Assignment - 3
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Ques 1 (a) find the z transform of following signals

(i) $x(n) = (n-3)\,u(n-3)$

$$u(n-3) = \begin{cases} 1 & n \geq 3 \\ 0 & n < 3 \end{cases}$$

$$f(z) = \sum_{n=-\infty}^{\infty} f(n) z^{-n}$$

$$f(z) = \sum_{n=3}^{\infty} 1\,(n-3) z^{-n}$$

$$f(z) = 0 + z^{-4} + 2z^{-5} + 3z^{-6} + \cdots \infty$$

$$z f(z) = z^{-3} + 2z^{-4} + 3z^{-5} + \cdots \infty$$

$$(z-1) f(z) = z^{-3} + z^{-4} + z^{-5} + \cdots \infty$$

$$f(z) = \frac{z^{-3}}{(1-z^{-1})(z-1)}$$

$$f(z) = \frac{z^{-2}}{(z-1)^2}$$

(ii) $x(n) = (n-3)\,u[n]$

$$= n u(n) - 3u(n)$$

$$X(z) = \frac{z}{(z-1)^2} - \frac{3z}{z-1}$$

$$X(z) = \frac{z}{z-1}\left[\frac{1}{z-1} - 3\right]$$

$$X(z) = \frac{z(4-3z)}{(z-1)^2}$$

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$x(n) = u(n) - u(n-3)$

$$X(z) = \frac{z}{z-1} - \frac{z^{-2}}{z-1} = \frac{z - z^{-2}}{(z-1)}$$

$$F\ n[u(n) - u(n-3)]$$

$$= n u[n] - n u(n-3)$$

$$= n u(n) + (n+3-3)(n-3)$$

$$= n u(n) - (n-3) u(n-3) - 3u(n-$$

$$= \frac{z}{(z-1)^2} - \frac{z^{-2}}{(z-1)^2} - \frac{3z^{-2}}{(z-1)}$$

verify parsevals theorem for DTFT

$$E = \sum_{n=-\infty}^{\infty} [x[n]]^2$$

$$= \sum_{n=-\infty}^{\infty} x(n) \frac{1}{2\pi} \int_{2\pi} x(\Omega) e^{j\omega n} d\Omega$$

$$= \frac{1}{2\pi} \int_{2\pi} X(\Omega) \sum_{n=-\infty}^{\infty} x(n) e^{jn\omega} d\Omega$$

(c) Show that if $x(n)$ is right side sequence and $x(z)$ converge for some value of z then ROC of $x(z)$ is in the form $|z| > r_{max}$

A right handed sequence is a sequence where $x[n] = 0$ for $n < n_1 < \infty$ looking at the positive time portion from derivation for $\sum r_{max}^{n} (|z|)^{-n}$ to converge $z > r_2$ and therefore ROC of R.H. Sequence is $z > r_{max}$.

Que 2(a) What is relationship b/w z transform and DTFT.

(a) If we restrict z-transform to unit circle in the complex plane then we get DTFT. z transform is generalisation of DTFT with similar advantages.

(b) State and prove following properties of z-transform

(i) Time shifting

we know that $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$ so time shifted signal $x(n-n_0)$.

$$X'(z) = \sum_{n=\infty}^{\infty} x(n-n_0) z^{-n}$$

Let $n - n_0 = m \Rightarrow n = n_0 + m$

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$n=-\infty$

$$= z^{-n_0} \sum_{n=-\infty}^{\infty} x(m) z^{-m}$$

$$X'(z) = z^{-n_0} X(z)$$

So, $x(n-n_0) \xrightarrow[\text{z tranform}]{} z^{-n_0} X(z)$

Differentiation : Preove that $n x(n) = -z \frac{d}{dz} X(z)$

we know that $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$$\frac{d X(z)}{dz} = \sum_{n=-\infty}^{\infty} x(n)(-n) z^{n-1}$$

$$= -\sum_{n=-\infty}^{\infty} n x(n) z^{-n} z^{-1}$$

$$\boxed{\sum_{n=-\infty}^{\infty} n x(n) z^{-n} = -z \frac{d}{dz} X(z)}$$

Determine the z-transform of following signal

$$x(n) = \frac{1}{2}(n^2+n)\left(\frac{1}{3}\right)^{n-1} u(n-1)$$

$$= \frac{1}{2}\left[n^2\left(\frac{1}{3}\right)^{n-1} u(n-1) + n\left(\frac{1}{3}\right)^{n-1} u(n-1)\right]$$

$$= \frac{1}{2}\left[z\frac{d}{dz} \frac{z^{-1} z}{z-1/3} + z^2 \frac{d^2}{dz^2} \frac{z^{-1} z}{z-1/3} - z\frac{d}{dz} \frac{z^{-1} z}{z-1/3}\right]$$

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$$= \frac{1}{2}\left[z^2 \frac{d^2}{dz^2}\left(\frac{1}{z-1/3}\right)\right]$$

$$= \frac{1}{2}\left[z^2\, 2\left(z-\frac{1}{3}\right)^{-3}\right]$$

$$= z^2\left(z-\frac{1}{\cancel{2}}\right)^{-3}$$

(d) Compute discrete fourier transform of

(i) $x[n] = 1 \qquad 0 \le n \le N/2$

$$X(\Omega) = \sum_{n=0}^{N/2-1} x(n)\, e^{-j\omega n}$$

$$= \sum_{n=0}^{N/2-1} e^{-j\omega n} = \frac{1-(e^{-j\omega})^{N/2}}{1-e^{-j\omega}}$$

$$= \frac{1-e^{-j2\pi kN/2}}{1-e^{-2j\pi k}}$$

Atte.

$x[n] = Z^{-1}[X(z)]$

There are, however a number of to perform the inverse z-transf the given expression. Some methods given as:

Partial fraction expansion method
Long division method
Residue method.

find out z-transform and ROC of following sequence.

$\alpha^n u(-n-1)$

we have, $u(-n-1) = \begin{cases} 1 ; & n \leq -1 \\ 0 ; & n \geq -1 \end{cases}$

$$F(z) = \sum_{n=-\infty}^{\infty} [a^n u(-n-1) z^{-n}]$$

$$F(z) = \sum_{n=-\infty}^{-1} \alpha^n z^{-n} \cdot 0 + \sum_{n=-1}^{\infty} \alpha^n \cdot z^{-n} \cdot 1$$

$= az^{-1} + a^2z^{-2} + a^3z^{-3} + \dots \infty$

$= \dfrac{az^{-1}}{1 - az^{-1}}$

ROC : $|az^{-1}| < 1$ or $|z| > \left|\dfrac{1}{a}\right|$

$\sin w_0 n\, u(n)$

$$F(z) = \sum_{n=-\infty}^{\infty} f(z) z^{-n}$$

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$F(z) = \sum_{n=-\infty}^{0} 0 \cdot \sin w_0 n z^{-n} + \sum_{n=0}^{\infty} 1 \cdot \sin w_0 n z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{e^{jw_0 n} - e^{-jw_0 n}}{2} \right) z^{-n}$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} (e^{jw_0} z^{-1})^n \mp \sum_{n=0}^{\infty} (e^{-jw_0} z^{-1})^n$$

$$= \frac{1}{2} \left[1 + (e^{jw_0} z^{-1})^1 + (e^{jw_0} z^{-1})^2 + (e^{jw_0} z^{-1})^3 + \dots \infty \right] \mp \frac{1}{2} \left[1 + (e^{-jw_0} z^{-1})^1 + (e^{-jw_0} z^{-1})^2 + (e^{-jw_0} z^{-1})^3 + \dots \infty \right]$$

$$= \frac{1}{2} \left(\frac{1}{1 - e^{jw_0} z^{-1}} \mp \frac{1}{1 - e^{-jw_0} z^{-1}} \right)$$

$$2\left(1-e^{-j\omega_0}z^{-1}-e^{j\omega_0}z^{-1}+z^{-2}\right.$$

$$= \frac{\sin\omega_0 z^{-1}}{1-2\cos\omega_0+z^{-2}}, \quad ROC: |z|>1$$

$x(n) = n u(n)$

$$u(n) = \begin{cases} 1 ; & n \geq 0 \\ 0 ; & n<0 \end{cases}$$

$$f(z) = \sum_{n=-\infty}^{0} 0 \cdot n z^{-n} + \sum_{n=0}^{\infty} 1 \cdot n z^{-n}$$

$$= 0 + z^{-1} + 2z^{-2} + 3z^{-3} + 4z^{-4}$$

$$z^{-1} f(z) = z^{-2} + 2z^{-3} + 3z^{-4} + 4z^{-5} +$$

$$(1-z^{-1}) f(z) = z^{-1} + z^{-2} + z^{-3} + z^{-4} + \cdots$$

$$f(z) = \frac{z^{-1}}{(1-z^{-1})^2}; \quad ROC: |z^{-1}| < 1 \text{ or}$$

Obtain the circular convalution the following two sequence:

$x_1(n) = \{1, 2, 3, 4\}$

$x_2(n) = \{2, 3, 4, 5\}$

Each being periodic with $N=4$

$$x_1(n) \circledast x_2(n) = \sum_{m=0}^{N-1} x_1(m)\, x_2((n-m))_N$$

DFT of $x_1(u) = \{1, 2, 3, 4\}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

DFT of $x_2(u) = \{2, 3, 4, 5\}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 14 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$\begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} \begin{bmatrix} 14 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} = [140 \quad -8j \quad 4 \quad 8j$$

$N = 4$

$$\Rightarrow \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 140 \\ -8j \\ 4 \\ 8j \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ [illegible] \\ 2 \end{bmatrix}$$

$$= [36, 38, 36, 30]$$

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a) Define ROC of z-transform.

the set of values of z in the z-plane for which of $X(z)$ is finite is called regi of converge for right handed ; $|z| > r$ for left handed ; $|z| < r_{min}$

Determine the sequence $x[n]$ associa with z-transform given below using method

$$X(z) = \frac{z^2+z}{(z-1)^2}$$

$$\frac{X(z)}{z} = \frac{z+1}{(z-1)^2} = \frac{A}{(z-1)} + \frac{B}{(z-1)^2}$$

$A = 1$, and $B = 2$

$$\frac{X(z)}{z} = \frac{1}{(z-1)} + \frac{2}{(z-1)^2}$$

$$X(z) = \frac{z}{z-1} + \frac{2z}{(z-1)^2}$$

$$\boxed{X(z) = u(n) + 2n\,u(n)}$$

Perform the circular convolution of two sequence given below and determine the relation b/w circular and linear convol

$x_1(n) = \{1, 3, 5, 3\}$

$x_2(n) = \{2, 3, 1, 1\}$

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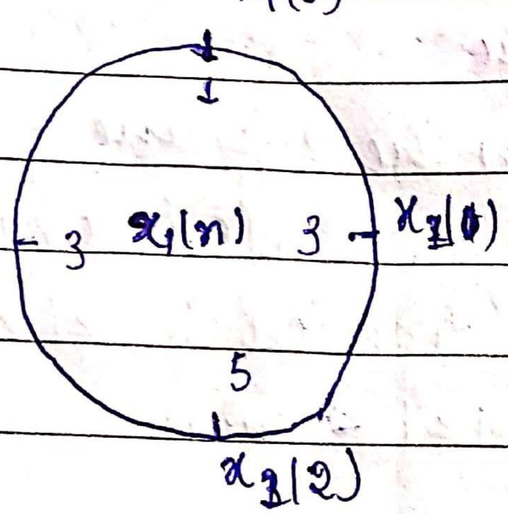
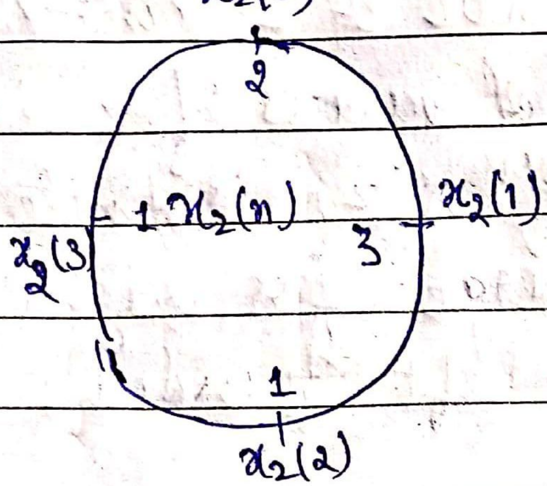
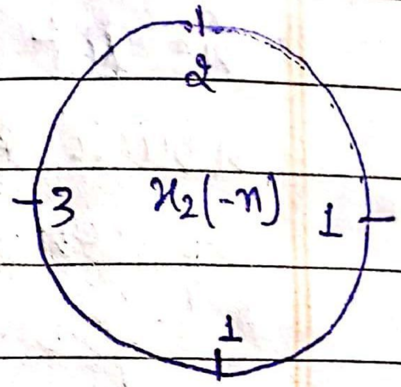




$x_2(-n)$



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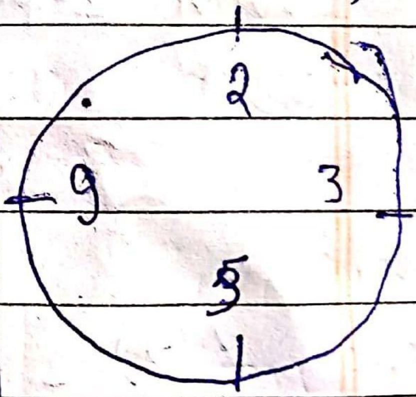
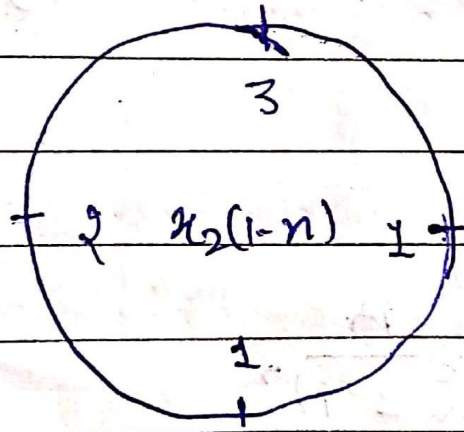
$x_1(n) \cdot x_2(-n)$

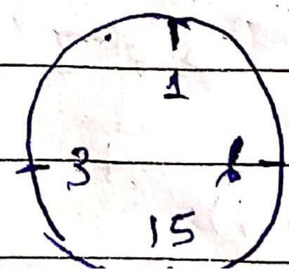


$2+9+5+3=19$

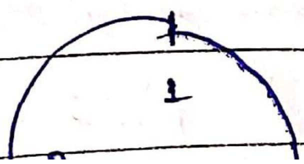
$x_2(1-n)$



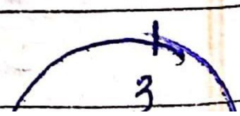
$x_1(n)\, x_2(3-n)$



$x_2(2-n)$

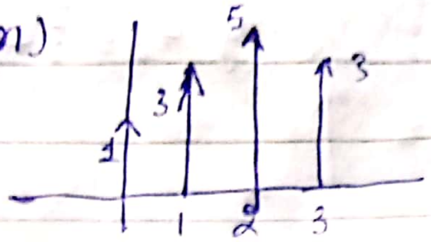


$x_1(n)\, x_2(1-n)$

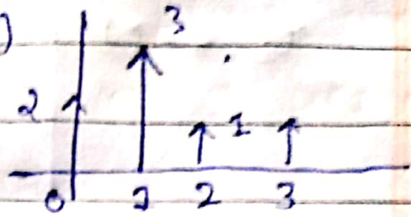


linear convolution

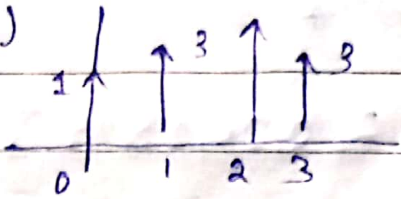
$x_1(n)$



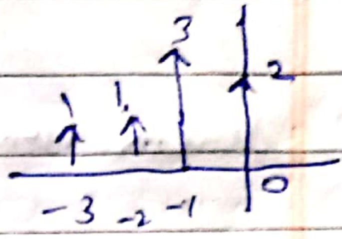
$x_2(n)$



$x_1(\tau)$



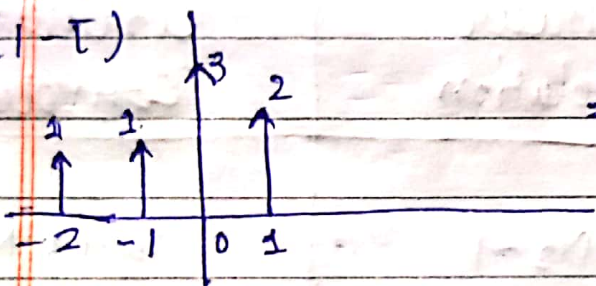
$x_2(-\tau)$



$y(0) = x_1(\tau)\, x_2(-\tau) = 2$

$y(1) = x_1(\tau)\, x_2(1-\tau)$

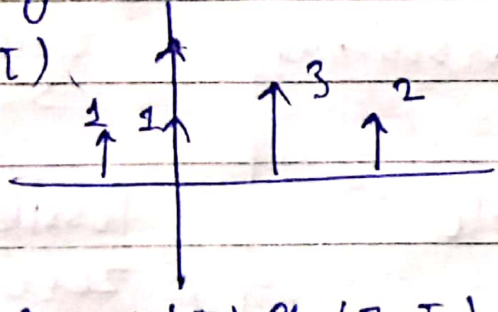
$x_2(1-\tau)$



$\Rightarrow \quad y(1) = 3+6 = 9$

$y(2) = x_1(\tau) \cdot x_2(2-\tau)$

$x_2(2-\tau)$

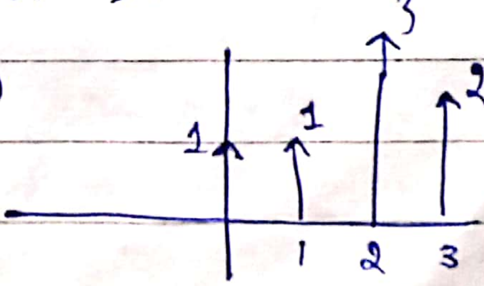


$y(2) = 1+9+10$

$= 20$

$y(3) = x_1(\tau)\, x_2(3-\tau)$

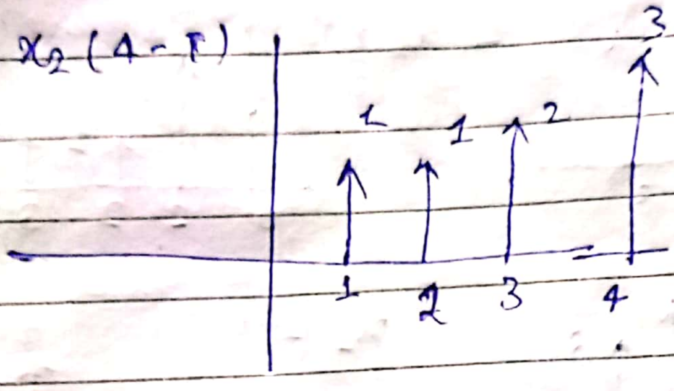
$x_2(3-\tau)$



$y(3) = 1+3+15+6$

$y(4) = x_1(\tau) \cdot x_2(4-\tau)$

$y(4) = 3+5+9 = 17$



linear convolution $= \{2, 9, 20, 25, 17\}$

Difference b/w linear and circular convolution

Camparision points	linear Convolution	Circular Convolution
i) Samples in ~~points~~ convolution result	N_1+N_2-1	$max(N_1,N_2)$
ii) shifting	linear shifting	Circular shifting
iii) finding response of filter	possible	Possible with zero passing

1) Define circular convolution:

A convolution operation that contains a circular shift is called the circular convolution and is given by

$$x_1(n) \circledast x_2(n) = \sum_{m=0}^{N-1} x_1(m)\, x_2((n-m))_N , \quad 0 \le n \le N-1$$

The DFT property for the circular convolution is

$$DFT\left[x_1(n) \, (N) \, x_2(n)\right] = X_1(K) \cdot X_2(K)$$

Explain any three properties of z transform along with their proofs

Initial Value theorem

If the limit exists, then

$$x[0] = \lim_{n \to 0} x[n] = \lim_{z \to \infty} X(z)$$

Proof: for a causal signal $x[n]$

$$X[z] = \sum_{n=0}^{\infty} x[n] z^{-n} = x[0] + \frac{x[1]}{z} + \frac{x[2]}{z^2} + \cdots$$

As $z \to \infty$, $z^{-n} \to 0$ for $n > 0$ where $n = 0$, $z^{-n} = 1$

therefore $\lim_{z \to \infty} X(z) = x[0]$

(ii) Final Value Theorem

If the limit exists, then

$$x[\infty] = \lim_{n\to\infty} x[n] = \lim_{z\to 1} (1-z^{-1}) X(z)$$

Proof: We have,

$$x[n] \xleftrightarrow{z} X(z)$$

$$[x[n] - x[n-1]] \xleftrightarrow{z} (1-z^{-1}) X(z)$$

let $z \to 1$ then

$$\lim_{n\to\infty} x[n] = \lim_{z\to 1} (1-z^{-1}) X(z)$$

(iii) Differentiation Property or Multiplica[tion] by n-

find z-transform :

$$x(n) = 2^n u(n-2)$$

$$u(n-2) = \begin{cases} 1 ; & n \geq 2 \\ 0 ; & n < 2 \end{cases}$$

$$F(z) = Z[f(n)] = \sum_{n=-\infty}^{\infty} f[n] z^{-n}$$

$$f(z) = \sum_{n=-\infty}^{2} f(n) z^{-n} + \sum_{n=2}^{\infty} f(n) z^{-n}$$

$$= \sum_{n=-\infty}^{2} 0 \cdot z^{-n} + \sum_{n=2}^{\infty} 1 \cdot 2^n z^{-n}$$

$$= 0 + 0 + 2^2 z^{-2} + 2^3 z^{-3} + 2^4 z^{-4} + \dots +$$

$$= \frac{2^2 z^{-2}}{1 - 2z^{-1}} = \frac{4z^{-2}}{1 - 2z^{-1}}$$

ROC: $|2z^{-1}| < 1$ OR $|z| > 2$

$$z^{-1}f(z) = z^{-2} + 2z^{-3} + 3z^{-4} + \cdots \infty$$

$$\cancel{(1-z^{-1})f(z) = z^{-1} + 3z^{-2} + 8z^{-3} + \cdots \infty}$$

$$\cancel{f(z) =}$$

$$f(z) = \frac{z(z+1)}{(z-1)^3}$$

ROC : $|z| > 1$

(d) find inverse Z-transform

(i) $X(z) = \dfrac{1+2z^{-1}}{1-2z^{-1}+4z^{-2}}$

by long division method:

$$
\begin{array}{r|l}
 & 1 + 4z^{-1} + 4z^{-2} - 8z^{-3} + 32z^{-5} \\
\hline
1-2z^{-1}+4z^{-2} & 1+2z^{-1} \\
 & 1-2z^{-1}+4z^{-2} \\
\hline
 & 4z^{-1}-4z^{-2} \\
 & 4z^{-1}-8z^{-2}+16z^{-3} \\
\hline
 & 4z^{-2}-16z^{-3} \\
 & 4z^{-2}-8z^{-3}+16z^{-4} \\
\hline
 & -8z^{-3}-16z^{-4} \\
 & -8z^{-3}-16z^{-4}-32z^{-5} \\
\hline
 & 0 \quad 0 \quad +32z^{-5}
\end{array}
$$

$$f(z) = 1 + 4z^{-1} + 4z^{-2} - 8z^{-3} + 32z^{-5}$$

$$f(n) = \{1, 4, 4, -8, 32, \cdots\}$$

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(ii) $$x(z) = \frac{1}{(1+z^{-1})(1-z^{-1})^2}$$

$$x(z) = \frac{A}{1+z^{-1}} + \frac{B}{1-z^{-1}} + \frac{C}{(1-z^{-1})^2}$$

$$A = \left.\frac{1}{(1-z^{-1})^2}\right|_{z^{-1}=-1} \Rightarrow A = \frac{1}{4}$$

$$C = \left.\frac{1}{(1+z^{-1})}\right|_{z^{-1}=1} \Rightarrow C = \frac{1}{2}$$

$$A+B+C = 1 \; ; \; \frac{1}{4} + B + \frac{1}{2} = 1 \Rightarrow B = \frac{1}{4}$$

$$x(z) = \frac{1}{4}\cdot\frac{1}{1+z^{-1}} + \frac{1}{4}\frac{1}{(1-z^{-1})} + \frac{1}{2}\frac{1}{(1-z^{-1})^2}$$

Taking inverse z transform

$$x(n) = \left[\frac{1}{4}(-1)^n + \frac{1^n}{4} + \frac{1}{2}n(1)^n\right]u[n]$$

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