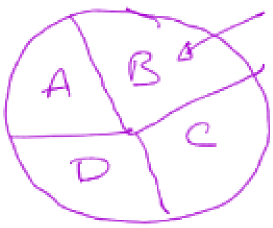
* Generate samples from Bayesian Network
* Use samples to answer inference question

(A) Random Sampling

- If we randomly select value for a random variable, the value with higher probability has higher chance of getting selected

$P(X) = P(\sim X) = \frac{1}{2}$

0 (x=1) ½ (x=0) 1

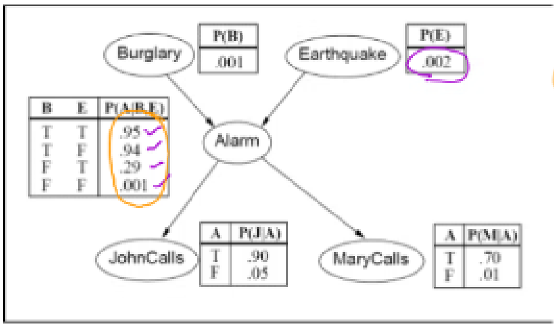


more example will fit in 'B' Region.

$P(X) = 0.8 \quad P(\sim X) = 0.2$

X = 1   X = 0

0   0.8   1



- Generate Random Numbers (0-10000)

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$

X = 5, 10, 1000, 2000, 3000

B=T, E=T, A=T, J=T, M=T

- Find values of R.V. using x_i & CPT.

J: 0 — J=T — 90% — J=F — 10000; A=T → M: T — 70% — F — 10000

J: 0.05% J=T — J=F; A=F → M: T — F

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$

100 10 5000 1000 8000

$x_6 \quad x_7 \quad x_8 \quad x_9 \quad x_{10}$

1000 2000 50 6000 7000

	B	E	A	J	M
1	T	T	T	T	T
2	F	T	F	F	F
3	F	F	F	F	F

$$P(E/\sim M) = \frac{\#(E=T, M=F)}{\#(M=F)} = \frac{1}{2}$$

$\boxed{P(E/\sim M) = \frac{1}{2}}$

Limitation of Random Sampling: $P(E=F/M=T)$?

$$P(E=F/M=T) = \frac{\#(E=F, M=T)}{\#(M=T)} = \frac{0}{1}$$

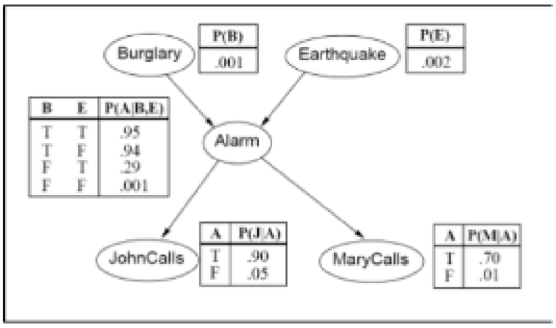
Evidence

Query variable

Variable

(B) Rejection Sampling

- "Reject the sample in which the "evidence variable's" value does not match with the value in the inference query."



$P(J / \sim M, \sim E)$?

x_1	x_2	x_3	x_4	x_5	x_6
2000	3000	4000	5000	6000	
B=F	E=F	A=F	J=F	M=F	
10	3000	100	2000	5000	
B=T	E=F	A=T	J=T	M=T	

B	E	A	J	M	
F	F	F	F	F	✓
T	F	T	T	T	X ← Reject because M=T

Limitation of Rejection Sampling

↳ "Sometime we need to reject after drawing values for many R.V."

B	E	A	J	M
T	F	F	F	F
T	F	T	F	F
F	F	T	F	F
T	F	F	T	F

$P(J / \sim M, \sim E)$

$= \frac{\#(J=T)}{\#\text{ Total Samples}} = \frac{1}{4}$

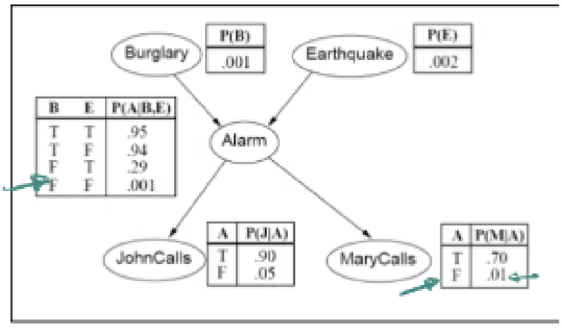
$= \frac{\#(J=T, M=F, E=F)}{\#(M=F, E=F)}$

(c) Likelihood weighting

(1) - Do not sample the evidence variable, fix the value given in the query.

- Weight the sample with the probability of the value assigned to the evidence variable.

(2) - Sample other variables using random sampling.



$P(J / \sim M, \sim E)$

B	E	A	J	M	Weight
F	F	F	F	F	(0.998×0.99)
F	F	T	T	F	(0.998×0.3)
T	F	T	F	F	(0.998×0.3)
T	F	F	T	F	(0.998×0.99)

$$P(J / \sim M, \sim E) = \frac{\#(J, \sim M, \sim E)}{\#(\sim M, \sim E)}$$

Sum of weights

$$= \frac{0.998 \times 0.3 + 0.998 \times 0.99}{0.998 \times 0.3 \times 2 + 0.998 \times 0.99 \times 2}$$

$$= 0.5$$

$P(B / \sim M, \sim E) = \frac{1}{2}$?

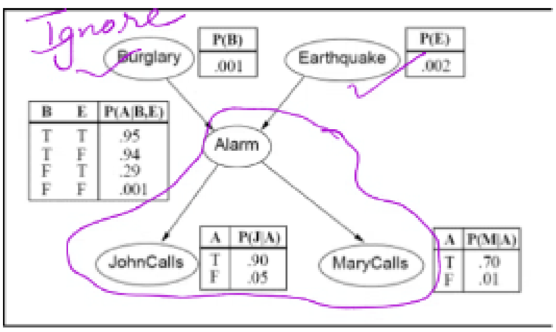
P) Gibbs Sampling

1- Fix evidence variable,

2- Randomly initialize other variable

- Sample one variable at a time
- Repeat sufficient number of time, (and samples would represent true distribution)

P(J / ~M, ~E)



B	E	A	J	M
F	F	F	F	F
	F			F
	F			F
	F			F

1

2

2

B	A	J
F	F	F

3 Sample one variable at a time

→ B — 2000

"P(B) = we will use knowledge of condition dependence & independence to calculate P(B)"

Markov Blanket : "parent, children, spouse"

MB(B) = A, E

MB(J) = A, M

$MB(A) = B, E, J, M$

"A node is conditionally independent of all other nodes in the graph, given its markov blanket.