Signals & Systems

ASSIGNMENT/TUTORIAL-3

Solve any five problems of your choice.

Q.1 (a) Find the z-transform of following signals:

(i) $x(n) = (n-3)u[n-3]$

(ii) $x(n) = (n-3)u[n]$

(iii) $x(n) = u(n) - u[n-3]$

(iv) $x(n) = n\{u[n] - x[n-3]\}$

(b) Verify parseval's theorem for the discrete time Fourier transform that is

$$\sum_{n=-x}^{x} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} |X(\Omega)|^2 \, d\Omega$$

(c) Show that if x(n) is right side sequence and x(z) Converge for some value of z then roc of x(z)

Is in the form $|z| > r_{max}$

Q.2 (a) What is the relationship between z-transform and discrete time Fourier transform?

(b) State and prove following properties of z-transform:

(i) Time shifting

(ii) Differentiation

(c) Determine the z-transform of the following signal:

$$x[n] = \frac{1}{2}(n^2+n)\left(\frac{1}{3}\right)^{n-1} u[n-1]$$

(d) Compute discrete Fourier transform of:

(i) $x[n] = 1 \quad 0 \le n \le N/2$

Q.3 (a)Explain the important properties of convolution integral

(b) For an LTI system with unit impulse response
$h(t) = e^{-2t}\, u(t)$ determine the output to the input
$x(t) = e^{-t}u(t)$

(c) Obtain the convolution of the two continuous time function given below

$$x(t) = e^{-t^2}$$
$$and\ h(t) = 3t^2\ for\ all\ values\ of\ t$$

(d) By using continuous time convolution find out the response of the system to unit step input signal. Impulse response is given as:

Q.4 (a) Given that

$$X(z) = \frac{z(z-4)}{(z-1)(z-2)(z-3)}$$

State all possible ROC (region of convergence) for which ROC the X(z) is stable?

(b) Obtain the convolution using time domain approach

$$of\ X_1[n]\ with\ X_2[n]$$
$$X_1[n] = \{1\ 2\ 3\ 4\}$$
$$X_2[n] = \{1\ 1\ 1\ 1\}$$

(c) Find the IDFT from x[k]

$$x[k] = \{6, -2+j2, -2, -2-j2\}$$

Q.5 (a) What are the methods by which inverse z-transformation can be found out?

(b) Find the z-transform & ROC of the following sequence

(i) $a^n u(-n-1)$ (ii) $\sin \omega_0 n u(n)$ (iii) $x(n) = nu(n)$

(c) Obtain the circular convolution of the following two sequences:

$$x_1(u) = \{1, 2, 3, 4\};\ x_2(u) = \{2, 3, 4, 5\}, \qquad x_1(u)\ \&\ x_2(u)$$

Each being periodic with period 4 (i.e.) N = 4

Q.6 (a) Define ROC of the z-transform

(b)Determine the sequence x[n] associated with z-transform given below using partial fraction expansion (PEE) method.

$$x(z) = \frac{z^2 + z}{(z-1)^2}$$

Right sided sequence

(c) Perform the circular convolution of the two sequences given below and determine the relation between circular convolution and linear convolution and linear convolution

$$X_1(n) = \{1, 3, 5, 3\}$$
$$X_2(n) = \{2, 3, 1,1\}$$

Q.7 (a) Explain properties of Z-transform in details

(b) Find inverse Z-transform

(i) $X(z) = -4 + 8z^{-1}/(1 + 6z^{-1} + 8z^{-2})$

(ii) $X(z) = z^3/(z+1)(z-1)^2$

(c) Find the Z-transform and the region of convergence (ROC) of the discrete-time signal

$$x(n) \begin{cases} a^n & for \geq 0 \\ 0\ for\ n < 0 \end{cases}$$

(d) The impulse response for a discrete time system is given as h(n) = {1, 2, 3} and output respone is given as $y(n) = \{1, 1, 2, -1, 3\}$. Determine the discrete-time input signal x(n) using the log division method.

Q.8 (a) Find the z-transform of following signal and sketch its ROC:

$$X(n) = \begin{cases} 2^n & ; \ n < 0 \\ \left(\frac{1}{2}\right)^n & ; \ n = 0, 2, 4 \\ \left(\frac{1}{3}\right)^n & ; \ n = 1, 3, 5 \end{cases}$$

(b) (i) Determine the 8-point DFT of the sequence

$$X(n) = \begin{cases} A, & |n| \leq N \\ 0, & |n| > N \end{cases}$$

(c) Find linear convolution and circular convolution of the two sequences:

$$x_1(n) = \delta(n) + \delta(n-1) + \delta(n-2)$$
$$x_2(n) = 2\delta(n) - \delta(n-1) + 2\delta(n-2)$$

Q.9 (a) Define circular convolution.

(b) Explain any three properties of Z-transform along with their proofs

(c) Find Z-transform

(i) $x(n) = 2^n u(n-2)$ (ii) $x(n) = n^2 u(n)$

(d) Find inverse Z-transform

(i) $x(z) = \frac{1+2z^{-1}}{1-2z^{-1}+4z^{-2}}$ (ii) $x(z) = \frac{1}{(1+z^{-1})(1-z^{-1})^2}$