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ASSIGNMENT-1
Signals and Systems

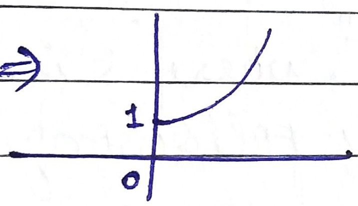
Q.1:- The signal $e^{t}u(t)$ belongs to which of the following classes of signals?

i) Analog Signals ✓
ii) Energy Signals
iii) Power Signals
iv) Deterministic Signals ✓

Sol.
$$E = \lim_{t\to\infty}\int_{-t}^{t} |e^{t}u(t)|^2\,dt = \lim_{t\to\infty}\int_{0}^{t} e^{2t}\,dt = \lim_{t\to\infty}\left[\frac{e^{2t}}{2}\right]_0^{t}$$

$$\cancel{E = \lim_{t\to\infty}\left[\frac{e^{t}-1}{2}\right]} \quad E = \lim_{t\to\infty}\left[\frac{e^{2t}}{2} - \frac{1}{2}\right] \Rightarrow \boxed{E = \infty}$$

$e^{t}u(t) \Rightarrow$



$$P = \lim_{t\to\infty}\frac{1}{2t}\int_{-t}^{t} e^{2t}\,dt = \lim_{t\to\infty}\frac{1}{2t}\int_{0}^{t} e^{2t}\,dt$$

$$P = \lim_{t\to\infty}\frac{1}{2t}\left[\frac{e^{2t}}{2} - \frac{1}{2}\right]_0^{t}$$

$$\boxed{P = 0.\infty = 0}$$

So, $e^{t}u(t)$ can't be energy or power signal. Therefore, This signal is Analog and deterministic signals.

Q.2. The value of $\int_{-\infty}^{\infty} \sin(t)\delta'(t)\,dt$ is

Sol. Using the property, $\boxed{\int_{-\infty}^{\infty} x(t)\delta'(t)\,dt = -x'(t)\Big|_{t=0}}$

$$\text{So, } \int_{-\infty}^{\infty} \sin(t)\delta'(t)\,dt = -\frac{d}{dt}\sin(t)\Big|_{t=0}$$

$$= -\cos t\Big|_{t=0}$$

$$= -\cos 0$$

$$= \boxed{-1} \text{ Ans}$$

Q.3. The signal $x(n) = e^{j\frac{2\pi kn}{N}}$, where k is an integer belongs to which of the following classes of signals?

i) Discrete time signals

ii) Power signals

iii) Energy signals

iv) Periodic signals

Sol.

$$E = \lim_{N\to\infty} \int_{-N}^{N} \left(e^{\frac{j2\pi kn}{N}}\right)^2 dt$$

$$= \lim_{N\to\infty} \int_{0}^{N} e^{\frac{4j\pi kn}{N}}\,dt$$

$$= \lim_{N\to\infty} \left[\frac{e^{\frac{4j\pi kn}{N}}\,N}{4j\pi k}\right]_0^N$$

$$= \lim_{N \to \infty} \frac{N}{4j\pi k}\left[e^{\frac{4j\pi k n}{N}} - 1\right]$$

$E = \infty$ Not a energy signal as E is ∞.

$$P = \lim_{N \to \infty} \frac{1}{2N} \int_{-N}^{N} \left(e^{\frac{j2\pi k n}{N}}\right)^2 dt$$

$$P = \lim_{N \to \infty} \frac{1}{2\cancel{N}} \times \frac{\cancel{N}}{4j\pi k}\left[e^{4j\pi k} - 1\right]$$

$$P = \frac{e^{4j\pi k} - 1}{8j\pi k}$$ It is a power signal as P is finite

$x(n)$ is a complex exponential signal, so complex signals are always periodic signal.

<u>Ans</u> i, ii, iv

<u>Q.4=</u> An LTI system has to satisfy:-

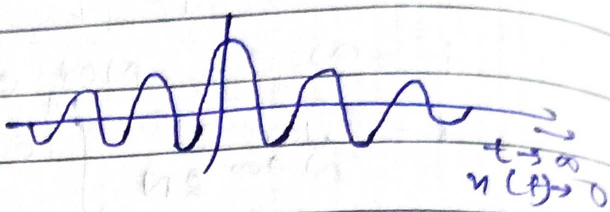
<u>Ans:-</u> (iv) Additivity, homogeneity and time invariance properties.

<u>Q.5=</u> The eigenfunction of LTI system is of the form.

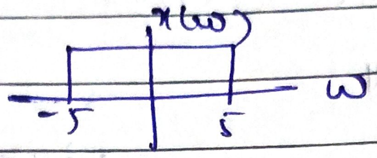
<u>Ans</u> The eigenfunction of ~~a~~ LTI system is of the form e^{at} because a signal for which the output is equal to the constant time input signal.

Q.6. The signal $x(t) = \frac{\sin 5t}{\pi t}$ is

Sol. $x_1(t) = \text{sinc}(t) = \frac{\sin \pi t}{\pi t}$



$\frac{\sin 5(t)}{\pi t} \rightarrow$



By using, Parsvel's theorem,

$$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} |x(\omega)|^2 d\omega$$

$$E = \frac{1}{2\pi}\int_{-5}^{5} 1\, d\omega = \frac{1}{2\pi}\left[\omega\right]_{-5}^{5}$$

$$E = \frac{1}{2\pi} \times 10 \qquad \boxed{E = \frac{5}{\pi}}$$

So, It is an energy signal with energy $\frac{5}{\pi}$.

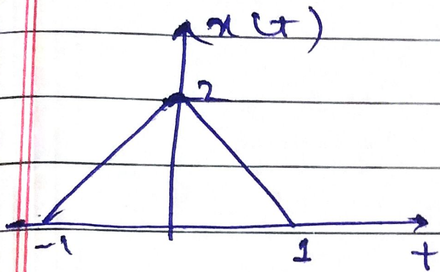
Q.7. The odd component of the complex exponential signal $e^{j\omega_0 t}$ is

Sol. The odd component of the complex exponential signal $e^{j\omega_0 t}$ is $\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2} = j\sin \omega_0 t$

Ans.: $j\sin(\omega_0 t)$

Q.8:- Consider the signal $x(t) = 1-2|t|$ for $|t| \le 1$ and 0 otherwise. Let $y(t) = x(4-t)$, which of the following statements is true?

Sol: $x(t) = 1-2|t|$, $|t| \le 1$ and 0 otherwise.
$x(t)$ is symmetric about 0 hence the signal $y(t) = x(4-t)$ is $x(t) = 1-2|t-4|$ for $|t-4| \le 1$
$3 \le t \le 5$ and 0 otherwise.



It can be seen that the true statement is $\frac{dy(t)}{dt} = 2$ for $3 \le t < 4$

Ans:- $\frac{d}{dt}y(t) = 2$ for $3 < t < 4$

Q.9:- The signal $\cos^2(\frac{\pi}{8}n)$ is

Sol: By using, $\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$

$$\cos^2\left(\frac{\pi}{8}n\right) = \frac{1}{2}\left(1 + \cos\left(\frac{\pi n}{4}\right)\right)$$

Fundamental Period is $\boxed{8}$

Q.10:- The value of $\int_{-\infty}^{\infty} e^{-\beta t}\delta(\tau - \alpha t)dt$ for $\alpha > 0$ is.

Sol: Let $t - \alpha t = u$

$$\int_{-\infty}^{\infty} e^{-\beta t}\delta(\tau-\alpha t)dt = \frac{1}{\alpha}\int_{-\infty}^{\infty} e^{-\beta\left(\frac{\tau-u}{\alpha}\right)}\delta(u)du$$

$$= \frac{1}{\alpha}e^{-\beta\frac{[\tau-u]}{\alpha}}\Big|_{u=0}$$

$$\boxed{\text{Ans} = \frac{1}{\alpha}e^{-\beta/\alpha\, t}}$$