Image Processing

Semester VII

Department of Computer Engineering

Dharmsinh Desai University

Digital Image Fundamentals

- Elements of Visual perception
- Light and Electromagnetic spectrum
- Image Sensing and application
- image sampling and quantization
- Basic relationships between pixels

Elements of Visual perception

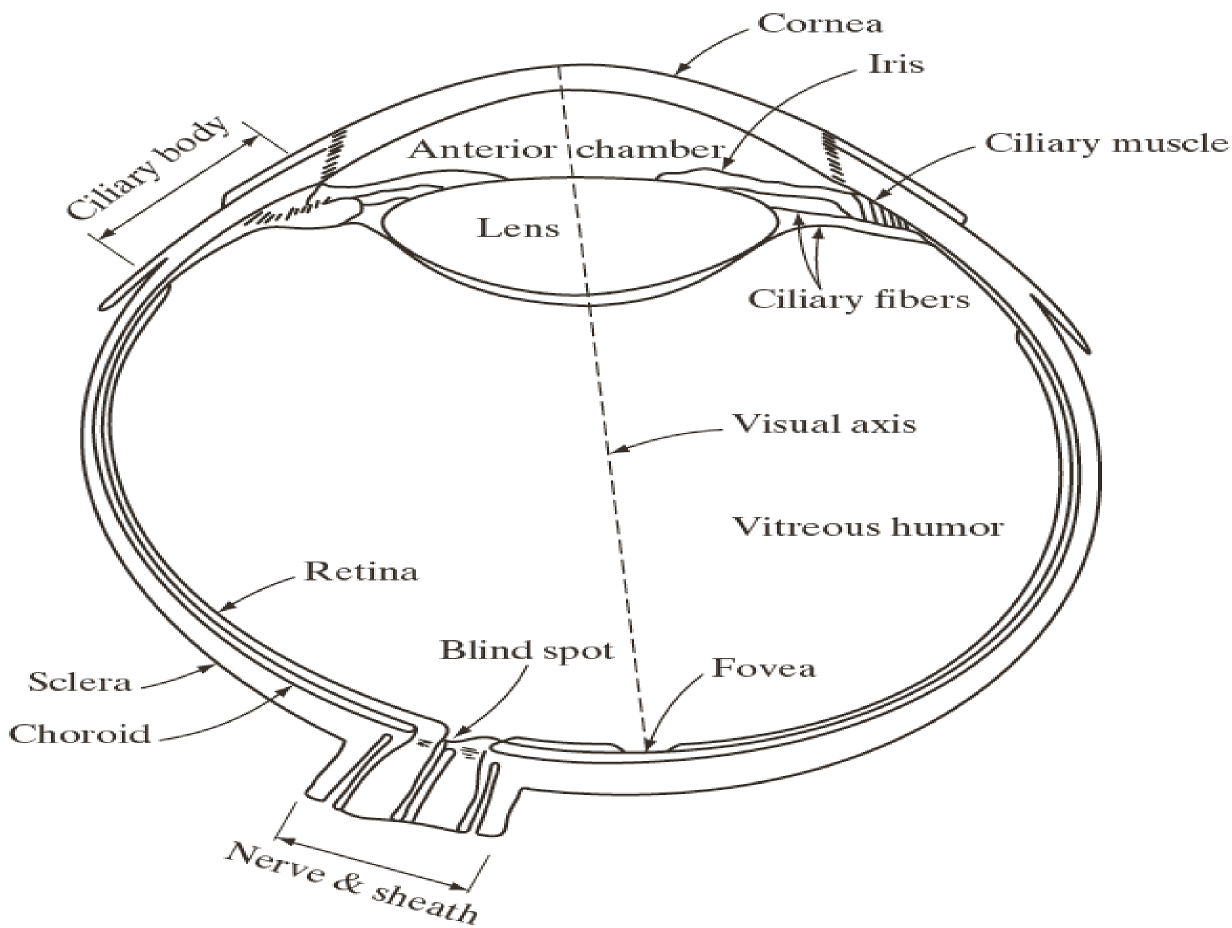
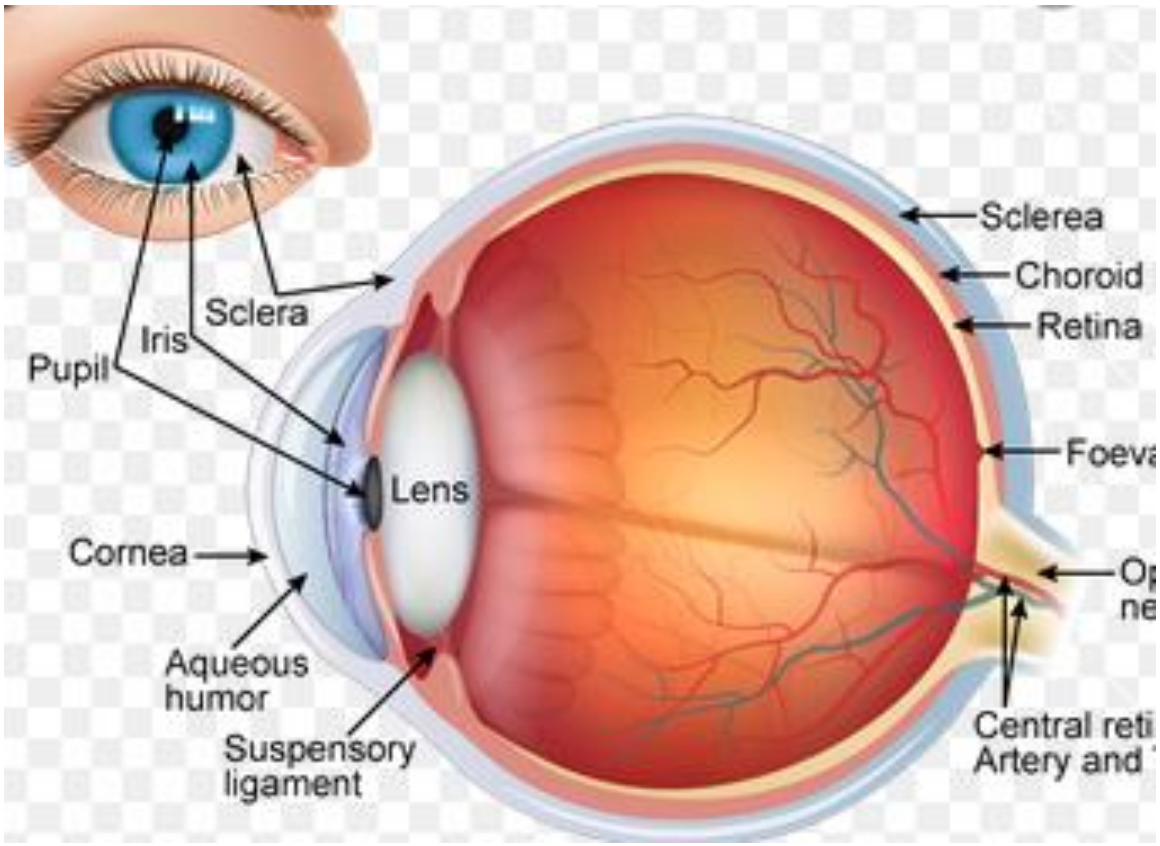


FIGURE 2.1 Simplified diagram of a cross section of the human eye.



Sclerea
Choroid
Retina
Sclera
Iris
Pupil
Foeva
Lens
Cornea
Aqueous
humor
Suspensory
ligament

Elements of Visual Perception

- The cornea and sclera outer cover
- The choroid
  - Ciliary body
  - Iris diaphragm
  - Lens
- Retina

Elements of Visual perception

- Iris: controls the size of the pupil which in turn controls the amount of light that enters the eye; it forms the colored portion of the eye
- Cornea: The transparent dome-shaped anterior portion of the outer covering of the eye; it covers the iris and pupil and is continuous with the sclera.
- The choroid contains the network of blood vessels that serve as the major source of nutrition to the eye.

- The lens contains 60-70% water, 6% of fat and more protein than any other tissue in eye.
- The iris diaphragm controls amount of light that enters the eye.
- Light receptors in the retina Elements of Visual Perception Structure of the human eye

Elements of Visual perception

- Receptors: An organ having nerve endings (in the skin, viscera, eye, ear, nose or mouth) that respond to stimulation
- Two classes of receptors: cones and rods

Cones

- Number between 6 and 7 million
- Located primarily in the central portion of the retina, called **fovea.**
- Highly sensitive to color
- Humans can resolve fine details with these cones largely because each one is connected to its own nerve end.
- Cone vision is called “photopic” or “bright-light vision”.

Rods

- Number between 75 to 150 million
- Distributed over the retinal surface
- Rods serve to give a general ,overall picture of the field of view.
- Not involved in color vision and are sensitive to low level illumination.
- Example. Objects that appear brightly colored in daylight when seen by moonlight appear as colorless forms only the rods are stimulated.
- Phenomenon is known as “scotopic” or dim-light vision

Distributions of rods and cones in retina

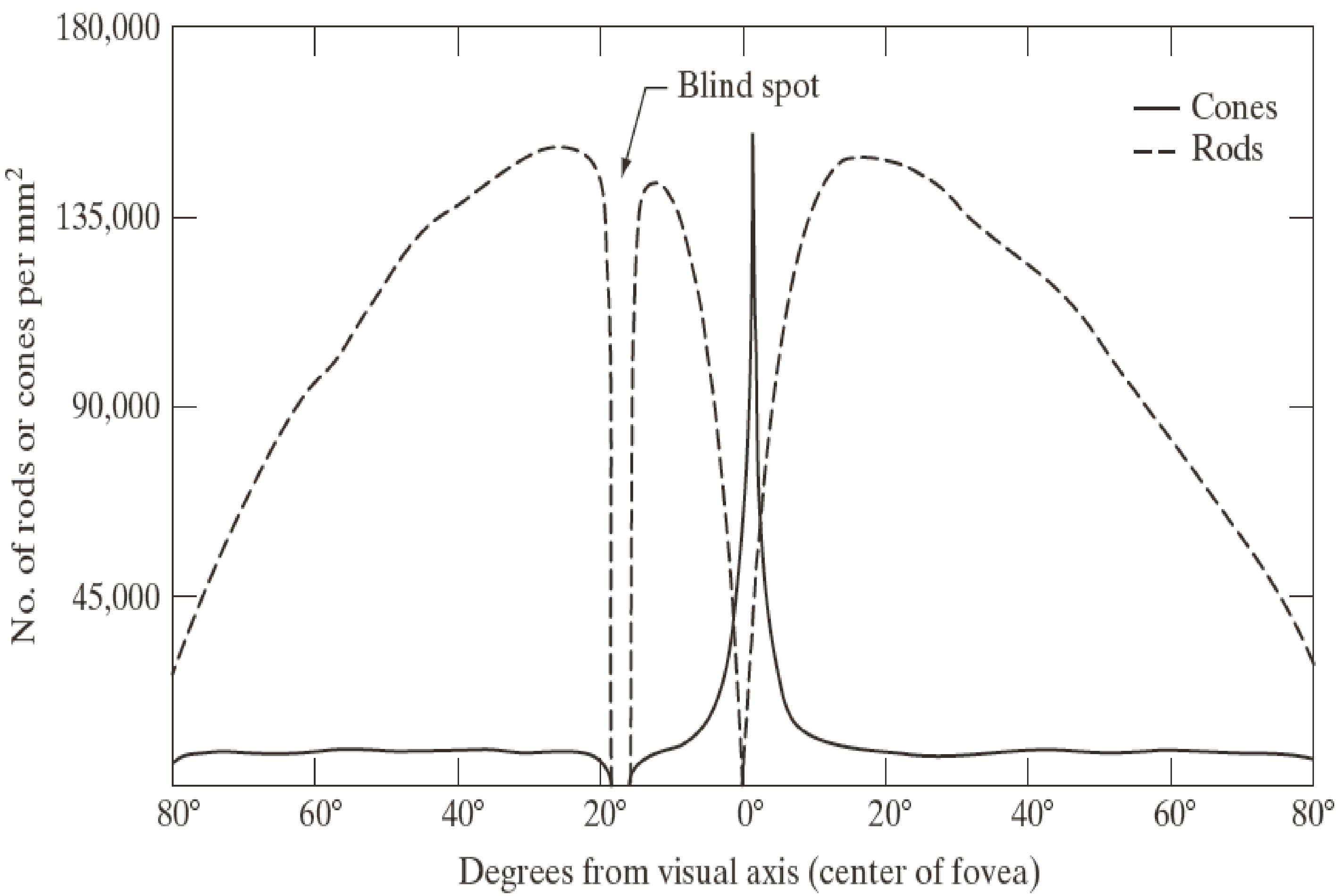


FIGURE 2.2 Distribution of rods and cones in the retina.

Blind spot

- Blind Spot: The absence of receptors in this area is called blind spot.

Image formation of a eye

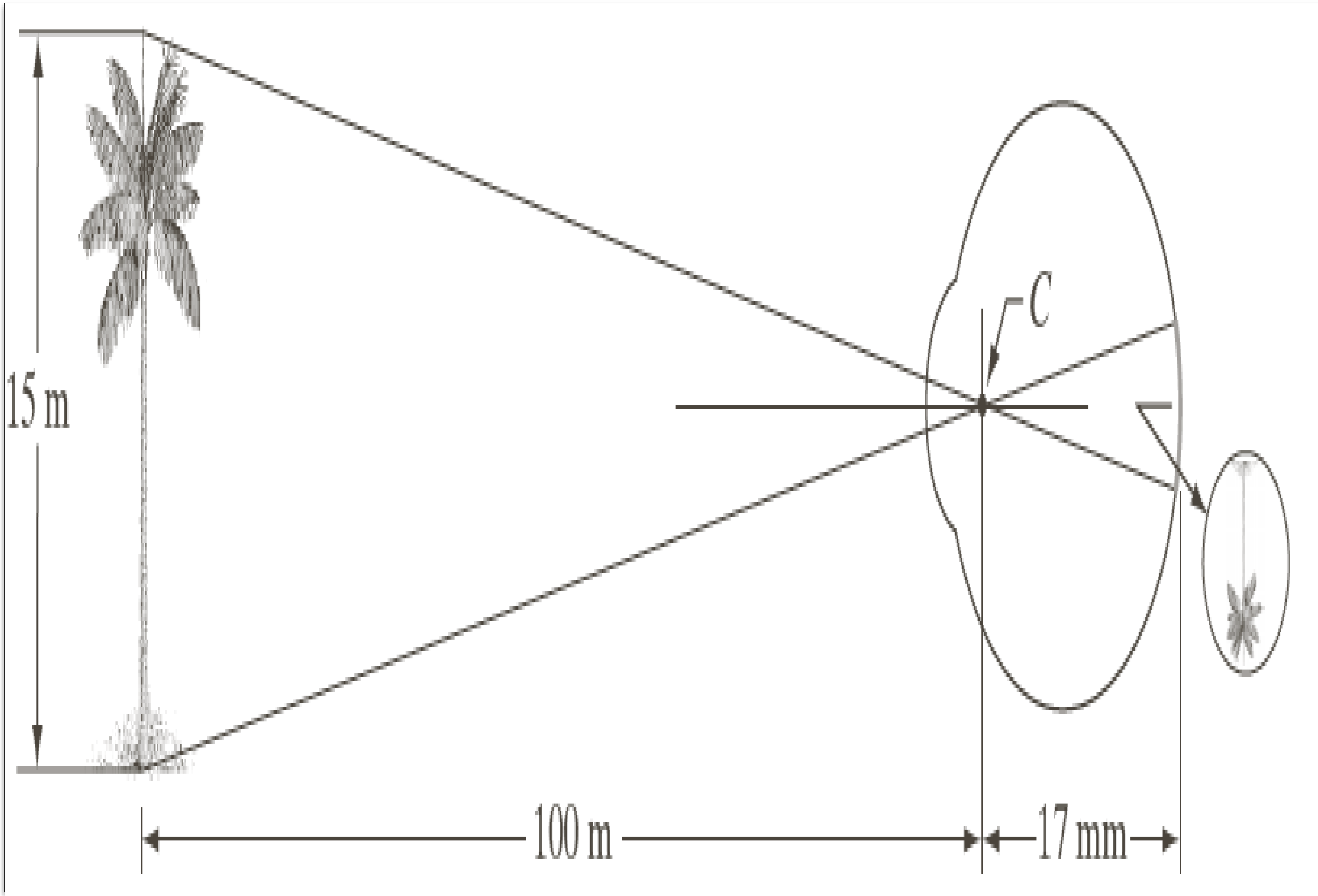


FIGURE 2.3 Graphical representation of the eye looking at a palm tree. Point C is the optical center of the lens.

Image formation of a eye

- Example: a person is looking at a in the retinal image
- Distance between the center of the lens and the retina along the visual axis is approximately 17 mm
- h denote the height of that object in the retinal image.
- $15/100 = h/17$
- $h = 2.55$ mm

Brightness Adaptation and Discrimination

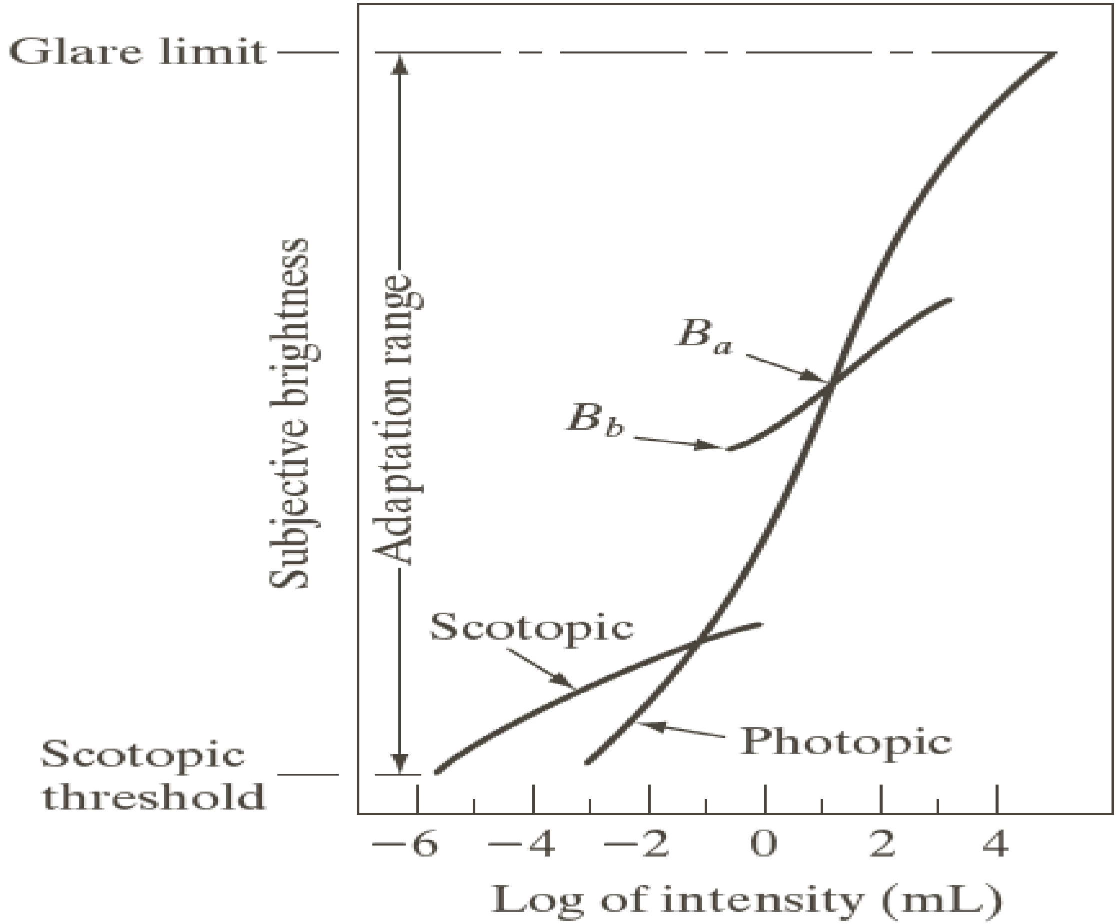


FIGURE 2.4 Range of subjective brightness sensations showing a particular adaptation level.

Brightness Adaptation and Discrimination

- Subjective brightness is a logarithmic function of the light intensity incident on the eye.
- For any given set of conditions, the current sensitivity level of the visual system is called the brightness adaption.
- Example brightness Ba in figure.
- The short intersecting curve represents the range of subjective brightness that the eye can perceive when adapted to this level.
- This range is rather restricted ,having a level Bb at and below which all stimuli are perceived as indistinguishable blacks.

Brightness Adaptation and Discrimination

- The ability of the eye to discriminate between changes in light intensity at any specific adaption level is also of considerable interest.
- A classic experiment is used to determine the capability of the human visual system for brightness discrimination

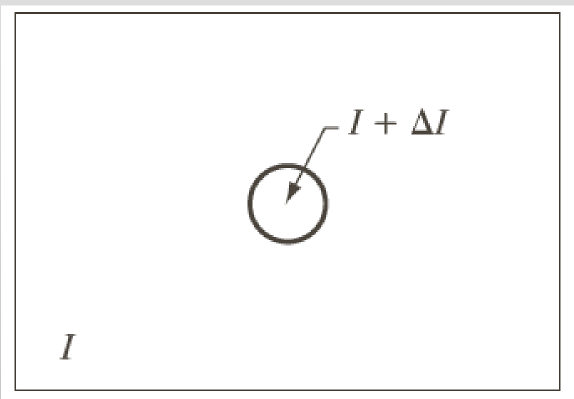


FIGURE 2.5 Basic experimental setup used to characterize brightness discrimination.

- Delta I short duration flash that appears as a circle in the center of uniformly illuminated field as given in figure.

Brightness Adaptation and Discrimination

- if ΔI is not bright enough , the subject says “no,” indicating perceivable change.
- As ΔI gets stronger ,the stronger , the subject may give positive response of “yes”, indicating perceivable change
- Weber ratio : $\Delta I_c/I$, where ΔI_c is the increment of illumination discriminable 50% of the time with the background illumination I.
- A small value of $\Delta I_c/I$ means that small percentage change in intensity is discriminable. This represents “good” brightness discrimination.
- Conversely large value of $\Delta I_c/I$ means that a large percentage change in intensity is required. This represents “poor” brightness discrimination.

Brightness Adaptation and Discrimination

➢It shows that brightness discrimination is poor (when weber ratio is large) at low levels of illumination.

➢It improves significantly (when weber ratio decreases).

➢The two branches in the curve reflect the fact that at low levels of illumination vision is carried out by rods whereas high levels vision is the function of cones.

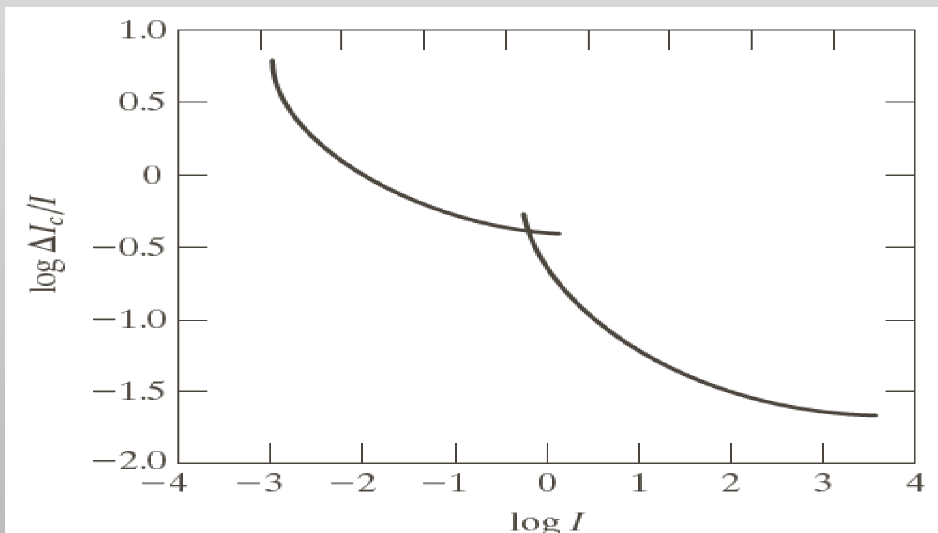


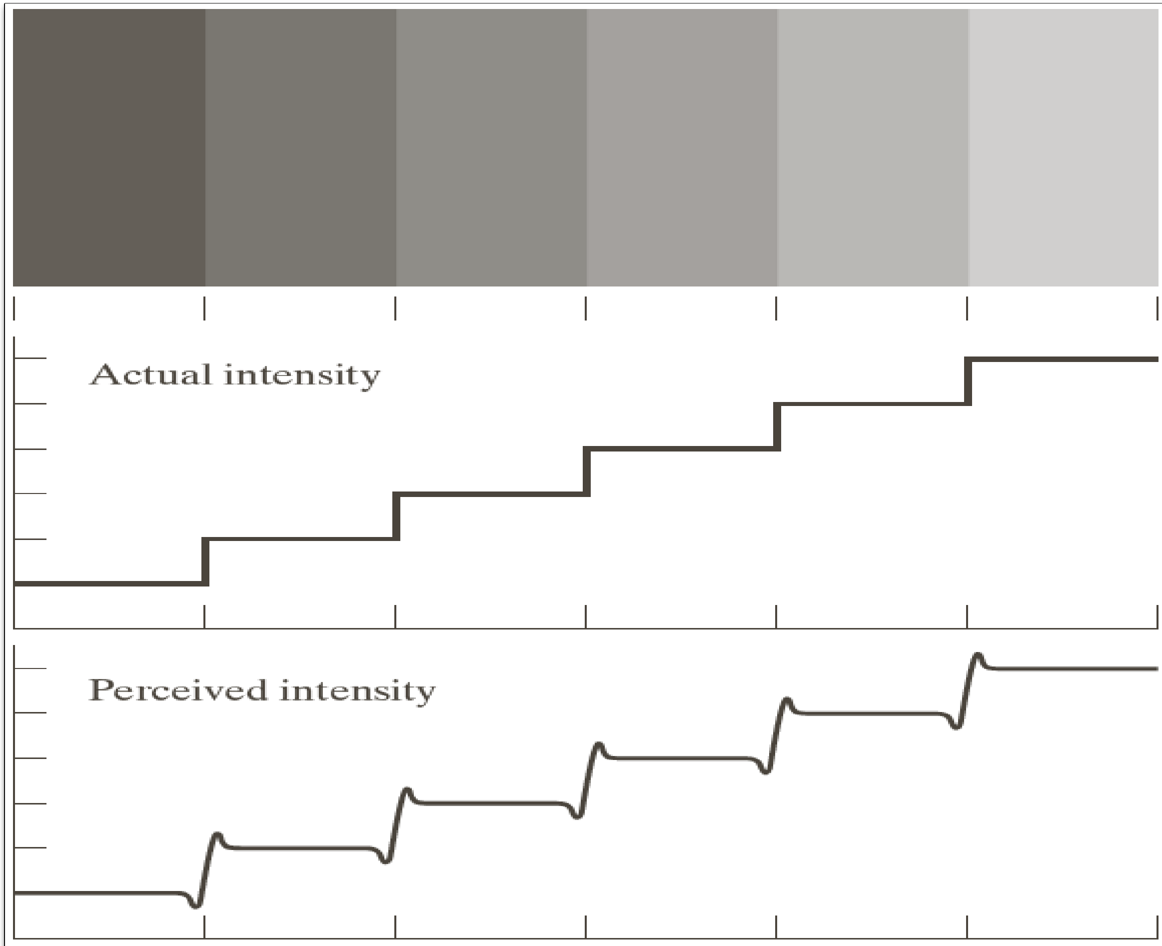
FIGURE 2.6 Typical Weber ratio as a function of intensity.

Brightness Adaptation and Discrimination

- Perceived brightness is not a simple function of intensity.
- Two phenomena clearly demonstrate it

1) Mach band effect
2) Simultaneous contrast

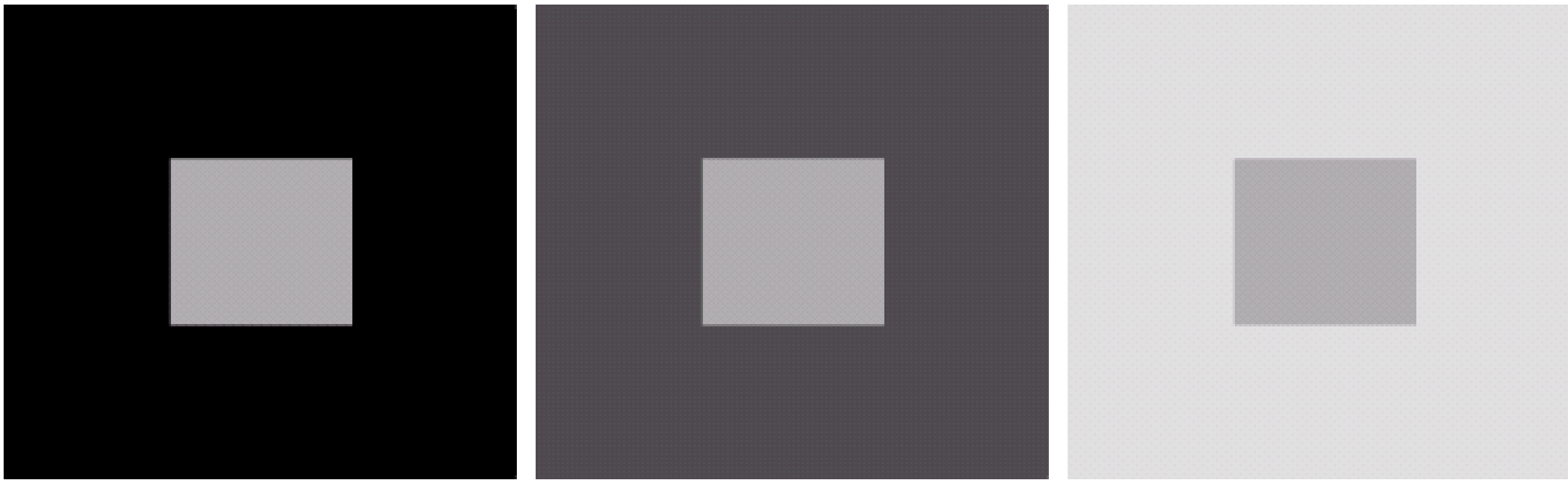
Match band effect



a
b
c

FIGURE 2.7 Illustration of the Mach band effect. Perceived intensity is not a simple function of actual intensity.

Simultaneous contrast



a b c

FIGURE 2.8 Examples of simultaneous contrast. All the inner squares have the same intensity, but they appear progressively darker as the background becomes lighter.

- Region's perceived brightness does not depend simply on its intensity

Optical illusions

- Eye fills in non existing information or wrongly perceives geometrical properties of objects.
- Figure C appears same length but shorter than the other.
- Figure D that are oriented at 45 degree are equidistant parallel.
- Yet the crosshatching creates the illusion that those lines are far from being parallel.

Optical illusions

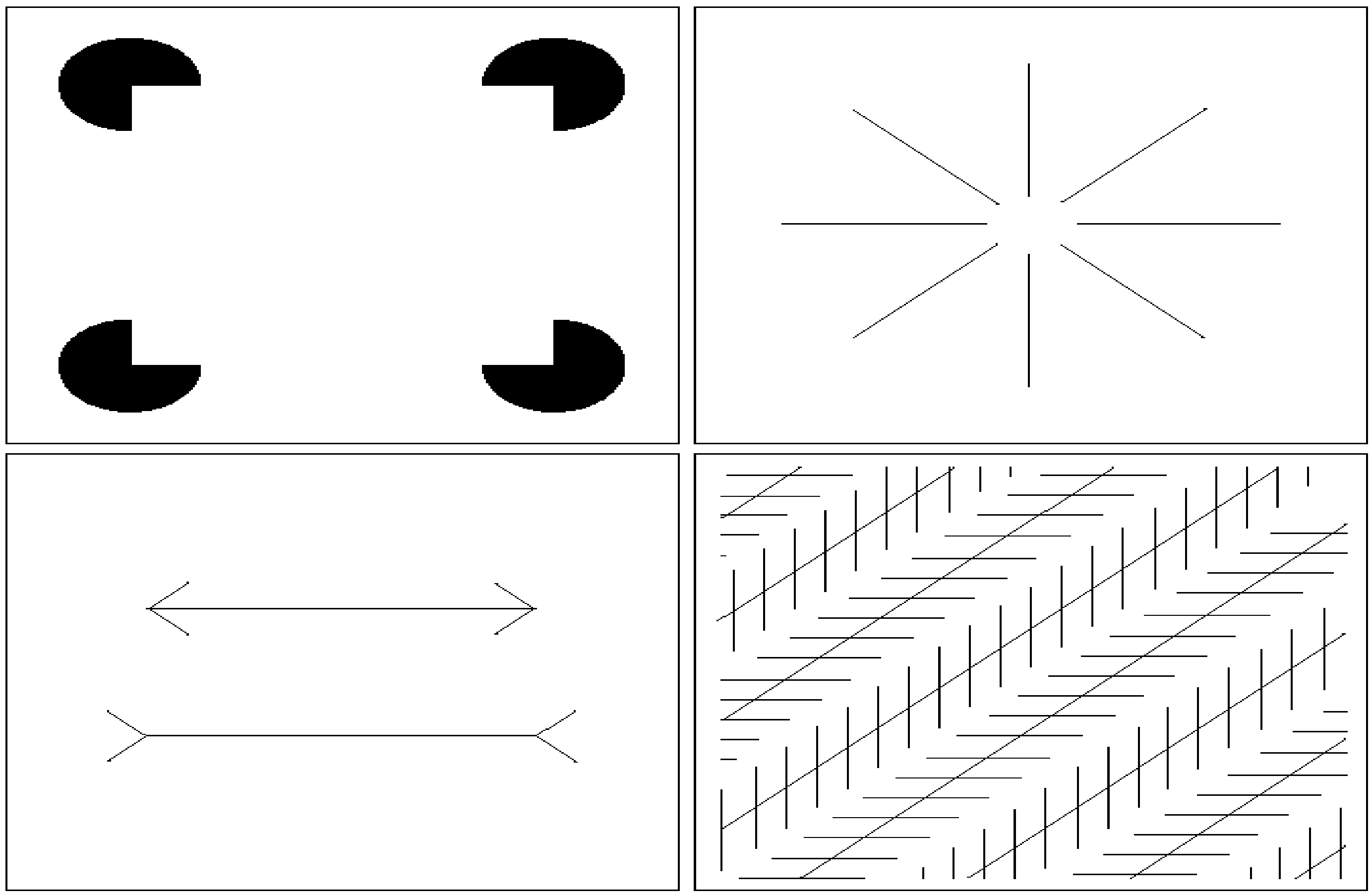


FIGURE 2.9 Some well-known optical illusions.

Electromagnetic spectrum

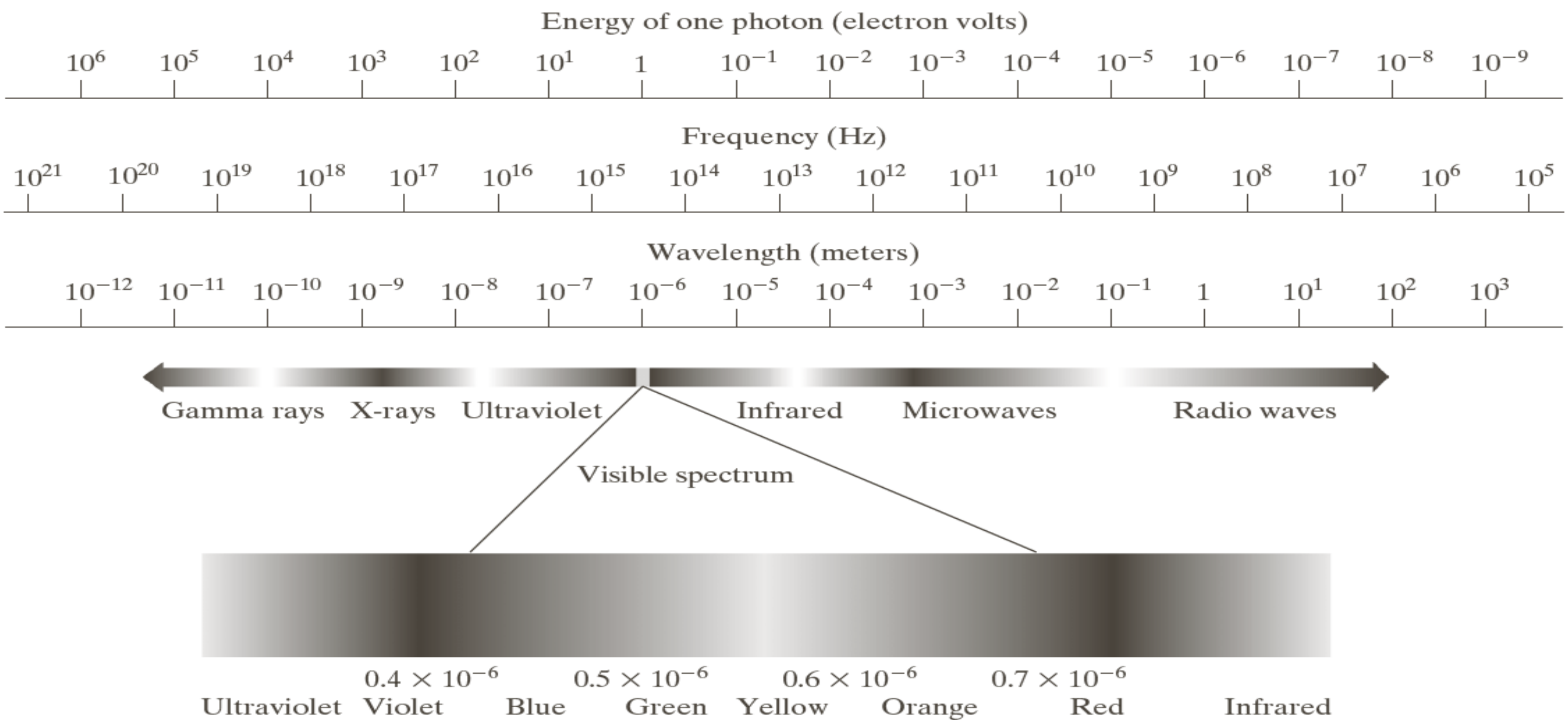
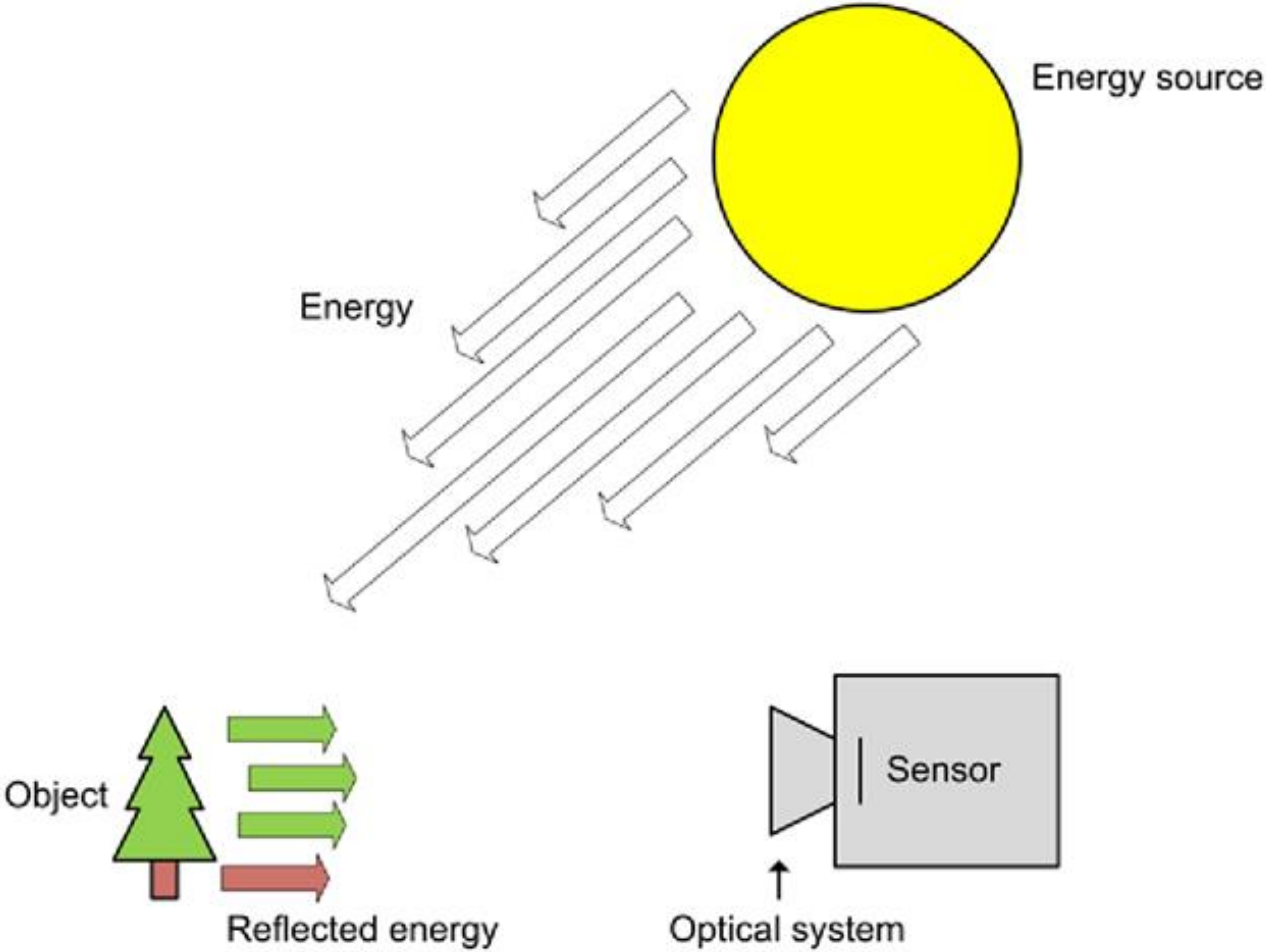


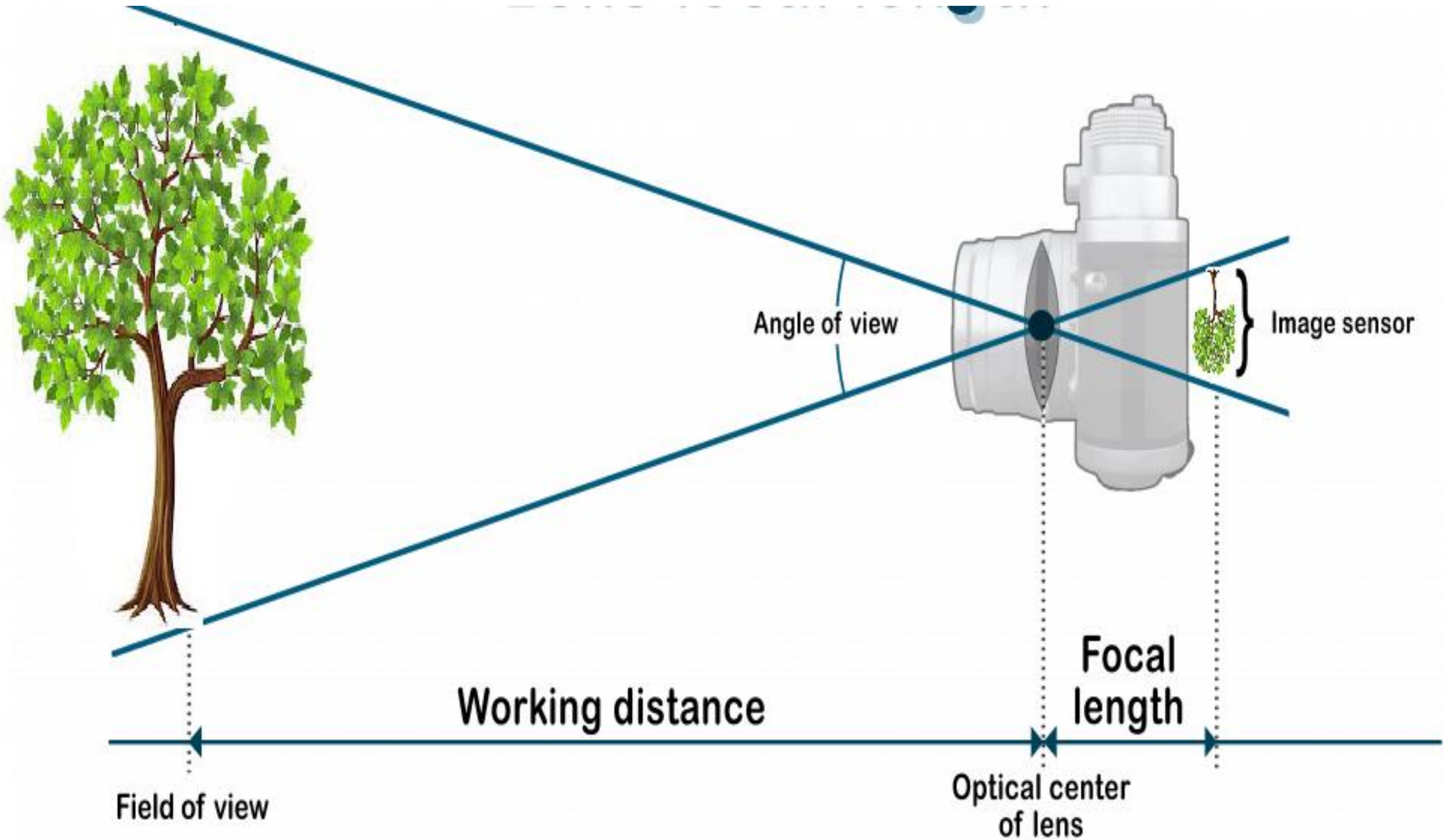
FIGURE 2.10 The electromagnetic spectrum. The visible spectrum is shown zoomed to facilitate explanation, but note that the visible spectrum is a rather narrow portion of the EM spectrum.

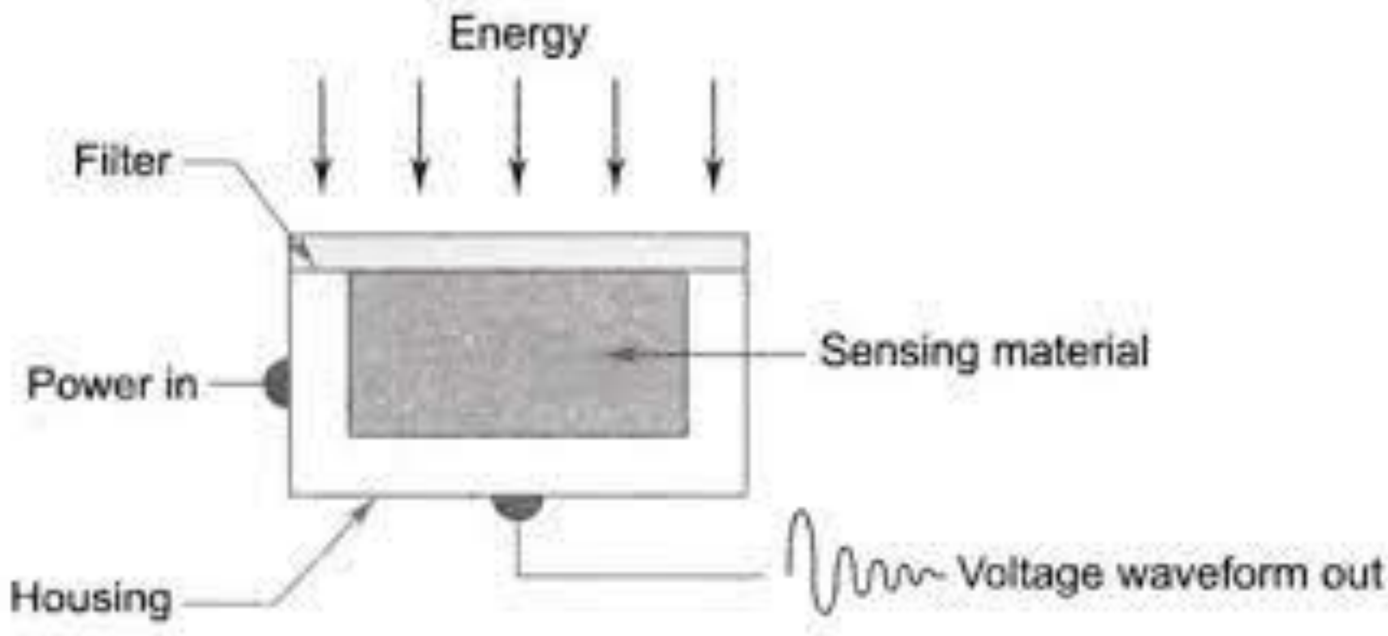
Electromagnetic spectrum

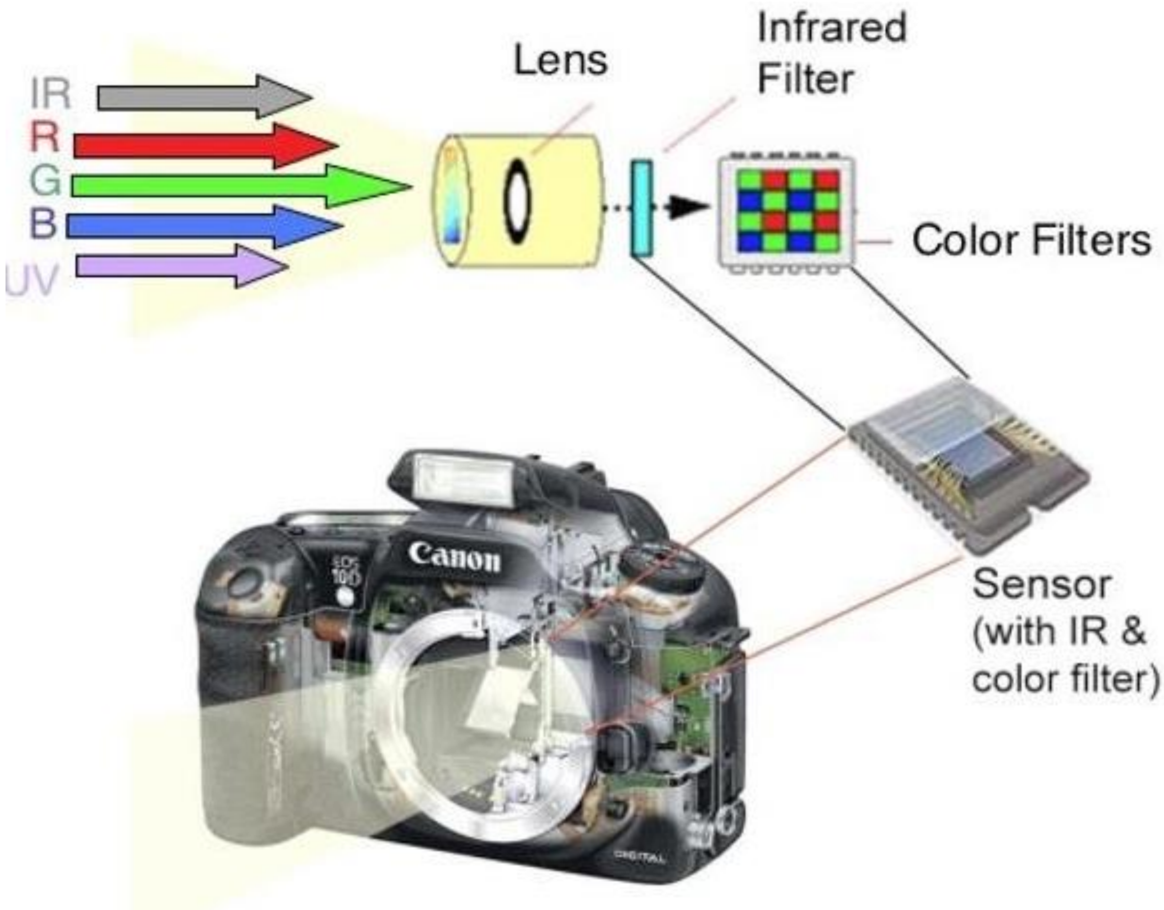
- Radiance is the total amount of energy that flows from the light source, and it is usually measured in watts(W).
- Luminance measured in lumens(lm), give a measure of the amount of energy an observer perceives from a light source.

Image sensing and acquisition


Energy source
Energy
Object
Reflected energy
Sensor
Optical system


Angle of view
Image sensor
Working distance
Focal
length
Field of view
Optical center
of lens


Energy
Filter
Power in
Sensing material
Housing
Voltage waveform out


IR
R
G
B
UV
Lens
Infrared
Filter
Color Filters
Sensor
(with IR &
color filter)
Canon

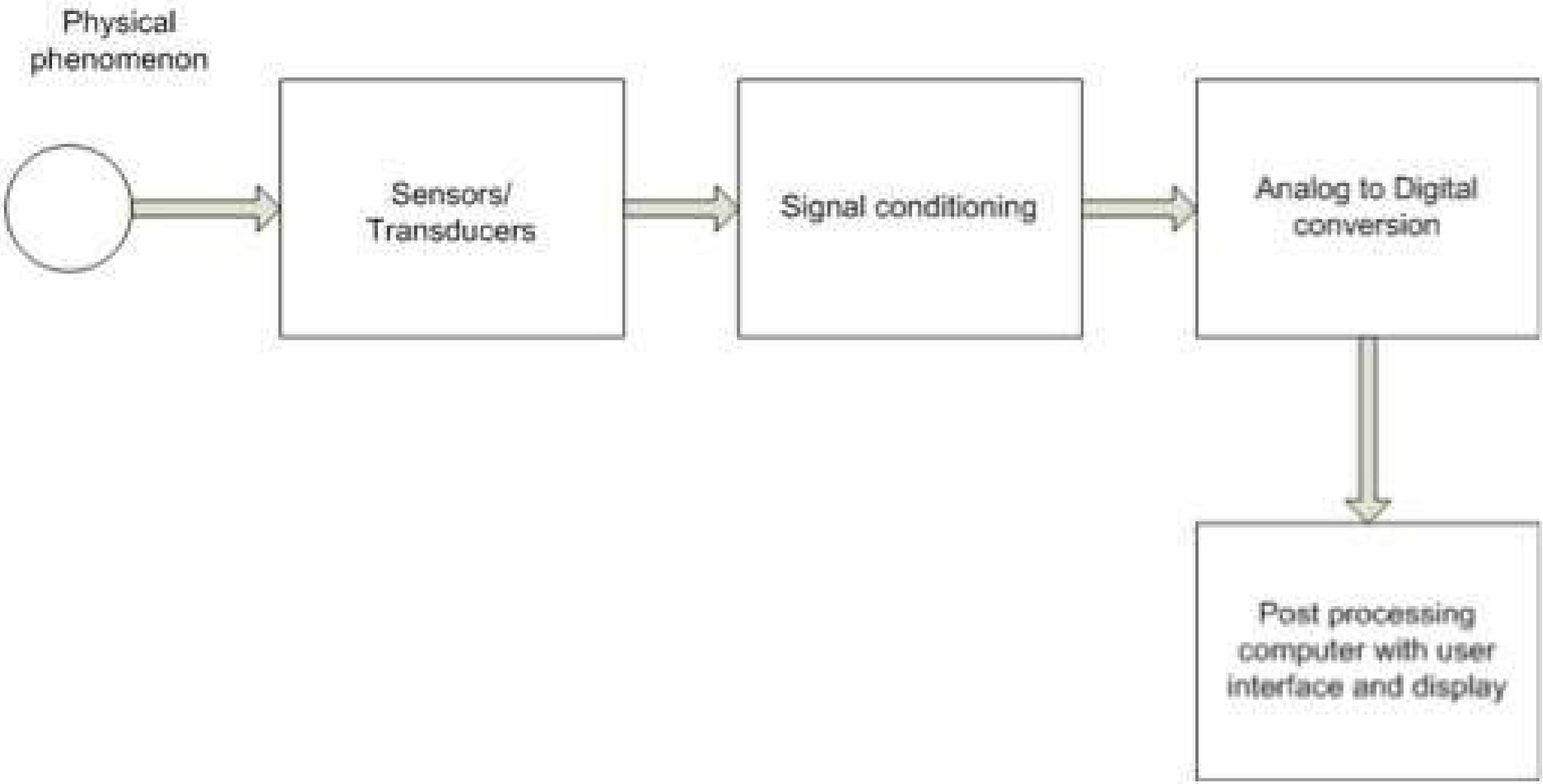


Figure-1: Data Acquisition system to acquire data from a single sensor

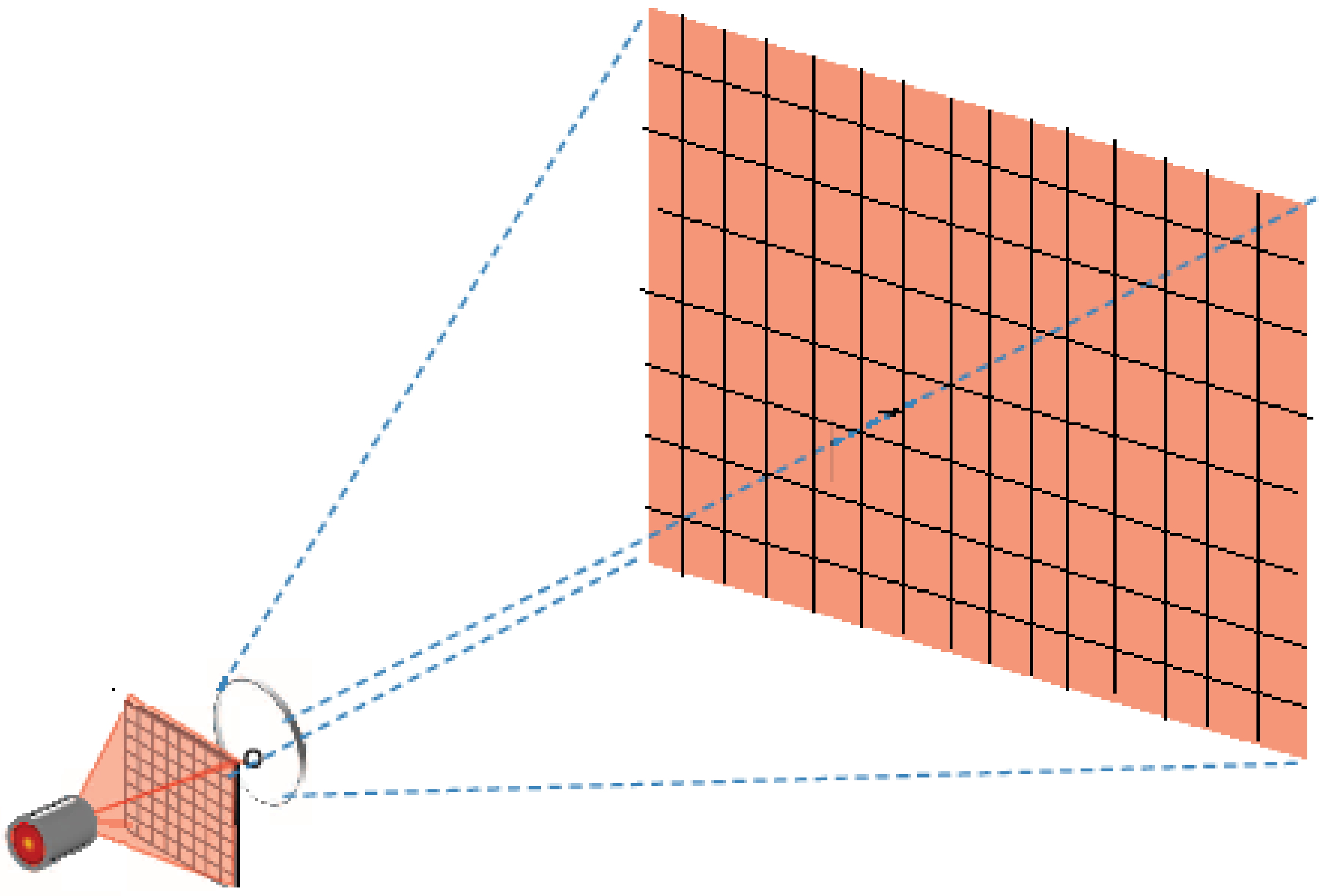
Image sensing and acquisition

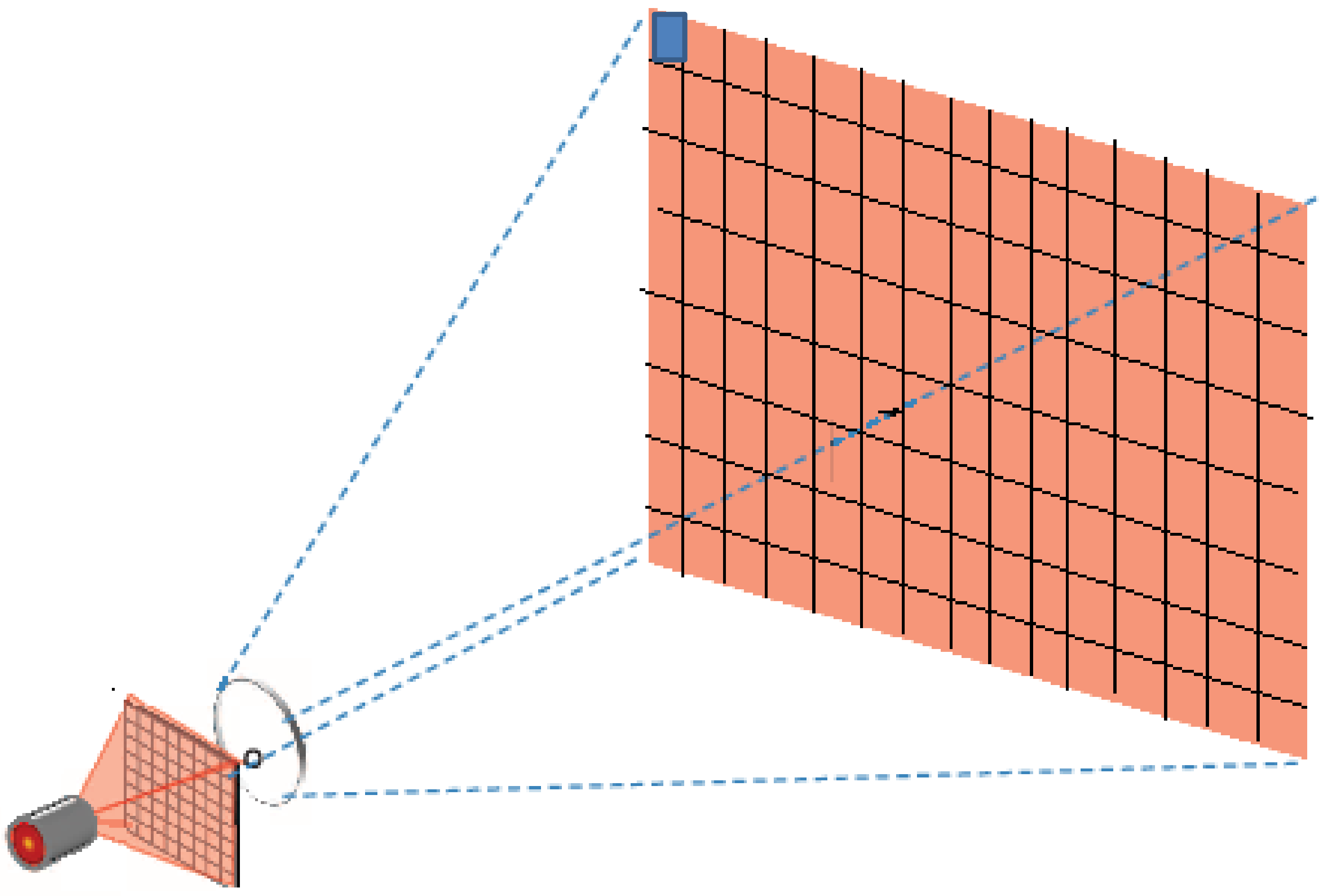
- Incoming energy is transformed into a voltage by the combination of input electrical power and sensor material that is responsive to the particular type of energy being detected.
- The output voltage waveform is the response of the sensor(s), and a digital quantity is obtained from each sensor by digitizing its response.

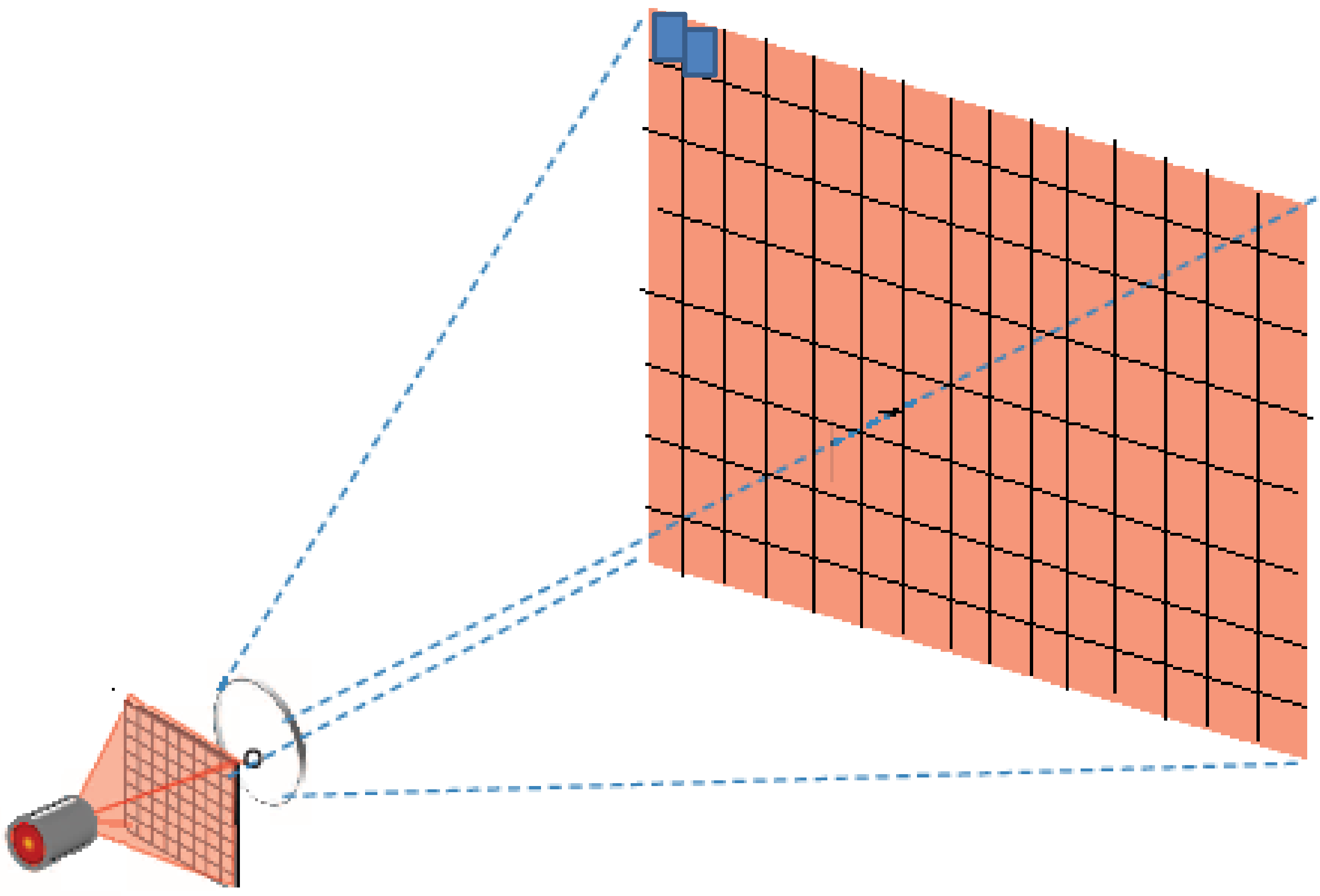
Image acquisition using a single sensor

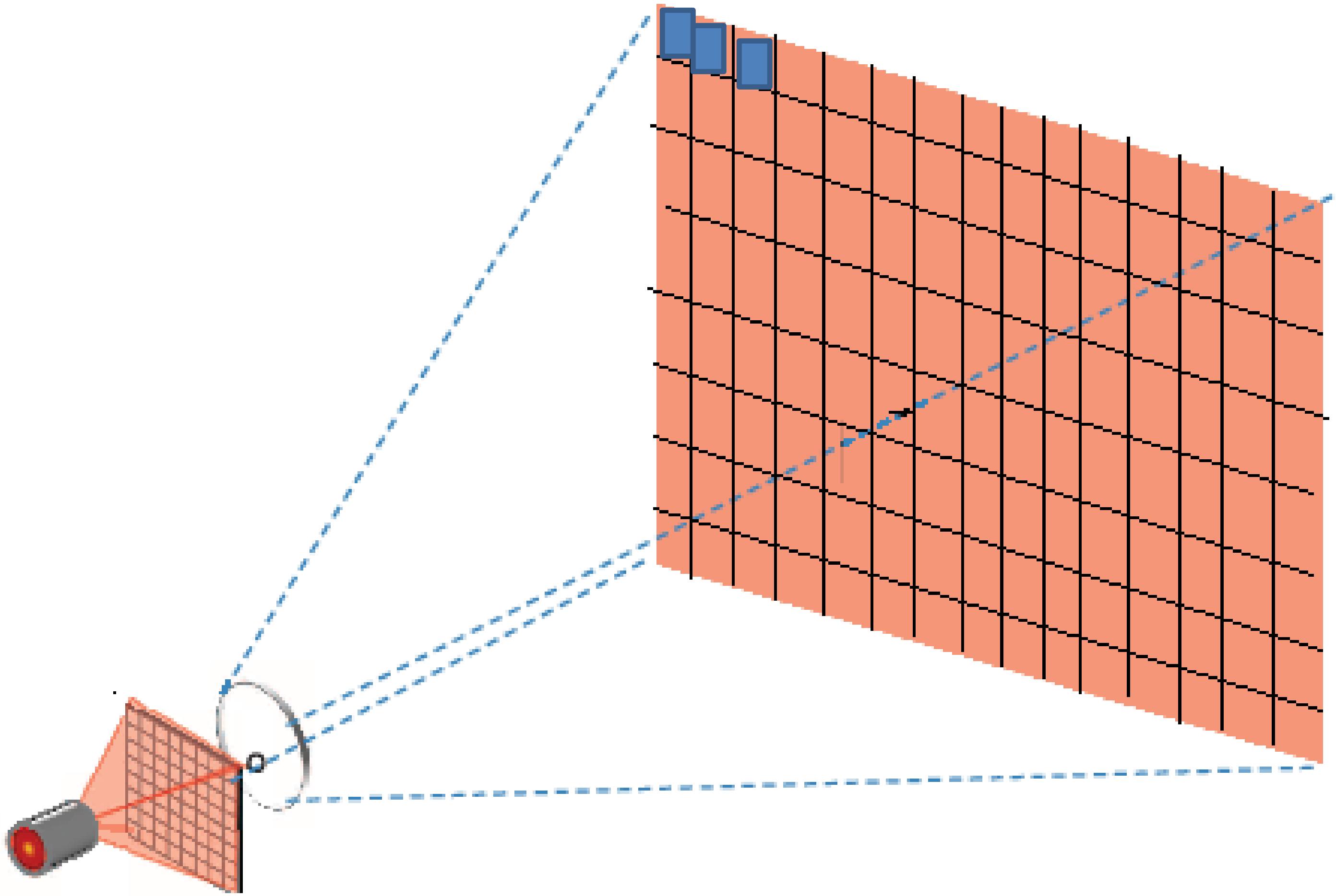
- Most familiar sensor of this type is the photodiode, which is constructed of silicon materials and whose output voltage waveform is proportional to light.

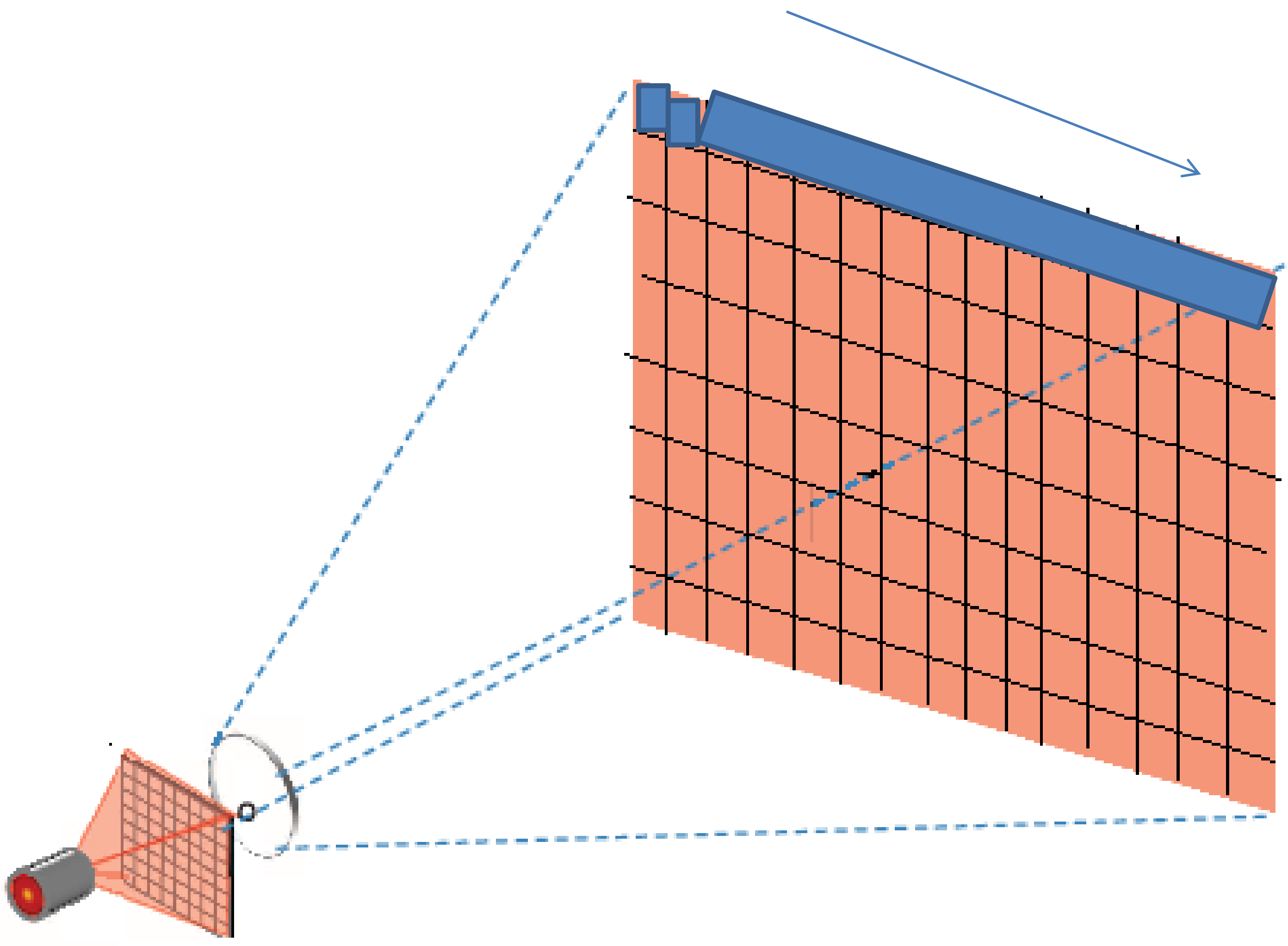
- Method : 1 Relative displacement of the sensor in both x and y direction and sensor is moving in two linear directions on a flat bed.











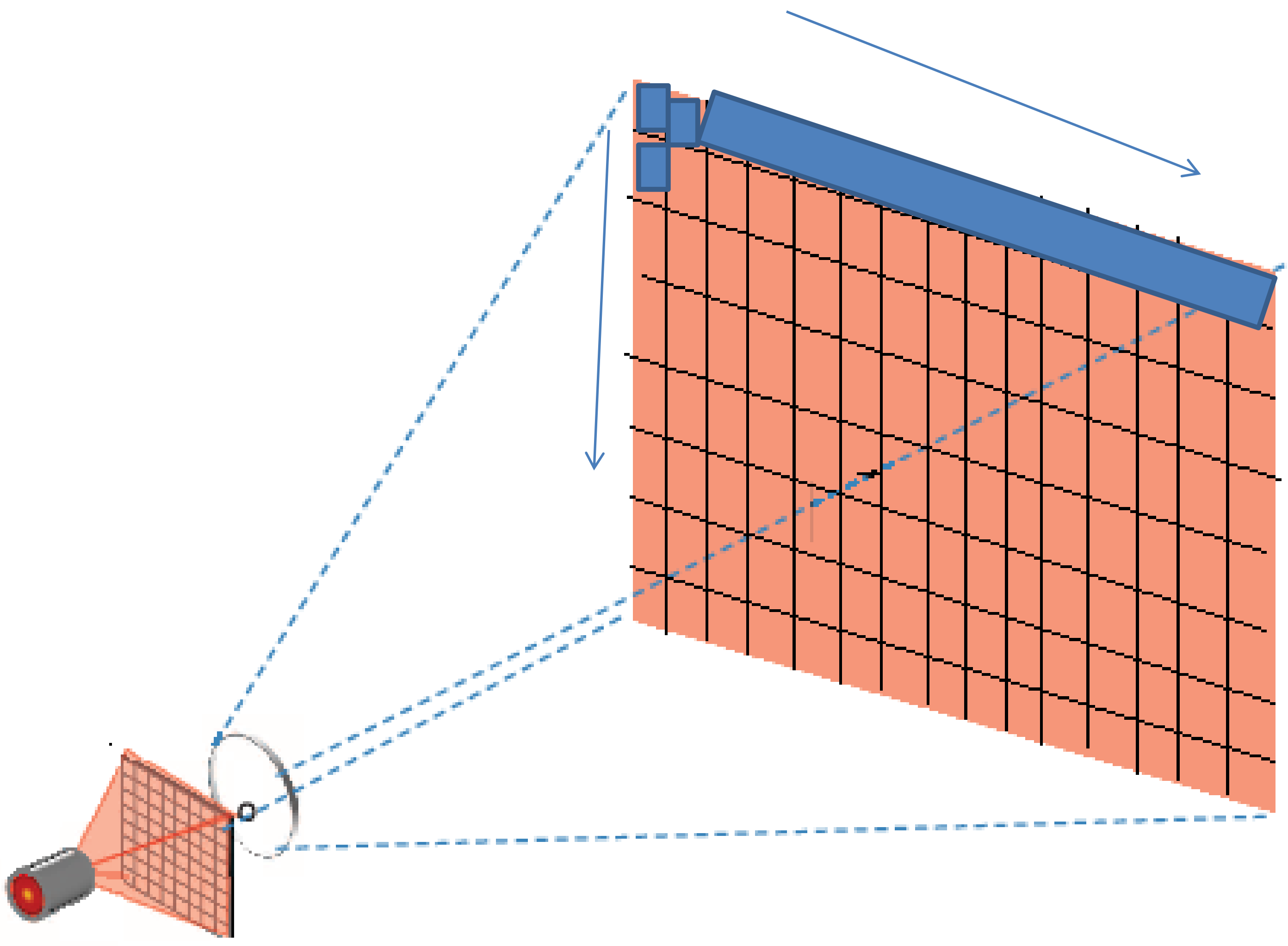


Image sensing and acquisition

Method: 2

- A film negative is mounted onto a drum whose mechanical rotation provides displacement in one dimension.
- The single sensor is mounted on a lead screw that provides motion in the perpendicular direction.

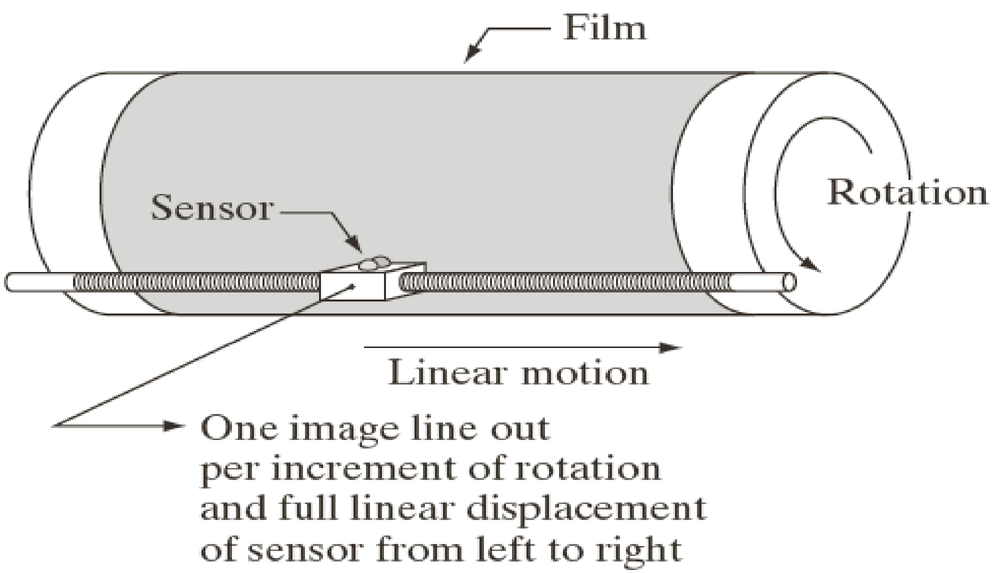


FIGURE 2.13 Combining a single sensor with motion to generate a 2-D image.

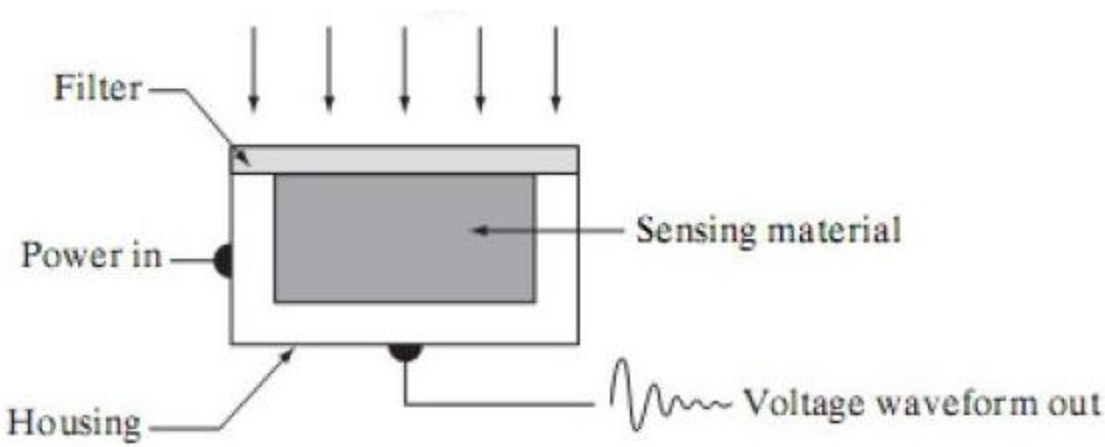
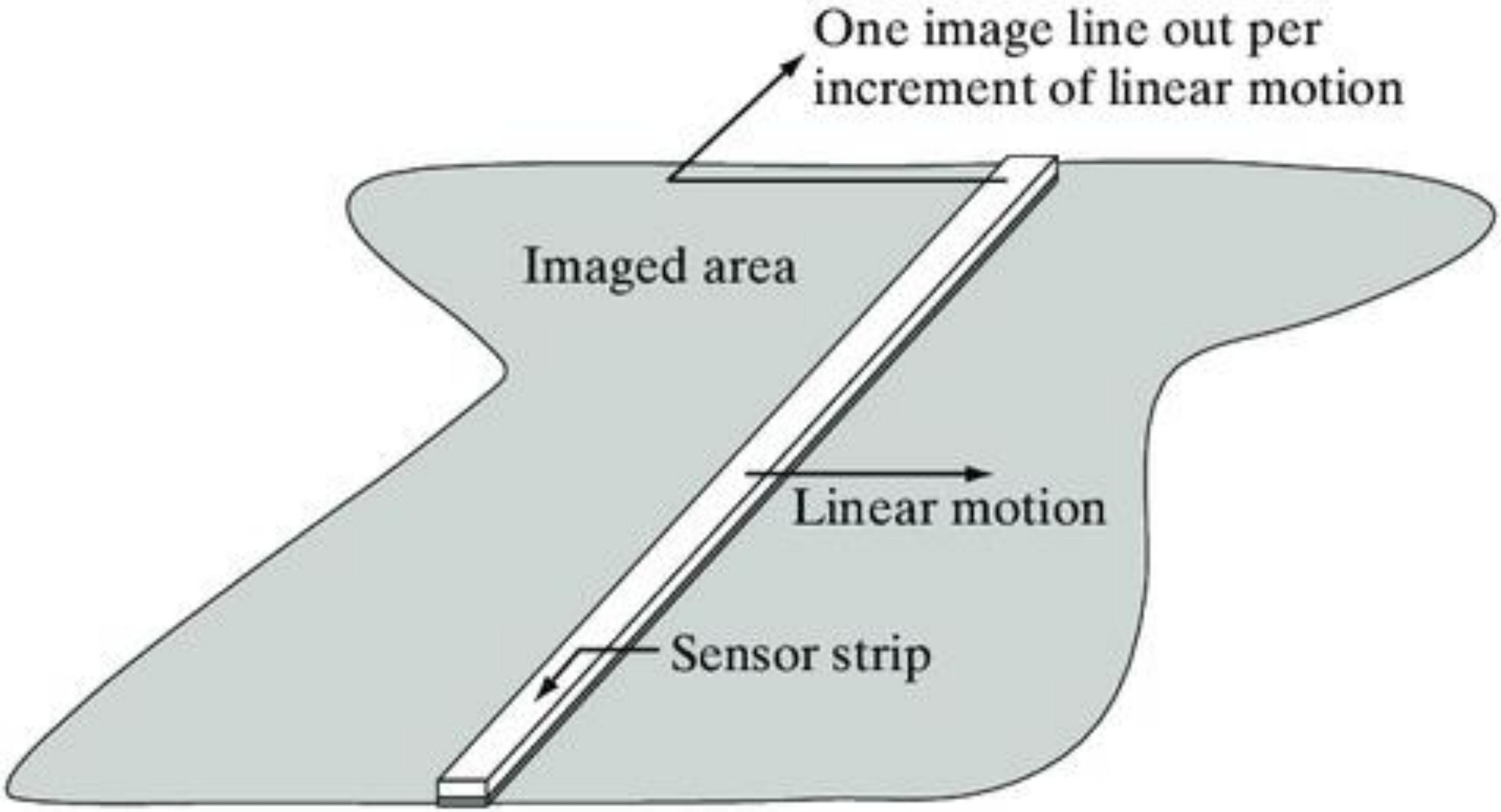

Filter
Power in
Sensing material
Housing
Voltage waveform out

Image acquisition using sensor trips

- This type of arrangement used in flat bed scanners.
- Sensing devices with 4000 or more in-line sensors are possible.
- The imaging strip gives one line of an image at a time, the motion of the strip completes the other dimension of a two-dimensional image.
- Lenses or other focusing schemes are used to project the area to be scanned onto the sensors.
- Inexpensive but slow methods.


One image line out per
increment of linear motion
Imaged area
Linear motion
Sensor strip

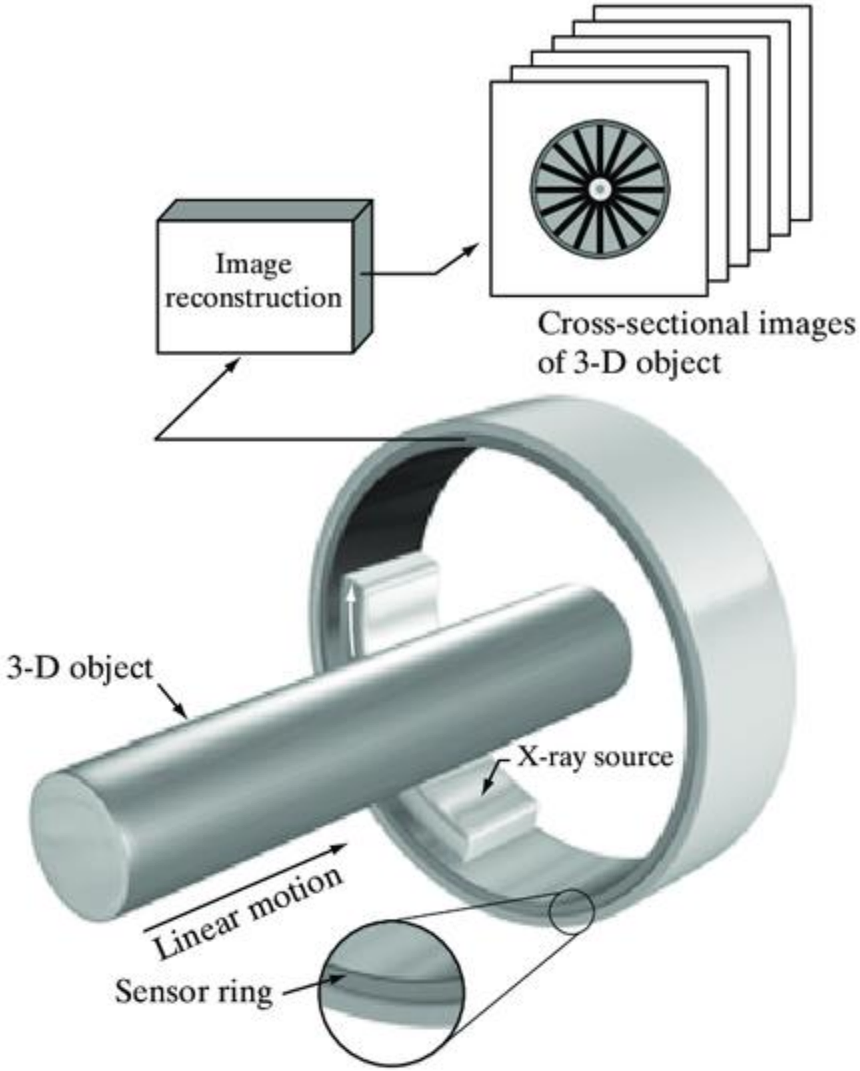


Image
reconstruction
Cross-sectional images
of 3-D object
3-D object
X-ray source
Linear motion
Sensor ring

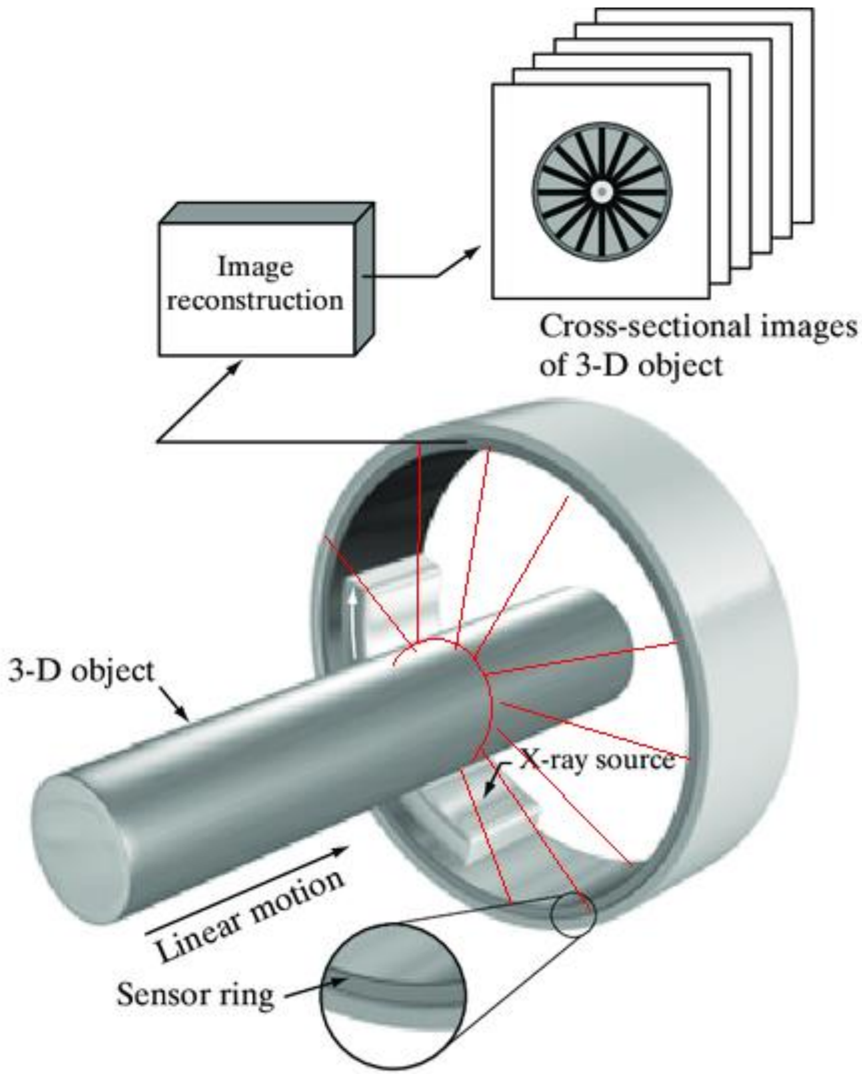


Image reconstruction
Cross-sectional images of 3-D object
3-D object
X-ray source
Linear motion
Sensor ring

Image acquisition using sensor trips

- Sensor strips mounted in a ring configuration are used in medical and industrial imaging to obtain cross-sectional(“slice”) images of 3-D objects.
- A rotating X-ray source provides illumination and the sensors opposite the source collect the X-ray energy that passes through object.
- This is the basics for medical and industrial computerized axial tomography (CAT) imaging, MRI(magnetic resonance imaging) and PET (Positron emission tomography).
- It is important to note that output of sensors must be processed by reconstruction algorithms whose objective is to transform the sensed data into meaningful cross-sectional images.

Image acquisition using sensor arrays

- Individual sensors arranged in the form of a 2-D array.
- Numerous electromagnetic and some ultrasonic sensing devices frequently are arranged in an array format.
- This is also the **predominant arrangement** found in **digital cameras**.
- A typical sensor for these cameras is a CCD (Charged Coupled Device)array, which can be manufactured with broad range of sensing properties and can be packaged in rugged arrays of 4000 * 4000 elements or more.
- CCD sensors are used widely in digital cameras and other light sensing instruments.

Image sensing and acquisition

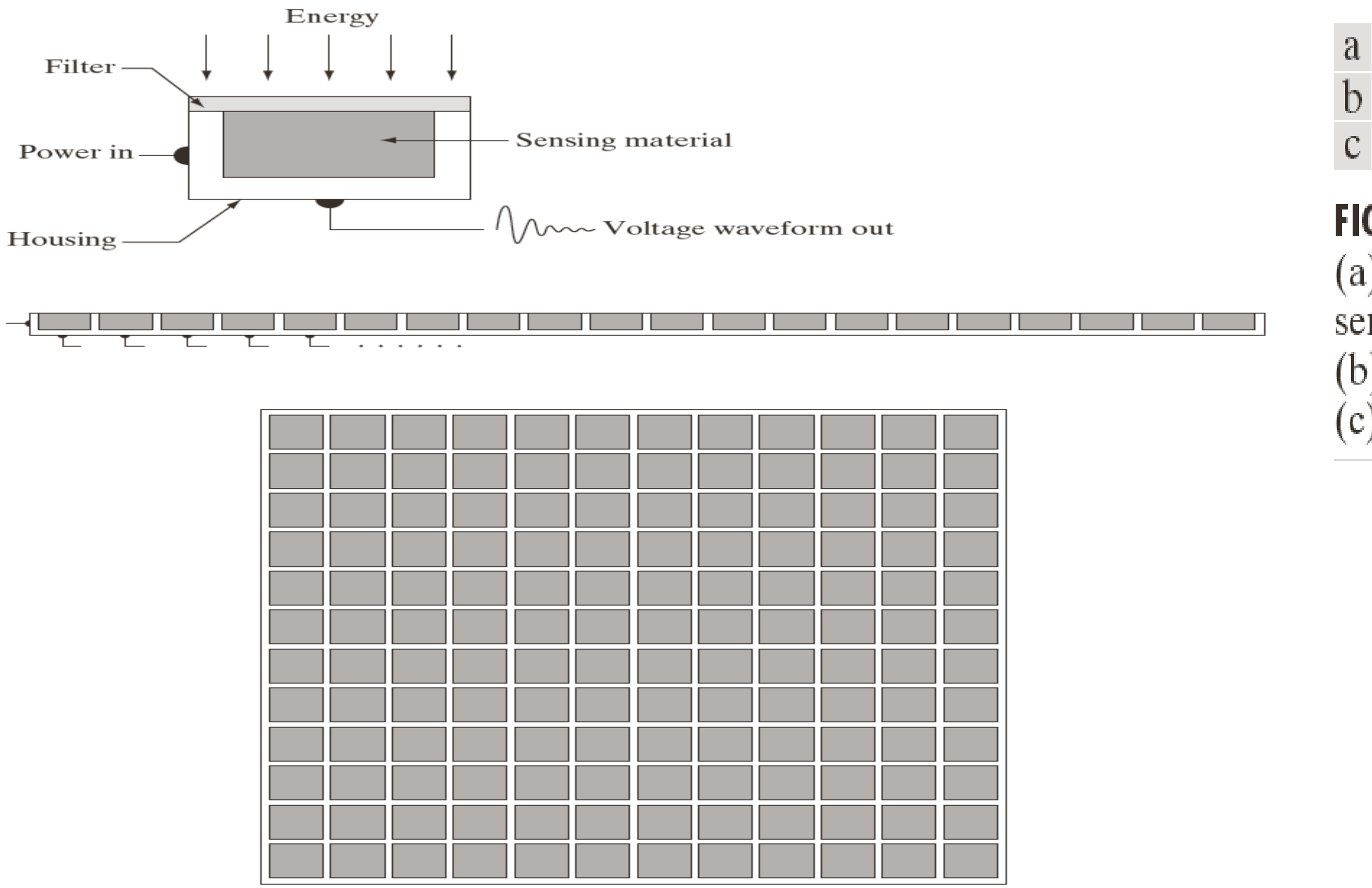
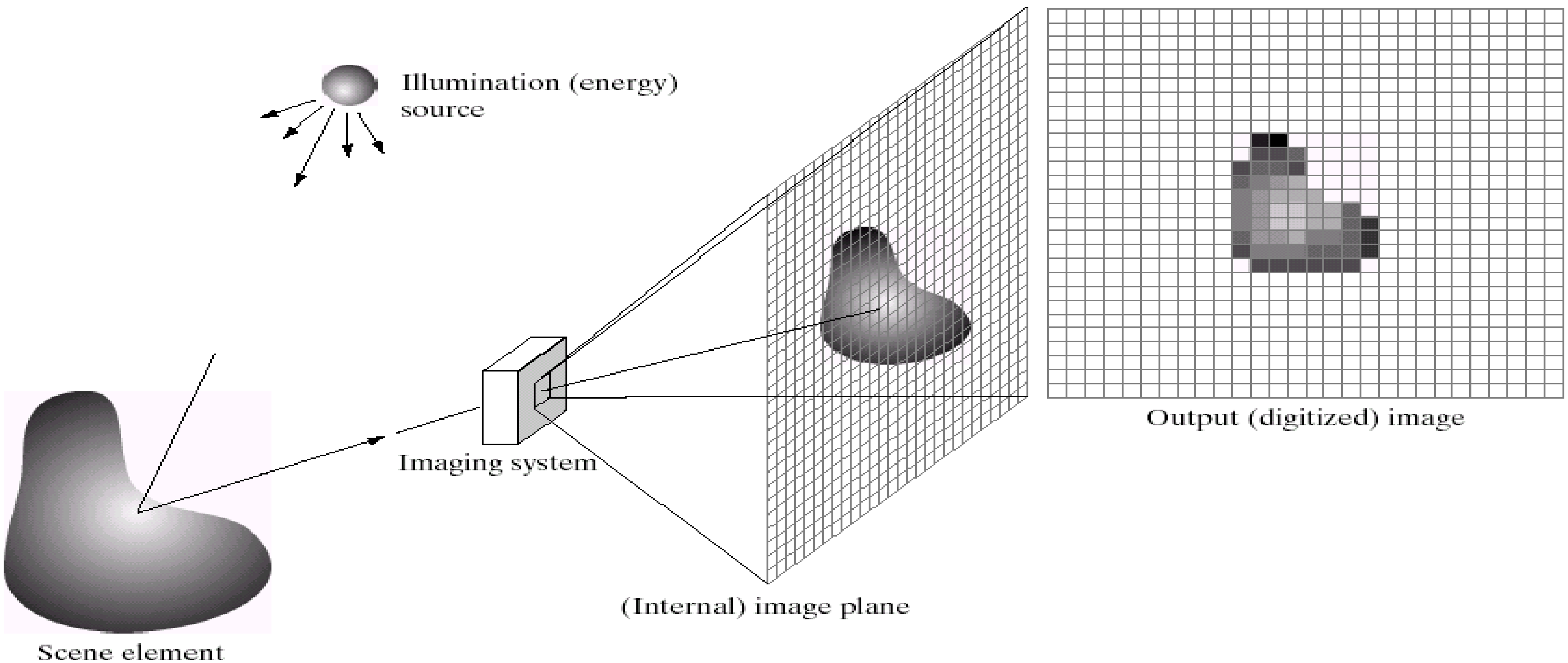


FIGURE 2.12
(a) Single imaging sensor.
(b) Line sensor.
(c) Array sensor.

Image acquisition using sensor arrays

- The response of each sensor is proportional to the integral of the light energy projected onto the surface of the sensor.
- Key advantage is that a complete image can be obtained by focusing the energy pattern onto the surface of the array.

Image acquisition using sensor arrays



a
b c d e

FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

Image acquisition using sensor arrays

- The figure shows that the energy from an illumination source being reflected from scene element.
- Figure 2.15(c), the first function performed by the imaging system in figure is to collect the incoming energy and focus it onto an image plane.
- If illumination is light , the front end of the imaging system is an optical lens that projects the viewed scene onto the lens focal plane, produces outputs proportional to the integral of the light received at each sensor as fig. 2.15(d) shows.
- The sensor array which is coincident with the focal plane, produces outputs proportional to the integral of the light received at each sensor.
- Fig. 2.15(e) conversion of an image into digital form.

Image formation model

Image formation model

- When image is generated from a physical process, its intensity values are proportional to energy radiated by a physical source(e.g. electromagnetic waves)
- As a conquence f(x,y) must be non zero and finite that is

$$0 < f(x,y) < \infty$$

- The function f(x,y) may be characterized by two components
- The amount of source illumination incident on the scene being viewed
- The amount of illumination reflected by the objects in the scene.
- Appropriately, these are called the illumination and reflectance components and are denoted by i(x,y) and r(x,y)

Image formation model

- The two functions combine as a product to form f(x, y)

$$f(x, y) = i(x, y) * r(x, y)$$

Where, $0 < i(x\ y) < \infty$ and $0 < r(x, y) < 1$

- Reflectance is bounded by 0 (total absorption) and 1 (total reflectance)
- Let the intensity (gray level) of a monochrome image at any coordinates (X_o, Y_o) be denoted by
- $L_{min} <= L <= L_{max}$
- $L_{min} = i_{min}.r_{min}$ and $L_{max} = i_{max}.r_{max}$
- The interval $[L_{min}, L_{max}]$ is called the gray (or intensity) scale
- Common practice is to shift this interval numerically to the interval [0 , L-1]

Image sampling and quantization

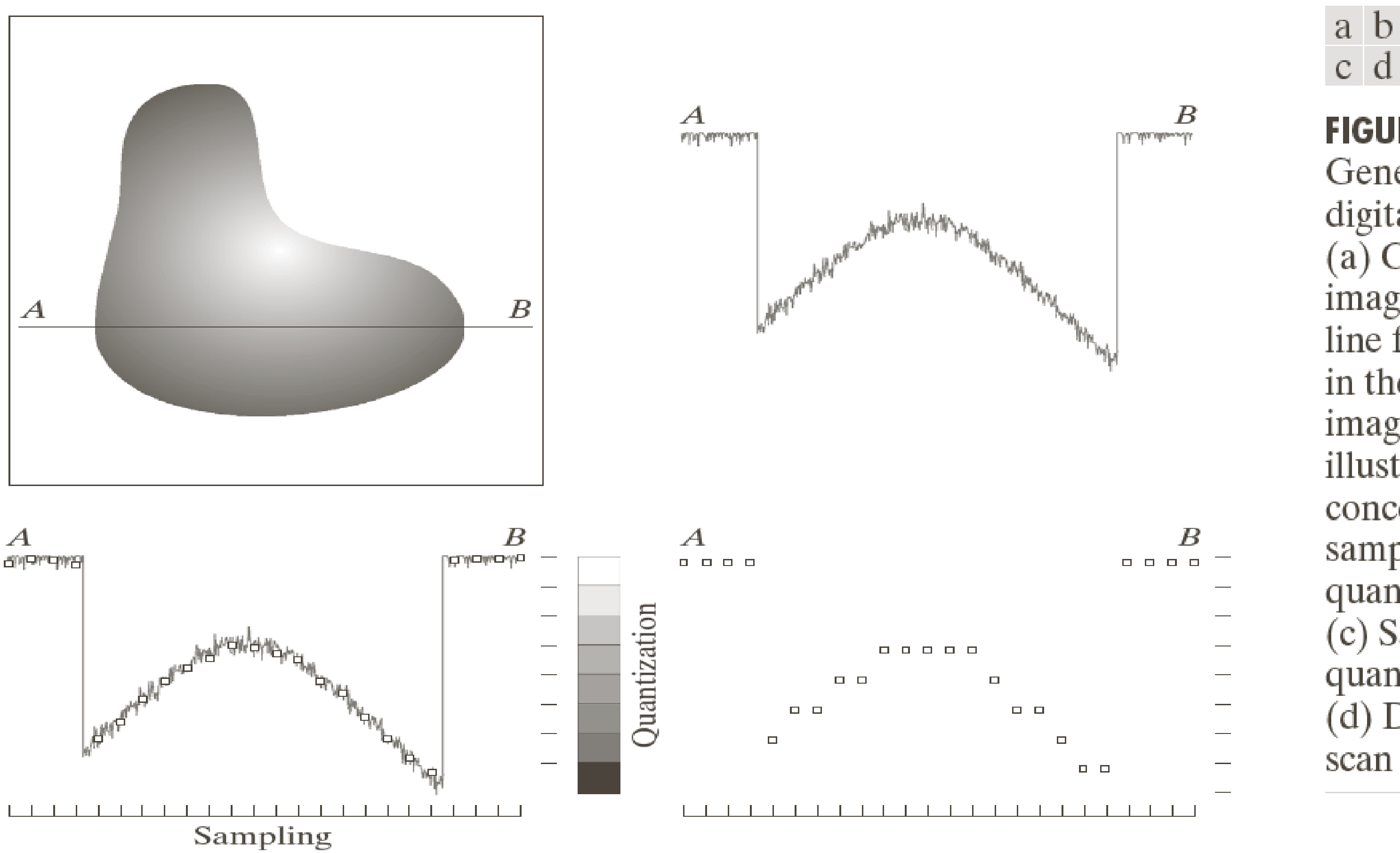
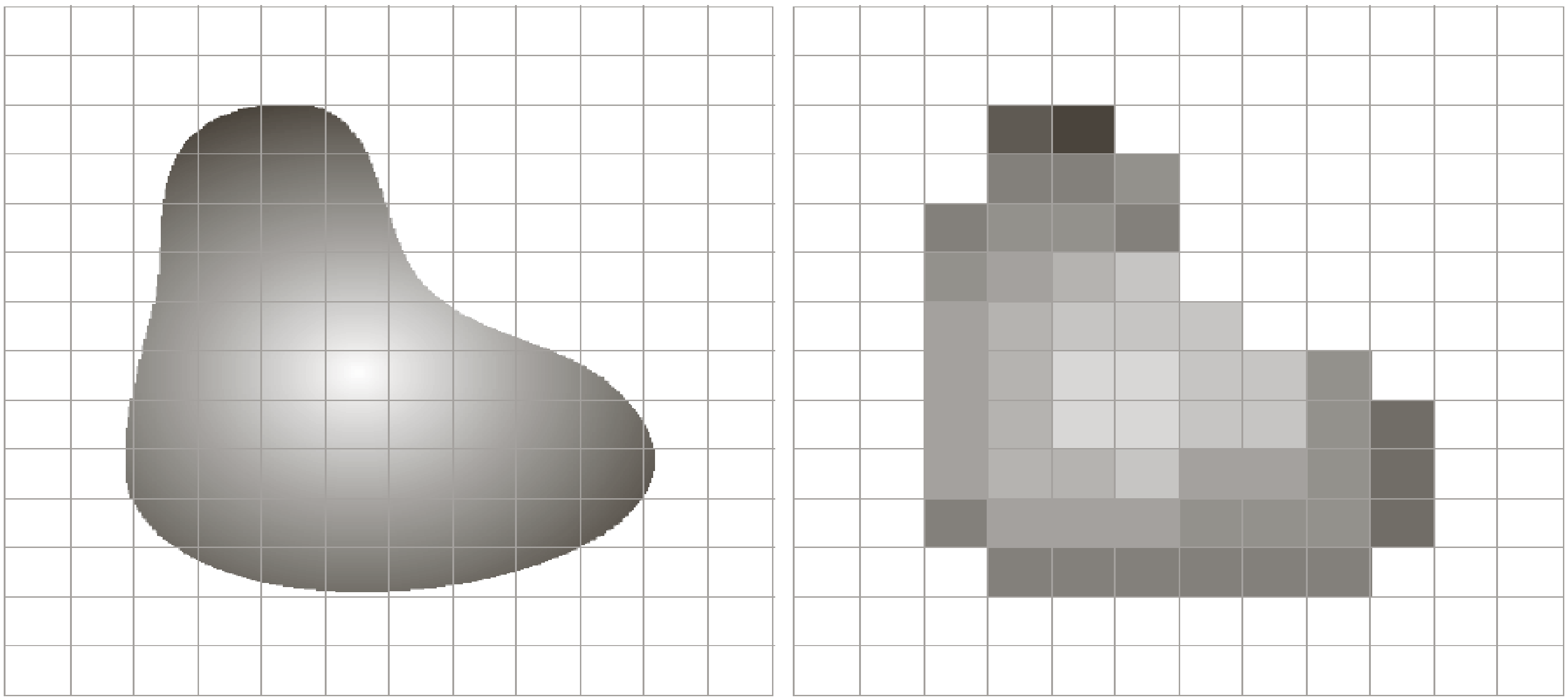


FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Image sampling and quantization



a b

FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

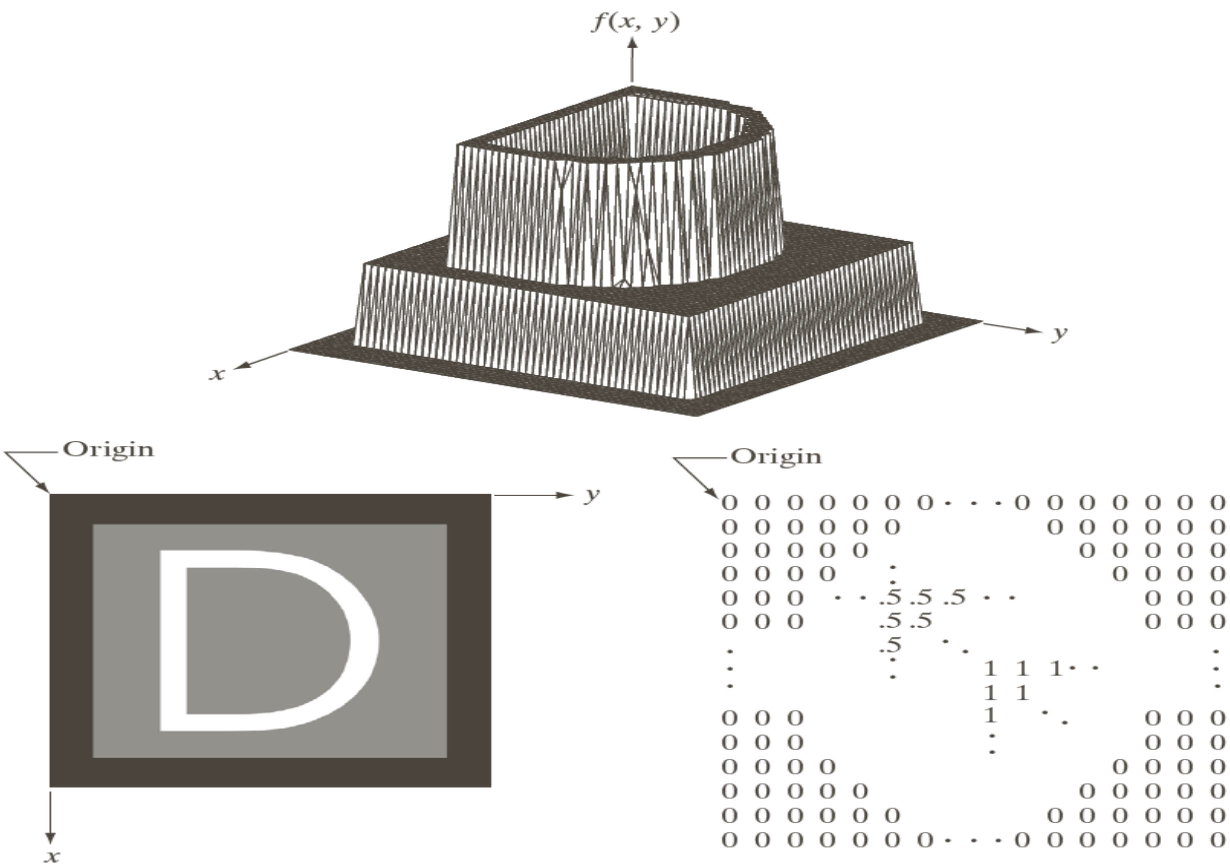
Image sampling and quantization

- Digitizing the coordinate values is called sampling .
- Digitizing the amplitude values is called quantization.
- fig. 2.16(a) shows a continuous image with respect to the x – and y- coordinates, and also in amplitude.
- The one –dimensional function in fig. 2.16(b) is a plot of amplitude (intensity level) values of the continuous image.
- The random variations are due to image noise.
- The spatial location of each sample is indicated by a vertical tick mark in the bottom part of the figure.
- The set of these discrete locations gives a sampled function.

Image sampling and quantization

- However the values of the samples still span (vertically) a continuous range of intensity values.
- In order to form a digital function, the intensity values also must be converted (quantized) into discrete quantities.
- The right side of fig. 2.16(c) shows the intensity scale divided into eight discrete intervals, ranging from black to white.
- The vertical tick marks indicate the specific value assigned to each of the eight intensity intervals.
- The continuous intensity level are quantized by assigning one of the eight values to each sample.
- Fig. 2.16(d) represents both sampling and quantization

Representation of digital images



a
b c

FIGURE 2.18
(a) Image plotted as a surface.
(b) Image displayed as a visual intensity array.
(c) Image shown as a 2-D numerical array (0, .5, and 1 represent black, gray, and white, respectively).

Representation of digital images

- This digitization process requires that decisions be made regarding the values for M , N and for the number L of discrete intensity levels.
- There are no restrictions placed on M and N other than they have to be positive integers.
- The number of intensity levels typically is an integer power of 2

$$L = 2^k$$

Representation of digital images

- The number of bits required to store a digitized image is

$$b = M * N * K$$

If M = N this equation becomes

$$b = N^2K$$

When an image can have 2^K intensity levels, it is common practice to refer to the images as a “k-bit image”

Ex. $b = 32^2.1=1024$, Where N =32,K=1.

Representation of digital images

TABLE 2.1
Number of storage bits for various values of N and k.

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

Saturation vs. Noise

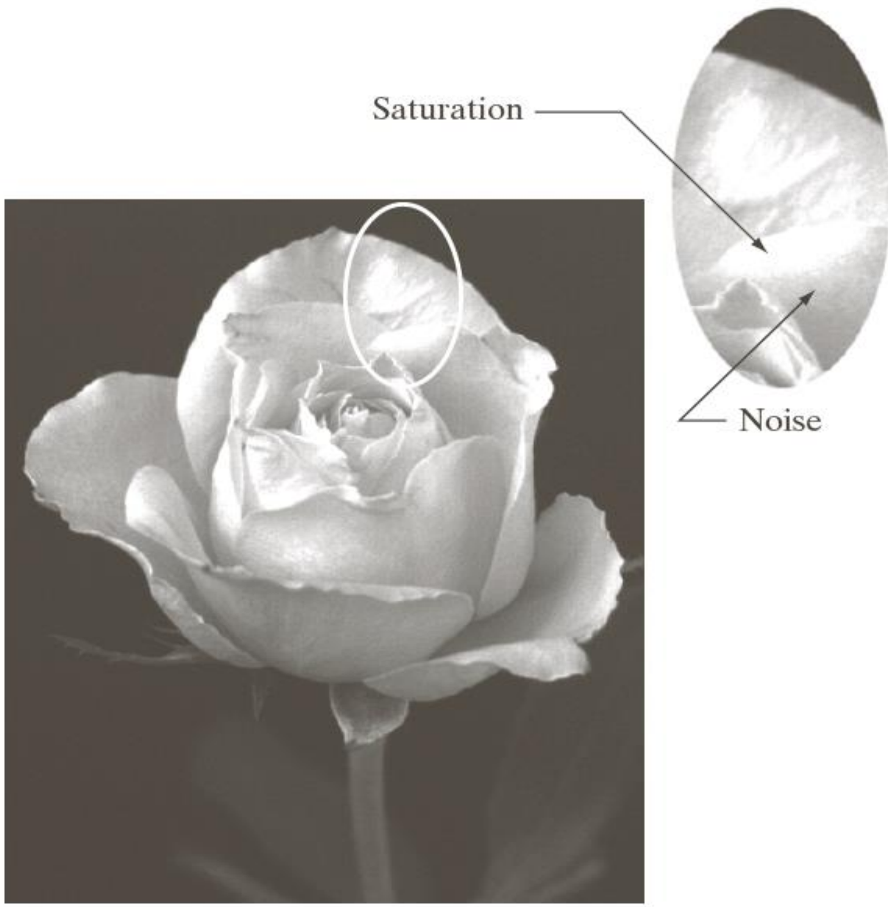


FIGURE 2.19 An image exhibiting saturation and noise. Saturation is the highest value beyond which all intensity levels are clipped (note how the entire saturated area has a high, *constant* intensity level). Noise in this case appears as a grainy texture pattern. Noise, especially in the darker regions of an image (e.g., the stem of the rose) masks the lowest detectable true intensity level.

Spatial resolution

- Spatial Resolution: measure of the smallest discernible detail in an image.
- Spatial resolution can be stated in a number of ways, with line pairs per unit distance, and dots (pixels) per unit distance being among the most common measures.
- In the U.S. this measure usually is expressed as dots per inch (dpi).
- Newspaper are printed with a resolution of 75 dpi, magazines at 133, glossy brochures at 175 dpi, and book of DIP (by gonzalez) is printed at 2400 dpi.

Spatial resolution

a b
c d

FIGURE 2.20 Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.

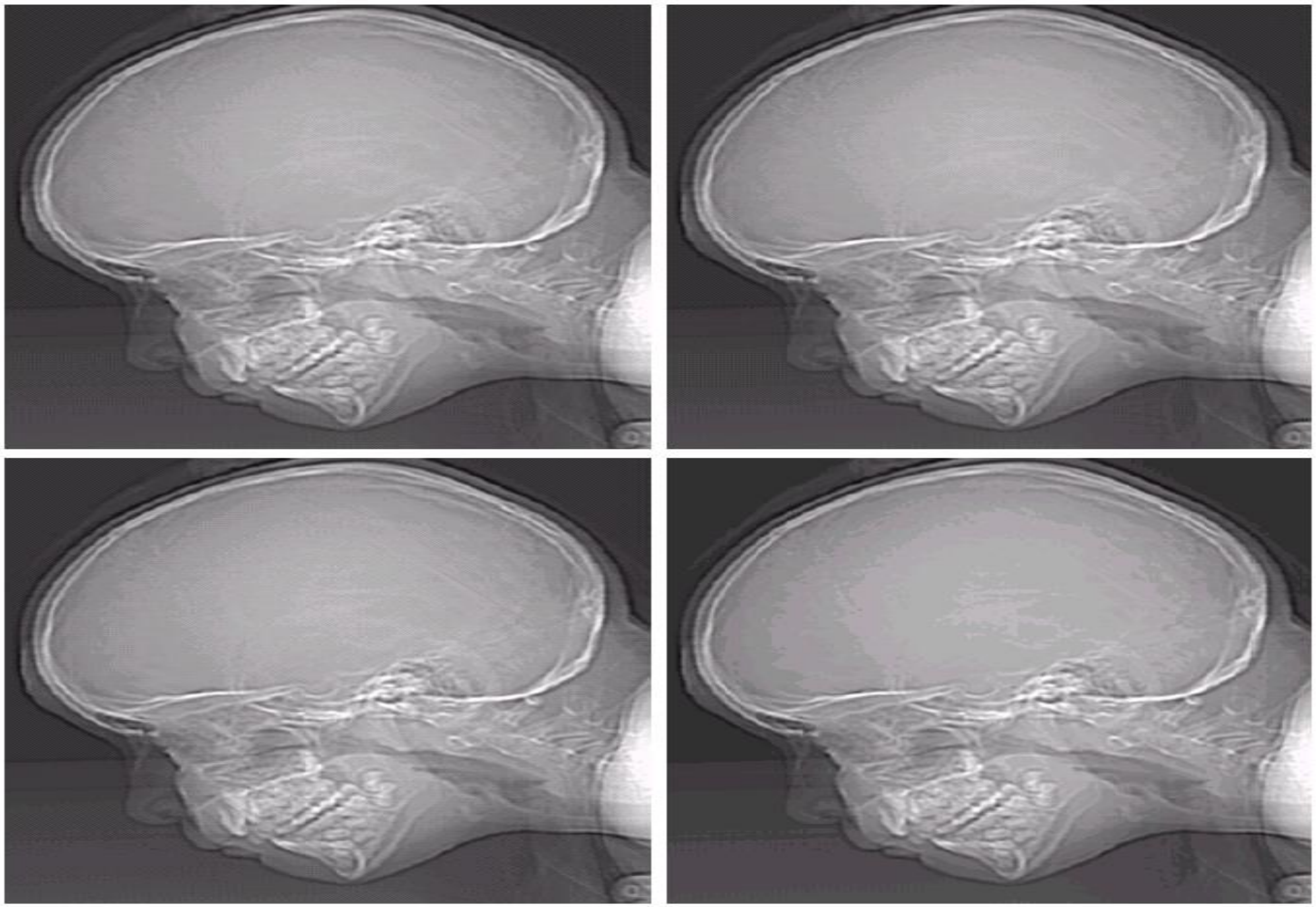
Intensity resolution

- Intensity Resolution: measure of the smallest discernible change in intensity level.
- Based on hardware consideration , the number of intensity levels usually is a integer power of two, as mentioned in previous slides.
- The most common number is 8 bits, with 16 bits being used in some applications in which enhancement of specific intensity ranges is necessary.

Intensity resolution

- Fig. 2.21(d) however , has an imperceptible set of very fine ridge-like structures in areas of constant or nearly constant intensity (Particularly in the skull)
- These effect is caused by the use of insufficient number of intensity levels in smooth areas of a digital image is called false contouring.
- False contouring generally is quite visible in images displayed using 16 or less uniformly spaced intensity levels as in fig. 2.21(e) through h

Intensity resolution



a b
c d

FIGURE 2.21
(a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

Intensity resolution

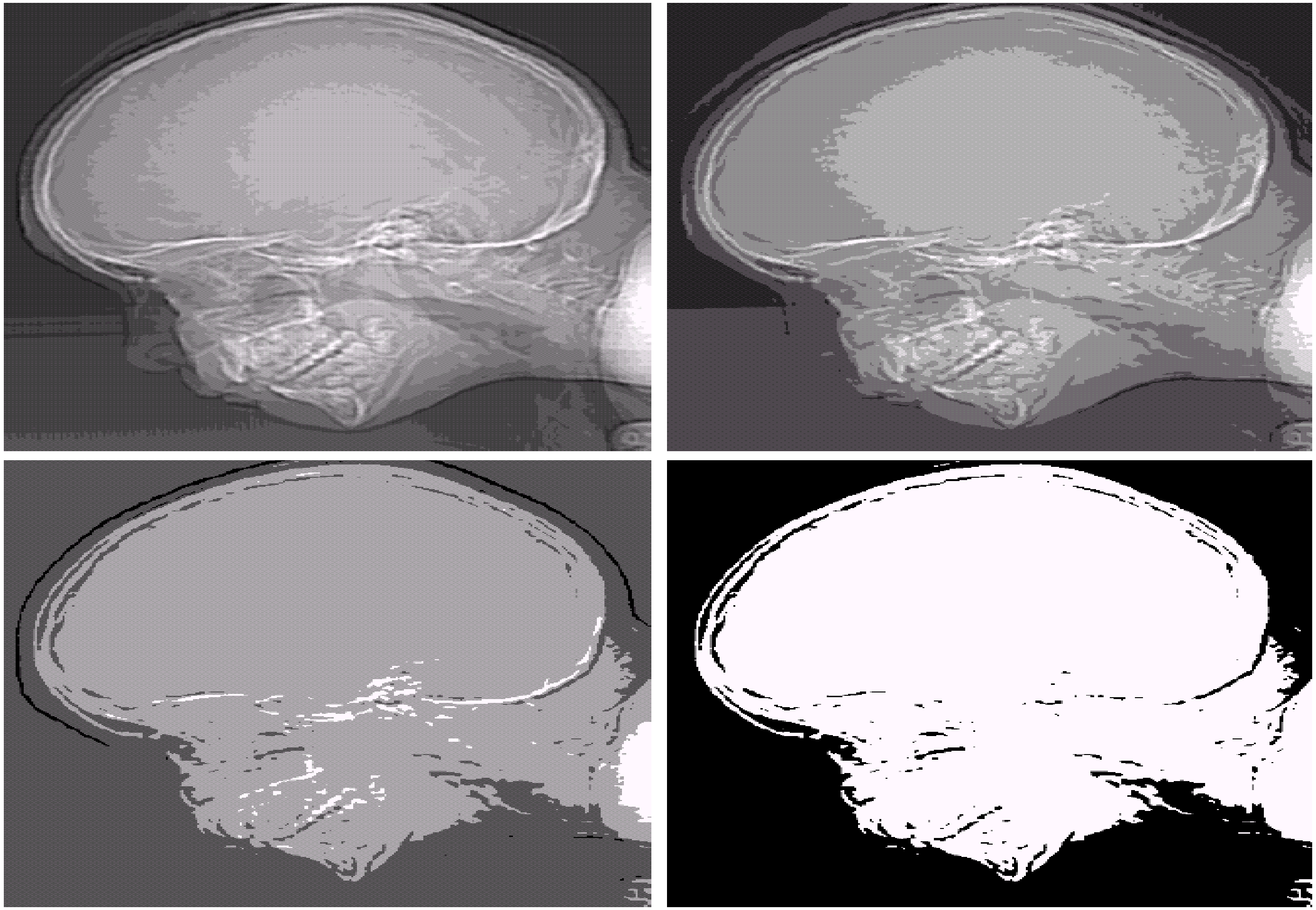


FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)

Isopreference curve

a b c

FIGURE 2.22 (a) Image with a low level of detail. (b) Image with a medium level of detail. (c) Image with a relatively large amount of detail. (Image (b) courtesy of the Massachusetts Institute of Technology.)

Isopreference curve

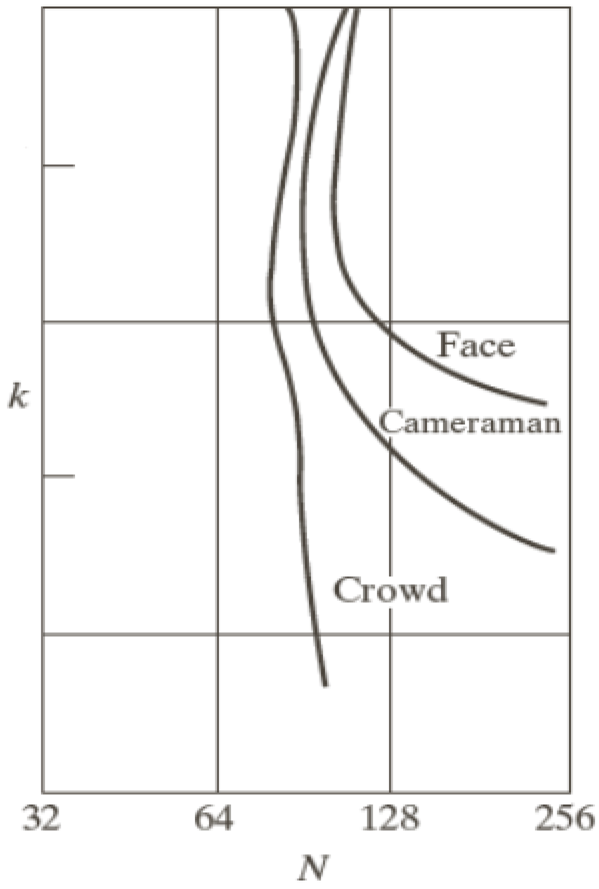
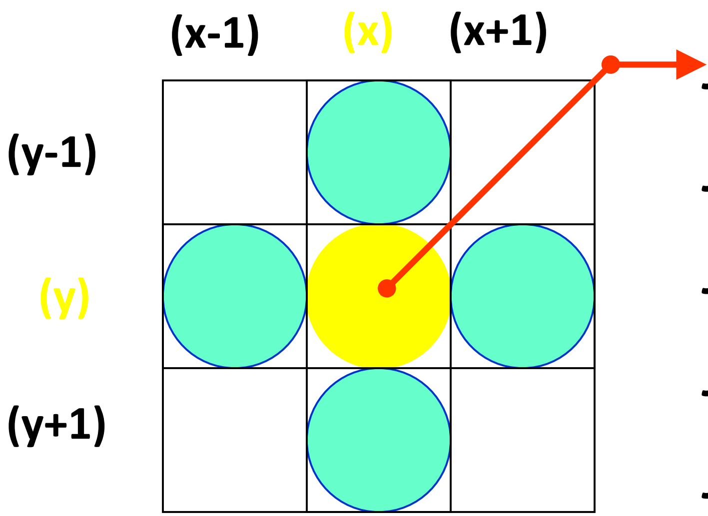


FIGURE 2.23 Typical isopreference curves for the three types of images in Fig. 2.22.

4-neighbors of pixel

- 4-neighbors of pixel is denoted by $N_4(p)$
- It is set of horizontal and vertical neighbors



f(x,y) is a yellow circle

f(x,y-1) is top one

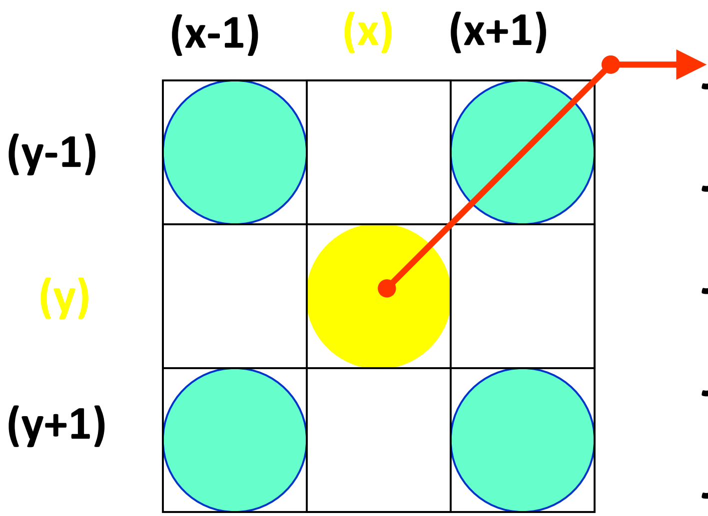
f(x-1,y) is left one

f(x+1,y) is right one

f(x,y+1) is bottom one

Diagonal neighbors of pixel

- Diagonal neighbors of pixel is denoted by $N_D(p)$
- It is set of diagonal neighbors



f(x,y) is a yellow circle

f(x-1,y-1) is top-left one

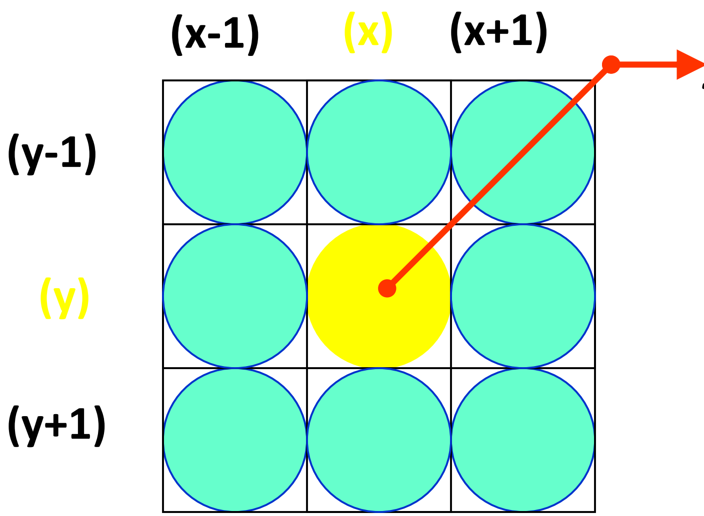
f(x+1,y-1) is top-right one

f(x-1,y+1) is bottom-left one

f(x+1,y+1) is bottom-right one

8-neighbors of pixel

- 8-neighbors of pixel is denoted by $N_8(p)$
- 4-neighbors and Diagonal neighbors of pixel



$f(x,y)$ is a yellow circle

$(x-1,y-1)$, $(x,y-1)$,$(x+1,y-1)$,

$(x-1,y)$, (x,y), $(x+1,y)$,

$(x-1,y+1)$,$(x,y+1)$, $(x+1,y+1)$

Connectivity

- Establishing boundaries of objects and components of regions in an image.
- Group the same region by assumption that the pixels being the same color or equal intensity will are the same region

Connectivity

- Let C is the set of colors used to define
- There are three type of connectivity:
  - 4-Connectivity : 2 pixels (*p* and *q*) with value in C are 4-connectivity if q is in the set $N_4(p)$
  - 8-Connectivity : 2 pixels (*p* and *q*) with value in C are 8-connectivity if q is in the set $N_8(p)$
  - M-Connectivity : 2 pixels (*p* and *q*) with value in C are 8-connectivity if

    (i) Q is in $N_4(p)$, or

    (ii) Q is in $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ is empty

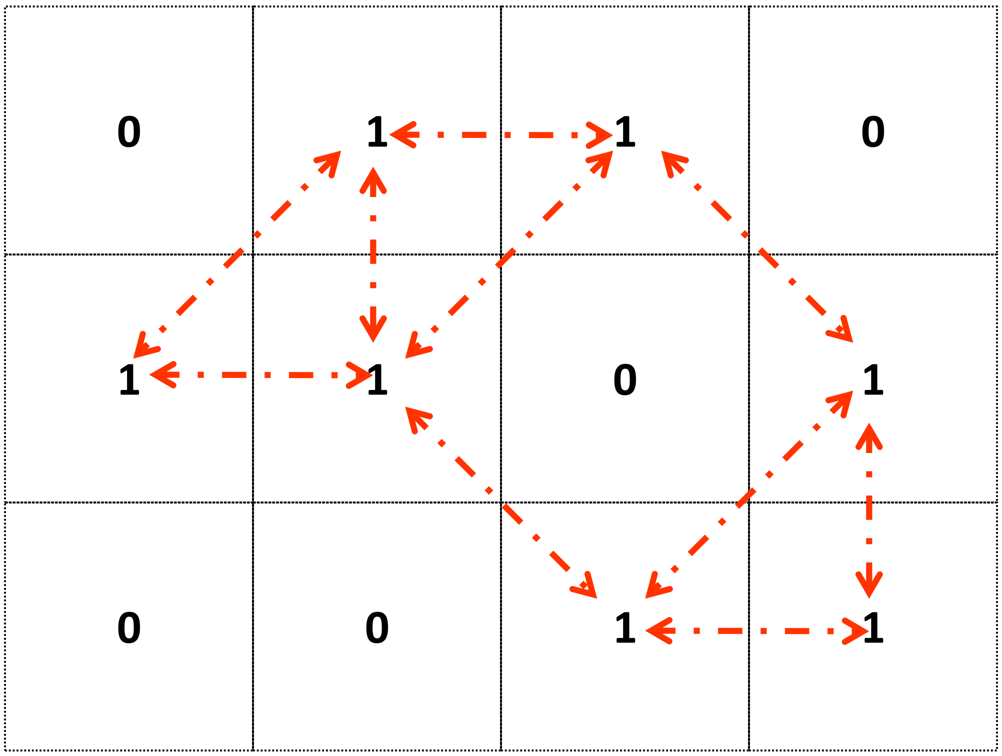
Example 4-Connectivity

- Set of color consists of color 1 ; C ={1}

0	**1**	**1**	**0**
1	**1**	**0**	**1**
0	**0**	**1**	**1**

Example 8-Connectivity

- Set of color consists of color 1 ; C ={1}



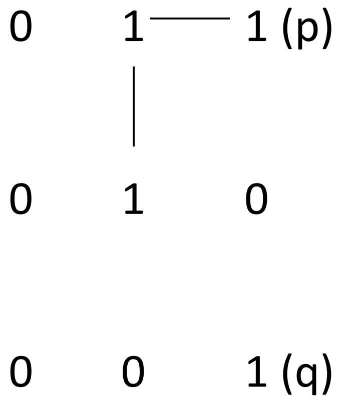
Example M-Connectivity

- Set of color consists of color 1 ; C ={1}

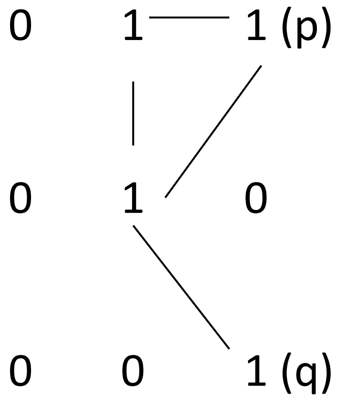
0	1	1	0
1	1	0	1
0	0	1	1

4- Path, 8- Path and m-path

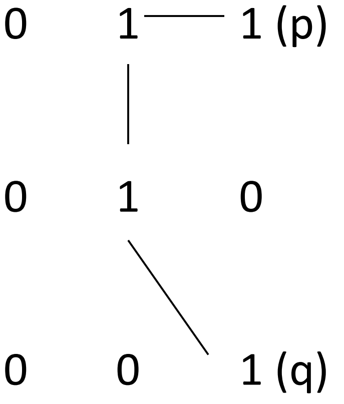
- Path: 4, 8, and m-paths
  - A sequence of distinct pixels from pixel p to q.



4-Path does not Exist



8-Path Exists



m-Path Exists

Pixel adjacencies and paths

- Pixel p is adjacent to q if they are connected
- We can define 4-, 8-, or m-adjacency depending on the specified type of connectivity
- Two image subsets S1 and S2 are adjacent if some pixel in S1 is adjacent to S2
- A path from p at (x,y) to q at (s,t) is a sequence of distinct pixels with coordinates $(X_{0,}\ Y_0)$, $(X_{1,}\ Y_1)$,….., $(X_{n,}\ Y_n)$
- Where $(X_{0,}\ Y_0)=(x,y)$ and $(X_{n,}\ Y_n)=(s,t)$ and $(X_{i,}\ Y_i)$ is adjacent to (x_{i-1},y_{i-1}) for 1<= i <= n - n is the length of the path
- If p and q are in S, then p is connected to q in S if there is a path from p to q consisting entirely of pixels in S

Connectivity

- Connectivity
  - Let S represent a subset of pixels in an image.

    Two pixels p and q are said to be connected in S if there exists a path between them entirely of pixels in S
  - There are 4, 8 and m-connectivity
  - Connect set: only has one connected component.

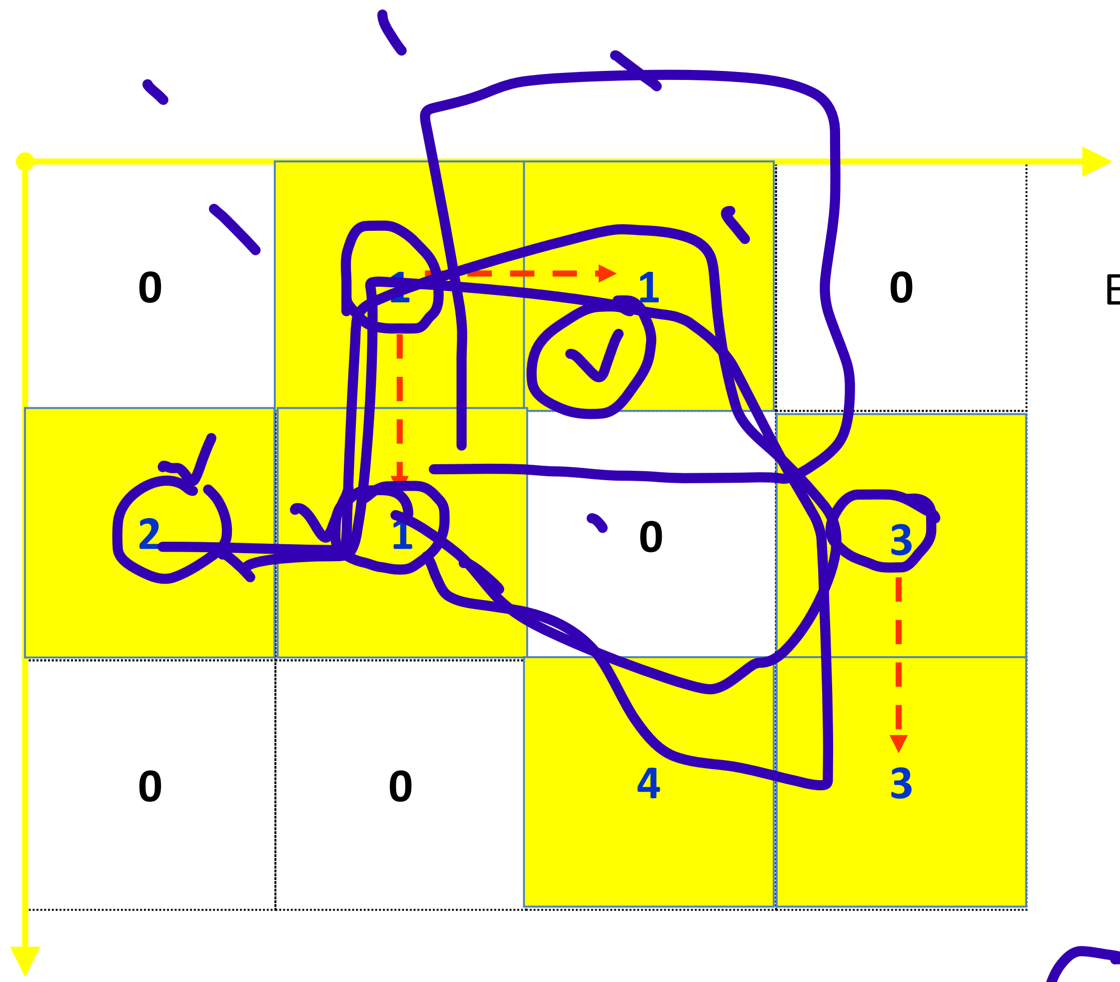
Labeling of connected Components

- Scan an image pixel by pixel from left to right and top to bottom
- Equivalent labeling

Labeling of connected Components

- P is pixel scanned process
- If pixel p is color value 0 move on the next scaning
- If pixel p is color value 1 examine pixel top and left
  - If top and left were 0, assign a new label to p
  - If only one of them’re 1, assign its label to p
  - If both of them’re 1 and have
    - the same number, assign their label to p
    - Different number, assign label of top to p and make note that two label is equivalent
- Sort all pairs of equivalent labels and assign each equivalent to be same type

Example connected components Labeling

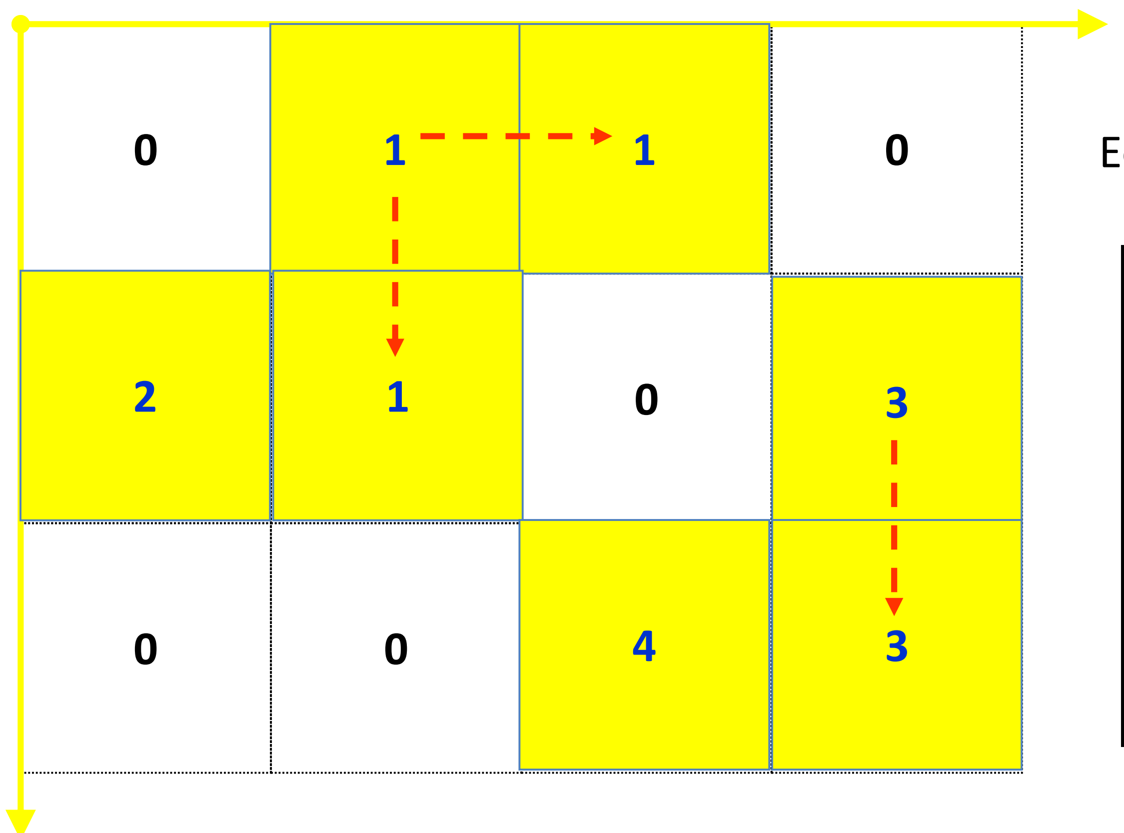


Equivalent Table

1	2
3	4

1

Connected components Labeling using **4-connectivity**



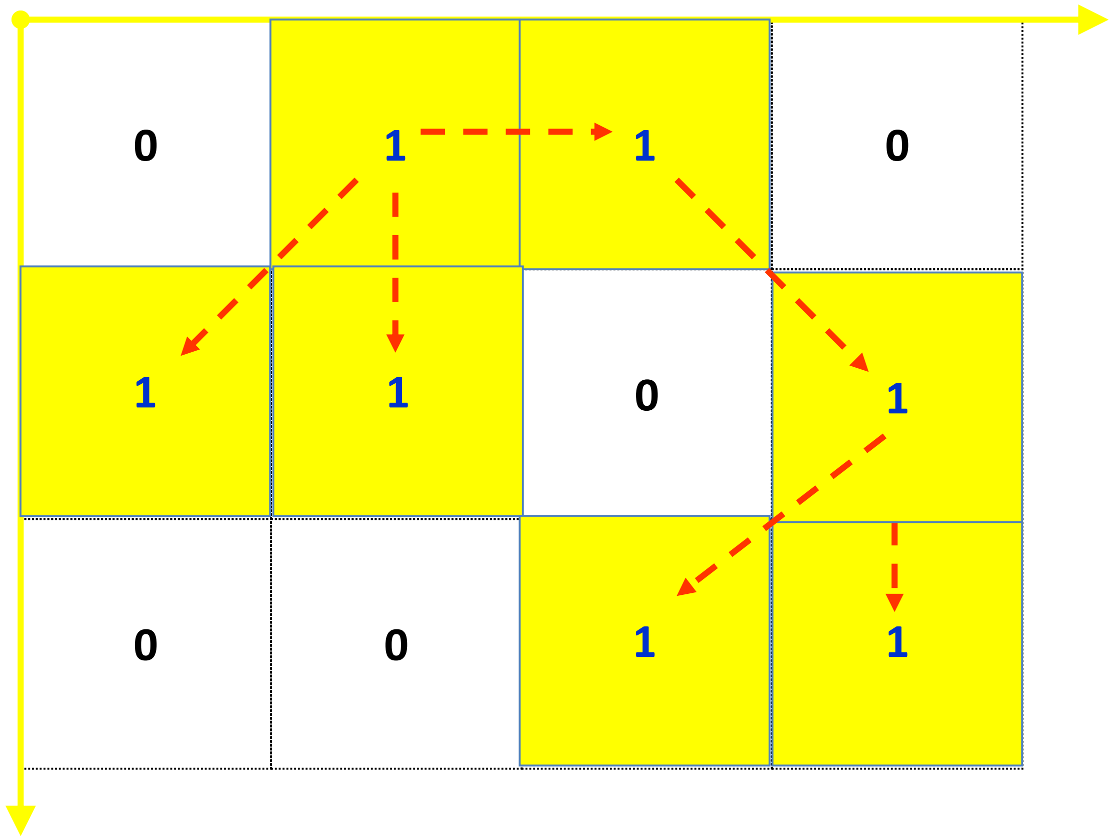
Equivalent Table

1	2
3	4

Connected components Labeling using 8-connectivity

- Steps are same as 4-connected components
- But the pixel that are consider is 4 previous pixels (top-left, top, top-right and left)

Example-components Labeling using **8-connectivity**



Find out the connected components of a given image using 8-connectivity

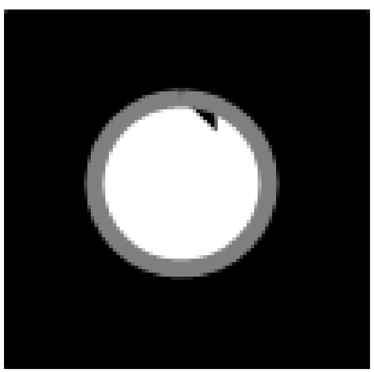
1	1	0	1	1	1	0	1
1	1	0	1	0	1	0	1
1	1	1	1	0	0	0	1
0	0	0	0	0	0	0	1
1	1	1	1	0	1	0	1
0	0	0	1	0	1	0	1
1	1	0	1	0	0	0	1
1	1	0	1	0	1	1	1

Labeling (answer)

1	1	0	1	1	1	0	2
1	1	0	1	0	1	0	2
1	1	1	1	0	0	0	2
0	0	0	0	0	0	0	2
3	3	3	3	0	4	0	2
0	0	0	3	0	4	0	2
5	5	0	3	0	0	0	2
5	5	0	3	0	2	2	2

Labeling connected components in non-binary images

- The 4-connected or 8-connected labeling schemes can be extended to gray level images
- The set *V* may be used to connect into a component only those pixels within a specified range of pixel values



000	000	000	000	000
000	000	128	000	000
000	128	255	128	000
000	128	255	128	000
000	000	128	000	000
000	000	000	000	000

	L0	
L0	L1	L0
L0	L1	L0
	L0	

Region

- Region
  - Region is a connected set.

$$\left.\begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{matrix}\right\} \quad \text{R1}$$

$$\begin{matrix} 0 & 0 & 1 \end{matrix}$$

$$\left.\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}\right\} \quad \text{R2}$$

Only 8-path between two regions exists

Boundary

- Boundary
  - The border of region is the set of pixels in the region that have at least one background neighbor.

0	0	0	0
0	0	1	0
0	1	1	0
0	1	1	0
0	0	0	0

0	0	0
0	1	0
0	1	0
0	1	0
0	0	0

Regions & Boundaries

Boundaries (border or contour): The boundary of a region R is the set of points that are adjacent to points in the compliment of R.

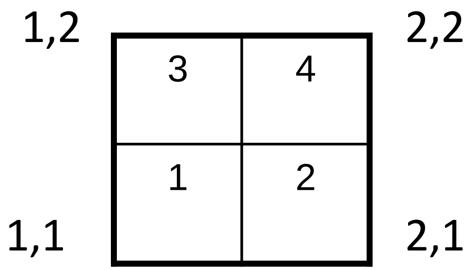
0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

RED colored 1 is NOT a member of border if 4-connectivity is used between region and background. It is if 8-connectivity is used.

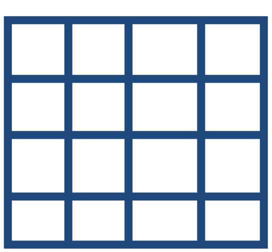
Image interpolation

- Interpolation is a basic tool used extensively in tasks such as zooming, shrinking, rotating and geometric corrections.
- In this section our principal objective is image resizing (shrinking and zooming) which are basically image resampling methods.
- Interpolation is the process of using known data to estimate values of unknown locations.
- Suppose image of size 500 * 500 pixels has to be enlarged 1.5 times to 750 * 750.

Image Zooming (1)
(Bi-linear Interpolation)



(x1,y1) = (1,1)
(x2,y1) = (2,1)
(x1,y2) = (1,2)
(x2,y2) = (2,2)
f(Q11) = 1
f(Q21) = 2
f(Q12) = 3
f(Q22) = 4



y

1,2	1.33,2	1.66,2	2,2
1,1,66	1.33,1.66	1.66,1.66	2,1.66
1,1.33	1.33,1.33	1.66,1.33	2,1.33
1,1	1.33,1	1.66,1	2,1

x

90	150
91	150

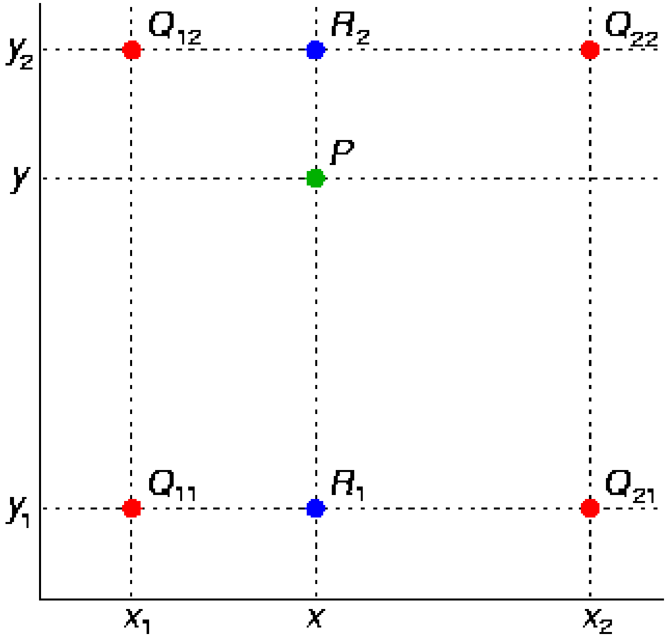
90			150
91			151

Image Zooming (2)
(Bi-linear Interpolation)

$$f(R_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21}) \quad \text{where} \quad R_1 = (x, y_1),$$

$$f(R_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22}) \quad \text{where} \quad R_2 = (x, y_2).$$

$$f(P) \approx \frac{y_2 - y}{y_2 - y_1} f(R_1) + \frac{y - y_1}{y_2 - y_1} f(R_2).$$



$$f(x,y) \approx \frac{f(Q_{11})}{(x_2 - x_1)(y_2 - y_1)}(x_2 - x)(y_2 - y) + \frac{f(Q_{21})}{(x_2 - x_1)(y_2 - y_1)}(x - x_1)(y_2 - y)$$

$$+ \frac{f(Q_{12})}{(x_2 - x_1)(y_2 - y_1)}(x_2 - x)(y - y_1) + \frac{f(Q_{22})}{(x_2 - x_1)(y_2 - y_1)}(x - x_1)(y - y_1).$$

Image Zooming (3)
(Bi-linear Interpolation)

$$f(1.33,1) = \frac{(2 - 1.33)}{(2-1)} *1 + \frac{(1.33 - 1)}{(2-1)} *2$$

$$f(1.33,1) = 0.67 + 0.66$$

$$= 1.33$$

$$f(1.33,2) = \frac{(2 - 1.33)}{(2-1)} * 3 + \frac{(1.33 - 1)}{(2-1)} *4$$

$$f(1.33,2) = 0.67 * 3 + 0.33 * 4$$

$$= 3.33$$

$$f(1.66,1) = \frac{(2 - 1.66)}{(2-1)} *1 + \frac{(1.66 - 1)}{(2-1)} *2$$

$$f(1.66,1) = 0.34 + 1.32$$

$$= 1.66$$

$$f(1.66,2) = \frac{(2 - 1.66)}{(2-1)} *3 + \frac{(1.66 - 1)}{(2-1)} *4$$

$$f(1.66,2) = 0.34 * 3 + 0.66 * 4$$

$$= 3..66$$

Image Zooming (4)
(Bi-linear Interpolation)

$$f(1,1.33) = \frac{(2-1.33)}{(2-1)} * 1 + \frac{(1.33-1) * 3}{(2-1)}$$

$$f(1,1.33) = 0.67 + 0.99 = 1.66$$

$$f(2,1.33) = \frac{(2-1.33)}{(2-1)} * 2 + \frac{(1.33-1) * 4}{(2-1)}$$

$$f(2,1.33) = 0.67 * 2 + 0.33 * 4 = 2.66$$

$$f(1,1.66) = \frac{(2-1.66) * 1}{(2-1)} + \frac{(1.66-1) * 3}{(2-1)}$$

$$f(1,1.66) = 0.34 + 1.99 = 2.33$$

$$f(2,1.66) = \frac{(2-1.66)}{(2-1)} * 2 + \frac{(1.66-1) * 4}{(2-1)}$$

$$f(2,1.66) = 0.34 * 2 + 0.66 * 4 = 3..33$$

Image Zooming (5)
(Bi-linear Interpolation)

$$f(1.33,1.33) = \frac{(2 - 1.33) * 1.66}{(2-1)} + \frac{(1.33 - 1) * 2.66}{(2-1)}$$

$$f(1,1.33) = 1.11 + 0.88$$

$$= 1.99$$

$$f(1.66,1.66) = \frac{(2 - 1.66) * 2.33}{(2-1)} + \frac{(1.66 - 1) * 3.33}{(2-1)}$$

$$f(2,1.66) = 0.34 * 2.33 + 0.66 * 3.33$$

$$= 0.79 + 2.19$$

$$= 2.99$$

Distance Measures

- Application of distance measure is arithmetic/logical operation on images.
- Neighborhood operation on images
- Shape matching

Types of Distance measures:

- Euclidean Distance
- City block Distance
- Chess board Distance

Euclidean Distance

- D is a distance function (or metric) if:
- $D(p,q) \geq 0$ ($D(p,q)=0$ iff $p=q$),
- $D(p,q) = D(q,p)$, and
- $D(p,z) \leq D(p,q) + D(q,z)$.

If p has coordinates (x,y) and q has coordinates (s,t) then,

Euclidean distance between p and q is

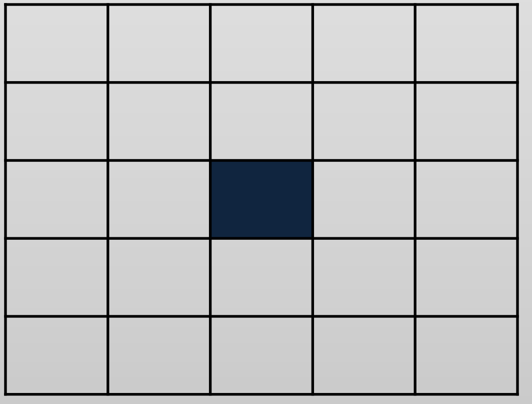
$$D_e(p,q) = [\,(x - s)^2 + (y - t)^2\,]^{1/2}$$

City block distance

The D4 distance (also called the city block distance) between p and q is given by: The pixels having a D4 distance less than some r from (x,y) form a diamond centered at (x,y)

- Example: pixels where D4 ≤ 2
- D4 (p, q) = |x − s| + |y − t|

$$
\begin{array}{ccccc}
 & & 2 & & \\
 & 2 & 1 & 2 & \\
2 & 1 & 0 & 1 & 2 \\
 & 2 & 1 & 2 & \\
 & & 2 & &
\end{array}
$$



Note: Pixels with D4=1 are the 4-neighbors of (x,y)

Chessboard distance

- The D8 distance (also called the chessboard distance) between p and q is given by:
- The pixels having a D8 distance less than some r from (x,y) form a square centered at (x,y)
- Example: pixels where D8 ≤ 2
- $D_8(p,q) = \max(|x - s|, |y - t|)$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Note: Pixels with D8=1
are the 8-neighbors
of (x,y)

Distance Measures

- The D4 and D8 distances between p and q are independent of any paths that might exist between the points because these distances involve only the coordinates of the points.

- If we elect to consider m-adjacency.
- In this case, the distance between two pixels will depend on the values of the pixels along the path , as well as the value of their neighbors.

	p3	p4
p1	p2	
p		

Distance Measures

- If we elect to consider m-adjacency.
- In this case, the distance between two pixels will depend on the values of the pixels along the path , as well as the value of their neighbors.

	p3	p4
p1	p2	
p		

	0	1 (q)
0	1	
(p)1		

Case 1

	0	1 (q)
1	1	
(p)1		

Case 2

	1	1 (q)
1	1	
(p)1		

Case 3

An Introduction to the Mathematical Tools Used in Digital Image Processing

- Array versus matrix operations
- Array Product

a11	a21	b11	b12	a11b11	a12b12
a12	a22	b21	b22	a21b21	a22b22

- Matrix Product
- [a11b11 +a12b21   a11b12+ a12b22 ; a21b11 +a22b21 a21b12 + a22b22]

An Introduction to the Mathematical Tools Used in Digital Image Processing

- Linear operation vs nonlinear operations
  - H is said to be a linear operator if, for any two images f and g and any two scalars a and b,

$$H(af + bg) = aH(f) + bH(g)$$

Example : max operation (Non Linear)

Example : Summation (Linear)

$$\sum\left[a_i f_i(x, y) + a_j f_j(x, y)\right] = \sum a_i f_i(x, y) + \sum a_j f_j(x, y)$$

$$= a_i \sum f_i(x, y) + a_j \sum f_j(x, y)$$

$$= a_i g_i(x, y) + a_j g_j(x, y)$$

$$H(af + bg) = aH(f) + bH(g)$$

An Introduction to the Mathematical Tools Used in Digital Image Processing

- f=[0 2; 2 3] g= [6 5 ; 4 7] a= 1 b = -1
- Example : Max (Nonlinear)

$$H(af + bg) = aH(f) + bH(g)$$

- L.H.S.   af + bg = [-6 -3 ; -2 -4]

Max(af +bg)   = **-2**

- R.H.S.   aH(f) + bH(g) = 1 (3) + (-1) (7)

=   **-4**

Arithmetic Operations

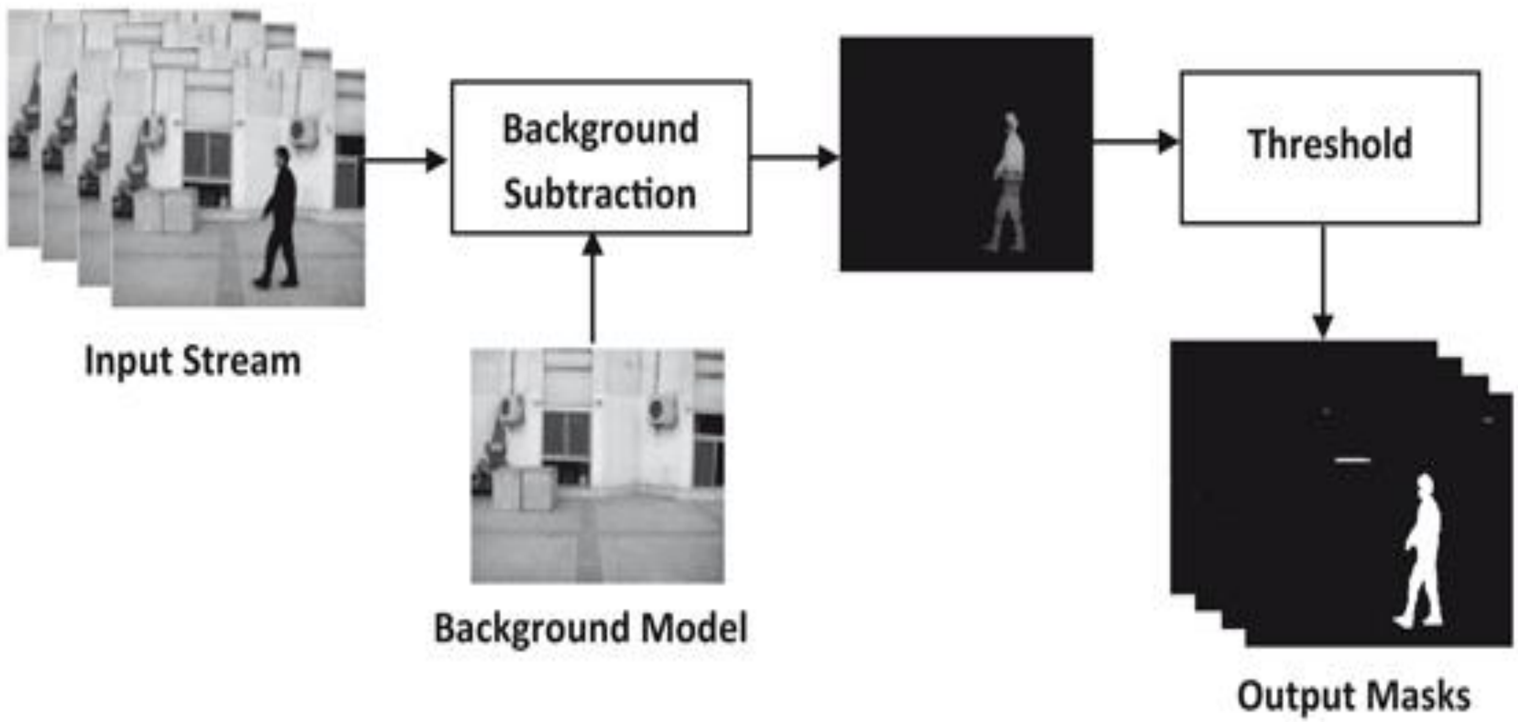
$$s(x, y) = f(x, y) + g(x, y)$$

$$d(x, y) = f(x, y) - g(x, y)$$

$$p(x, y) = f(x, y) \times g(x, y)$$

$$v(x, y) = f(x, y) \div g(x, y)$$

Arithmetic Operations



Arithmetic operations

FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

Logical operations

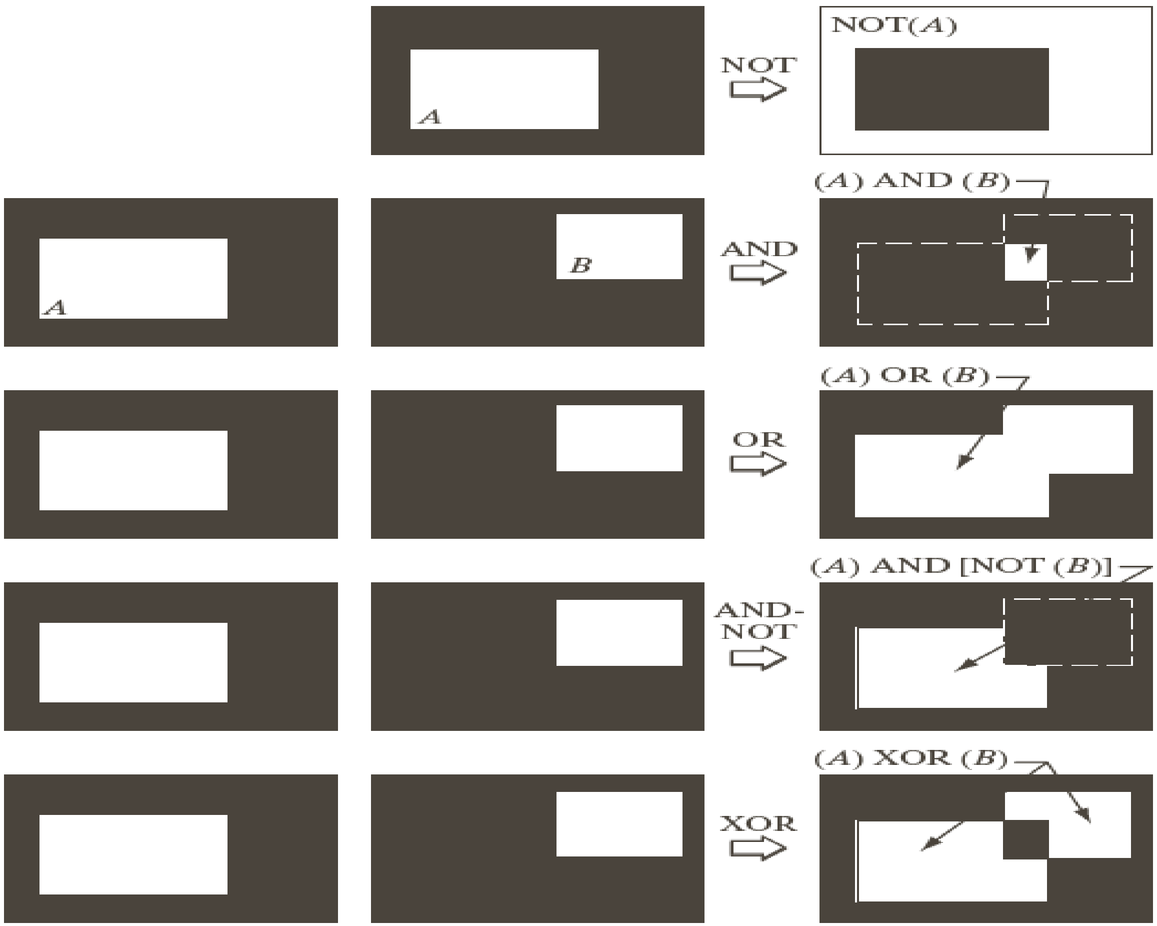
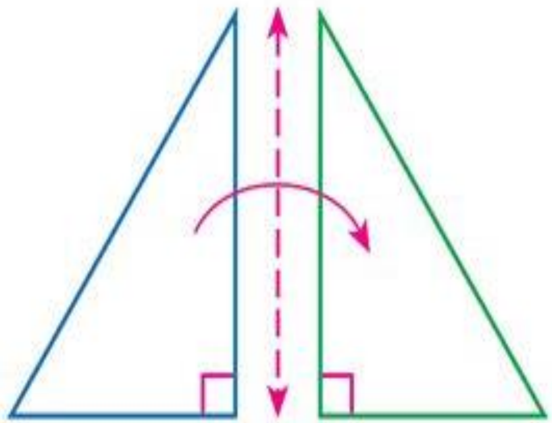


FIGURE 2.33 Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.

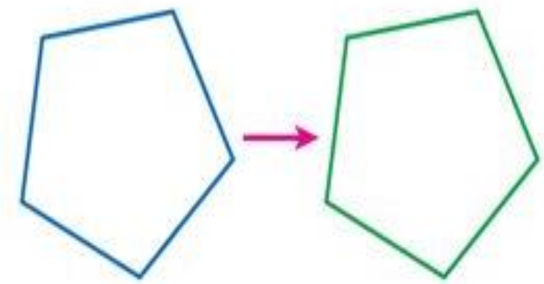
reflection

a figure is flipped over a line



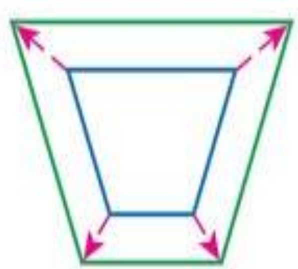
translation

a figure is slid in any direction



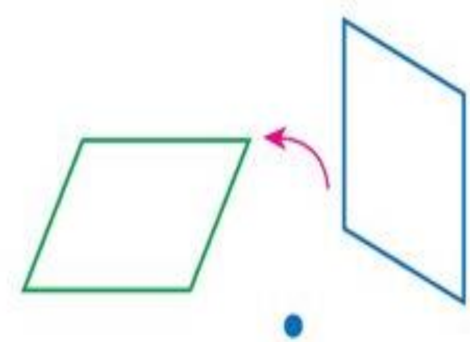
dilation

a figure is enlarged or reduced



rotation

a figure is turned around a point



The transformation of coordinates may be expressed as

$$(x, y) = T\{(v, w)\}$$

where (v, w) are pixel coordinates in the original image and (x, y) are the corresponding pixel coordinates in the transformed image.

One of the most commonly used spatial coordinate transformations is the *affine transform* (Wolberg [1990]), which has the general form

$$[x \;\; y \;\; 1] = [v \;\; w \;\; 1]\,\mathbf{T} = [v \;\; w \;\; 1]\begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}$$

This transformation can scale, rotate, translate, or sheer a set of coordinate points, depending on the value chosen for the elements of matrix $\mathbf{T}$.

Identity

$$[x \;\; y \;\; 1] = [v \;\; w \;\; 1]\,\mathbf{T} = [v \;\; w \;\; 1]\begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}$$

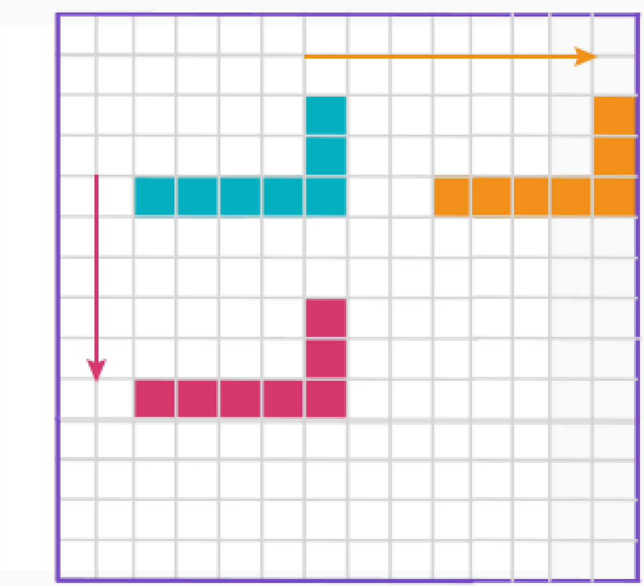
$$[x \;\; y \;\; 1] = [v \;\; w \;\; 1]\,\mathbf{T} = [v \;\; w \;\; 1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x = v$$
$$y = w$$

Translation

$$[x \ \ y \ \ 1] = [v \ \ w \ \ 1]\,\mathbf{T} = [v \ \ w \ \ 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$$

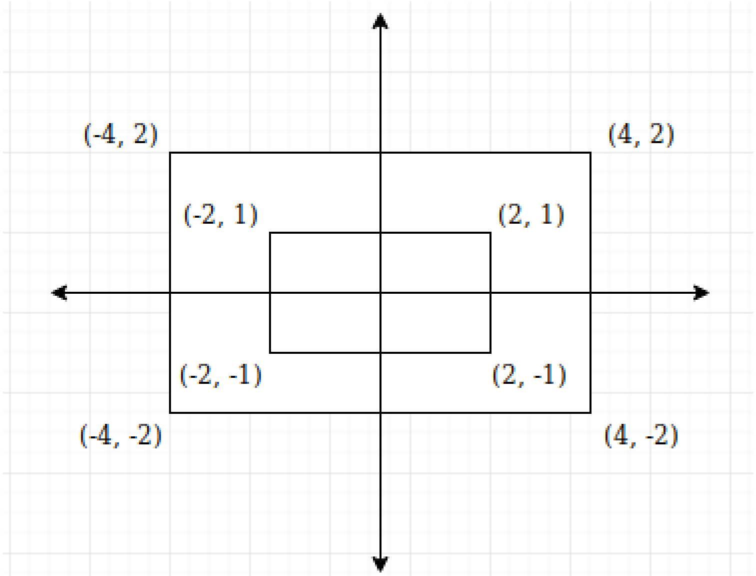
$$x = v + t_x$$
$$y = w + t_y$$



Scaling

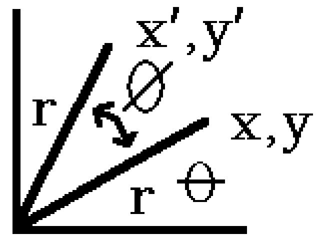
$$[x \ \ y \ \ 1] = [v \ \ w \ \ 1]\,\mathbf{T} = [v \ \ w \ \ 1] \begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x = c_x v$$
$$y = c_y w$$



Geometric spatial transformations

- **Rotation**



- (old coordinates are (x, y) and the new coordinates are (x', y'))
- ɵ = initial angle, Φ = angle of rotation.
- $x = r \cos \theta$
  $y = r \sin \theta$
- $x' = r \cos(\theta + \Phi) = r \cos\theta \cos\Phi - r \sin\theta \sin\Phi$
  $y' = r \sin(\theta + \Phi) = r \sin\theta \cos\Phi + r \cos\theta \sin\Phi$
- hence:
  $x' = x \cos\Phi - y \sin\Phi$
  $y' = y \cos\Phi + x \sin\Phi$
- Example: ɵ = 0 and 30 , x =10, y =5
- $x' = x \cos 30 – y \sin 30 = 6.16$
- $y' = y \cos 30 + x \sin 30 = 9.33$

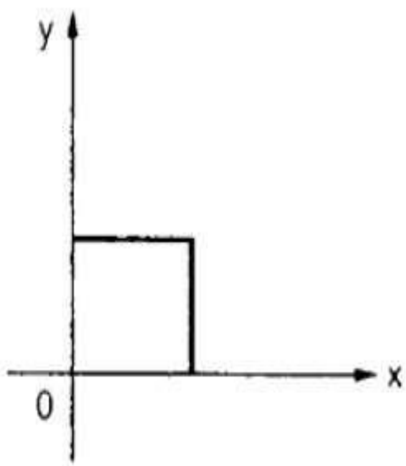
Shear

$$[x \;\; y \;\; 1] = [v \;\; w \;\; 1]\,\mathbf{T} = [v \;\; w \;\; 1] \begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

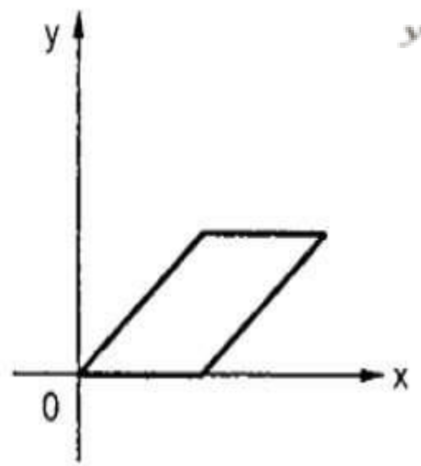
$$x = v + s_v w$$
$$y = w$$

$$[x \;\; y \;\; 1] = [v \;\; w \;\; 1]\,\mathbf{T} = [v \;\; w \;\; 1] \begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x = v$$
$$y = s_h v + w$$



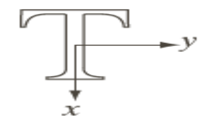
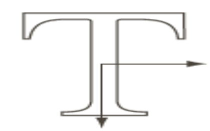
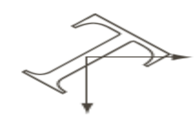
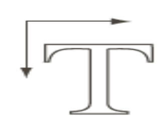
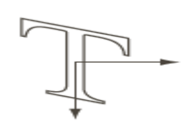
(a) Original object



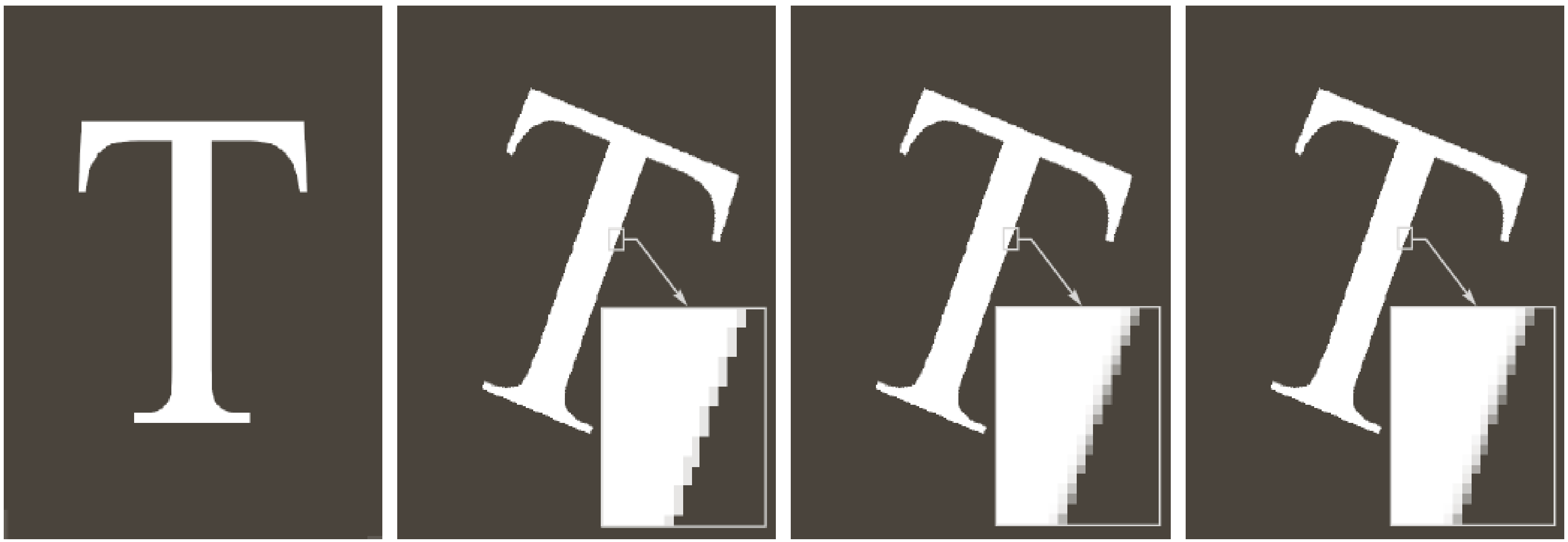
(b) Object after x shear

Geometric spatial transformations

TABLE 2.2
Affine transformations based on Eq. (2.6.–23).

Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\cos\theta + w\sin\theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	

Geometric spatial transformations



a b c d

FIGURE 2.36 (a) A 300 dpi image of the letter T. (b) Image rotated 21° clockwise using nearest neighbor interpolation to assign intensity values to the spatially transformed pixels. (c) Image rotated 21° using bilinear interpolation. (d) Image rotated 21° using bicubic interpolation. The enlarged sections show edge detail for the three interpolation approaches.

Geometric spatial transformations

- Nearest neighbor interpolation produced the most jagged (Having a sharply uneven surface or outline) edges and bilinear interpolation produced yielded significantly improved results.

- Bicubic interpolation produced slightly sharper results.