

## Discrete convolution

If  $x_1(n)$  and  $x_2(n)$  are two ~~sig~~ sequences then the convolution is given by

$$y(n) = x_1(n) * x_2(n)$$

$$= \sum x_1(k) \cdot x_2(n-k) \quad \forall k, \text{ at constant } n$$

To understand the meaning of this equation let us break it down step by step

Step-1 change variable of the sequences from  $n$  to  $k$ . therefore sequences are now  $x_1(k)$  and  $x_2(k)$

Step 2: flip one of the sequence. ~~to be~~ for example of convolution of  $x_1(k)$  and  $x_2(k)$  then create  $x_2(-k)$ .

Step 3: Shift the flipped sequence to the leftmost position where the two sequences start overlapping by selecting proper value of  $n$  for  $x_2(n-k)$

Step 4 Multiply the sequence  $x_1(k)$  and  $x_2(n-k)$  at each value of  $k$  this will create a new sequence  
$$z(m) = x_1(k) \cdot x_2(n-k)$$

Step 5 Then calculate  
$$y(n) = \sum z(m)$$

Repeat step (3), (4) and (5) for all values of  $n$  where there is overlapping in signals.

Example 1  $x_1(n) = \{1, 2, 3\}$   $x_2(n) = \{1, -1, 1\}$

Calculate  $y(n) = x_1(n) * x_2(n)$

Step 1  $x_1(k) = \{1, 2, 3\}$   $x_2(k) = \{1, -1, 1\}$

Step 2  $x_1(k) = \{1, 2, 3\}$   $x_2(-k) = \{1, -1, \underset{\uparrow}{1}\}$

Step 3 if  $n=0$

$$y(0) = \sum x_1(k) x_2(-k) = \sum \{0 + 0 + 1 + 0 + 0\}$$

$$= 1$$

Step 4 if  $n=1$   $x_1(k) = \{1, 2, 3\}$ ,  $x_2(1-k) = \{1, \underset{\uparrow}{-1}, 1\}$

$$y(1) = \sum x_1(k) x_2(1-k) = \sum \{0, -1, 2, 0\}$$

$$= 1$$

Step 5 if  $n=2$

$$x_1(k) = \{1, 2, 3\}$$
  $x_2(2-k) = \{1, \underset{\uparrow}{-1}, 1\}$

$$y(2) = \sum x_1(k) x_2(2-k) = \sum \{1, -2, 3\}$$

$$= 2$$

Step 6  $n=3$

$$x_1(k) = \{1, 2, 3\} \quad x_2(3-k) = \{0, 1, -1, 1\}$$

$$y(3) = \sum x_1(k) x_2(3-k) = \sum \{0, 2, -3, 0\} \\ = -1$$

Step 7  $n=4$

$$x_1(k) = \{1, 2, 3\} \quad x_2(4-k) = \{0, 0, 1, -1, 1\}$$

$$y(4) = \sum x_1(k) x_2(4-k) = \{0, 0, 3, 0, 0\} \\ = 3$$

Step 8  $n=5$

$$x_1(k) = \{1, 2, 3\} \quad x_2(5-k) = \{0, 0, 0, 1, -1, 1\}$$

$$y(5) = \sum x_1(k) x_2(5-k) = \{0, 0, 0, 0, 0, 0\} \\ = 0$$

Therefore

$$y(n) = x_1(n) * x_2(n)$$

$$y(n) = \{1, 1, 2, -1, 3\}$$



### Practice problem

① Find convolution of  $x_1(n) = \{1, 2, 3\}$  and  $x_2(n) = \{1, 2, 1\}$

Ans:  $y(n) = \{1, 4, 8, 8, 3\}$

② Find convolution of  $x_1(n) = \{1, 1, 2\}$   $x_2(n) = \{1, 2, 1\}$

Ans  $y(n) = \{1, 3, 5, 5, 2\}$

③ Find convolution of  $x_1(n) = \{1, 1, 2\}$  ,  $x_2(n) = \{1, 1, 1\}$

Ans  $y(n) = \{1, 2, 4, 3, 2\}$