Reconstruction using First order hold



By

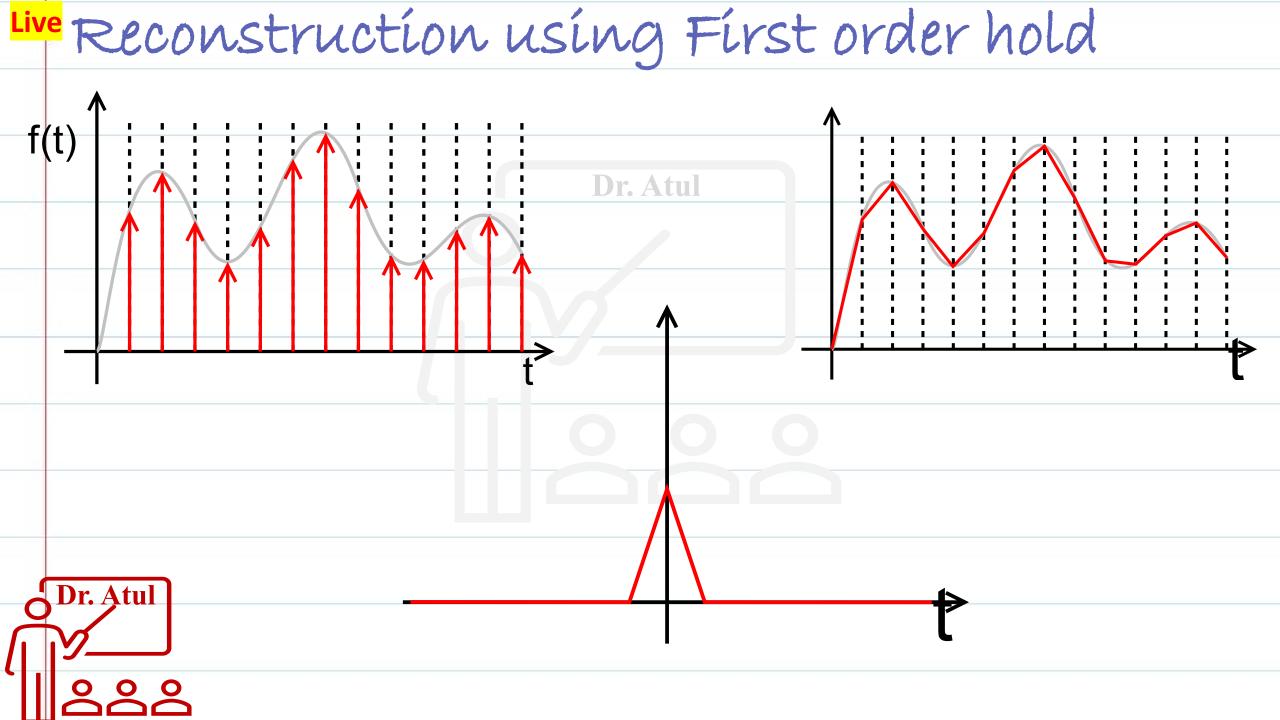
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Interpolation using First order hold



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First-order hold is the hypothetical filter or LTI system that converts the ideally sampled signal

$$x_s(t) = x(t) T \sum_{n=-\infty}^{\infty} \delta(t - nT)$$
 $= T \sum_{n=-\infty}^{\infty} x(nT) \delta(t - nT)$

to the piecewise linear signal

$$x_{ ext{FOH}}(t) = \sum_{n=-\infty}^{\infty} x(nT) ext{tri}\left(rac{t-nT}{T}
ight)$$

resulting in an effective impulse response of

$$h_{ ext{FOH}}(t) = rac{1}{T} ext{tri} \left(rac{t}{T}
ight) = egin{cases} rac{1}{T} \left(1 - rac{|t|}{T}
ight) & ext{if } |t| < T \ 0 & ext{otherwise} \end{cases}$$

where $\operatorname{tri}(x)$ is the triangular function.



Live The effective frequency response is the continuous Fourier transform of the impulse response.

$$egin{aligned} H_{ ext{FOH}}(f) &= \mathcal{F}\{h_{ ext{FOH}}(t)\} \ &= \left(rac{e^{i\pi fT} - e^{-i\pi fT}}{i2\pi fT}
ight)^2 & ext{Dr. Atul} \ &= ext{sinc}^2(fT) \end{aligned}$$

where
$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$
 is the normalized sinc function.

The Laplace transform transfer function of FOH is found by substituting $s = i \ 2 \ \pi f$.

$$H_{ ext{FOH}}(s) = \mathcal{L}\{h_{ ext{FOH}}(t)\}$$

$$=\left(rac{e^{sT/2}-e^{-sT/2}}{sT}
ight)^2$$



