# Inverse Z Transform

By

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# 1. Direct Method

$$X(z) = 1 - z^{-1} + z^{-2}$$

taking inverse z transform

$$x(n) = \{1, -1, 1\}$$

## 2. Long Division Method

**Que** using long division method, determine the inverse z transform of

$$X(z) = \frac{1+2z^{-1}}{1-2z^{-1}+z^{-2}}$$

(a) $x(n)$ is causal. (b) $x(n)$ is anticausal.

# 2. Long Division Method

Sol$^n$: If $x(n)$ is causal then its $z$ transform contains negative powers of $z$.

$$
\begin{array}{r|l}
 & 1+4z^{-1}+7z^{-2}+10z^{-3} \\
\hline
1-2z^{-1}+z^{-2} & 1+2z^{-1} \\
 & \underline{1-2z^{-1}+z^{-2}} \\
 & \quad 4z^{-1}-z^{-2} \\
 & \quad \underline{4z^{-1}-8z^{-2}+4z^{-3}} \\
 & \qquad 7z^{-2}-4z^{-3} \\
 & \qquad \underline{7z^{-2}-14z^{-3}+7z^{-4}} \\
 & \qquad\quad 10z^{-3}-7z^{-4} \\
 & \qquad\quad \underline{10z^{-3}-20z^{-4}+10z^{-5}}
\end{array}
$$

$$X(z) = 1+4z^{-1}+7z^{-2}+10z^{-3}+\ldots\ldots$$

$$x(n) = \{\underset{\uparrow}{1}, 4, 7, 10, 13, 16, 19, \ldots\}$$

(b) If $x(n)$ is anticausal sequence then $X(z)$ contains all positive powers of $z$.

$$
\begin{array}{r|l}
 & 2z + 5z^2 + 8z^3 + 11z^4 + \ldots \\
\hline
z^{-2} - 2z^{-1} + 1 & 2z^{-1} + 1 \\
 & -\,(2z^{-1} - 4 + 2z) \\
 & \quad 5 - 2z \\
 & -\,(5 - 10z + 5z^2) \\
 & \quad 8z - 5z^2 \\
 & -\,(8z - 16z^2 + 8z^3) \\
 & \quad 11z^2 - 8z^2 \\
 & \quad \ldots
\end{array}
$$

$$X(z) = 2z + 5z^2 + 8z^3 + 11z^4 + \ldots\ldots$$

$$x(n) = \{\ldots\ldots 14, 11, 8, 5, 2, \underset{\uparrow}{0}\}$$

(b) If $x(n)$ is anticausal sequence then $X(z)$ contains all positive powers of $z$.



$$
\begin{array}{r|l}
 & 2z + 5z^2 + 8z^3 + 11z^4 + \cdots \\
\hline
z^{-2} - 2z^{-1} + 1 & 2z^{-1} + 1 \\
 & -\left(2z^{-1} - 4 + 2z\right) \\
\hline
 & 5 - 2z \\
 & -\left(5 - 10z + 5z^2\right) \\
\hline
 & 8z - 5z^2 \\
 & -\left(8z - 16z^2 + 8z^3\right) \\
\hline
 & 11z^2 - 8z^2 \\
 & \cancel{11z^2} \quad \cdots\cdots
\end{array}
$$

$$x(n) = \{\ldots\ldots 14, 11, 8, 5, 2, \underset{\uparrow}{0}\}$$

$$X(z) = 2z + 5z^2 + 8z^3 + 11z^4 + \cdots\cdots$$

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# Practice Problem

Que:- using long division method, determine the inverse z transform of $X(z) = \dfrac{4z^{-1}}{2-2z^{-1}+z^{-2}}$

if ROC of $X(z)$ is $|z|>1$

**Ans** : $x(n) = \{0, 2, 2, 1, 0, -\frac{1}{2}, -\frac{1}{2}, \ldots\}$

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# Partial Fraction Method : Useful z Transform Table

| $x(z)$ | $x(n)$ | for ROC $\|z\|>a$ |
|---|---|---|
| $\frac{z}{z-a}$ | $a^n u(n)$ | |
| $\frac{z}{(z-a)^2}$ | $n a^{n-1} u(n)$ | |
| $\frac{z}{(z-a)^3}$ | $\frac{n(n-1)a^{n-2}}{2!} u(n)$ | |
| const | $\text{Const.}\,\delta(n)$ | |


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# Partial Fraction Method : Example 1

Que:- find the Inverse z transform of

$$X(z) = \frac{\frac{1}{4}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)} \qquad \text{ROC}: |z| > \frac{1}{2}$$

Sol$^n$

$$X(z) = \frac{\frac{1}{4}z}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}$$

$$\frac{X(z)}{z} = \frac{A}{\left(z-\frac{1}{2}\right)} + \frac{B}{\left(z-\frac{1}{4}\right)}$$

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# Partial Fraction Method : Example 1

$$A = \cancel{\left(z-\tfrac{1}{2}\right)} \cdot \frac{1/4}{\cancel{\left(z-\frac{1}{2}\right)}\left(z-\frac{1}{4}\right)}\Bigg|_{z=1/2} = 1$$

$$B = \left(z-\tfrac{1}{4}\right) \frac{\frac{1}{4}}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}\Bigg|_{z=\frac{1}{4}} = 1$$

therefore

$$\frac{X(z)}{z} = \frac{1}{z-\frac{1}{2}} - \frac{1}{z-\frac{1}{4}}$$

$$X(z) = \frac{z}{z-\frac{1}{2}} - \frac{z}{z-\frac{1}{4}} \qquad \text{ROC } |z| > \tfrac{1}{2}$$

$$\boxed{x(n) = \left(\tfrac{1}{2}\right)^n u(n) - \left(\tfrac{1}{4}\right)^n u(n)}$$

# Practice Problem

: find the inverse z transform of $X(z) = \dfrac{z}{(z+2)(z+3)}$

when ROC: $2<|z|3$

**Ans** $\dfrac{1}{5}\left[(-3)^n u(-n-1) - (-2)^n u(n)\right]$

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*Thank you for your attention!*

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