

Inverse Z Transform



By

Dr. Atul Kumar Dwivedi



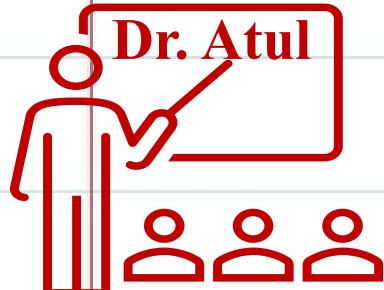
1. Direct Method

$$X(z) = 1 - z^{-1} + z^{-2}$$

Dr. Atul

taking inverse z transform

$$x(n) = \{1, -1, 1\}$$



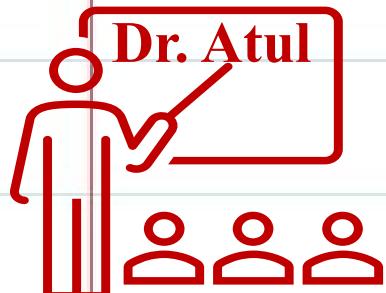
2. Long Division Method

~~Ques~~ using long division method, determine
the inverse Z transform of

$$X(z) = \frac{1+2z^{-1}}{1-2z^{-1}+z^{-2}}$$

(a) $x(n)$ is causal, (b) $x(n)$ is anticausal.

Dr. Atul



2. Long Division Method

SOLⁿ: If $x(n)$ is called α then its Z transform contains negative powers of Z .

Dr. Atul

$$\begin{array}{r}
 1 + 4z^{-1} + 7z^{-2} + 10z^{-3} \\
 \hline
 1 - 2z^{-1} + z^{-2}) \overline{1 + 2z^{-1}} \\
 \underline{-} \quad \underline{+} \\
 \hline
 1 - 2z^{-1} + z^{-2} \\
 \hline
 4z^{-1} - z^{-2} \\
 \underline{-} \quad \underline{+} \\
 \hline
 4z^{-1} - 8z^{-2} + 4z^{-3} \\
 \underline{-} \quad \underline{+} \quad \underline{-} \\
 \hline
 7z^{-2} - 4z^{-3} \\
 \underline{-} \quad \underline{+} \\
 \hline
 7z^{-2} - 14z^{-3} + 7z^{-4} \\
 \underline{-} \quad \underline{+} \quad \underline{-} \\
 \hline
 10z^{-3} - 7z^{-4} \\
 \underline{-} \quad \underline{+} \\
 \hline
 10z^{-3} - 20z^{-4} + 10z^{-5} \\
 \underline{-} \quad \underline{+} \quad \underline{-}
 \end{array}$$

$$X(z) = 1 + 4z^{-1} + 7z^{-2} + 10z^{-3} + \dots$$

$$x(n) = \{1, 4, 7, 10, 13, 16, 19, \dots\}$$

↑



(b) If $x(n)$ is antisymmetric sequence then
 $X(z)$ contains all positive powers of z .

$$\begin{array}{r} 2z + 5z^2 + 8z^3 + 11z^4 + \dots \\ \hline z^2 - 2z^{-1} + 1 \end{array}$$

$$\begin{array}{r} -2z^{-1} - 4 + 2z \\ \hline 5 - 2z \end{array}$$

$$\begin{array}{r} 5 - 10z + 5z^2 \\ \hline - \end{array}$$

$$\begin{array}{r} 8z + 5z^2 \\ \hline - \end{array}$$

$$\begin{array}{r} 8z - 16z^2 + 8z^3 \\ \hline 11z^2 - 8z^2 \\ \hline \cancel{z^2} \dots \end{array}$$



$$X(z) = 2z + 5z^2 + 8z^3 + 11z^4 + \dots$$

$$x(n) = \{ \dots, 14, 11, 8, 5, 2, 0 \}$$

Live

(b) If $x(n)$ is antisymmetric sequence then
 $X(z)$ contains all positive powers of z .

$$\begin{array}{r} 2z + 5z^2 + 8z^3 + 11z^4 + \dots \\ \hline z^2 - 2z^{-1} + 1 \Big) \quad 2z^{-1} + 1 \\ \underline{-} \quad \underline{2z^{-1} + 4} \\ \hline \quad \quad \quad 5 - 2z \\ \underline{-} \quad \underline{5 - 10z + 5z^2} \\ \hline \quad \quad \quad 8z + 5z^2 \\ \underline{-} \quad \underline{8z - 16z^2 + 8z^3} \\ \hline \quad \quad \quad 11z^2 - 8z^2 \\ \hline \quad \quad \quad \cancel{z^2} - \dots \end{array}$$

Dr. Atul

$$x(n) = \{ \dots, *14, 11, 8, 5, 2, 0 \}$$



$$X(z) = 2z + 5z^2 + 8z^3 + 11z^4 + \dots$$

Practice Problem

~~Ques:-~~ using long division method, determine
the Inverse Z transform of

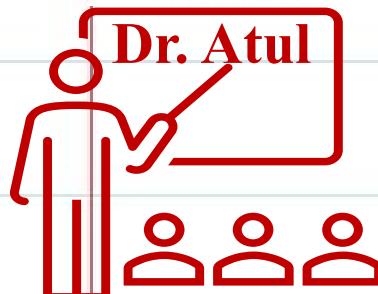
Dr. Atul

$$x(z) = \frac{4z^{-1}}{2 - 2z^{-1} + z^{-2}}$$

if ROC of $x(z)$ is $|z| > 1$

Ans : $x(n) = \{0, 2, 2, 1, 0, -\frac{1}{2}, -\frac{1}{2}, \dots\}$

Dr. Atul



Live

Partial Fraction Method : Useful z Transform Table

$$x(z)$$

$$\frac{z}{z-a}$$

$$\frac{z}{(z-a)^2}$$

$$\frac{z}{(z-a)^3}$$

const

$$x(n)$$

Dr. Atul

$$a^n u(n)$$

$$n a^{n-1} u(n)$$

$$\frac{n(n-1)a^{n-2}}{2!} u(n)$$

Const. $\delta(n)$

for ROC $|z| > a$

Partial Fraction Method : Example 1

Ques:- find the Inverse z transform of

$$x(z) = \frac{\frac{1}{4}z^{-1}}{(z - \frac{1}{2})^2 (z - \frac{1}{4})} \quad \text{ROC: } |z| > \frac{1}{2}$$

sol

$$x(z) = \frac{\frac{1}{4}z}{(z - \frac{1}{2})^2 (z - \frac{1}{4})}$$

$$\frac{x(z)}{z} = \frac{A}{(z - \frac{1}{2})} + \frac{B}{(z - \frac{1}{4})}$$



Partial Fraction Method : Example 1

$$A = \frac{(z-\frac{1}{2})}{(z-\frac{1}{2})(z-\frac{1}{4})} \Big|_{z=\frac{1}{2}} = 1$$

Dr. Atul

$$B = \frac{(z-\frac{1}{4})}{(z-\frac{1}{2})(z-\frac{1}{4})} \Big|_{z=\frac{1}{4}} = 1$$

therefore

$$\frac{x(z)}{z} = \frac{1}{z-\frac{1}{2}} - \frac{1}{z-\frac{1}{4}}$$

$$x(z) = \frac{z}{z-\frac{1}{2}} - \frac{z}{z-\frac{1}{4}} \quad \text{ROC } |z| > \frac{1}{2}$$

$$x(n) = \left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{4}\right)^n u(n)$$



Practice Problem

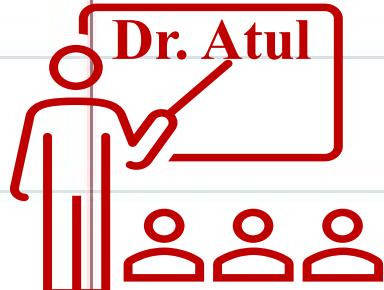
→ : find the inverse z transform of $X(z)$

Dr. Atul

When ROC: $2 < |z| < 3$

$$= \frac{z}{(z-2)(z+3)}$$

Ans $= \frac{1}{5} \left[(-3)^n u(-n-1) - (-2)^n u(n) \right]$



Live

Dr. Atul

Thank you for your
attention!

Dr. Atul