Image Restoration

IMAGE RESTORATION

The main objective of restoration is to improve the quality of a digital image which has been degraded due to Various phenomena like:

- Motion
- Improper focusing of Camera during image acquisition.
- Atmospheric turbulence
- Noise

Image Restoration

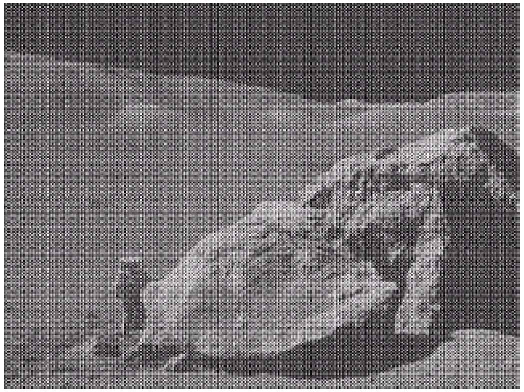
- In many applications (***e.g.***, satellite imaging, medical imaging, astronomical imaging, poor-quality family portraits) the imaging system introduces a slight distortion

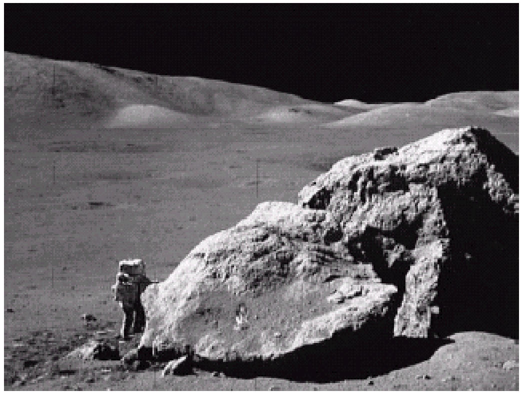
- Image Restoration attempts to reconstruct or recover an image that has been degraded by using a priori knowledge of the degradation phenomenon.

- Restoration techniques try to model the degradation and then apply the inverse process in order to recover the original image.

Image Restoration

- Image restoration attempts to restore images that have been degraded
  - Identify the degradation process and attempt to reverse it
  - Similar to image enhancement, but more objective





- Difference:

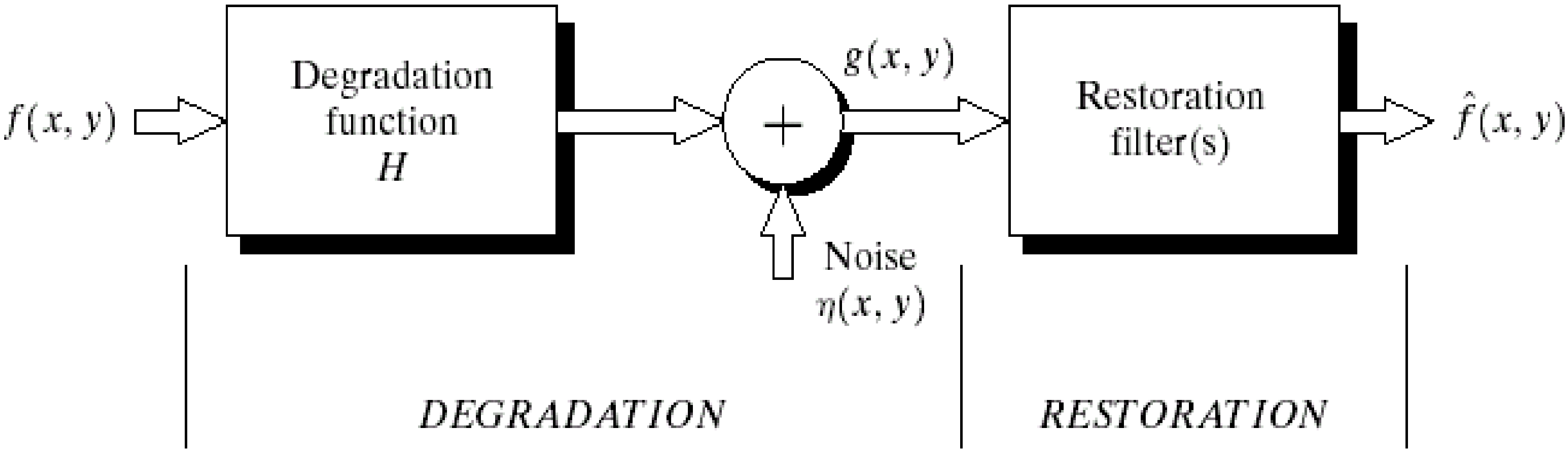
Image Enhancement --- Subjective process Image Restoration --- Objective Process

Restoration tries to recover / restore degraded image by using a prior knowledge of the degradation phenomenon.

Restoration techniques focuses on:

1. Modeling the degradation
2. Applying inverse process in order to recover the original image.

A model of the image degradation/restoration process



$$\begin{cases} g(x,y)=f(x,y)*h(x,y)+\eta(x,y) \\ G(u,v)=F(u,v)H(u,v)+N(u,v) \end{cases}$$

Image Restoration

- If we are provided with the following information
  - The degraded image g(x,y)
  - Some knowledge about the degradation function *H* , and
  - Some knowledge about the additive noise η(x,y)
- Then the objective of restoration is to obtain an estimate $\hat{f}(x,y)$ of the original image

Principle Sources of Noise

- Image Acquisition
  - Image sensors may be affected by Environmental conditions (light levels etc)
  - Quality of Sensing Elements (can be affected by e.g. temperature)
- Image Transmission
  - Interference in the channel during transmission e.g. lightening and atmospheric disturbances

White Noise

- When the Fourier Spectrum of noise is constant the noise is called White Noise
- The terminology comes from the fact that the white light contains nearly all frequencies in the visible spectrum in equal proportions
- The Fourier Spectrum of a function containing all frequencies in equal proportions is a constant

Gaussian Noise

Spatial noise descriptor based on the statistical behavior of the gray-level values ⇨ consider the gray-level values as random variables characterized by a probability density function (PDF)

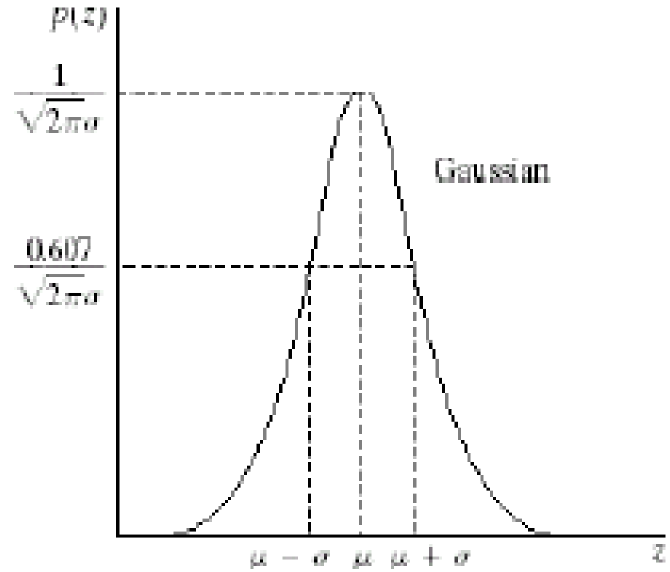
- Gaussian noise (also called "normal noise model")

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\mu)^2 / 2\sigma^2}$$

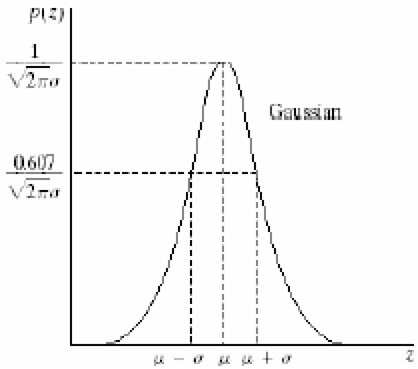
z: gray level

μ: mean of random variable z

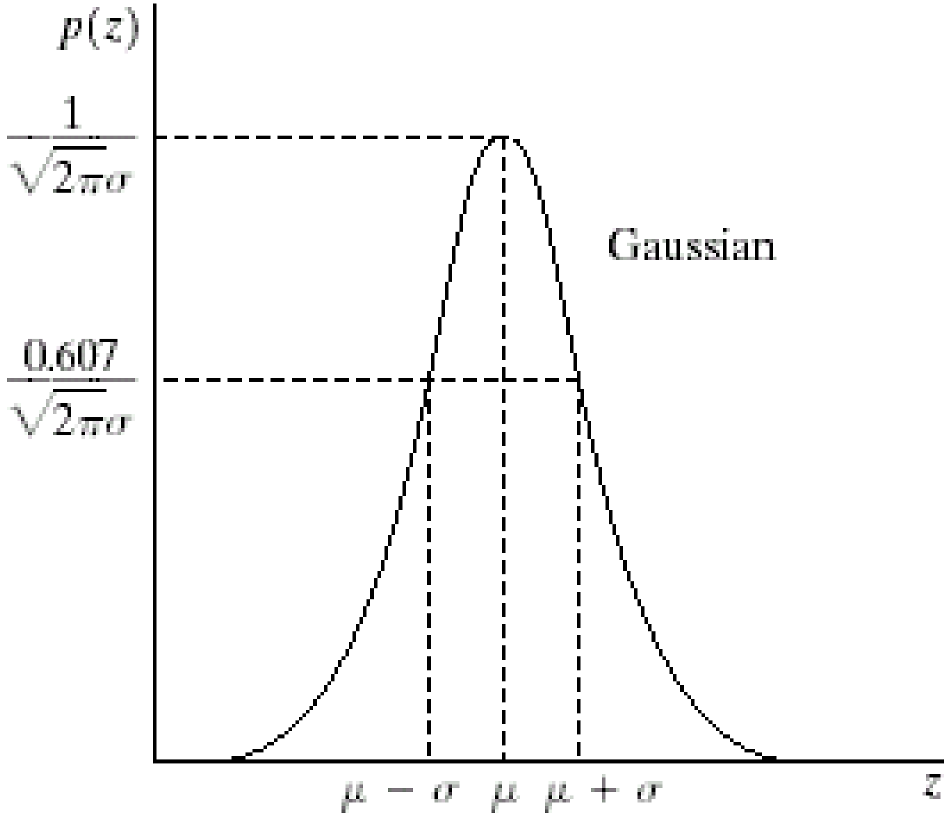
σ^2: variance of z



- Approximately 70% of its value will be in the range [(μ-σ), (μ+σ)] and about 95% within range [(μ-2σ), (μ+2σ)]
- Gaussian Noise is used as approximation in cases such as Imaging Sensors operating at low light levels



Gaussian Noise



Other Noise Models

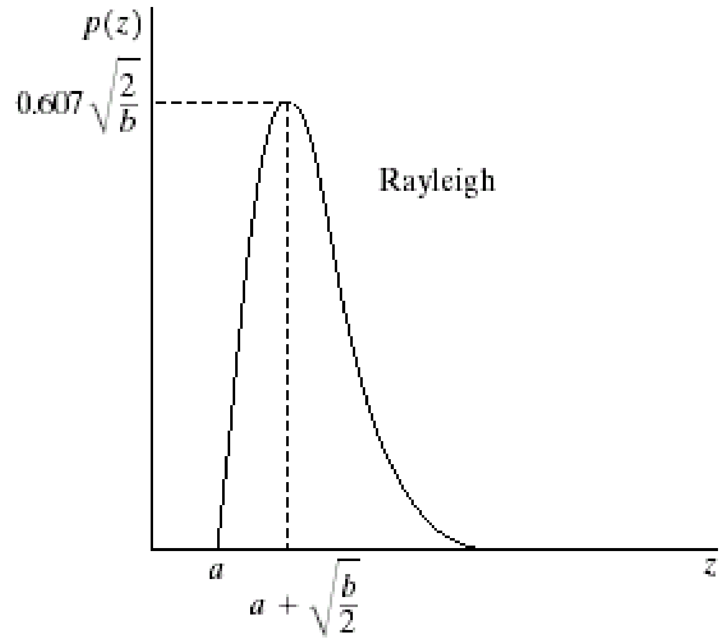
- **Rayleigh** noise

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

- The mean and variance of this density are given by

$$\mu = a + \sqrt{\pi b/4} \text{ and } \sigma^2 = \frac{b(4-\pi)}{4}$$

- a and b can be obtained through mean and variance



Other Noise Models

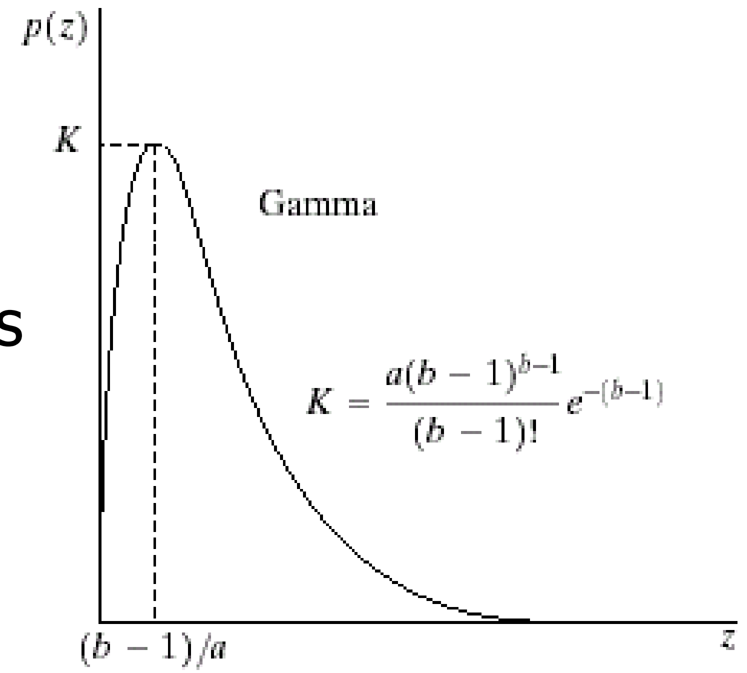
- **Erlang (Gamma)** noise

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

  - The mean and variance of this density are given by

$$\mu = b/a \ \text{ and } \sigma^2 = \frac{b}{a^2}$$

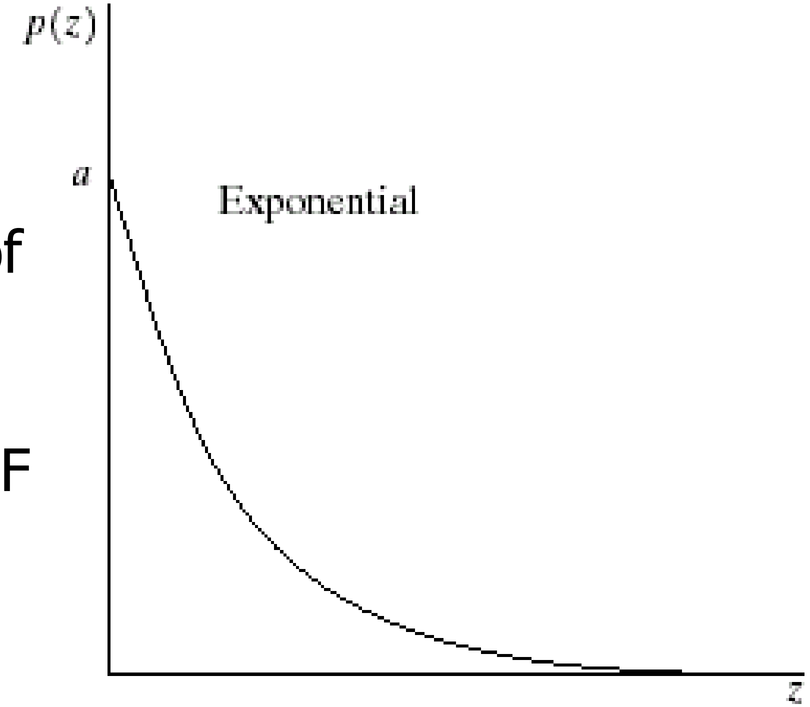
  - a and b can be obtained through mean and variance



Other Noise Models

- **Exponential** noise
  - $$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$
  - The mean and variance of this density are given by
  - Special case of Erlang PDF with b=1

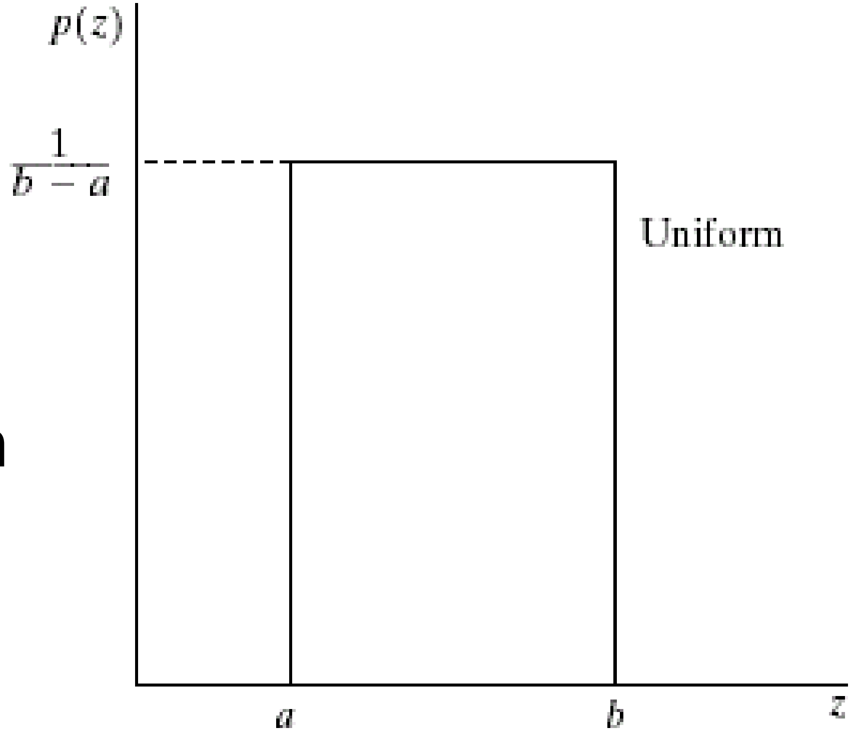
$$\mu = 1/a \text{ and } \sigma^2 = \frac{1}{a^2}$$



Other Noise Models

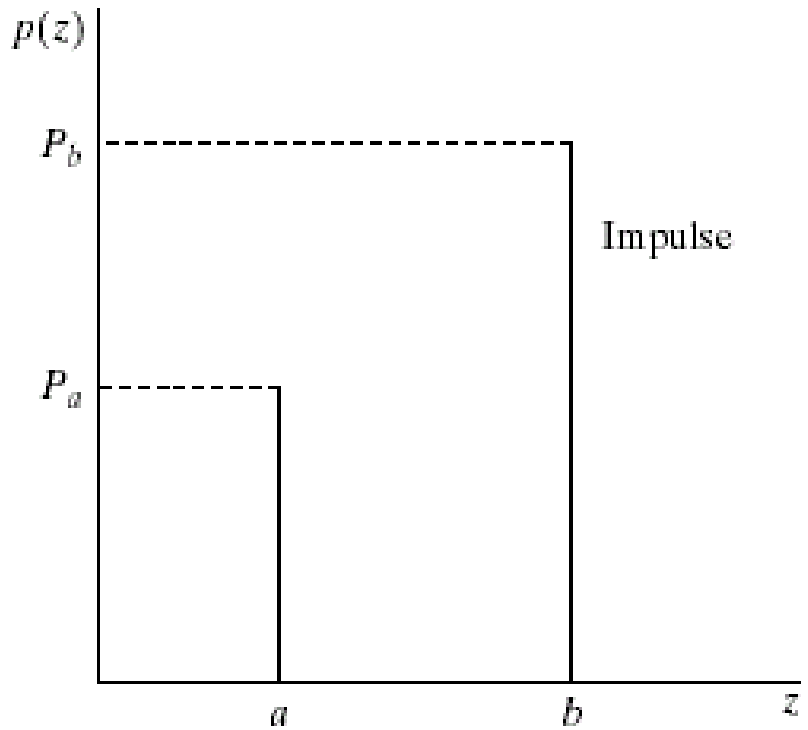
- **Uniform** noise
  - $$p(z) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$
  - The mean and variance of this density are given by

$$\mu = (a+b)/2 \text{ and } \sigma^2 = \frac{(b-a)^2}{12}$$



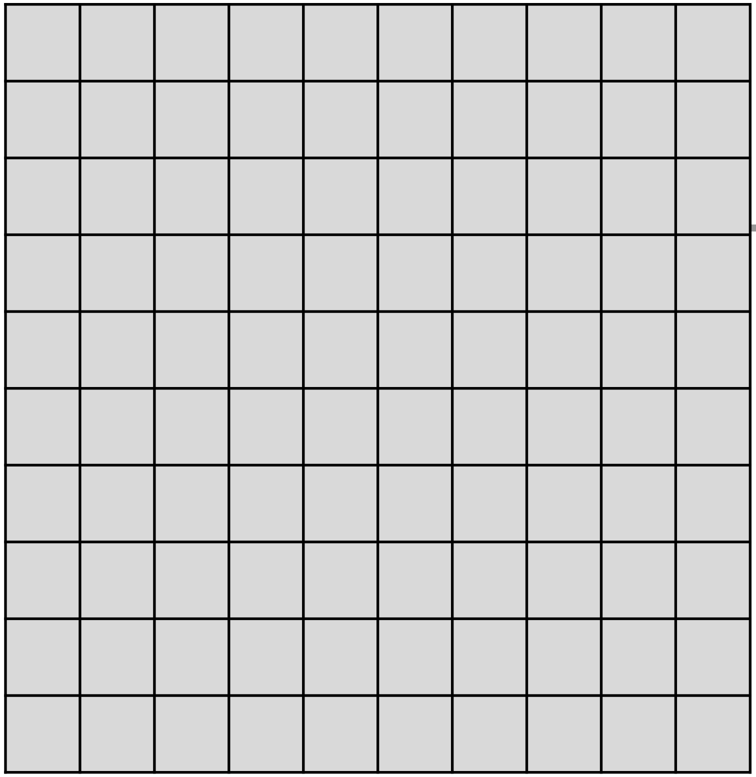
Other Noise Models

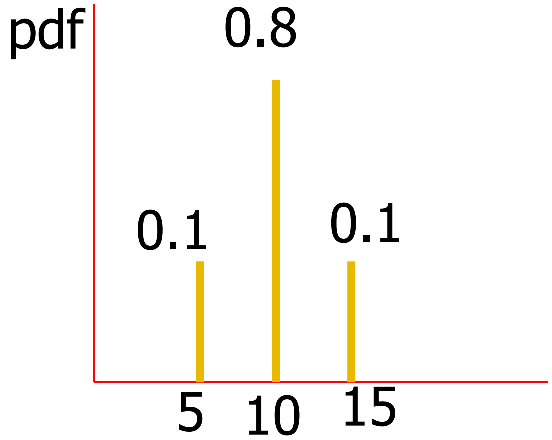
- **Impulse** (salt-and-pepper) noise
  - $$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$
  - If either Pa or Pb is zero, the impulse noise is called unipolar
  - a and b usually are extreme values because impulse corruption is usually large compared with the strength of the image signal
  - It is the only type of noise that can be distinguished from others visually



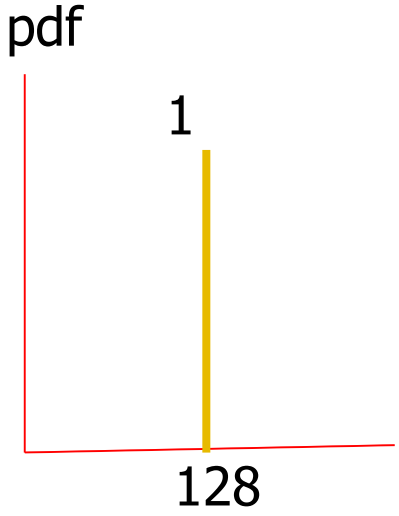
- **Gaussian noise** arises in an image due to factors such as electronic circuit noise and sensor noise due to poor illumination and/or high temperature.
- **Rayleigh noise** is helpful in characterizing noise phenomena in range imaging.
- **Range imaging** is the name for a collection of techniques that are used to produce a 2D image showing the **distance to points in a scene from a specific point.**
- The resulting image, the *range image*, has pixel values that correspond to the distance. If the sensor that is used to produce the range image is properly calibrated the pixel values can be given directly in physical units, such as meters.

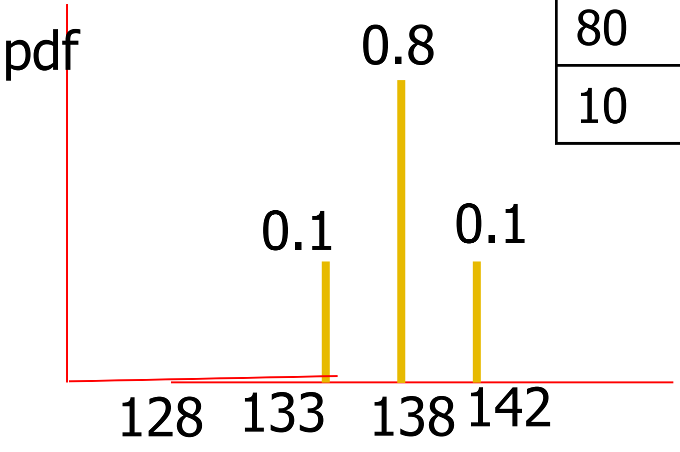
- Exponential and gamma noise find application laser imaging.
- Impulse noise is found in situations where quick transients ,such as faulty switching ,take place during imaging.





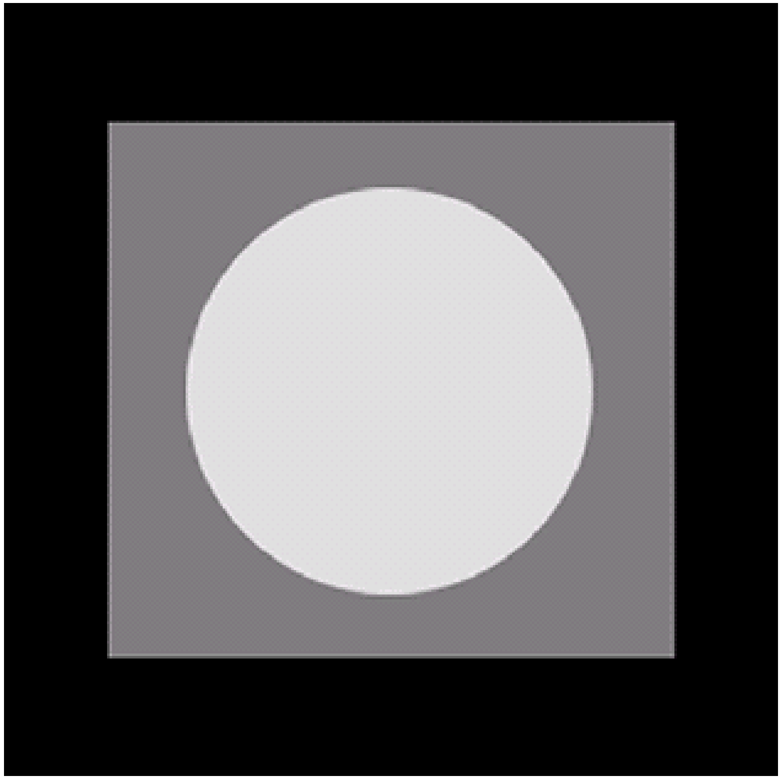
No. pixel	Noise Value	Pixel value
10	5	128+5 =133
80	10	128+10=138
10	15	128+15=142



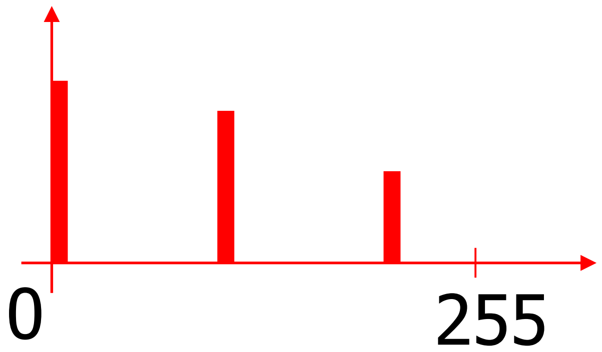


Uniform noise implementation

- Test pattern



Its histogram:



Effect of Adding Noise to Sample Image

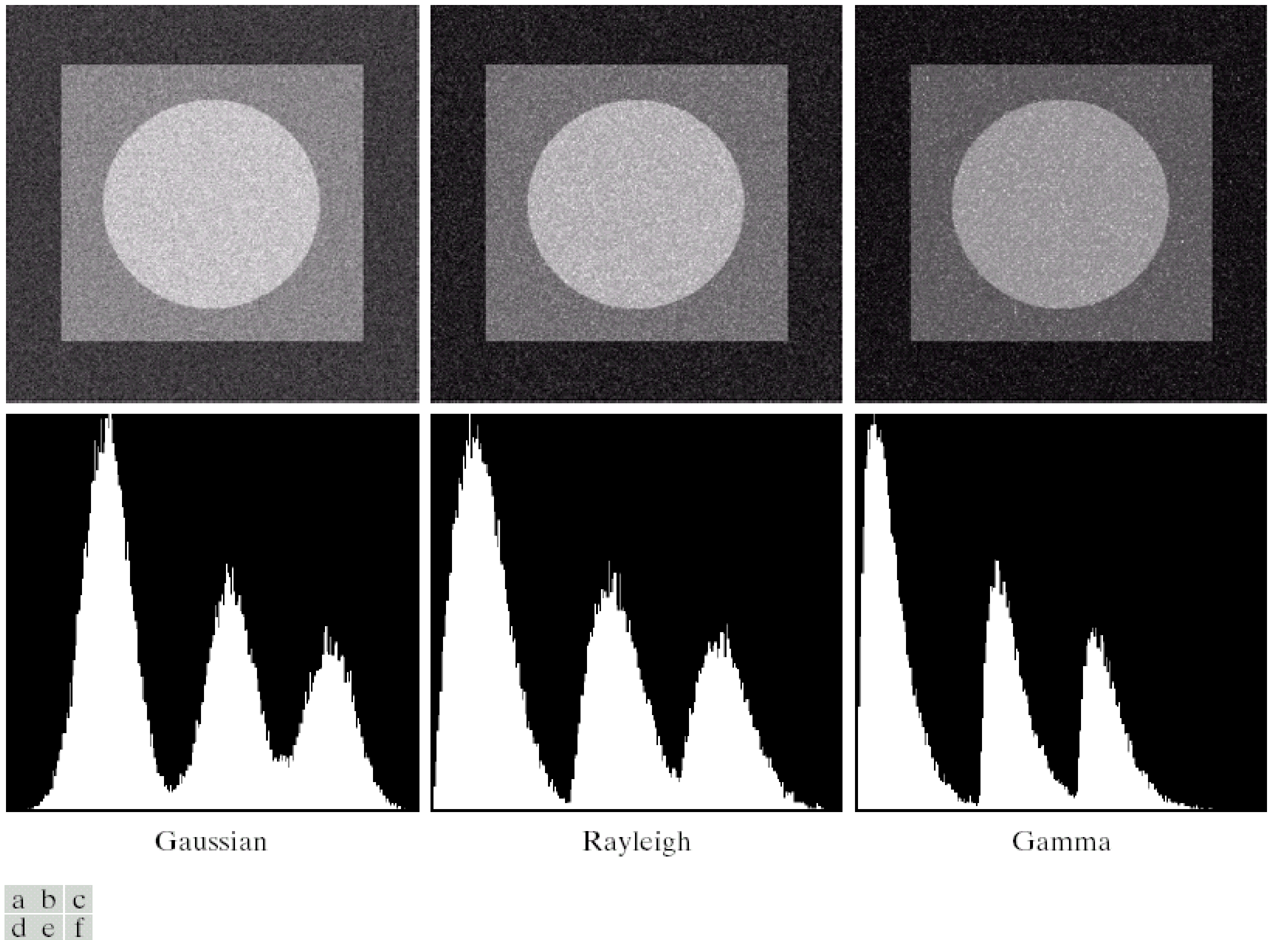


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

Effect of Adding Noise to Sample Image

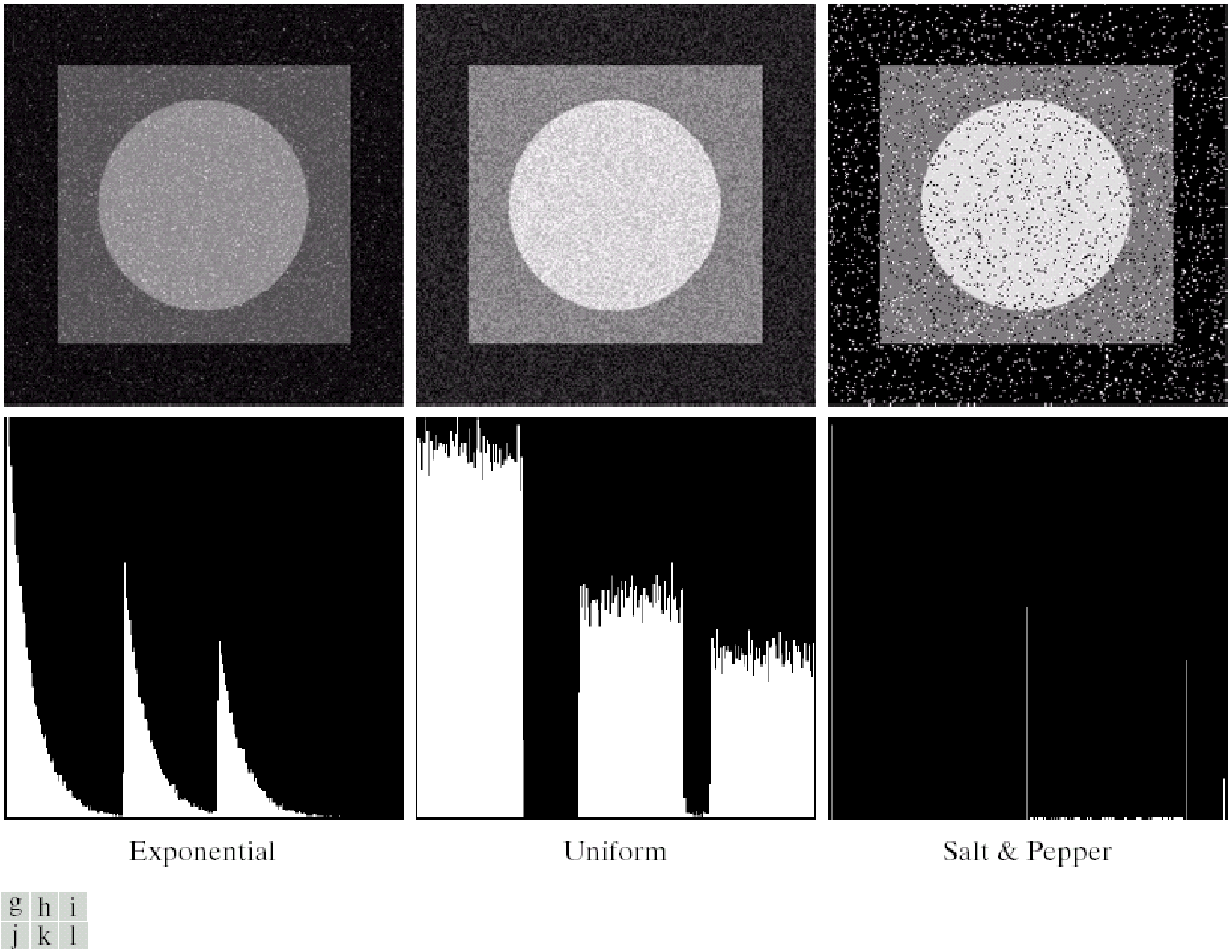
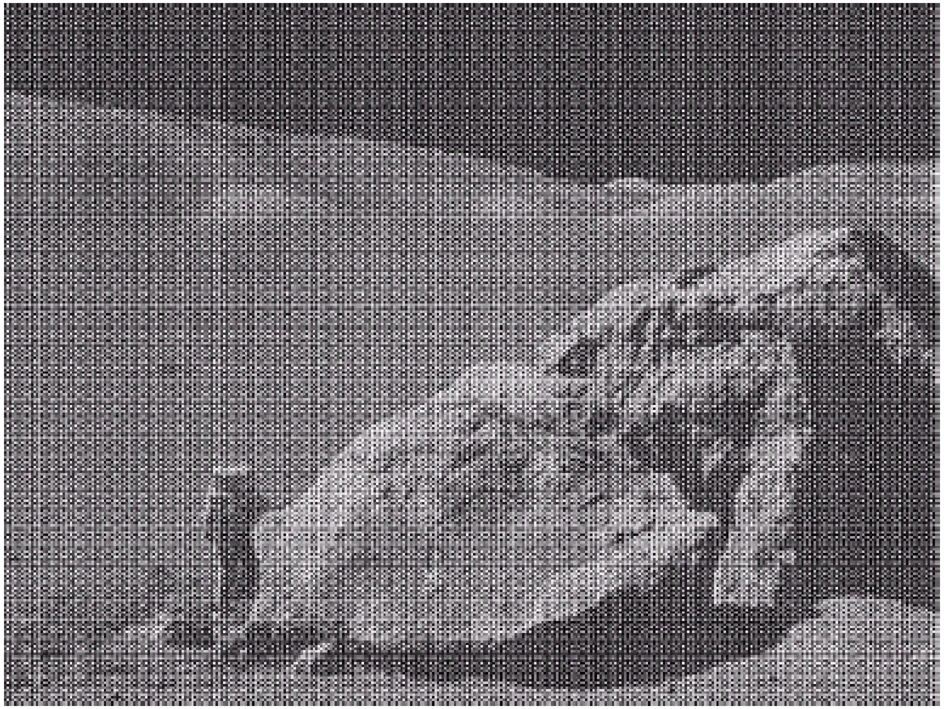
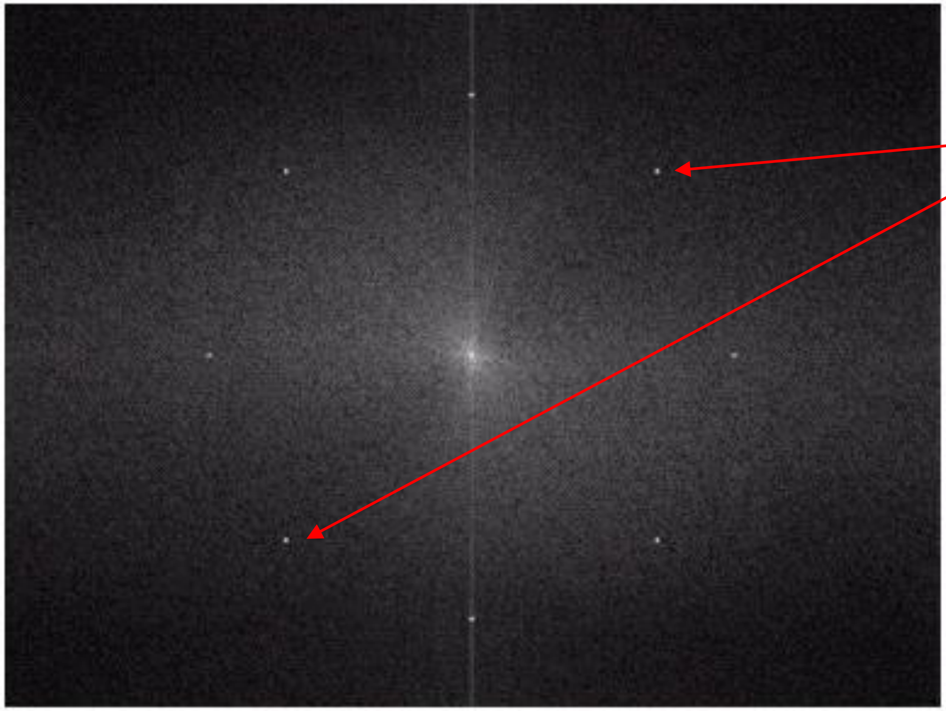


FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and impulse noise to the image in Fig. 5.3.

Periodic noise

- Arise from electrical or electromechanical interference during image acquisition
- Observed in the frequency domain



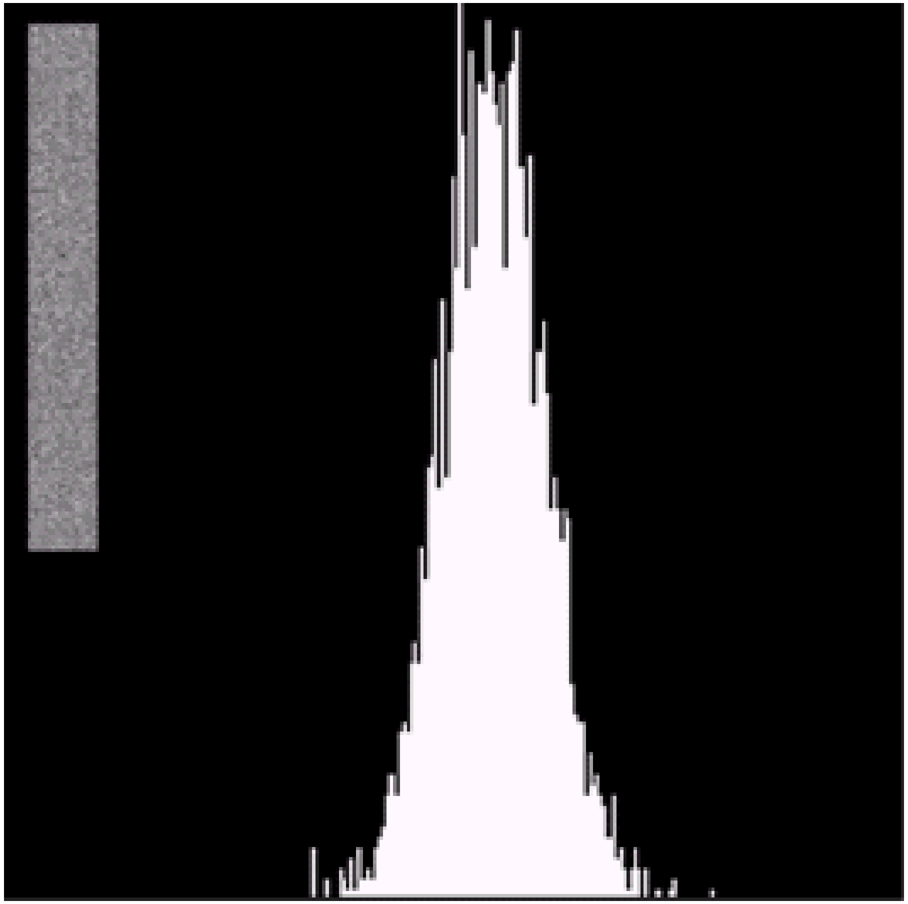


Sinusoidal noise: Complex conjugate pair in frequency domain

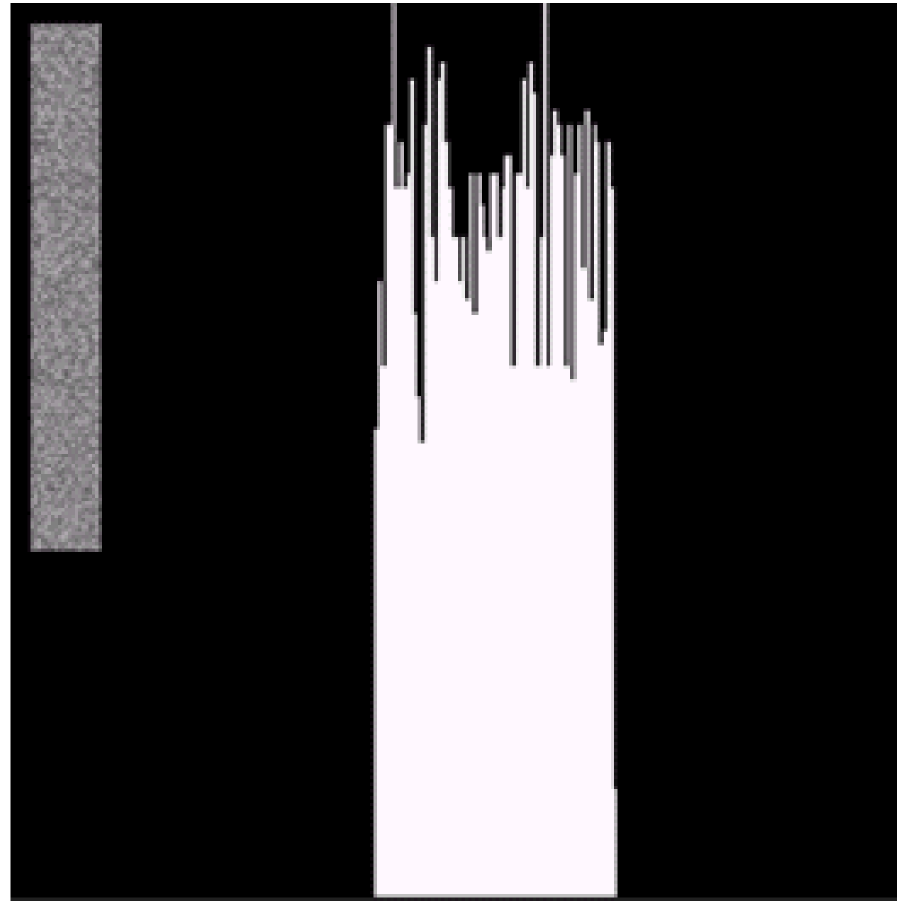
Estimation of noise parameters

- Periodic noise
  - Observe the frequency spectrum
- Random noise with unknown PDFs
  - Case 1: imaging system is available
    - Capture images of "flat" environment
  - Case 2: noisy images available
    - Take a strip from constant area
    - Draw the histogram and observe it
    - Measure the mean and variance

Observe the histogram



Gaussian



uniform

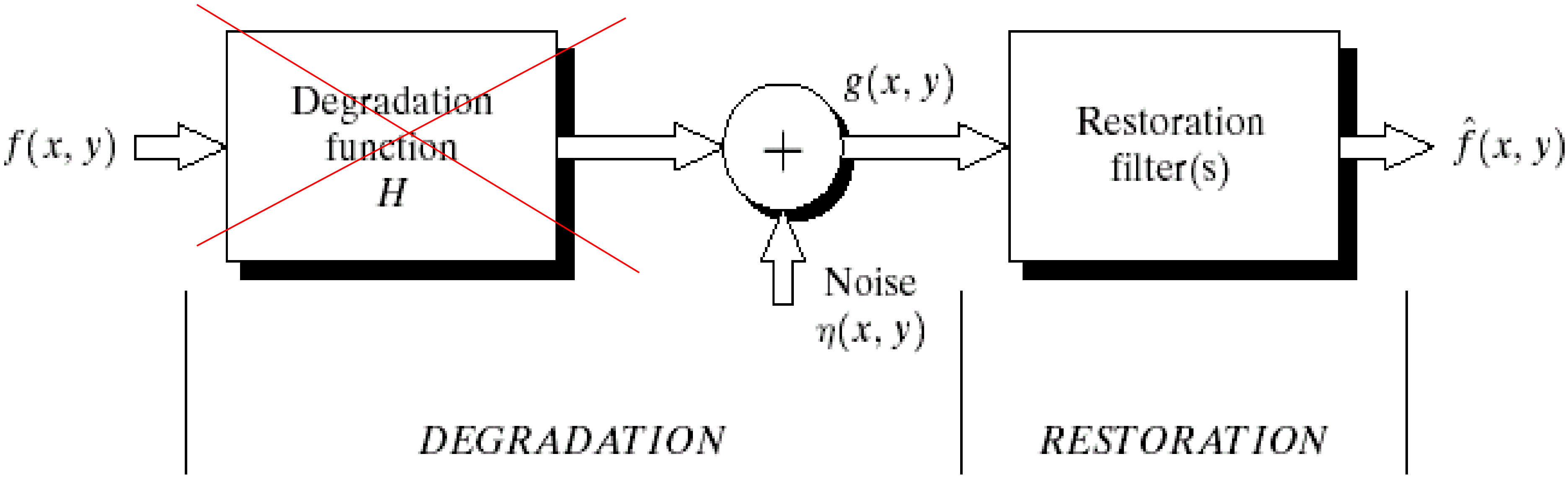
Measure the mean and variance

- Histogram is an estimate of PDF

$$\begin{cases} \mu = \sum_{z_i \in S} z_i \, p(z_i) \\ \sigma^2 = \sum_{z_i \in S} (z_i - \mu)^2 \, p(z_i) \end{cases}$$

$\Leftrightarrow$ $\begin{cases} \text{Gaussian: } \mu, \sigma \\ \text{Uniform: } a, b \end{cases}$

Additive noise only



$$
\begin{cases}
g(x,y)=f(x,y)+\eta(x,y) \\
G(u,v)=F(u,v)+N(u,v)
\end{cases}
$$

Spatial filters for de-noising additive noise

- Skills similar to image enhancement
- Mean filters
- Order-statistics filters
- Adaptive filters

Mean filters

- Arithmetic mean

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

Window centered at (x,y)

- It smooth's local variations in an image and noise is reduced as a result of blurring
- m * n spatial filter size
- S_{xy} is the set of coordinates in a rectangular sub image window

Mean filters

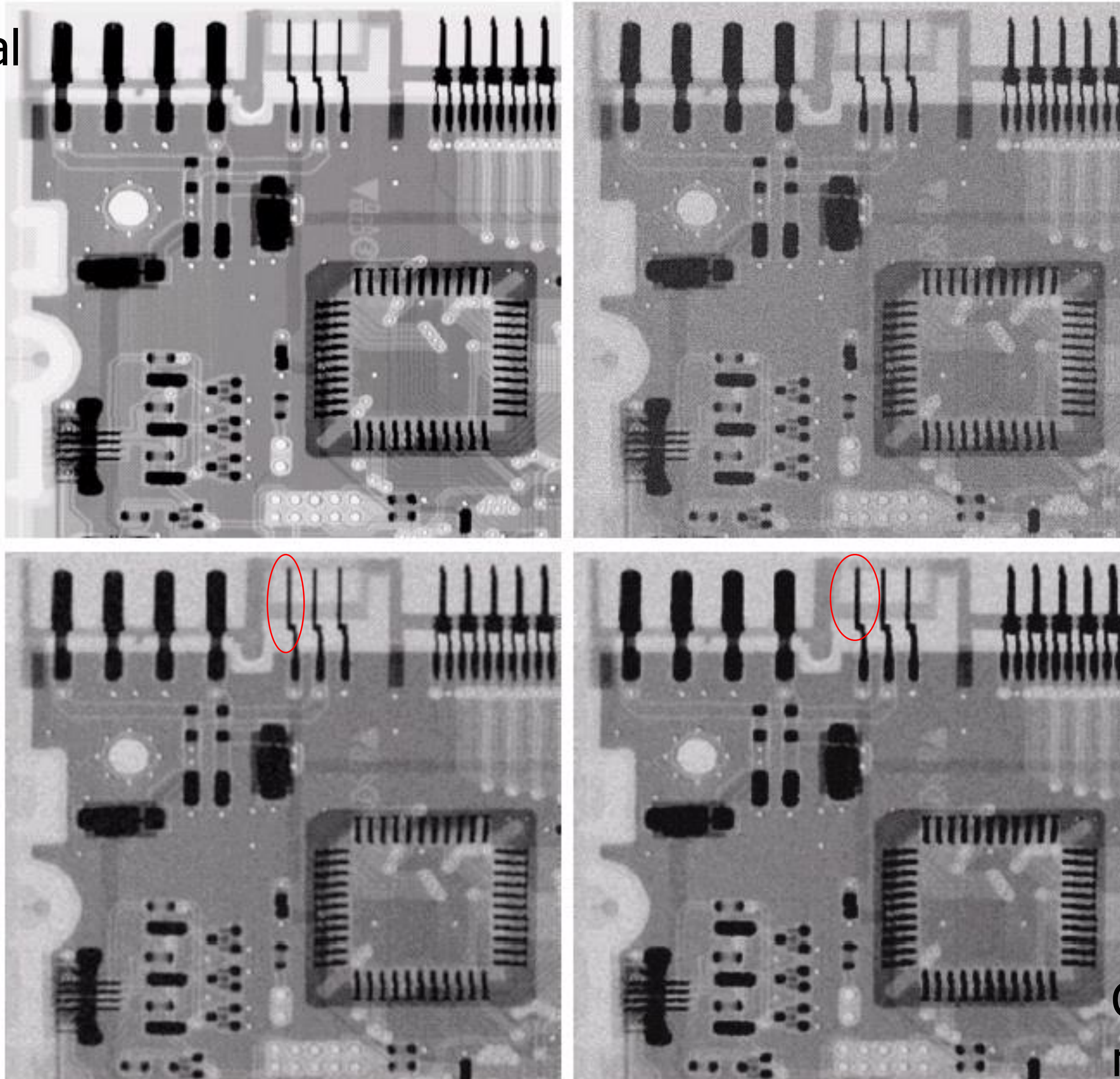
- Geometric mean

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{1/mn}$$

- Each restored pixel is given by the product of the pixels in the sub image window, raised to the power 1/mn.
- It achieves smoothing comparable to the arithmetic mean filter, but it tends to **lose less image detail** in the process.

original

Noisy Gaussian
$\mu=0$
$\sigma=20$



Arith. mean

Geometric mean

Mean filters (cont.)

- Harmonic mean filter

$$\hat{f}(x,y)=\frac{mn}{\sum_{(s,t)\in S_{xy}}\frac{1}{g(s,t)}}$$

- It works well for salt noise , but fails to pepper noise.
- It does well also with other types of noise like Gaussian noise.

The formula is:

$$\text{Harmonic Mean} = \frac{n}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \ldots}$$

Where **a,b,c,...** are the values, and **n** is how many values.

The harmonic mean is also good at handling large outliers.

Example: 2, 4, 6 and 100

The arithmetic mean is $\frac{2+4+6+100}{4} = 28$

The harmonic mean is $4/(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{100}) = 4.32$ (to 2 places)

Mean filters (cont.)

- Contra-harmonic mean filter

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Q=-1, harmonic

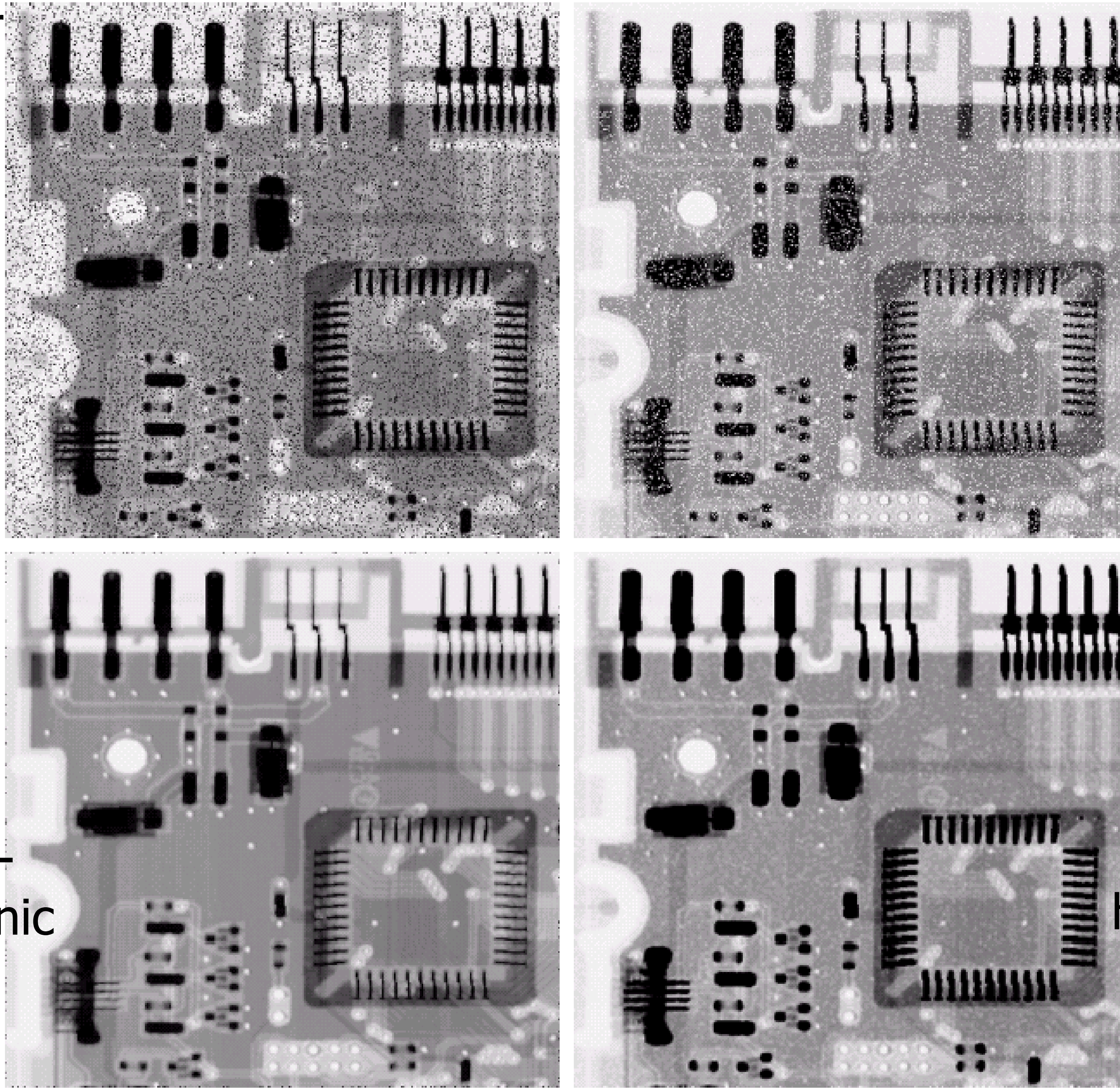
Q=0, airth. mean

Q=+

- Q is the order of the filter
- It is well suited for reducing or virtually eliminating the effects of salt-and-pepper noise.
- For positive values of Q , it eliminates pepper noise. It can not do both simultaneously.
- For negative values of Q , it eliminates salt noise.

Pepper Noise
黑點

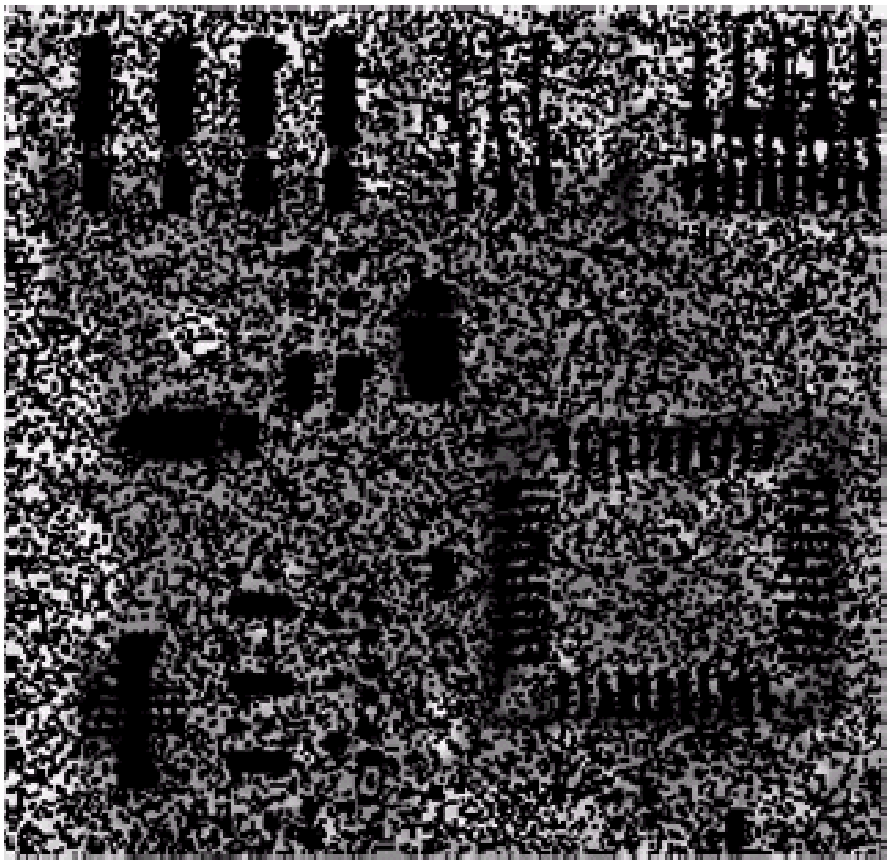
Salt Noise
白點



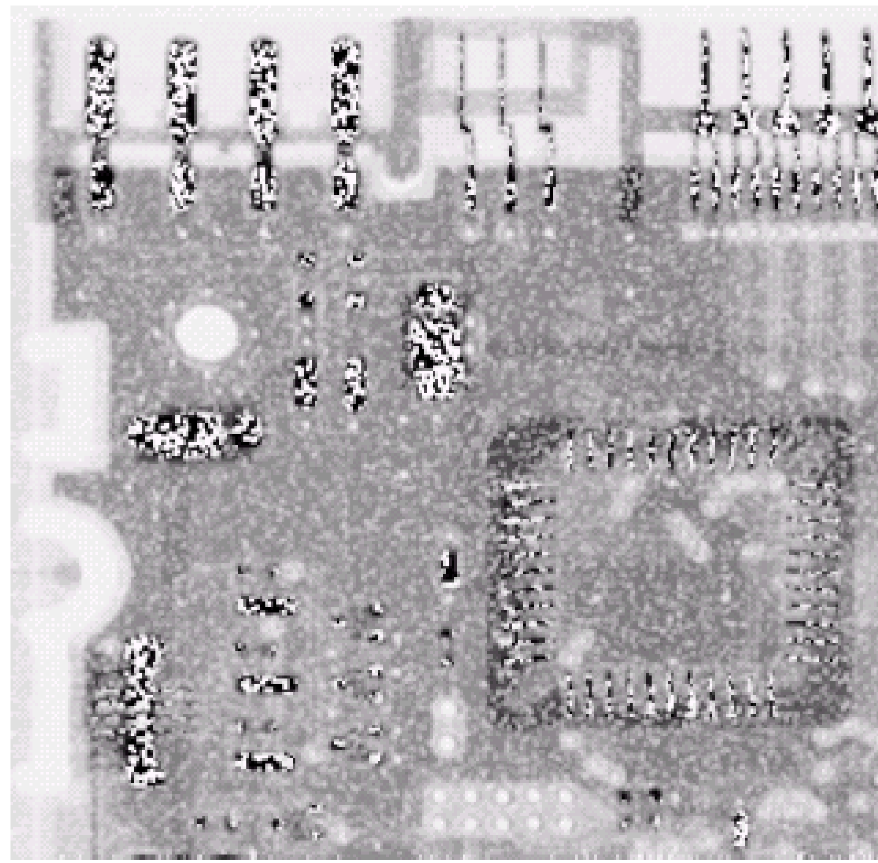
Contra-harmonic Q=1.5

Contra-harmonic Q=-1.5

Wrong sign in contra-harmonic filtering



Q=-1.5



Q=1.5

Order-statistics filters

- Based on the ordering(ranking) of pixels
  - Suitable for unipolar or bipolar noise (salt and pepper noise)
- Median filters
- Max/min filters
- Midpoint filters
- Alpha-trimmed mean filters

Order-statistics filters

- Median filter

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

- It is effective in the presence of bipolar and unipolar noise.

Order-statistics filters

- Max filter

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

- It is used to find brightest points in the image.
- It alleviates pepper noise because it has very low values.

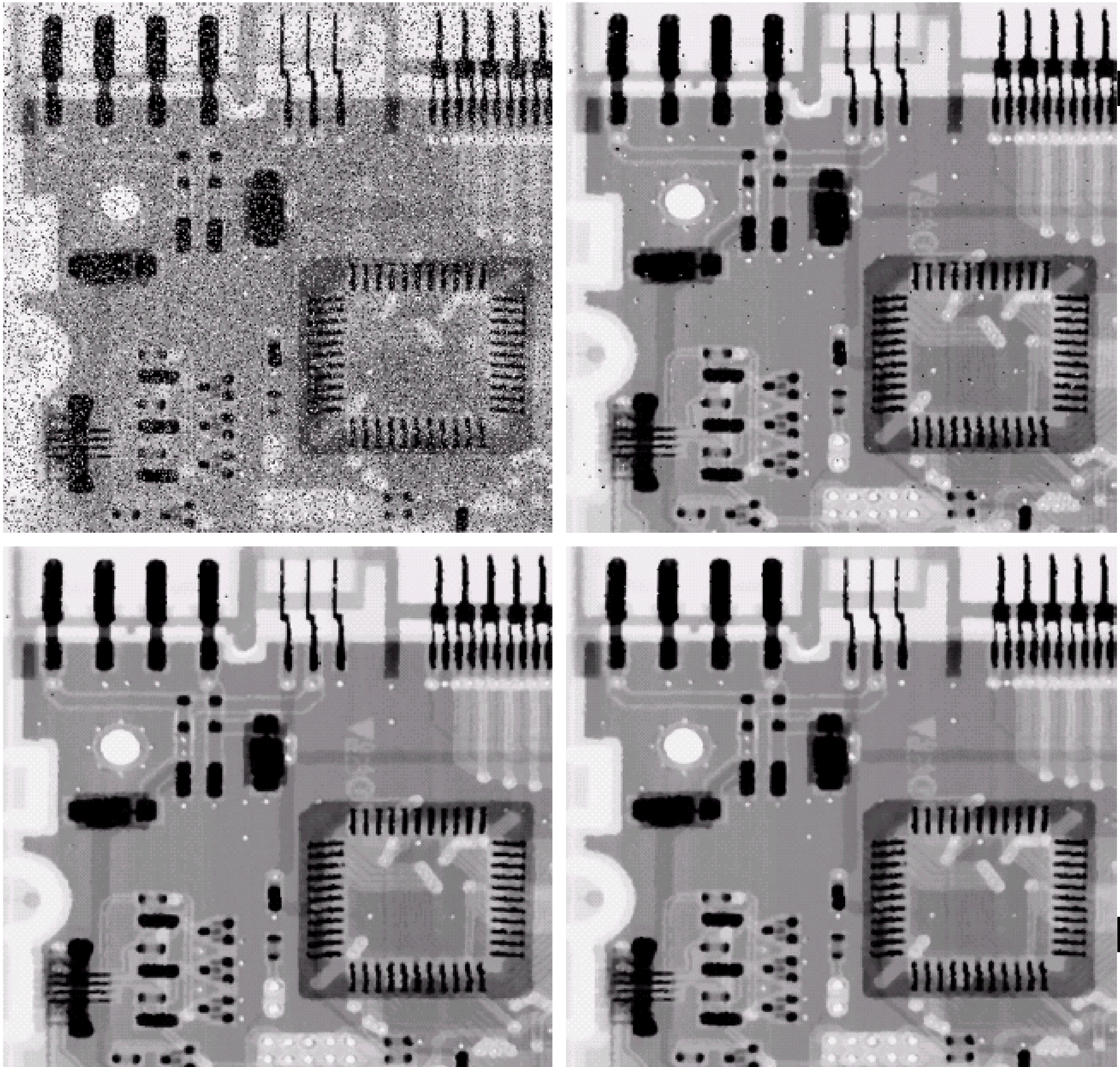
Order-statistics filters

- Min filter

$$\hat{f}(x, y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

- It is used to find darkest points in the image.
- It alleviates salt noise because it has very high values.

bipolar Noise $P_a = 0.1$ $P_b = 0.1$

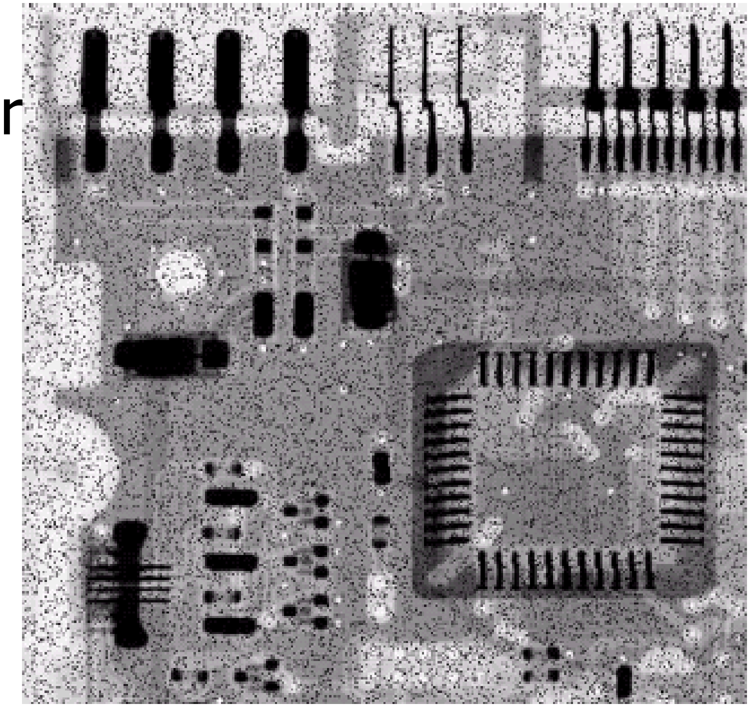


3x3 Median Filter Pass 1

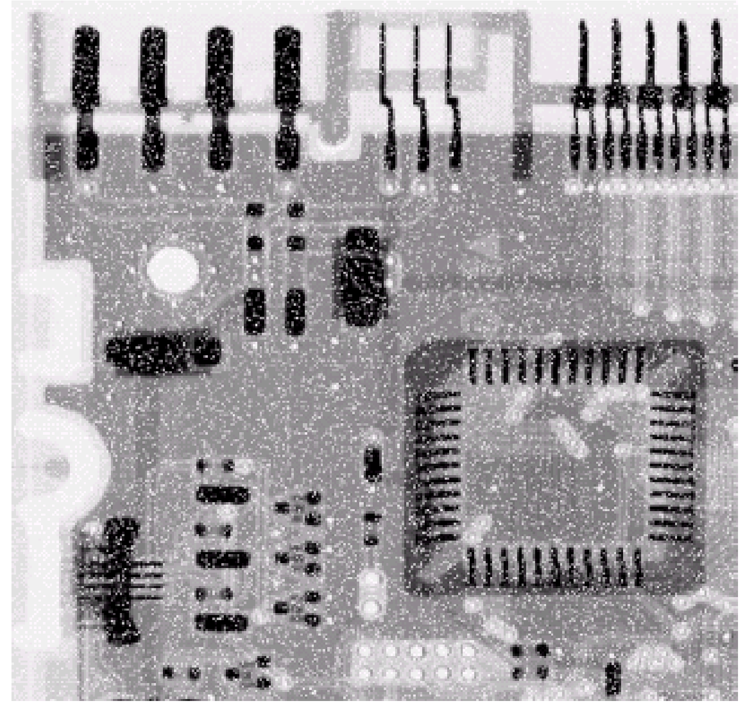
3x3 Median Filter Pass 2

3x3 Median Filter Pass 3

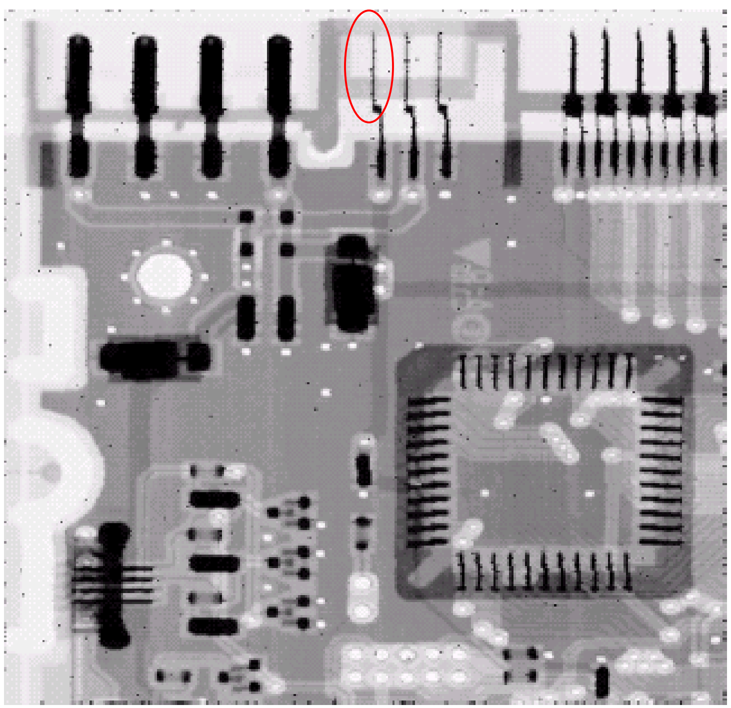
Pepper noise



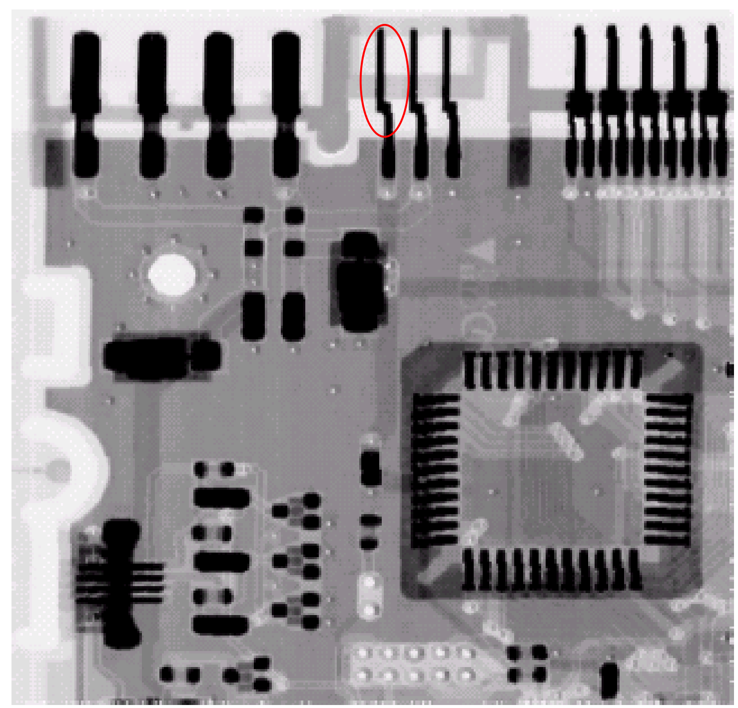
Salt noise



Max filter



Min filter



Order-statistics filters (cont.)

- Midpoint filter

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

- This filter combines order statistics and averaging.
- It works best for randomly distributed noise ,like gausssian or uniform noise.

Order-statistics filters (cont.)

- Alpha-trimmed mean filter
  - Delete the d/2 lowest and d/2 highest gray-level pixels

$$\hat{f}(x,y)=\frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

Middle (mn-d) pixels

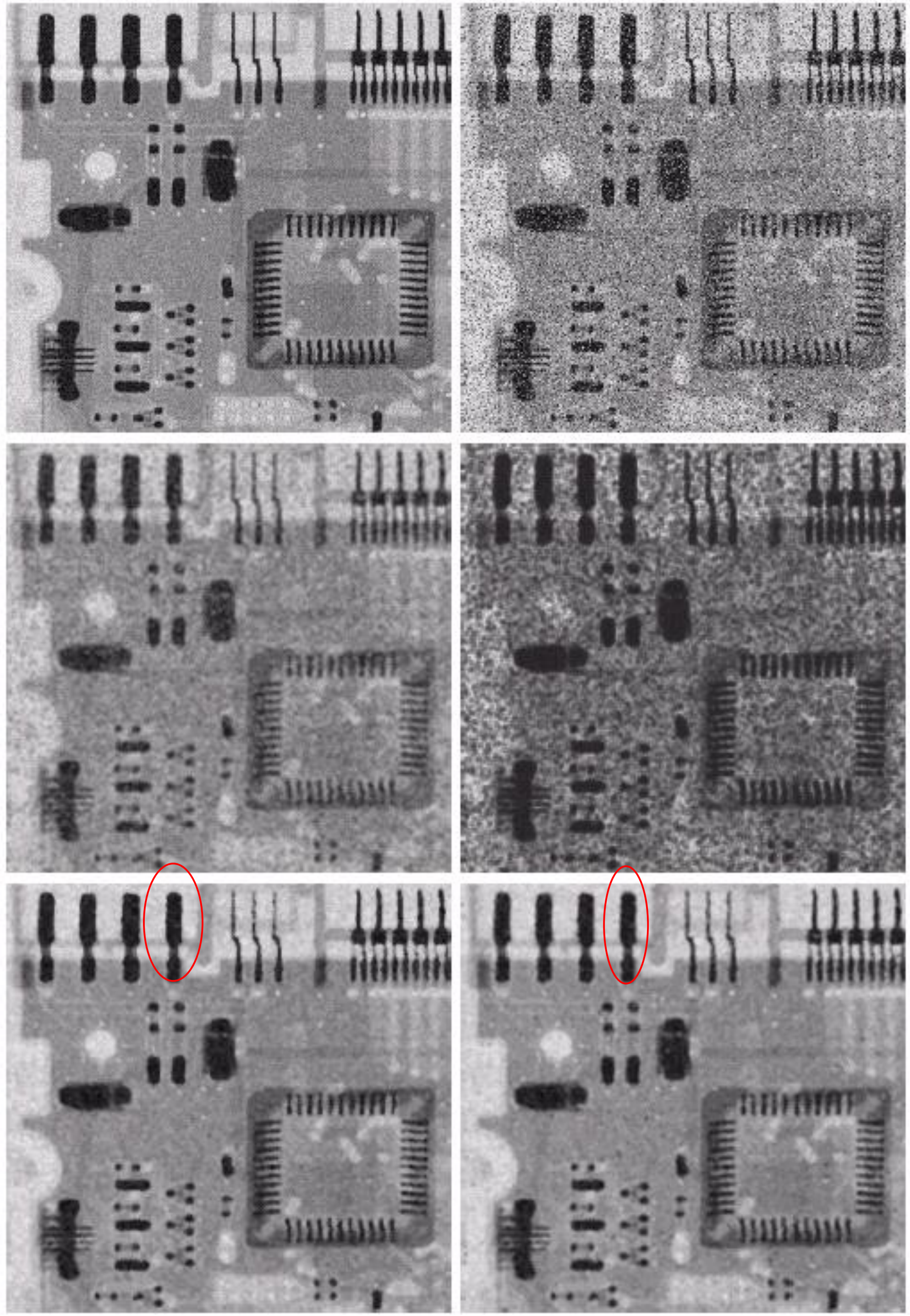
  - d= 0 , it reduces to arithmetic mean filter
  - d = mn-1 , the filter becomes median filter.
  - It useful in the situations involving multiple types of noise , such as a combination if salt-and-pepper and Gaussian noise.

Uniform noise

$\mu=0$

$\sigma^2=800$

Left +
Bipolar Noise
$P_a = 0.1$
$P_b = 0.1$



5x5
Arith. Mean
filter

5x5
Geometric
mean

5x5
Median
filter

5x5
Alpha-trim.
Filter
d=5

Adaptive filters

- Adapted to the behavior based on the statistical characteristics of the image inside the filter region S_{xy}
- Improved performance v.s increased complexity
- Example: Adaptive local noise reduction filter

Adaptive local noise reduction filter

- Simplest statistical measurement
  - Mean and variance
- Known parameters on local region S_{xy}
  - g(x,y): noisy image pixel value
  - σ^2_η: noise variance (assume known a prior)
  - m_L : local mean
  - σ^2_L : local variance

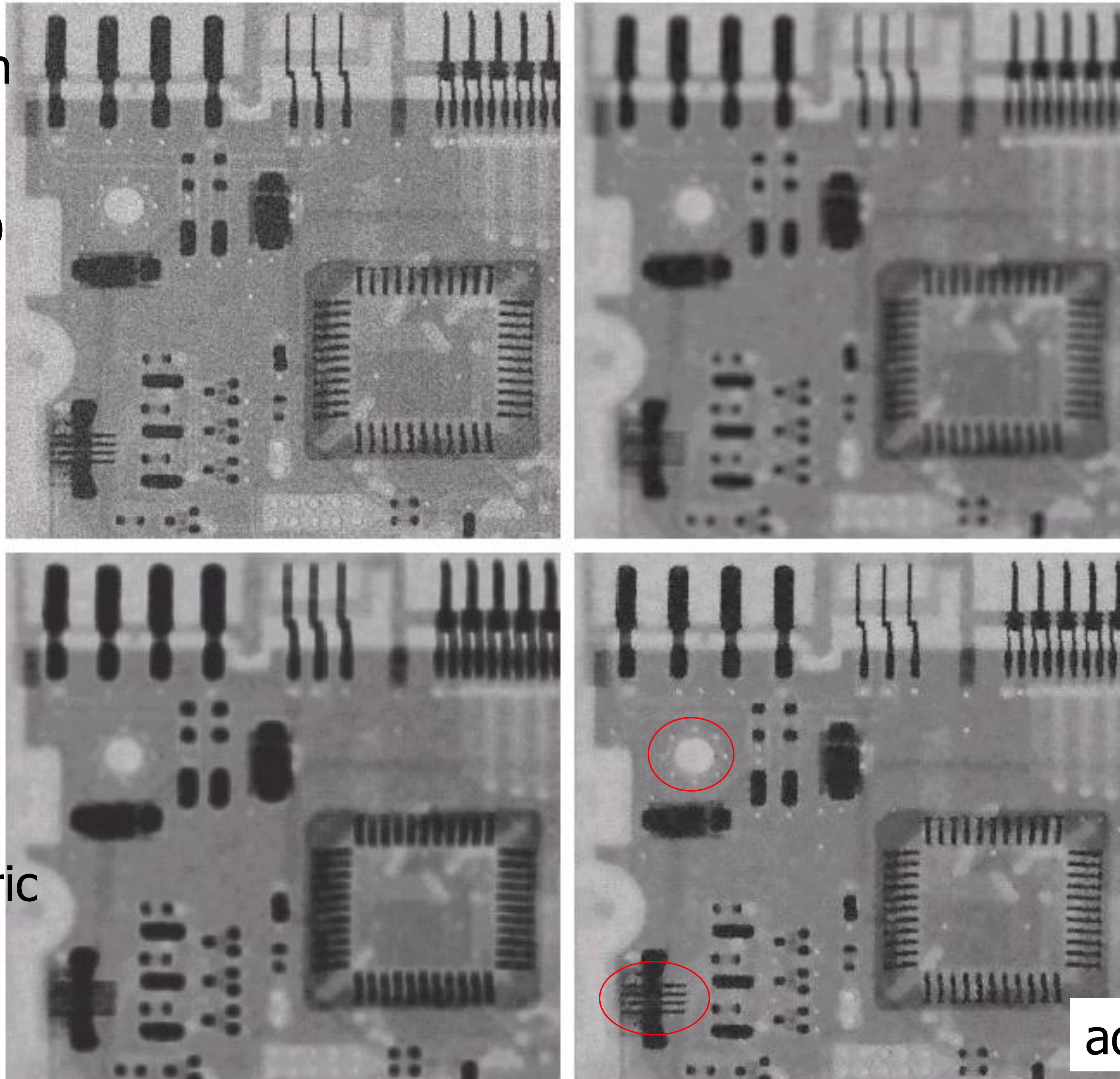
Adaptive local noise reduction filter (cont.)

- Analysis: we want to do
  - If σ^2_η is zero, return g(x,y) (zero Noise)
  - If $\sigma^2_L > \sigma^2_\eta$, return value close to g(x,y)
  - it is associtated with edges and it should be preserved
  - If $\sigma^2_L = \sigma^2_\eta$, return the arithmetic mean m_L
  - It occurs when local area has same properties as the overall image and local noise is to be reduced by averaging.
- Formula

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

Gaussian noise
$\mu=0$
$\sigma^2=1000$

Arith. mean 7x7



Geometric mean 7x7

adaptive

Adaptive Filters: Adaptive Median Filters

- Median filter is effective for removing salt-and pepper noise if spatial density of the impulse noise is not large (as a rule of thumb Pa, Pb = 0.2)
- Adaptive median filter can handle impulse noise with probabilities larger than these.
- In addition, it seek to preserve detail while smoothing non-impulse noise, something that traditional filter does not do.
- Three main purposes
  - To remove salt pepper noise
  - To provide smoothing of other noise that may not be impulsive
  - To reduce distortion, such as excessive thinning or thickening of object boundaries.

Adaptive Filters: Adaptive Median Filters

- Adaptive median filter

  Notation

  Z_{min}: minimum gray value in S_{xy}
  Z_{max}: maximum gray value in S_{xy}
  Z_{med}: median of gray levels in S_{xy}
  Z_{xy}: gray value of the image at (x,y)
  S_{max}: maximum allowed size of S_{xy}

Adaptive Median Filter (De-Noising)

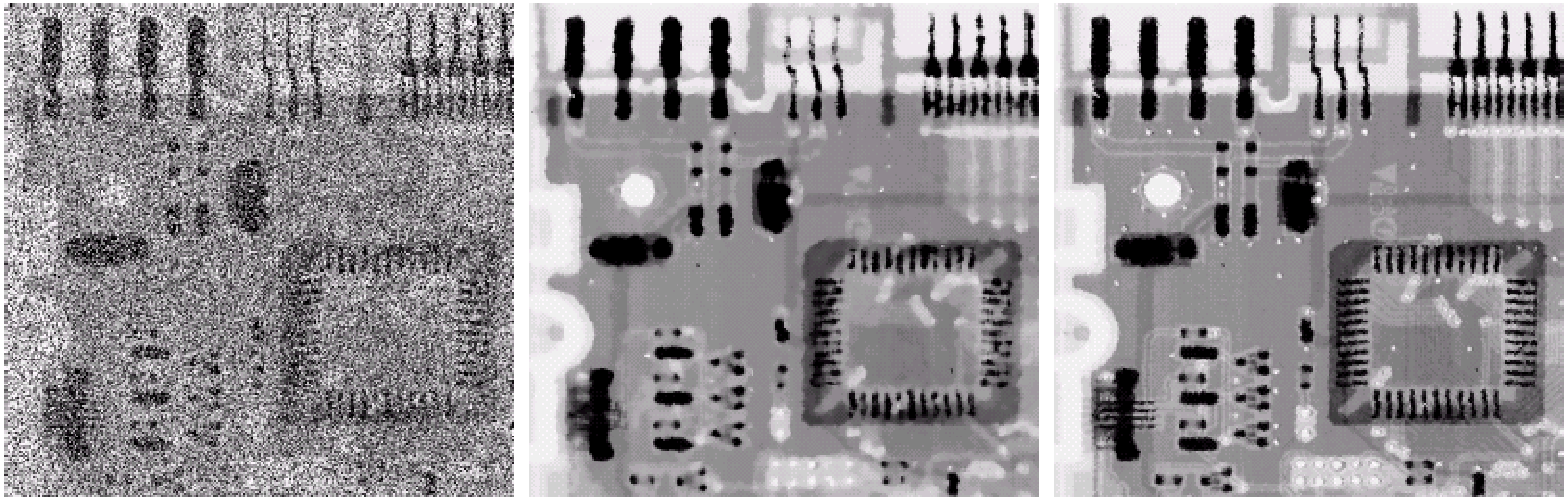
- Two levels of operations

Level A:

- $A1 = Z_{med} - Z_{min}$
- $A2 = Z_{med} - Z_{max}$
- If A1 > 0 AND A2 < 0, Go to level B
  else increase the window size by 2
- If window size <= S_{max} repeat level A
  else output Z_{xy}

Level B:

- $B1 = Z_{xy} - Z_{min}$
- $B2 = Z_{xy} - Z_{max}$
- If B1 > 0 AND B2 < 0, output Z_{xy}
  else output Z_{med}



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{\max} = 7$.

Periodic noise reduction (cont.)

- Bandreject filters
- Bandpass filters
- Notch filters
- Optimum notch filtering

Bandreject filters

* Reject an isotropic frequency

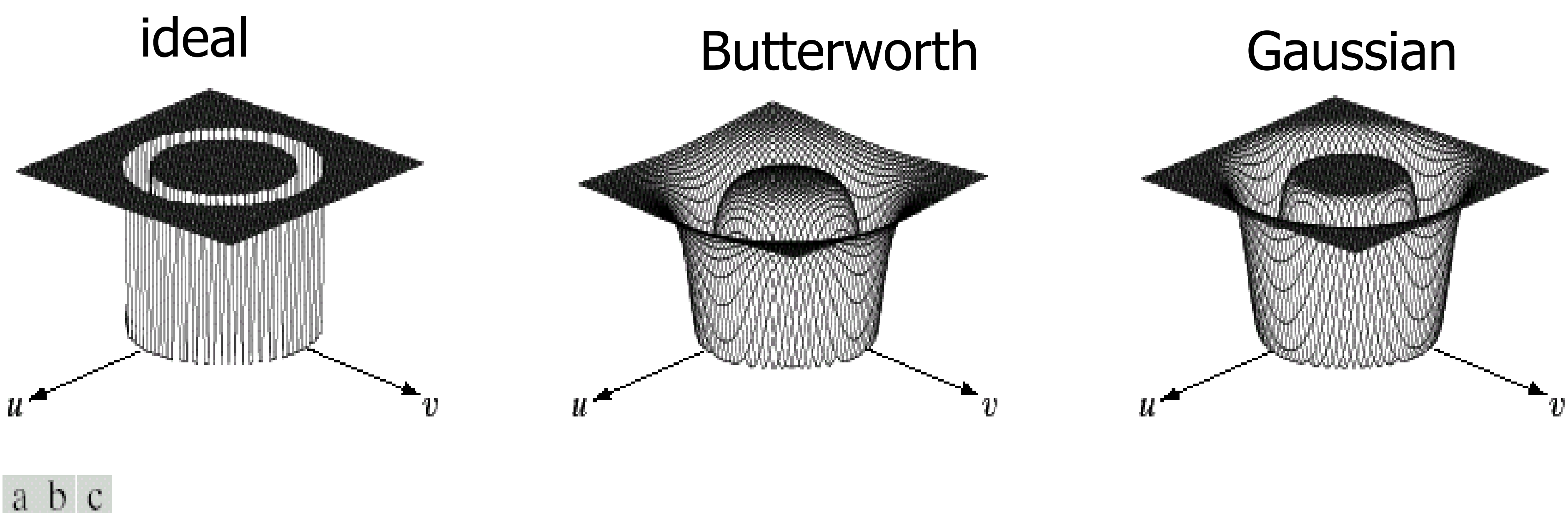
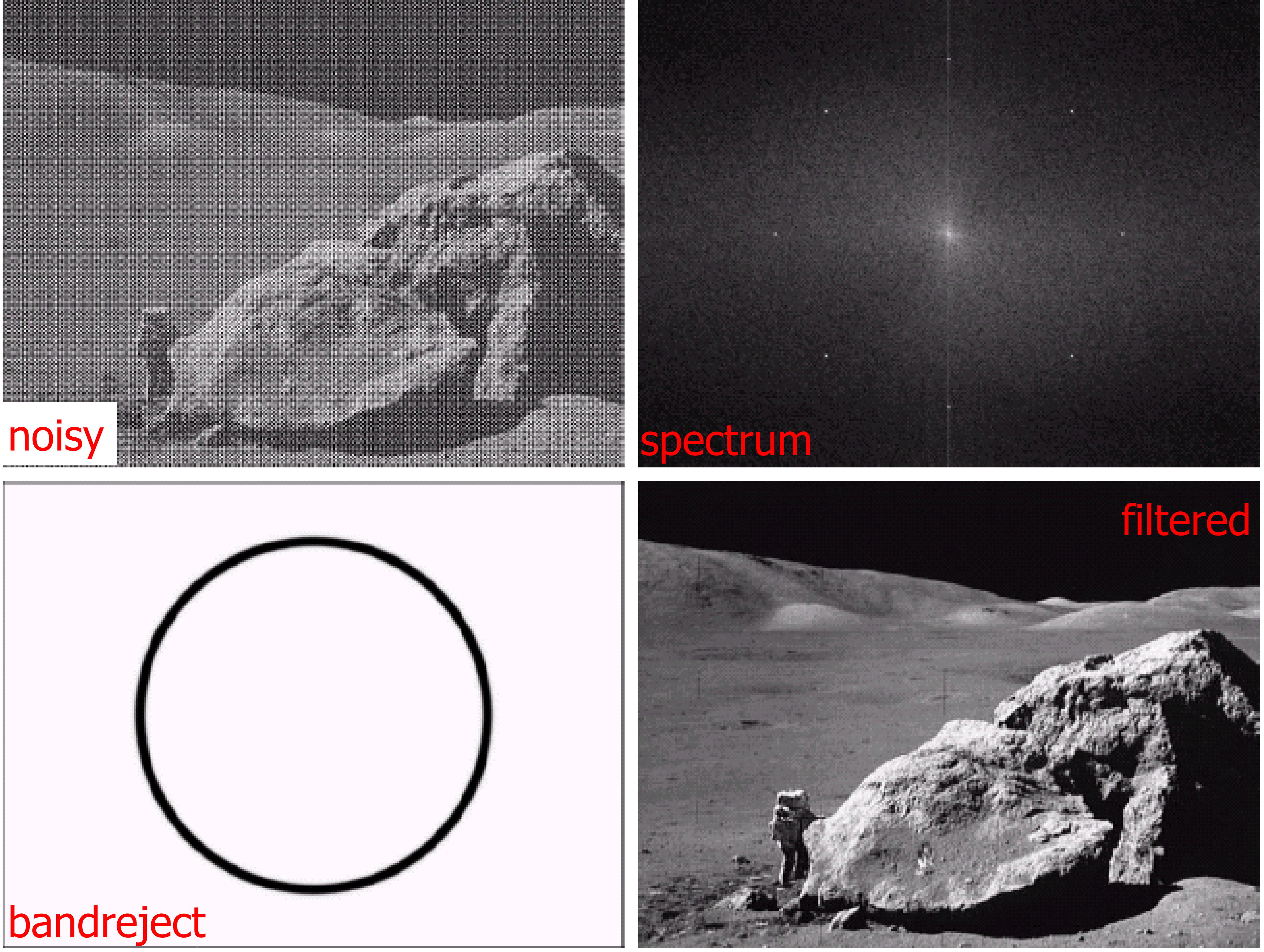
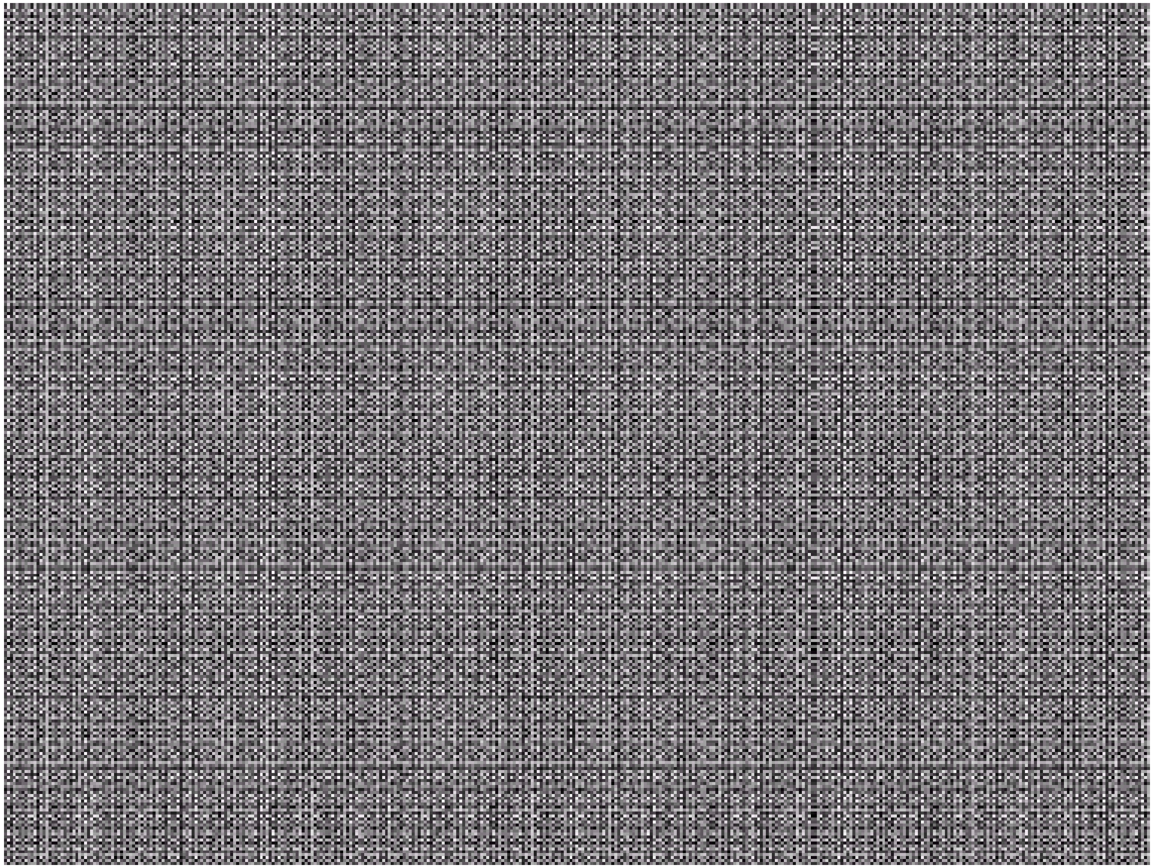


FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.


noisy
spectrum
bandreject
filtered

Bandpass filters

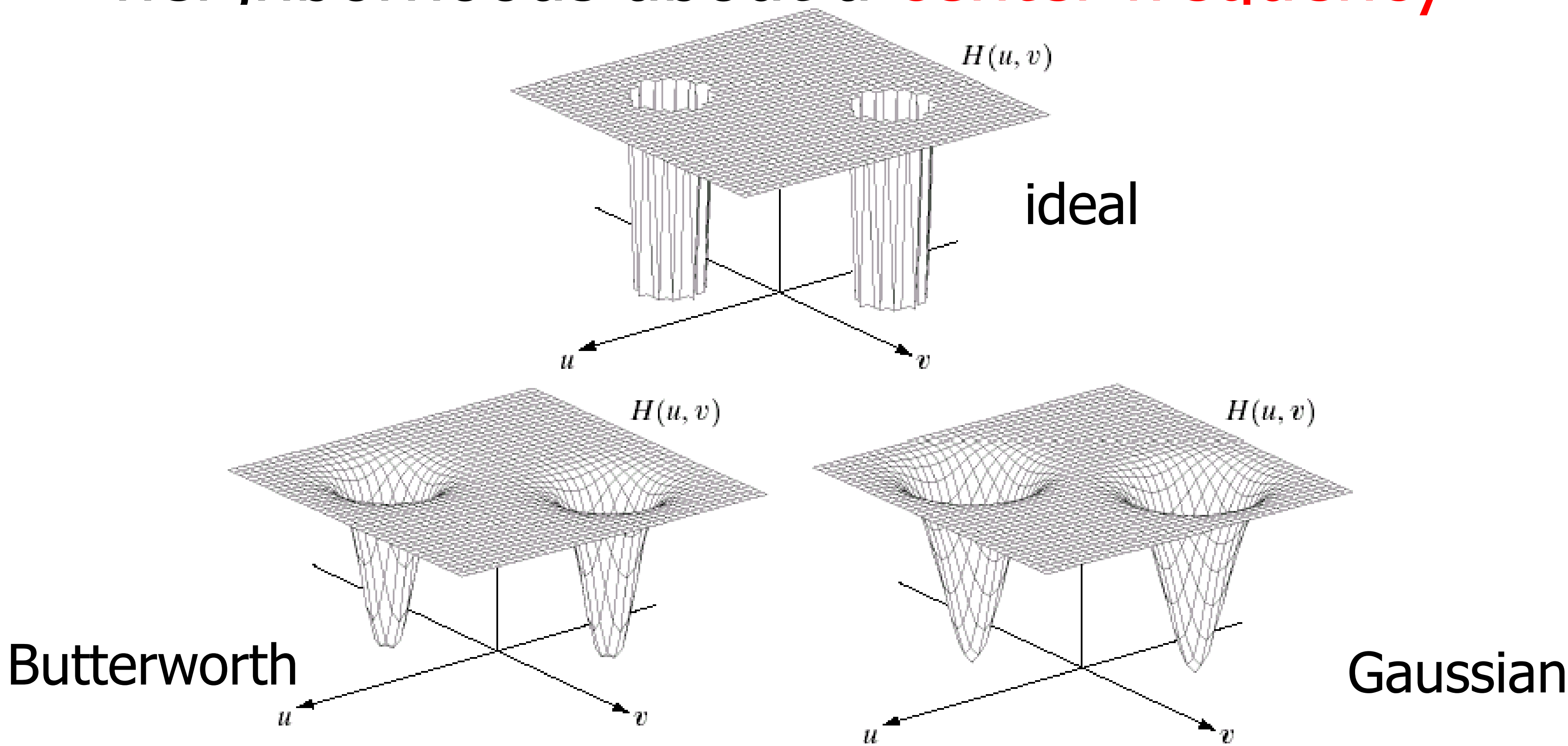
- $H_{bp}(u,v)=1- H_{br}(u,v)$

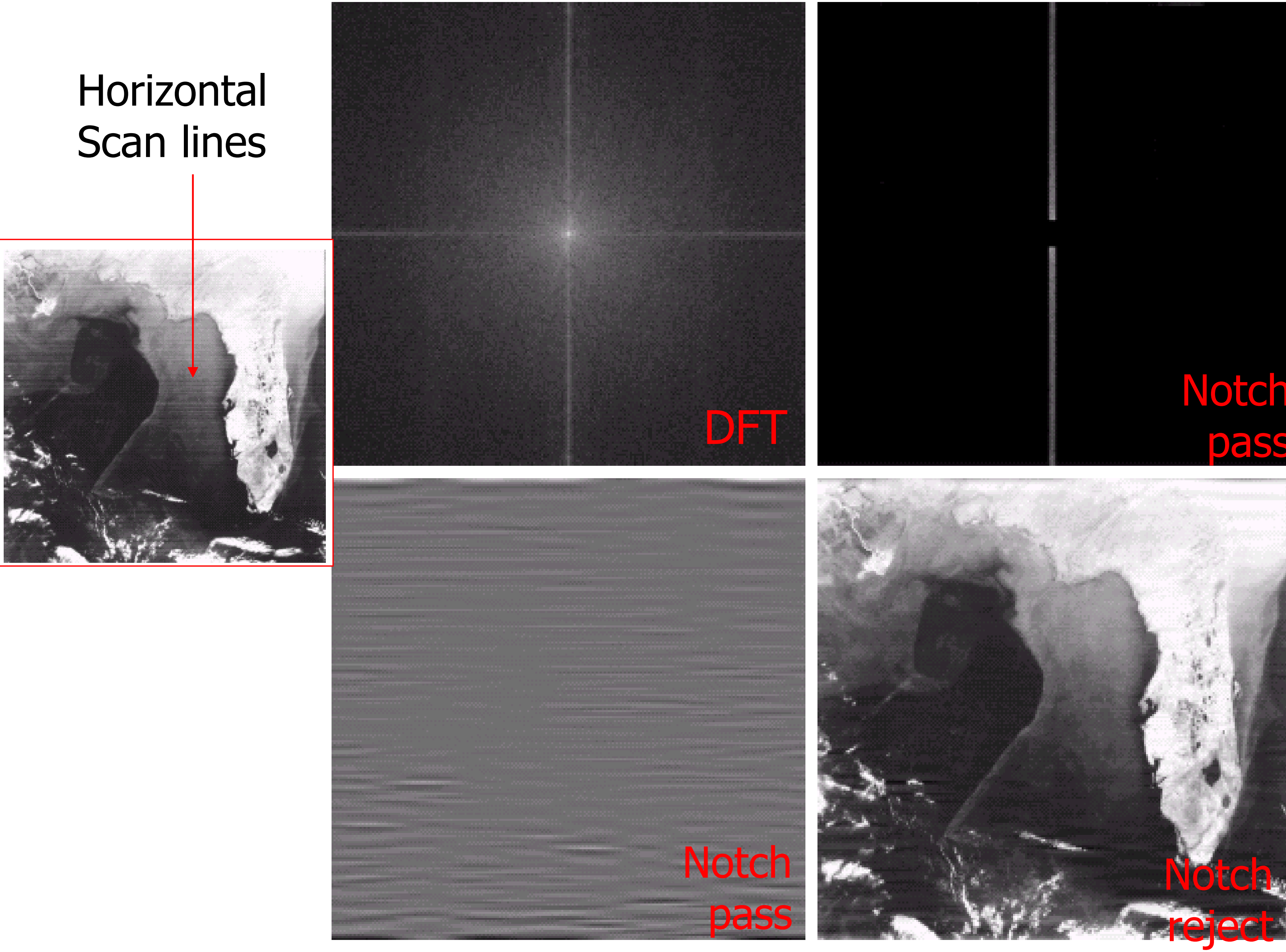


$$\Im^{-1}\left\{G(u,v)H_{bp}(u,v)\right\}$$

Notch filters

- Reject(or pass) frequencies in predefined neighborhoods about a center frequency




Horizontal
Scan lines
DFT
Notch
pass
Notch
pass
Notch
reject