Discrete Time Fourier- Transform

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By

Dr. Atul Kumar Dwivedi

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Descrete time fourier transform (DTFT)

CTFT

If $x(t)$ is an aperiodic signal then

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

Inverse fourier transfor.

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

DTFT.

If $x(n)$ is a descrete sequence then

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

Inverse fourier transform.

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

Existance of DTFT

The descrete time fourier transform does not exist for every aperiodic sequence. A sufficient condition for the existance of DTFT for an aperiodic sequence $x(n)$ is

$$\sum_{n=-\infty}^{\infty} |x(n)| < \infty$$

It means that the sequence must be absolutely summable.

The necessary condition for existance of DTFT is the sequence should be bounded sequence.

$$\min[x(n)] \leq \infty$$

$$\max[x(n)] < \infty.$$

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Que:- find the descrete time fourier transform of the following

(1) $x(n) = \{1, 2, 3, 4\}$ Ans $1 + 2e^{-j\omega} + 3e^{-2j\omega} + 4e^{-3j\omega}$

(2) $x_2(n) = \{1, \underset{\uparrow}{2}, 3, 4\}$ Ans $e^{j\omega} + 2 + 3e^{-j\omega} + 4e^{-2j\omega}$

(3) $x_3(n) = (0.5)^n u(n)$ Ans $\dfrac{1}{1 - 0.5e^{-j\omega}}$

(4) $x_4(n) = (2)^n u(n)$ NA because not absolutely summable

(5) $x_5(n) = \delta(n)$ Ans $= 1$

(6) $x_6(n) = u(n)$ Ans $\dfrac{1}{1 - e^{-j\omega}}$

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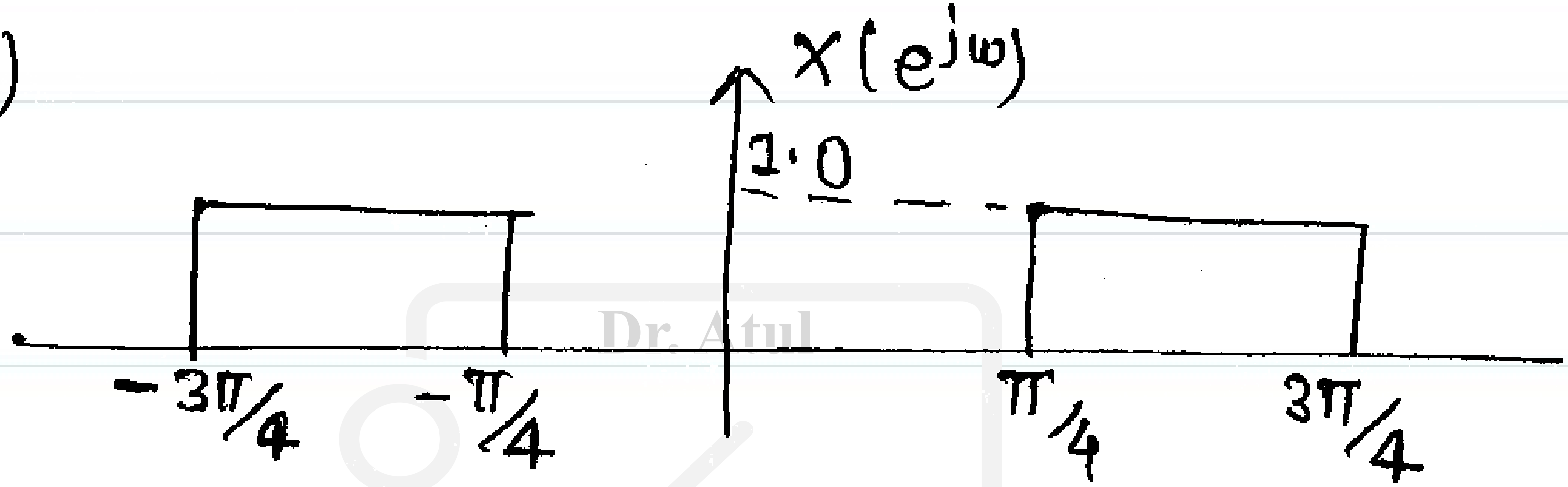
Que: find the inverse fourier transform of

(i) $X(e^{j\omega}) = 1 + 2e^{-j\omega} + 2e^{-j2\omega} + 3e^{-j3\omega}$

(ii) $X(e^{j\omega}) = e^{-j\omega}\left[0.5 + 0.5\cos\omega\right]$

(iii) $X(e^{j\omega}) = \begin{cases} 1 & \frac{\pi}{4} \leq |\omega| \leq \frac{3\pi}{4} \\ 0 & \text{otherwise} \end{cases}$

Sol (iii)



$$x(n) = \frac{1}{2\pi}\left[\int_{-3\pi/4}^{-\pi/4} e^{j\omega n}\, d\omega + \int_{\pi/4}^{\frac{3\pi}{4}} e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-3\pi/4}^{-\pi/4} + \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{\pi/4}^{3\pi/4}$$

$$x(n) = \frac{1}{\pi n}\left[\sin\frac{3\pi}{4}n - \sin\frac{\pi}{4}n\right]$$

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Thank you for your attention!

Dr. Atul