

Probability

	R	G	B
Y : Actual probabilities	[0.35	0.25	0.4]
Y-hat: Predicted probabilities	[0.25	0.45	0.3]

Error In the prediction:

$$MSE = \frac{1}{n} \sum \underbrace{\left(y - \hat{y} \right)^2}_{\substack{\text{The square of the difference} \\ \text{between actual and} \\ \text{predicted}}}$$

Can we do better ??

With Certain Events

Certain event-> can still be written as Probability distribution
where whole mass located at one label

A	B	C	D
[0	1	0	0]
[0.2	0.1	0.6	0.1]

Classification Problem:

Probability distribution of true label possible

Expectation

- Expectation: For every event there is some value associated with it

A	B	C	D
0	1	0	0

 True Y

0.2	0.1	0.6	0.1
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 Predicted Y

10k	20k	5k	2k
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 Value[Rewards]

Expectation

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A	B	C	D
[0	1	0	0]

 True Y

[0.2	0.1	0.6	0.1]
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 Predicted Y

[10k	20k	5k	2k]
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 Value[Rewards]

$$\sum_{i=A,B,C,D} P(i) V(i)$$

Probability and Information Content

- High probability – Low Information Content
- Low probability – High Information Content
- A: Sun rises in east (Prob=1 means IC =?)
- B: there will be a cyclone tomorrow

$$I(x) = -\log P(x)$$

Entropy

- For every events we have IC[Info. content] (Value associated with that event)

$$H(x) = -\sum_x P(x) \cdot \log P(x)$$

$$= \sum_x P(x) \cdot \log \left(\frac{1}{P(x)} \right)$$

The prob. of event x

WHAT IS THIS?

- If we have $P(X) = 1$, the entropy is 0. It has 0 bits of uncertainty. ($-\log 1 = 0$)
- Note that “entropy” in information theory capture increasing randomness.
- **Cross Entropy??**