\quad x_3$ = Test

C = Infection

$P(x_1, x_2, x_3, c)$

Parameters

$P(x_1/x_3) \cdot P(x_2/x_3) \cdot P(x_3|c) \cdot P(c)$



Bayesian Network

	1	2	3	4	5	6	7	8	9	10
contact X_1	1	1	1	0	0	1	1	0	1	1
Symptom X_2	1	1	1	1	0	0	0	1	0	0
Test X_3	1	0	0	1	1	1	0	1	1	0
Infected C	1	1	0	1	0	1	1	1	1	1

CPT:

$P(c)$, $P(x_3|c)$, $P(x_1/x_3)$, $P(x_2/x_3)$

$P(c)$

1	8/10
0	2/10

x_3 \ c	1	0
1	5/8	1/2
0	3/8	1/2

x_1 \ x_3	1	0
1	3/6	$4^{+1}/4^{+2}$
0	3/6	$0^{+1}/0^{+2}$

x_2 \ x_3	1	0
1	3/6	2/4
0	3/6	2/4

Ex: 1 Inference

$$P(c=1, x_1=1, x_2=1, x_3=0)$$

$$= P(\overset{=1}{x_1}/\overset{=0}{x_3}) \cdot P(\overset{=1}{x_2}/\overset{=0}{x_3}) \cdot P(\overset{=0}{x_3}/\overset{=1}{c}) \cdot P(\overset{=1}{c})$$

$$= (5/6)(2/4)(3/8)(8/10)$$

$$\approx 3/20$$

Inference Ex: 2

$P(c=1, x_1=1, x_2=1)$?

$$= \sum_{x_i \in X_3} P(c=1, x_1=1, x_2=1)$$

$$= P(c=1, x_1=1, x_2=1, x_3=0) + P(c=1, x_1=1, x_2=1, x_3=1)$$

$$= P(x_1=1/x_3=0) \cdot P(x_2=1/x_3=0) \cdot P(x_3=0/c=1) \cdot P(c=1)$$
$$+ P(x_1=1/x_3=1) \cdot P(x_2=1/x_3=1) \cdot P(x_3=1/c=1) \cdot P(c=1)$$

$$= 5/40 + 1/8 = 1/8$$

Inference Example 3

$P(c=0, x_1=0)$?

$$= \sum_{\forall i \in X_2} \sum_{\forall j \in X_3} P(c=0, x_1=0, x_2=i, x_3=j)$$

$$= P(c=0, x_1=0, x_2=0, x_3=0)$$
$$+ P(c=0, x_1=0, x_2=0, x_3=1)$$
$$+ P(c=0, x_1=0, x_2=1, x_3=0)$$
$$+ P(c=0, x_1=0, x_2=1, x_3=1)$$

$\Rightarrow$ DIY

Inference Ex: 4

$P(c=1/x_1=1)$

$$= \frac{P(c=1,\ x_1=1)}{P(x_1=1)} \quad \begin{matrix} 1 \\ 2 \end{matrix}$$

$$= \frac{\sum_{x_2} \sum_{x_3} P(c=1,\ x_1=1,\ x_2,\ x_3)}{\sum_{x_2} \sum_{x_3} \sum_{c} P(c,\ x_1=1,\ x_2,\ x_3)}$$

$$= \frac{P(c=1, x_1=1, x_2=0, x_3=0) \cdot P(c=1, x_1=1, x_2=0, x_3=1) \cdot P(c=1, x_1=1, x_2=1, x_3=0) \cdot P(c=1, x_1=1, x_2=1, x_3=1)}{P(c=1, x_1=1, \ldots}$$