L-99: Ninety-Nine Lisp Problems

Based on a Prolog problem list by werner.hett@hti.bfh.ch

Working with lists

P01 (*) Find the last box of a list.

Example:
* (my-last '(a b c d))
(D)

P02 (*) Find the last but one box of a list.

Example:
* (my-but-last '(a b c d))
(C D)

P03 (*) Find the K'th element of a list.

The first element in the list is number 1.
Example:
* (element-at '(a b c d e) 3)
C

P04 (*) Find the number of elements of a list.

P05 (*) Reverse a list.

P06 (*) Find out whether a list is a palindrome.

A palindrome can be read forward or backward; e.g. (x a m a x).

P07 (**) Flatten a nested list structure.

Transform a list, possibly holding lists as elements into a `flat' list by replacing each list with its elements (recursively).

Example:
* (my-flatten '(a (b (c d) e)))
(A B C D E)

Hint: Use the predefined functions list and append.

P08 (**) Eliminate consecutive duplicates of list elements.

If a list contains repeated elements they should be replaced with a single copy of the element. The order of the elements should not be changed.

Example:
* (compress '(a a a a b c c a a d e e e e))
(A B C A D E)

P09 (**) Pack consecutive duplicates of list elements into sublists.

If a list contains repeated elements they should be placed in separate sublists.

Example:
* (pack '(a a a a b c c a a d e e e e))
((A A A A) (B) (C C) (A A) (D) (E E E E))

P10 (*) Run-length encoding of a list.

Use the result of problem P09 to implement the so-called run-length encoding data compression method. Consecutive duplicates of elements are encoded as lists (N E) where N is the number of duplicates of the element E.

Example:
* (encode '(a a a a b c c a a d e e e e))
((4 A) (1 B) (2 C) (2 A) (1 D)(4 E))

P11 (*) Modified run-length encoding.

Modify the result of problem P10 in such a way that if an element has no duplicates it is simply copied into the result list. Only elements with duplicates are transferred as (N E) lists.

Example:
* (encode-modified '(a a a a b c c a a d e e e e))
((4 A) B (2 C) (2 A) D (4 E))

P12 (**) Decode a run-length encoded list.

Given a run-length code list generated as specified in problem P11. Construct its uncompressed version.

P13 (**) Run-length encoding of a list (direct solution).

Implement the so-called run-length encoding data compression method directly. I.e. don't explicitly create the sublists containing the duplicates, as in problem P09, but only count them. As in problem P11, simplify the result list by replacing the singleton lists (1 X) by X.

Example:
* (encode-direct '(a a a a b c c a a d e e e e))
((4 A) B (2 C) (2 A) D (4 E))

P14 (*) Duplicate the elements of a list.

Example:
* (dupli '(a b c c d))
(A A B B C C C C D D)

P15 (**) Replicate the elements of a list a given number of times.

Example:
* (repli '(a b c) 3)
(A A A B B B C C C)

P16 (**) Drop every N'th element from a list.

Example:
* (drop '(a b c d e f g h i k) 3)
(A B D E G H K)

P17 (*) Split a list into two parts; the length of the first part is given.

Do not use any predefined functions.

Example:
* (split '(a b c d e f g h i k) 3)
((A B C) (D E F G H I K))

P18 (**) Extract a slice from a list.

Given two indices, I and K, the slice is the list containing the elements between the I'th and K'th element of the original list (both limits included). Start counting the elements with 1.

Example:
* (slice '(a b c d e f g h i k) 3 7)
(C D E F G)

P19 (**) Rotate a list N places to the left.

Examples:
* (rotate '(a b c d e f g h) 3)
(D E F G H A B C)

* (rotate '(a b c d e f g h) -2)
(G H A B C D E F)

Hint: Use the predefined functions length and append, as well as the result of problem P17.

P20 (*) Remove the K'th element from a list.

Example:
* (remove-at '(a b c d) 2)
(A C D)

P21 (*) Insert an element at a given position into a list.

Example:
* (insert-at 'alfa '(a b c d) 2)
(A ALFA B C D)

P22 (*) Create a list containing all integers within a given range.

If first argument is smaller than second, produce a list in decreasing order.
Example:
* (range 4 9)
(4 5 6 7 8 9)

P23 (**) Extract a given number of randomly selected elements from a list.

The selected items shall be returned in a list.
Example:
* (rnd-select '(a b c d e f g h) 3)
(E D A)

Hint: Use the built-in random number generator and the result of problem P20.

P24 (*) Lotto: Draw N different random numbers from the set 1..M.

The selected numbers shall be returned in a list.
Example:
* (lotto-select 6 49)
(23 1 17 33 21 37)

Hint: Combine the solutions of problems P22 and P23.

P25 (*) Generate a random permutation of the elements of a list.

Example:
* (rnd-permu '(a b c d e f))
(B A D C E F)

Hint: Use the solution of problem P23.

P26 (**) Generate the combinations of K distinct objects chosen from the N elements of a list

In how many ways can a committee of 3 be chosen from a group of 12 people? We all know that there are C(12,3) = 220 possibilities (C(N,K) denotes the well-known binomial coefficients). For pure mathematicians, this result may be great. But *we* want to really generate all the possibilities in a list.

Example:
* (combination 3 '(a b c d e f))
((A B C) (A B D) (A B E) ...)

P27 (**) Group the elements of a set into disjoint subsets.

a) In how many ways can a group of 9 people work in 3 disjoint subgroups of 2, 3 and 4 persons? Write a function that generates all the possibilities and returns them in a list.

Example:
* (group3 '(aldo beat carla david evi flip gary hugo ida))
(((ALDO BEAT) (CARLA DAVID EVI) (FLIP GARY HUGO IDA))
...)

b) Generalize the above function in a way that we can specify a list of group sizes and the function will return a list of groups.

Example:
* (group '(aldo beat carla david evi flip gary hugo ida) '(2 2 5))
(((ALDO BEAT) (CARLA DAVID) (EVI FLIP GARY HUGO IDA))
...)

Note that we do not want permutations of the group members; i.e. ((ALDO BEAT) ...) is the same solution as ((BEAT ALDO) ...). However, we make a difference between ((ALDO BEAT) (CARLA DAVID) ...) and ((CARLA DAVID) (ALDO BEAT) ...).

You may find more about this combinatorial problem in a good book on discrete mathematics under the term "multinomial coefficients".

[P28](#) **(**) Sorting a list of lists according to length of sublists**

a) We suppose that a list contains elements that are lists themselves. The objective is to sort the elements of this list according to their **length**. E.g. short lists first, longer lists later, or vice versa.

Example:
* (lsort '((a b c) (d e) (f g h) (d e) (i j k l) (m n) (o)))
((O) (D E) (D E) (M N) (A B C) (F G H) (I J K L))

b) Again, we suppose that a list contains elements that are lists themselves. But this time the objective is to sort the elements of this list according to their **length frequency**; i.e., in the default, where sorting is done ascendingly, lists with rare lengths are placed first, others with a more frequent length come later.

Example:
* (lfsort '((a b c) (d e) (f g h) (d e) (i j k l) (m n) (o)))
((I J K L) (O) (A B C) (F G H) (D E) (D E) (M N))

Note that in the above example, the first two lists in the result have length 4 and 1, both lengths appear just once. The third and forth list have length 3 which appears twice (there are two list of this length). And finally, the last three lists have length 2. This is the most frequent length.

Arithmetic

[P31](#) **(**) Determine whether a given integer number is prime.**

Example:
* (is-prime 7)
T

[P32](#) **(**) Determine the greatest common divisor of two positive integer numbers.**

Use Euclid's algorithm.
Example:
* (gcd 36 63)

9

P33 (*) Determine whether two positive integer numbers are coprime.

Two numbers are coprime if their greatest common divisor equals 1.
Example:
* (coprime 35 64)
T

P34 (**) Calculate Euler's totient function phi(m).

Euler's so-called totient function phi(m) is defined as the number of positive integers r (1 <= r < m) that are coprime to m.

Example: m = 10: r = 1,3,7,9; thus phi(m) = 4. Note the special case: phi(1) = 1.

* (totient-phi 10)
4

Find out what the value of phi(m) is if m is a prime number. Euler's totient function plays an important role in one of the most widely used public key cryptography methods (RSA). In this exercise you should use the most primitive method to calculate this function (there are smarter ways that we shall discuss later).

P35 (**) Determine the prime factors of a given positive integer.

Construct a flat list containing the prime factors in ascending order.
Example:
* (prime-factors 315)
(3 3 5 7)

P36 (**) Determine the prime factors of a given positive integer (2).

Construct a list containing the prime factors and their multiplicity.
Example:
* (prime-factors-mult 315)
((3 2) (5 1) (7 1))

Hint: The problem is similar to problem P10.

P37 (**) Calculate Euler's totient function phi(m) (improved).

See problem P34 for the definition of Euler's totient function. If the list of the prime factors of a number m is known in the form of problem P36 then the function phi(m) can be efficiently calculated as follows: Let ((p1 m1) (p2 m2) (p3 m3) ...) be the list of prime factors (and their multiplicities) of a given number m. Then phi(m) can be calculated with the following formula:

phi(m) = (p1 - 1) * p1 ** (m1 - 1) * (p2 - 1) * p2 ** (m2 - 1) * (p3 - 1) * p3 ** (m3 - 1) * ...

Note that a ** b stands for the b'th power of a.

P38 (*) Compare the two methods of calculating Euler's totient function.

Use the solutions of problems P34 and P37 to compare the algorithms. Take the number of basic operations, including CARs, CDRs, CONSes, and arithmetic operations, as a measure for efficiency. Try to calculate phi(10090) as an example.

P39 (*) A list of prime numbers.
Given a range of integers by its lower and upper limit, construct a list of all prime numbers in that range.

P40 (**) Goldbach's conjecture.
Goldbach's conjecture says that every positive even number greater than 2 is the sum of two prime numbers. Example: 28 = 5 + 23. It is one of the most famous facts in number theory that has not been proved to be correct in the general case. It has been *numerically* confirmed up to very large numbers (much larger than we can go with our Lisp system). Write a function to find the two prime numbers that sum up to a given even integer.

Example:
* (goldbach 28)
(5 23)

P41 (**) A list of Goldbach compositions.
Given a range of integers by its lower and upper limit, print a list of all even numbers and their Goldbach composition.

Example:
* (goldbach-list 9 20)
10 = 3 + 7
12 = 5 + 7
14 = 3 + 11
16 = 3 + 13
18 = 5 + 13
20 = 3 + 17

In most cases, if an even number is written as the sum of two prime numbers, one of them is very small. Very rarely, the primes are both bigger than say 50. Try to find out how many such cases there are in the range 2..3000.

Example (for a print limit of 50):
* (goldbach-list 1 2000 50)
992 = 73 + 919
1382 = 61 + 1321
1856 = 67 + 1789
1928 = 61 + 1867

Logic and Codes

P46 (**) Truth tables for logical expressions.
Define functions and, or, nand, nor, xor, impl and equ (for logical equivalence) which return the result of the respective operation on boolean values.

A logical expression in two variables can then be written in prefix notation, as in the following example: (and (or A B) (nand A B)).

Write a function table which prints the truth table of a given logical expression in two variables.

Example:
* (table 'A 'B '(and A (or A B))).
```
true true true
true nil true
nil true nil
nil nil nil
```

P47 (*) Truth tables for logical expressions (2).

Modify problem P46 by accepting expressions written in infix notation, with all parenthesis present. This allows us to write logical expression in a more natural way, as in the example: (A and (A or (not B))).

Example:
* (table 'A 'B '(A and (A or (not B)))).
```
true true true
true nil true
nil true nil
nil nil nil
```

P48 (**) Truth tables for logical expressions (3).

Generalize problem P47 in such a way that the logical expression may contain any number of logical variables. Define table in a way that (table List Expr) prints the truth table for the expression Expr, which contains the logical variables enumerated in List.

Example:
* (table '(A B C) '((A and (B or C)) equ ((A and B) or (A and C)))).
```
true true true true
true true nil true
true nil true true
true nil nil true
nil true true true
nil true nil true
nil nil true true
nil nil nil true
```

P49 (**) Gray code.

An n-bit Gray code is a sequence of n-bit strings constructed according to certain rules. For example,
n = 1: C(1) = ("0" "1").
n = 2: C(2) = ("00" "01" "11" "10").
n = 3: C(3) = ("000" "001" "011" "010" "110" "111" "101" "100").

Find out the construction rules and write a function with the following specification:

(gray N) returns the N-bit Gray code

Can you apply the method of "result caching" in order to make the function more efficient, when it is to be used repeatedly?

P50 (***) Huffman code.

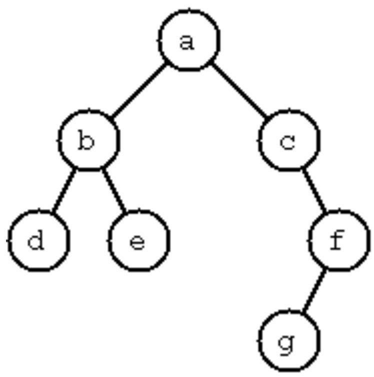
First of all, consult a good book on discrete mathematics or algorithms for a detailed description of Huffman codes!

We suppose a set of symbols with their frequencies, given as a list of (S F) elements. Example: ((a 45) (b 13) (c 12) (d 16) (e 9) (f 5)). Our objective is to construct a list of (S C) elements, where C is the Huffman code word for symbol S. In our example, the result could be ((A "0") (B "101") (C "100") (D "111") (E "1101") (F "1100")). The task shall be performed by a function huffman defined as follows:

(huffman F) returns the Huffman code table for the frequency table F

Binary Trees

A binary tree is either empty or it is composed of a root element and two successors, which are binary trees themselves.



In Lisp we represent the empty tree by 'nil' and the non-empty tree by the list (X L R), where X denotes the information at the root node and L and R denote the left and right subtrees, respectively. The example tree depicted opposite is therefore represented by the following list:

(a (b (d nil nil) (e nil nil)) (c nil (f (g nil nil) nil)))

Other examples are a binary tree that consists of a root node only:

(a nil nil) or an empty binary tree: nil.

You can check your functions using these example trees. They are given as test cases in p54.lisp.

P54A (*) Check whether a given expression represents a binary tree

Write a function istree which returns true if and only if its argument is a list representing a binary tree.
Example:
```
* (istree '(a (b nil nil) nil))
T
* (istree '(a (b nil nil)))
NIL
```

P55 (**) Construct completely balanced binary trees

In a completely balanced binary tree, the following property holds for every node: The number of nodes in its left subtree and the number of nodes in its right subtree are almost equal, which means their difference is not greater than one.

Write a function cbal-tree to construct completely balanced binary trees for a given number of nodes. The function should generate all solutions. Put the symbol 'x' as information into all nodes of the tree.
Example:
```
* (cbal-tree-print 4)
(X (X NIL NIL) (X NIL (X NIL NIL)))
(X (X NIL NIL) (X (X NIL NIL) NIL))
etc......
```

Note: you can either print the trees or return a list with them all.

```
* (cbal-tree 4)
((X (X NIL NIL) (X NIL (X NIL NIL))) (X (X NIL NIL) (X (X NIL NIL) NIL)) ......)
```

[P56](#) (**) Symmetric binary trees

Let us call a binary tree symmetric if you can draw a vertical line through the root node and then the right subtree is the mirror image of the left subtree. Write a function symmetric to check whether a given binary tree is symmetric. We are only interested in the structure, not in the contents of the nodes.

[P57](#) (**) Binary search trees (dictionaries)

Write a function to construct a binary search tree from a list of integer numbers.
Example:
```
* (construct '(3 2 5 7 1))
(3 (2 (1 nil nil) nil) (5 nil (7 nil nil)))
```

Then use this function to test the solution of the problem P56.
Example:
```
* (symmetric '(5 3 18 1 4 12 21))
T
* (symmetric '(3 2 5 7 1))
T
* (symmetric '(3 2 5 7))
NIL
```

[P58](#) (**) Generate-and-test paradigm

Apply the generate-and-test paradigm to construct all symmetric, completely balanced binary trees with a given number of nodes. Example:
```
* (sym-cbal-trees-print 5)
(X (X NIL (X NIL NIL)) (X (X NIL NIL) NIL))
(X (X (X NIL NIL) NIL) (X NIL (X NIL NIL)))
...
```

How many such trees are there with 57 nodes? Investigate about how many solutions there are for a given number of nodes. What if the number is even? Write an appropriate function.

P59 (**) Construct height-balanced binary trees

In a height-balanced binary tree, the following property holds for every node: The height of its left subtree and the height of its right subtree are almost equal, which means their difference is not greater than one.

Write a function hbal-tree to construct height-balanced binary trees for a given height. The function should generate all solutions. Put the letter 'x' as information into all nodes of the tree.
Example:
* (hbal-tree 3)
(X (X (X NIL NIL) (X NIL NIL)) (X (X NIL NIL) (X NIL NIL)))
= (X (X (X NIL NIL) (X NIL NIL)) (X (X NIL NIL) NIL))
etc......

P60 (**) Construct height-balanced binary trees with a given number of nodes

Consider a height-balanced binary tree of height H. What is the maximum number of nodes it can contain?
Clearly, MAXN = 2**H - 1. However, what is the minimum number MINN? This question is more difficult. Try to find a recursive statement and turn it into a function minnodes defined as follows:

(min-nodes H) returns the minimum number of nodes in a height-balanced binary tree of height H.

On the other hand, we might ask: what is the maximum height H a height-balanced binary tree with N nodes can have?

(max-height N) returns the maximum height of a height-balanced binary tree with N nodes

Now, we can attack the main problem: construct all the height-balanced binary trees with a given number of nodes.

(hbal-tree-nodes N) returns all height-balanced binary trees with N nodes.

Find out how many height-balanced trees exist for N = 15.

P61 (*) Count the leaves of a binary tree

A leaf is a node with no successors. Write a function count-leaves to count them.

(count-leaves tree) returns the number of leaves of binary tree tree

P61A (*) Collect the leaves of a binary tree in a list

A leaf is a node with no successors. Write a function leaves to return them in a list.

(leaves tree) returns the list of all leaves of the binary tree tree

P62 (*) Collect the internal nodes of a binary tree in a list

An internal node of a binary tree has either one or two non-empty successors. Write a function internals to collect them in a list.

(internals tree) returns the list of internal nodes of the binary tree tree.

P62B (*) Collect the nodes at a given level in a list

A node of a binary tree is at level N if the path from the root to the node has length N-1. The root node is at level 1. Write a function atlevel to collect all nodes at a given level in a list.

(atlevel tree L) returns the list of nodes of the binary tree tree at level L

Using atlevel it is easy to construct a function levelorder which creates the level-order sequence of the nodes. However, there are more efficient ways to do that.

P63 (**) Construct a complete binary tree

A *complete* binary tree with height H is defined as follows: The levels 1,2,3,...,H-1 contain the maximum number of nodes (i.e 2**(i-1) at the level i, note that we start counting the levels from 1 at the root). In level H, which may contain less than the maximum possible number of nodes, all the nodes are "left-adjusted". This means that in a levelorder tree traversal all internal nodes come first, the leaves come second, and empty successors (the nil's which are not really nodes!) come last.

Particularly, complete binary trees are used as data structures (or addressing schemes) for heaps.

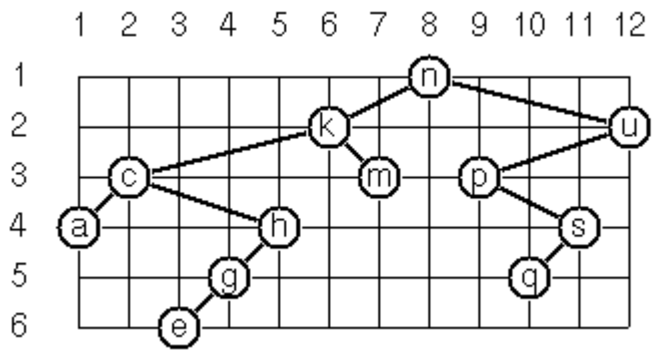
We can assign an address number to each node in a complete binary tree by enumerating the nodes in levelorder, starting at the root with number 1. In doing so, we realize that for every node X with address A the following property holds: The address of X's left and right successors are 2*A and 2*A+1, respectively, supposed the successors do exist. This fact can be used to elegantly construct a complete binary tree structure. Write a function complete-binary-tree with the following specification:

(complete-binary-tree N) returns a complete binary tree with N nodes

Test your function in an appropriate way.

P64 (**) Layout a binary tree (1)

Consider a binary tree as the usual symbolic expression (X L R) or nil. As a preparation for drawing the tree, a layout algorithm is required to determine the position of each node in a rectangular grid. Several layout methods are conceivable, one of them is shown in the illustration below.



In this layout strategy, the position of a node *v* is obtained by the following two rules:

- *x(v)* is equal to the position of the node *v* in the **inorder** sequence
- *y(v)* is equal to the depth of the node *v* in the tree

In order to store the position of the nodes, we extend the symbolic expression representing a node (and its successors) as follows:

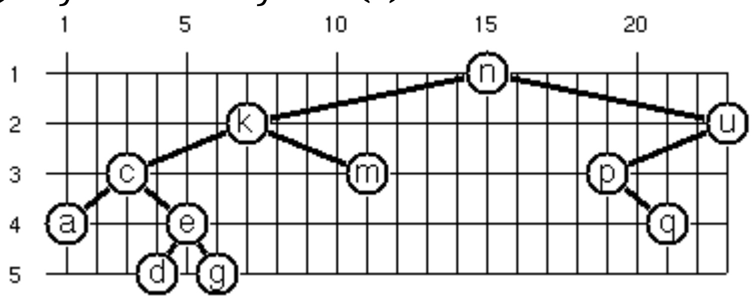
nil represents the empty tree (as usual)
(W X Y L R) represents a (non-empty) binary tree with root W "positioned" at (X,Y), and subtrees L and R

Write a function layout-binary-tree with the following specification:

(layout-binary-tree tree) returns the "positioned" binary tree obtained from the binary tree tree

Test your function in an appropriate way.

P65 (**) Layout a binary tree (2)

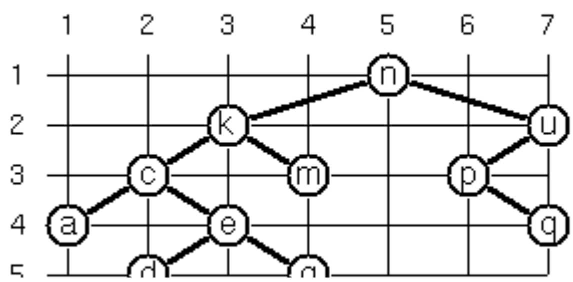


An alternative layout method is depicted in the illustration opposite. Find out the rules and write the corresponding Lisp function. Hint: On a given level, the horizontal distance between neighboring nodes is constant.

Use the same conventions as in problem P64 and test your function in an appropriate way.

P66 (***) Layout a binary tree (3)

Yet another layout strategy is shown in the illustration opposite. The method yields a very compact layout while maintaining a certain symmetry in every node. Find out the rules and write the corresponding Lisp function. Hint:

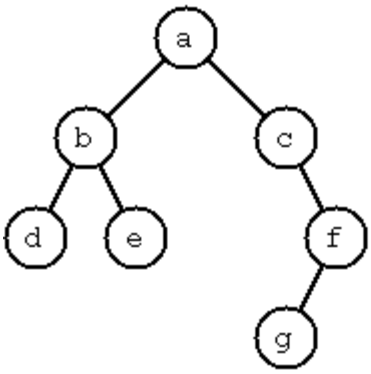


Consider the horizontal distance between a node and its successor nodes. How tight can you pack together two subtrees to construct the combined binary tree?

Use the same conventions as in problem P64 and P65 and test your function in an appropriate way. Note: This is a difficult problem. Don't give up too early!

Which layout do you like most?

[P67](#) (**) A string representation of binary trees



Somebody represents binary trees as strings of the following type (see example opposite):

a(b(d,e),c(,f(g,)))

[a)](#) Write a Lisp function which generates this string representation, if the tree is given as usual (as nil or (X L R) expression). Then write a function which does this inverse; i.e. given the string representation, construct the tree in the usual form.

[P68](#) (**) Preorder and inorder sequences of binary trees

We consider binary trees with nodes that are identified by single lower-case letters, as in the example of problem P67.

[a)](#) Write functions preorder and inorder that construct the preorder and inorder sequence of a given binary tree, respectively. The results should be lists, e.g. (A B D E C F G) for the preorder sequence of the example in problem P67.

[b)](#) Can you write the inverse of preorder from problem part a) ; i.e. given a preorder sequence, construct a corresponding tree?

[c)](#) If both the preorder sequence and the inorder sequence of the nodes of a binary tree are given, then the tree is determined unambiguously. Write a function pre-in-tree that does the job.

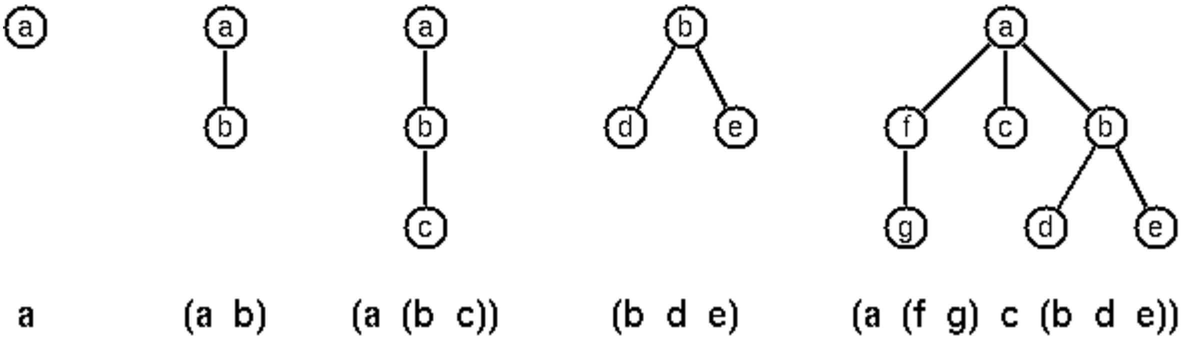
[P69](#) (**) Dotstring representation of binary trees

We consider again binary trees with nodes that are identified by single lower-case letters, as in the example of problem P67. Such a tree can be represented by the preorder sequence of its nodes in which dots (.) are inserted where an empty subtree (nil) is encountered during the tree traversal. For example, the tree shown in problem P67 is represented as `"ABD..E..C.FG..."`. First, try to establish a syntax (BNF or syntax diagrams) and then write functions tree and dotstring which do the conversion.

Multiway Trees

A multiway tree is composed of a root element and a (possibly empty) set of successors which are multiway trees themselves. A multiway tree is never empty. The set of successor trees is sometimes called a forest.

In Lisp we represent a multiway tree by either a symbol (root with no children) or by an expression (X C1 C2 ... CN), where X denotes the root node and Ci denote each of the children. The following pictures show how multiway tree structures are represented in Lisp.



Note that in this Lisp notation a node with successors (children) in the tree is always the first element in a list, followed by its children.

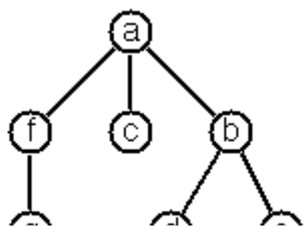
P70B (*) Check whether a given expression represents a multiway tree

Write a function istree which succeeds if and only if its argument is a Lisp expression representing a multiway tree.
Example:
* (istree '(a (f g) c (b d e)))
T

P70C (*) Count the nodes of a multiway tree

Write a function nnodes which counts the nodes of a given multiway tree.
Example:
* (nnodes '(a f))
2

P70 (**) Tree construction from a node string



We suppose that the nodes of a multiway tree contain single characters. In the depth-first order sequence of its nodes, a special character ^ has been inserted whenever, during the tree traversal, the move is a backtrack to the previous level.

By this rule, the tree in the figure opposite is represented as: `afg^^c^bd^e^^^`

Define the syntax of the string and write a function (tree string) to construct the tree when the string is given. Work with lists (instead of strings). Write also an inverse function.

P71 (*) Determine the internal path length of a tree

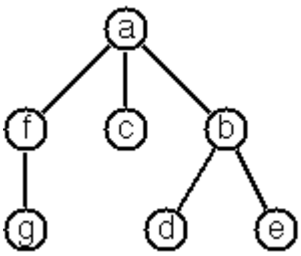
We define the internal path length of a multiway tree as the total sum of the path lengths from the root to all nodes of the tree. By this definition, the tree in the figure of problem P70 has an internal path length of 9. Write a function (ipl tree) to compute it.

P72 (*) Construct the bottom-up order sequence of the tree nodes

Write a function (bottom-up mtree) which returns the bottom-up sequence of the nodes of the multiway tree mtree as a Lisp list.

P73 (**) Prolog-like tree representation

There is a particular notation for multiway trees in **Prolog**. Prolog is a prominent functional programming language, which is used primarily for artificial intelligence problems. As such, it is one of the main competitors of Lisp. In Prolog everything is a term, just as in Lisp everything is a symbolic expression.



In Prolog we represent a multiway tree by a term t(X,F), where X denotes the root node and F denotes the forest of successor trees (a Prolog list). The example tree depicted opposite is represented by the following Lisp expression:

`t(a,[t(f,[t(g,[])]),t(c,[]),t(b,[t(d,[]),t(e,[])])])`

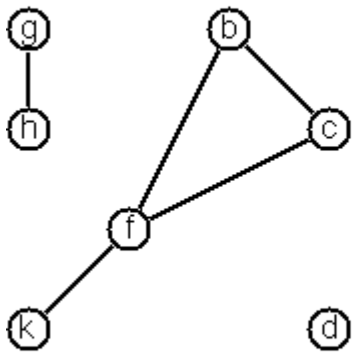
The Prolog representation of a multiway tree is a sequence of atoms, commas, parentheses "(" and ")", and brackets "[" and "]".which we shall collectively call "tokens". We can represent this sequence of tokens as a Lisp list; e.g. the Prolog expression t(a,[t(b,[]),t(c,[])]) could be represented as the Lisp list (t "(" a "," "[" t "(" b "," "[" "]" ")" "," t "(" c "," "[" "]" ")" "]" ")"). Write a function (tree-ptl expr) which returns the "Prolog token list" if the tree is given as an expression expr in the usual Lisp notation.

Example:
* (tree-ptl '(a b c))
(T "(" A "," "[" T "(" B "," "[" "]" ")" "," T "(" C "," "[" "]" ")" "]" ")")

As a second, even more interesting exercise try to write the inverse conversion: Given the list PTL, construct the corresponding Lisp tree.

Graphs

A graph is defined as a set of *nodes* and a set of *edges*, where each edge is a pair of nodes.



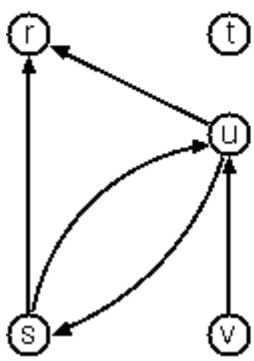
There are several ways to represent graphs in Lisp. One method is to represent the whole graph as one data object. According to the definition of the graph as a pair of two sets (nodes and edges), we may use the following Lisp expression to represent the example graph:

```
((b c d f g h k) ( (b c) (b f) (c f) (f k) (g h) ))
```

We call this *graph-expression form*. Note, that the lists are kept sorted, they are really *sets*, without duplicated elements. Each edge appears only once in the edge list; i.e. an edge from a node x to another node y is represented as (x y), the expression (y x) is not present. **The graph-expression form is our default representation.** In Common Lisp there are predefined functions to work with sets.

A third representation method is to associate with each node the set of nodes that are adjacent to that node. We call this the *adjacency-list form*. In our example:

```
( (b (c f)) (c (b f)) (d ()) (f (b c k)) ...)
```



When the edges are *directed* we call them *arcs*. These are represented by *ordered* pairs. Such a graph is called **directed graph**. To represent a directed graph, the forms discussed above are slightly modified. The example graph opposite is represented as follows:

Graph-expression form

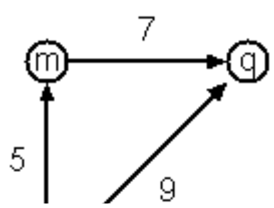
```
( (r s t u v) ( (s r) (s u) (u r) (u s) (v u) ) )
```

Adjacency-list form

```
( (r ()) (s (r u)) (t ()) (u (r)) (v (u)) )
```

Note that the adjacency-list does not have the information on whether it is a graph or a digraph.

Finally, graphs and digraphs may have additional information attached to nodes and edges (arcs). For the nodes, this is no problem, as we can easily replace the single symbol identifiers with arbitrary symbolic expressions, such as `("London" 4711)`. On the other hand, for edges we have to extend our notation. Graphs with additional information attached to edges are called **labelled graphs**.



Graph-expression form

```
( (k m p q) ( (m p 7) (p m 5) (p q 9) ) )
```

Adjacency-list form

```
( (k ()) (m ((q 7))) (p ((m 5) (q 9))) (q ()) )
```

Notice how the edge information has been packed into a list with two elements, the corresponding node and the extra information.

The notation for labelled graphs can also be used for so-called **multi-graphs**, where more than one edge (or arc) are allowed between two given nodes.

P80 (***) Conversions

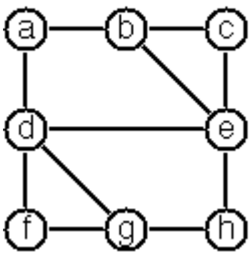
Write functions to convert between the different graph representations. With these functions, all representations are equivalent; i.e. for the following problems you can always pick freely the most convenient form. The reason this problem is rated (***) is not because it's particularly difficult, but because it's a lot of work to deal with all the special cases.

P81 (**) Path from one node to another one

Write a function (path g a b) to return an acyclic path from node a to node b in the graph g. The function should return all paths.

P82 (*) Cycle from a given node

Write a function (cycle g a) to find a closed path (cycle) starting at a given node a in the graph g. The function should return all cycles.

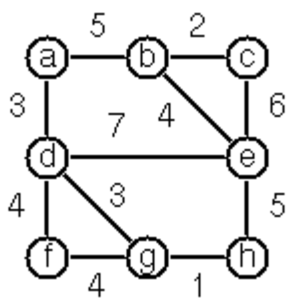


P83 (**) Construct all spanning trees

Write a function (s-tree graph) to construct (by backtracking) all spanning trees of a given graph. With this function, find out how many spanning trees there are for the graph depicted to the left. The data of this example graph can be found in the file p83.dat. When you have a correct solution for the s-tree function, use it to define two other useful functions: (is-tree graph) and (is-connected graph). Both are five-minutes tasks!

P84 (**) Construct a minimum spanning tree

Write a function (ms-tree graph) to construct a minimum spanning tree of a given labelled graph. The function must also return the minimum weight. Hint: Use the algorithm of Prim. A small modification of the solution of P83 does the trick. The data of the example graph to the right can be found in the file p84.dat.



P85 (**) Graph isomorphism

Two graphs (n1 e1) and (n2 e2) are isomorphic if there is a bijection f: n1 -> n2 such that for any nodes x,y of n1, x and y are adjacent if and only if f(x) and f(y) are adjacent.

Write a function that determines whether two graphs are isomorphic. Hint: Use an open-ended list to represent the function f.

P86 (**) Node degree and graph coloration

a) Write a function (degree graph node) that determines the degree of a given node.

b) Write a function that generates a list of all nodes of a graph sorted according to decreasing degree.

c) Use Welch-Powell's algorithm to paint the nodes of a graph in such a way that adjacent nodes have different colors.

P87 (**) Depth-first order graph traversal (alternative solution)

Write a function that generates a depth-first order graph traversal sequence. The starting point should be specified, and the output should be a list of nodes that are reachable from this starting point (in depth-first order).

P88 (**) Connected components (alternative solution)

Write a function that splits a graph into its connected components.

P89 (**) Bipartite graphs

Write a function that finds out whether a given graph is bipartite.

Miscellaneous Problems

P90 (**) Eight queens problem

This is a classical problem in computer science. The objective is to place eight queens on a chessboard so that no two queens are attacking each other; i.e., no two queens are in the same row, the same column, or on the same diagonal.

Hint: Represent the positions of the queens as a list of numbers 1..N. Example: (4 2 7 3 6 8 5 1) means that the queen in the first column is in row 4, the queen in the second column is in row 2, etc. Use the generate-and-test paradigm.

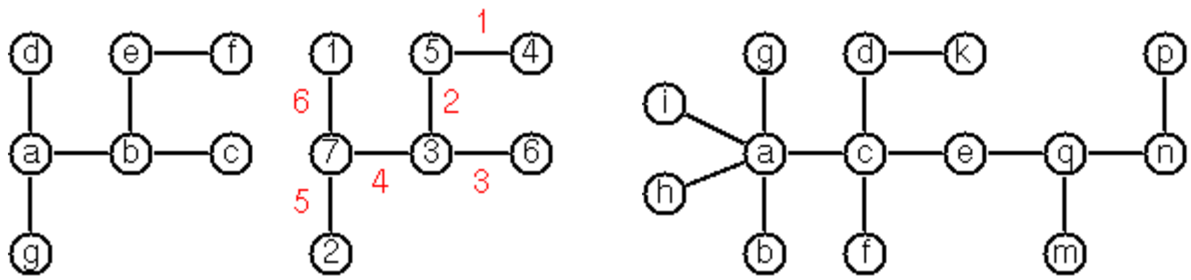
P91 (**) Knight's tour

Another famous problem is this one: How can a knight jump on an NxN chessboard in such a way that it visits every square exactly once?

Hints: Represent the squares by pairs of their coordinates of the form (X Y), where both X and Y are integers between 1 and N. Define a function (jump N (X Y)) that returns a list of the positions (U V) such that a knight can jump from (X Y) to (U V) on a NxN chessboard. And finally, represent the solution of our problem as a list of N*N knight positions (the knight's tour).

P92 (***) Von Koch's conjecture

Several years ago I met a mathematician who was intrigued by a problem for which he didn't know a solution. His name was Von Koch, and I don't know whether the problem has been solved since.



Anyway the puzzle goes like this: given a tree with N nodes (and hence N-1 edges), find a way to enumerate the nodes from 1 to N and, accordingly, the edges from 1 to N-1 in such a way that, for each edge K, the difference of its node numbers equals K. The conjecture is that this is always possible.

For small trees the problem is easy to solve by hand. However, for larger trees, and 14 is already very large, it is extremely difficult to find a solution. And remember, we don't know for sure whether there is always a solution!

Write a function that calculates a numbering scheme for a given tree. What is the solution for the larger tree pictured above?

P93 (***) An arithmetic puzzle

Given a list of integer numbers, find a correct way of inserting arithmetic signs (operators) such that the result is a correct equation. Example: With the list of numbers (2 3 5 7 11) we can form the equations 2-3+5+7 = 11 or 2 = (3*5+7)/11 (and ten others!).

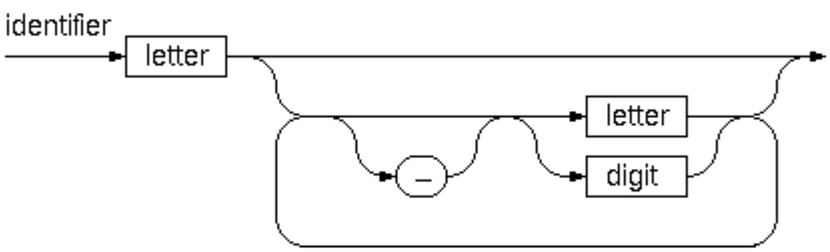
P94 (***) Generate K-regular simple graphs with N nodes

In a K-regular graph all nodes have degree K; i.e. the number of edges incident to each node is K. How many (non-isomorphic!) 3-regular graphs with 6 nodes are there? See also a table of results.

P95 (**) English number words

On financial documents, like cheques, numbers must sometimes be written in full words. Example: 175 must be written as one-seven-five. Write a function (full-words n) to print (non-negative) integer numbers in full words.

P96 (**) Syntax checker (alternative solution with difference lists)



In a certain programming language (Ada) identifiers are defined by the syntax diagram (railroad chart) opposite. Transform the syntax diagram into a system of syntax diagrams which do not contain loops; i.e. which are purely recursive. Using these modified diagrams, write a function (identifier str) that can check whether or not a given string s is a legal identifier.

* (identifier str) returns t when str is a legal identifier.

P97 (**) Sudoku

Sudoku puzzles go like this:

```
   Problem statement                 Solution

    .  .  4 | 8  .  . | .  1  7      9  3  4 | 8  2  5 | 6  1  7
            |         |                      |         |
    6  7  . | 9  .  . | .  .  .      6  7  2 | 9  1  4 | 8  5  3
            |         |                      |         |
    5  .  8 | .  3  . | .  .  4      5  1  8 | 6  3  7 | 9  2  4
    --------+---------+--------      --------+---------+--------
    3  .  . | 7  4  . | 1  .  .      3  2  5 | 7  4  8 | 1  6  9
            |         |                      |         |
    .  6  9 | .  .  . | 7  8  .      4  6  9 | 1  5  3 | 7  8  2
            |         |                      |         |
    .  .  1 | .  6  9 | .  .  5      7  8  1 | 2  6  9 | 4  3  5
    --------+---------+--------      --------+---------+--------
    1  .  . | .  8  . | 3  .  6      1  9  7 | 5  8  2 | 3  4  6
            |         |                      |         |
    .  .  . | .  .  6 | .  9  1      8  5  3 | 4  7  6 | 2  9  1
            |         |                      |         |
    2  4  . | .  .  1 | 5  .  .      2  4  6 | 3  9  1 | 5  7  8
```

Every spot in the puzzle belongs to a (horizontal) row and a (vertical) column, as well as to one single 3x3 square (which we call "square" for short). At the beginning, some of the spots carry a single-digit number between 1 and 9. The problem is to fill the missing spots with digits in such a way that every number between 1 and 9 appears exactly once in each row, in each column, and in each square.

P98 (***) Nonograms

Around 1994, a certain kind of puzzles was very popular in England. The "Sunday Telegraph" newspaper wrote: "Nonograms are puzzles from Japan and are currently published each week only in The Sunday Telegraph. Simply use your logic and skill to complete the grid and reveal a picture or diagram." As a Lisp programmer, you are in a better situation: you can have your computer do the work! Just write a little program ;-).

The puzzle goes like this: Essentially, each row and column of a rectangular bitmap is annotated with the respective lengths of its distinct strings of occupied cells. The person who solves the puzzle must complete the bitmap given only these lengths.

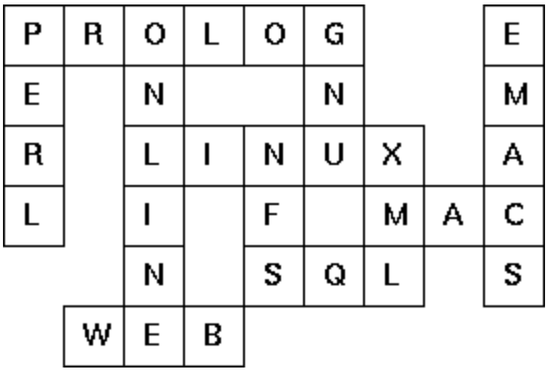
```
          Problem statement:          Solution:

          |_|_|_|_|_|_|_|_| 3         |_|X|X|X|_|_|_|_| 3
          |_|_|_|_|_|_|_|_| 2 1       |X|X|_|X|_|_|_|_| 2 1
          |_|_|_|_|_|_|_|_| 3 2       |_|X|X|X|_|_|X|X| 3 2
          |_|_|_|_|_|_|_|_| 2 2       |_|_|X|X|_|_|X|X| 2 2
          |_|_|_|_|_|_|_|_| 6         |_|_|X|X|X|X|X|X| 6
          |_|_|_|_|_|_|_|_| 1 5       |X|_|X|X|X|X|X|_| 1 5
          |_|_|_|_|_|_|_|_| 6         |X|X|X|X|X|X|_|_| 6
          |_|_|_|_|_|_|_|_| 1         |_|_|_|_|X|_|_|_| 1
          |_|_|_|_|_|_|_|_| 2         |_|_|_|X|X|_|_|_| 2
           1 3 1 7 5 3 4 3             1 3 1 7 5 3 4 3
           2 1 5 1                     2 1 5 1
```

For the example above, the problem can be stated as the two lists ((3) (2 1) (3 2) (2 2) (6) (1 5) (6) (1) (2)) and ((1 2) (3 1) (1 5) (7 1) (5) (3) (4) (3)) which give the "solid" lengths of the rows and columns, top-to-bottom and left-to-right, respectively. Published puzzles are larger than this example, e.g. 25 x 20, and apparently always have unique solutions.

P99 (***) Crossword puzzle

Given an empty (or almost empty) framework of a crossword puzzle and a set of words. The problem is to place the words into the framework.



The particular crossword puzzle is specified in a text file which first lists the words (one word per line) in an arbitrary order. Then, after an empty line, the crossword framework is defined. In this framework specification, an empty character location is represented by a dot (.). In order to make the solution easier, character locations can also contain predefined character values. The puzzle opposite is defined in the file p99a.dat, other examples are p99b.dat and p99d.dat. There is also an example of a puzzle (p99c.dat) which does not have a solution.

Words are strings (character lists) of at least two characters. A horizontal or vertical sequence of character places in the crossword puzzle framework is called a *site*. Our problem is to find a compatible way of placing words onto sites.

Hints: (1) The problem is not easy. You will need some time to thoroughly understand it. So, don't give up too early! And remember that the objective is a clean solution, not just a quick-and-dirty hack!
(2) Reading the data file is a tricky problem for which a solution is provided in the file p99-readfile.lisp. Use the function read-lines, which returns the words and the grid in a 2-element list.
(3) For efficiency reasons it is important, at least for larger puzzles, to sort the words and the sites in a particular order. For this part of the problem, the solution of P28 may be very helpful.

Last modified: Thu Sep 19 07:02:13 BRT 2013