Reconstruction using zero order hold

By

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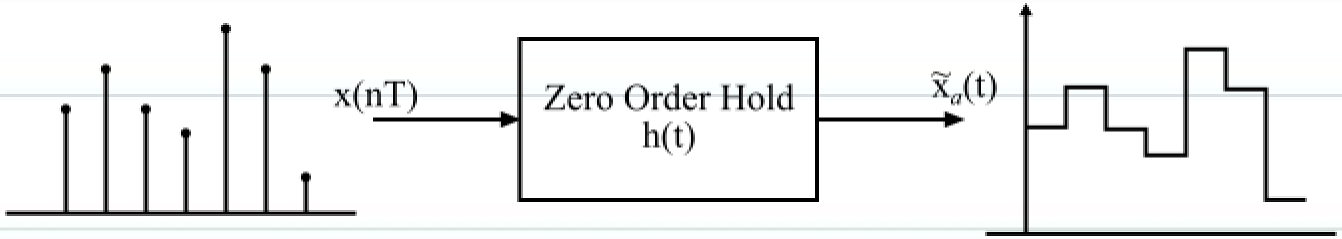
The zero-order hold is a method which is widely used to reconstruct the signals in real time.

In the zero-order hold reconstruction method, the continuous signal is reconstructed from its samples by holding the given sample for an interval until the next sample is received.

Therefore, the zero-order hold generates the step approximations. The process of reconstruction by zero-order hold is shown in the figure.



The output of a reconstruction after zero order hold can be given by

$$x_a(t) = x(n) \quad \text{for} \quad nT \leq n \leq (n+1)T$$

therefore

$$x_a(t) = x(0) \; ; \quad \text{for} \quad 0 \leq t \leq T$$

$$x_a(t) = x(T) \; ; \quad \text{for} \quad T < t < 2T$$

$$x_a(t) = x(2T) \; ; \quad \text{for} \quad 2T < t < 3T \quad \text{and so on}$$

also, the impulse response of zero order is given by

$$h(t) = \begin{cases} 1 & \text{for} \quad 0 \leq t \leq T \\ 0 & \text{otherwise} \end{cases}$$

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Derivation for transfer function

The output of zero order hold

$$x_a(t) = \text{input } x(nT) * \text{impulse response } h(t)$$

$$x_a(t) = x(nT) * h(t)$$

$$x_a(t) = \sum_{n=-\infty}^{\infty} x(nT)\, h(t-nT)$$

Since the impulse response of the zero order hold is given by

$$h(t) = u(t) - u(t-T)$$

$$h(t-nT) = u(t-nT) - u[t-(n+1)T]$$

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$$\therefore x_a(t) = \sum_{n=-\infty}^{\infty} x(nT)\left\{u(t-nT) - u[t-(n+1)T]\right\}$$

By taking Laplace transform of both sides, we get

$$L[x_a(t)] = L\left[\sum_{n=-\infty}^{\infty} x(nT)\left\{u(t-nT) - u[t-(n+1)T]\right\}\right]$$

$$X_a(s) = \sum_{n=-\infty}^{\infty} x(nT)\left[\frac{e^{-nTs}}{s} - \frac{e^{-(n+1)Ts}}{s}\right]$$

$$= \sum_{n=-\infty}^{\infty} x(nT)\left(\frac{1-e^{-Ts}}{s}\right)$$

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$$X_a(s) = \left(\frac{1-e^{-Ts}}{s}\right) X^*(s)$$

$$\boxed{TF = \frac{X_a(s)}{X^*(s)} = \frac{1-e^{-Ts}}{s}}$$

The output of the zero order hold consists of higher order harmonics because it consists of steps. These harmonics can be removed by applying the output of ZOH to a low pass filter. This LPF tends to smooth the corners on the step approximations. This LPF is also known as smoothing filter.

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Thank you for your attention!

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