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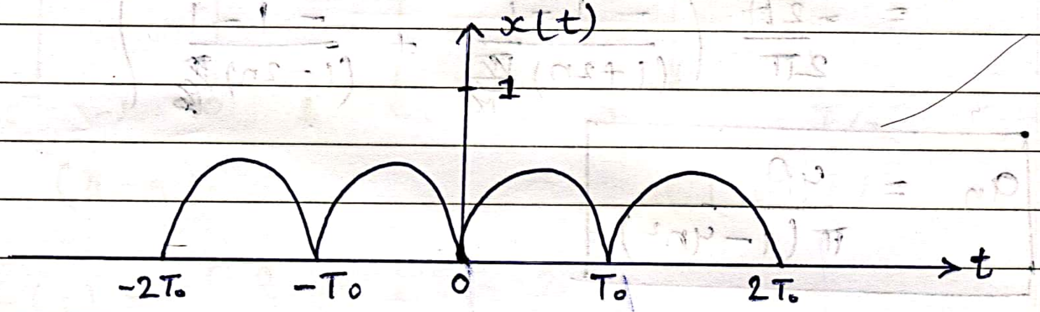
SIGNALS & SYSTEMS

Assignment - 02

Q.1. (a) find the trigonometric Fourier series representation for the Rectifier sine wave shown in figure :



Let $f(t) = \left\{ A\sin\frac{\pi t}{T_0} \quad \text{for} \quad 0 \leq t < T_0 \right\}$

and $f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{2n\pi t}{T_0} + b_n \sin\frac{2n\pi t}{T_0}\right)$ be its fourier series.

$$a_0 = \frac{1}{T_0}\int_0^{T_0} A\sin\frac{\pi t}{T_0}\,dt$$

$$= \frac{A}{T_0}\left|\frac{-\cos\frac{\pi}{T_0}t}{\left(\pi/T_0\right)}\right|_0^{T_0}$$

$$\boxed{a_0 = \frac{2A}{\pi}}$$

$$a_n = \frac{2}{T_0}\int_0^{T_0} A\sin\frac{\pi t}{T_0}\cdot\cos\frac{2n\pi t}{T_0}\,dt$$

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$$= \frac{2A}{2T_0}\int_0^{T_0}\left(\sin(1+2n)\frac{\pi t}{T_0} + \sin(1-2n)\frac{\pi t}{T_0}\right)$$

$$= -\frac{2A}{2T_0}\left[\frac{\cos(1+2n)\frac{\pi t}{T_0}}{(1+2n)\frac{\pi}{T_0}} + \frac{\cos(1-2n)\frac{\pi t}{T_0}}{(1-2n)\frac{\pi}{T_0}}\right]_0^{T_0}$$

$$= \frac{-2A}{2\pi}\left(\frac{-1-1}{(1+2n)\cancel{\frac{\pi}{T_0}}} + \frac{-1-1}{(1-2n)\cancel{\frac{\pi}{T_0}}}\right)$$

$$\boxed{a_n = \frac{4A}{\pi(1-4n^2)}}$$

$$b_n = \frac{2}{T_0}\int_0^{T_0} A\sin\frac{\pi t}{T_0}\cdot\sin\frac{2n\pi t}{T_0}\,dt$$

$$= \frac{2A}{2T_0}\int_0^{T_0}\left(\cos(1-2n)\frac{\pi t}{T_0} - \cos(1+2n)\frac{\pi t}{T_0}\right)dt$$

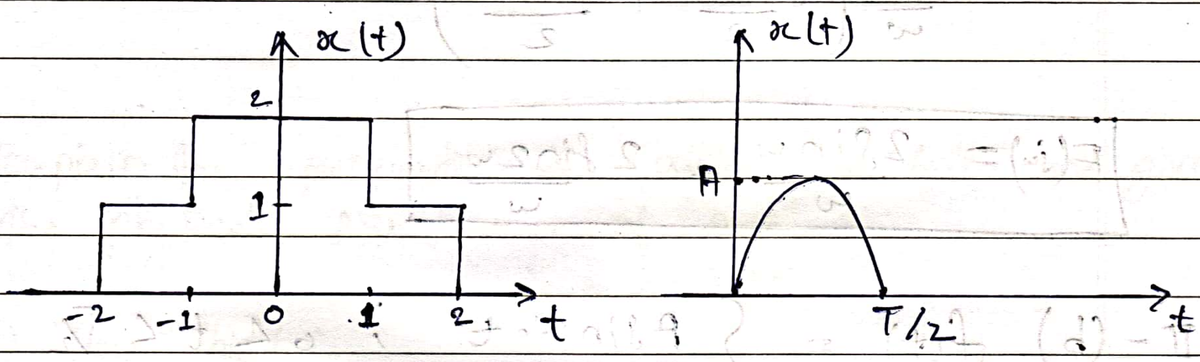
$$= \frac{A}{T_0}\left[\frac{\sin(1-2n)\frac{\pi t}{T_0}}{(1-2n)\frac{\pi}{T_0}} - \frac{\sin(1+2n)\frac{\pi t}{T_0}}{(1+2n)\frac{\pi}{T_0}}\right]_0^{T_0}$$

$$\boxed{b_n = 0}$$

Hence the given signal can be expressed as:

$$\boxed{s(t) = \frac{2A}{\pi} + \sum_{n=1}^{\infty}\frac{4A}{\pi(1-4n^2)}\cos\frac{2n\pi t}{T_0}}$$

(b) find the fourier transform of the signals shown in figure (II - a) and (II - b)



(II - a) (II - b)

II-(a) $f(t) = \begin{cases} 0 & ; t < -2 \\ 1 & ; -2 < t < -1 \\ 2 & ; -1 < t < 1 \\ 1 & ; 1 < t < 2 \\ 0 & ; t > 2 \end{cases}$

~~Lap~~ Fourier transform of $f(t)$ ~~is~~ is given as

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{-2} 0 \cdot e^{-j\omega t}\, dt + \int_{-2}^{-1} 1 \cdot e^{-j\omega t}\, dt + \int_{-1}^{1} 2 \cdot e^{-j\omega t}\, dt$$

$$+ \int_{1}^{2} 1 \cdot e^{-j\omega t}\, dt + \int_{2}^{\infty} 0 \cdot e^{-j\omega t}\, dt$$

$$= \frac{-1}{j\omega}\left[e^{-j\omega t}\right]_{-2}^{-1} + \frac{-2}{j\omega}\left[e^{-j\omega t}\right]_{-1}^{1} + \frac{-1}{j\omega}\left[e^{-j\omega t}\right]_{1}^{2}$$

$$= \frac{-2}{\omega}\sin\omega + \frac{2}{\omega}\sin 2\omega + \frac{4}{\omega}\sin\omega$$

$$= \frac{4}{\omega}\left(\sin\omega + \sin\frac{2\omega}{2} - \sin\frac{\omega}{2}\right)$$

$$= \frac{4}{\omega}\left(\frac{\sin\omega}{2} + \frac{\sin 2\omega}{2}\right)$$

$$\boxed{F(j\omega) = \frac{2\sin\omega}{\omega} + \frac{2\sin 2\omega}{\omega}}$$

II - (b) $f(t) = \begin{cases} A\sin\omega_0 t & ; 0 < t < T/2 \\ 0 & ; \text{otherwise} \end{cases}$

fourier transform of the $f(t)$ is given as:

$$F(j\omega) = \int_{-\infty}^{0} 0 \cdot e^{-j\omega t}\,dt + \int_{0}^{T/2} A\sin\omega_0 t \cdot e^{-j\omega t}\,dt + \int_{T/2}^{\infty} 0 \cdot e^{-j\omega t}\,dt$$

$$= A\int_{0}^{T/2} 2\left[\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right] e^{-j\omega t}\,dt$$

$$= \frac{A}{2j}\left[\frac{e^{j(\omega_0-\omega)}}{j(\omega_0-\omega)}\right]_0^{T/2} + \frac{A}{2j}\left[\frac{e^{-j(\omega_0+\omega)t}}{j(\omega_0+\omega)}\right]_0^{T/2}$$

$$= \frac{-A}{2(\omega_0-\omega)}\left[e^{-\frac{j\omega T}{4}} + e^{\frac{j\omega T}{4}}\right] + \frac{2e^{\frac{j\omega T}{4}}}{2(\omega_0+\omega)}\left[e^{-\frac{j\omega T}{4}} + e^{\frac{j\omega T}{4}}\right]$$

$$= \frac{Ae^{-\frac{j\omega T}{4}}}{\omega_0-\omega}\left[\cos\left(\frac{\omega T}{4}\right)\right] + \frac{Ae^{-\frac{j\omega T}{4}}}{\omega_0+\omega}\left[\cos\left(\frac{\omega T}{4}\right)\right]$$

$$= Ae^{-\frac{j\omega T}{4}}\cos\left(\frac{\omega T}{4}\right)\left[\frac{1}{\omega_0-\omega} + \frac{1}{\omega_0+\omega}\right]$$

$$= \left[\frac{2A\omega_0}{\omega_0^2-\omega^2}\cos\left(\frac{\omega T}{4}\right)\right] e^{-j\omega T}$$

(C) Explain the properties of Fourier transform and give the physical significance of each?

i) Time Shift:

$$s(t-t_0) \leftrightarrow S(f)\, e^{-j2\pi f t_0}$$

ii) Frequency Shift:

$$S(f+f_0) \leftrightarrow s(t)\, e^{-j2\pi f_0 t}$$

iii) Linearity

$$a s_1(t) + b s_2(t) \longleftrightarrow a S_1(f) + b S_2(f)$$

iv) Modulation

$$s(t)\cos(2\pi f_0 t) \longleftrightarrow \frac{1}{2}\left[S(f-f_0) + S(f+f_0)\right]$$

v) Time Scaling

$$s(at) \longleftrightarrow \frac{1}{|a|} S\left(\frac{f}{a}\right)$$

vi) Frequency Scaling:

$$s(af) \longleftrightarrow \frac{1}{|a|} s\left(\frac{t}{a}\right)$$

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vii) Differentiation

$$\frac{d^n s(t)}{dt^n} \longleftrightarrow (j2\pi f)^n S(f)$$

viii) Integration

$$\int_{-\infty}^{t} s(\lambda)\, d\lambda = (j2\pi f)^{-1} S(f) + \frac{1}{2} S(0)\, \delta(f)$$

(ix) Multiplication

$$s_1(t)\, s_2(t) \longleftrightarrow S_1(f) * S_2(f)$$

(x) Duality

$$S(t) \longleftrightarrow s\{(-f)\}$$

Q.2. (a) Define Fourier transform of signals $x(t)$. What is the condition for existence of fourier transform of signal $x(t)$?

Fourier Transform: The Fourier transform is a mathematical function that ~~takes~~ takes a time-based pattern as input and decomposes it into ~~the~~ its constituent frequencies scaled with suitable amplitudes which add up to form the original input signal. The result ~~of~~

produced by the Fourier transform is a complex valued function of frequency. The absolute value of the fourier transform is a complex valued function of frequency. The absolute value of the fourier transform represents the frequency value present in the original function and its complex argument represents the phase offset of the basic sinusoidal in that frequency.

Conditions for existance of Fourier Transform:

→ The signal should have a finite number of maximas and minimas over any finite interval.

→ The signal should have finite number of discontinuities over any finite interval.

→ The signal should be absolutely integrable.

(b) State and prove following properties of Fourier transform:

(i) Linearity

$$a s_1(t) + b s_2(t) \longleftrightarrow a S_1(f) + b S_2(f)$$

Proof:- Let $s_1(t)$ and $s_2(t)$ be two signals such that

$$s_1(t) \longleftrightarrow S_1(f)$$

$$s_2(t) \longleftrightarrow S_2(f)$$

The Fourier transform of $(a s_1(t) + b s_2(t))$ is given as:

$$S(f) = \int_{-\infty}^{\infty} (a s_1(t) + b s_2(t)) \cdot e^{-j2\pi f t} dt$$

$$= \int_{-\infty}^{\infty} a s_1(t) e^{-j2\pi f t} dt + \int_{-\infty}^{\infty} b s_2(t) e^{-j2\pi f t} dt$$

$$= a\left(\int_{-\infty}^{\infty} s_1(t) e^{-j2\pi f t} dt\right) + b\left(\int_{-\infty}^{\infty} s_2(t) e^{-j2\pi f t} dt\right)$$

$$\boxed{S(f) = a S_1(f) + b S_2(f)}$$

Hence proved!

(ii) Time shifting

$$s(t - t_0) \longleftrightarrow S(f) e^{-2\omega}$$

~~Proof: Fourier transform of $s(t-t_0)$ is given as:~~

$\cancel{S(f)} =$

Proof: Let $s(t)$ be a signal such that

$$s(t) \longleftrightarrow S(f)$$

Fourier transform of $s(t - t_0)$ is given as:

$$\mathcal{F}\left(s(t-t_0)\right) = \int_{-\infty}^{\infty} s(t-t_0)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{\infty} s(\tau)\, e^{-j2\pi f(\tau + t_0)}\, d\tau$$

by change of variable,
let $\tau = t - t_0$
$\Rightarrow d\tau = dt$

$$= e^{-j2\pi f t_0} \int_{-\infty}^{\infty} s(\tau)\, e^{-j2\pi f \tau}\, d\tau$$

$$= e^{-j2\pi f t_0}\, S(f)$$

Hence proved!

(iii) Time Scaling

$$s(at) \longleftrightarrow \frac{1}{|a|} S\left(\frac{f}{a}\right)$$

Proof: Let $s(t)$ be a waveform such that

$$s(t) \longleftrightarrow S(f)$$

Fourier Transform of $s(at)$ is given as:

$$\mathcal{F}\left(s(at)\right) = \int_{-\infty}^{\infty} s(at)\, e^{-j\omega f t}\, dt$$

$$= \frac{1}{|a|} \int_{-\infty}^{\infty} s(\tau)\, e^{-j\omega\left(\frac{f}{a}\right)\tau}\, d\tau$$

Let $\tau = at$
$d\tau = a\,dt$
or $dt = \frac{d\tau}{a}$

$$= \frac{1}{|a|} S\left(\frac{f}{a}\right)$$ Hence proved!

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~~(c) For the continuous time periodic signal;~~

$$\cancel{x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right)}$$

(c) For the transfer function:

$$f(s) = \frac{s+10}{s^2+3s+2}$$

find the response due to input $x(t)\ \sin 2t - u(t)$

$$X(s) = \frac{2}{s^2+4} - \frac{1}{s}$$

$$= \frac{2s - s^2 - 4}{s(s^2+4)}$$

$$Y(s) = X(s) \cdot F(s)$$

$$= \frac{2s - s^2 - 4}{s(s^2+4)} \times \frac{s+10}{s^2+3s+2}$$

$$\cancel{= \frac{-s^3 + 8s^2 + 6s - 40}{s(s}}$$

$$= -\frac{(s-2)^2 (s+10)}{s(s^2+4)(s^2+3s+2)}$$

$=$

~~Q~~

(d) For the continuous time periodic signal

$$x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right)$$

Determine the fundamental frequency Ω_0 and fourier series coefficient C_n such that

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j\Omega_0 n t}$$

$$\Omega_0 = HCF(\text{constituent frequencies of } x(t))$$

$$= HCF\left(\frac{2\pi}{3}, \frac{5\pi}{3}\right)$$

$$\boxed{\Omega_0 = \frac{\pi}{3}} \quad , \quad T = \frac{2\pi}{\Omega_0} = \frac{2\pi}{\pi/3} = 6$$

$$C_n = \frac{1}{T}\int_{t_0}^{t_0+T} x(t) \cdot e^{-j\Omega_0 n t}\, dt$$

$$= \frac{1}{6}\int_{0}^{6} 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right) e^{-j\Omega_0 n t}\, dt$$

Q.3. State the multiplication theorem of Fourier Transform

Multiplication theorem of Fourier Transform can be stated as follows ;

"Multiplication of two signals in time domain is equivalent to the convolution of the Fourier Transforms of the two signals"

Mathematically,

$$\boxed{x_1(t) \cdot x_2(t) \longleftrightarrow X_1(S) * X_2(S)}$$

(b) Find the trigonometric series representation of the rectified wave shown:

~~{ALREADY DONE in Q 1.(a)}~~

{ALREADY DONE in Q 1.(a)}

(c) Determine the Fourier Transform of the signal

$$x(t) = t \cos At$$

$$= j\omega \left[\frac{A}{S^2 + A^2} \right]$$

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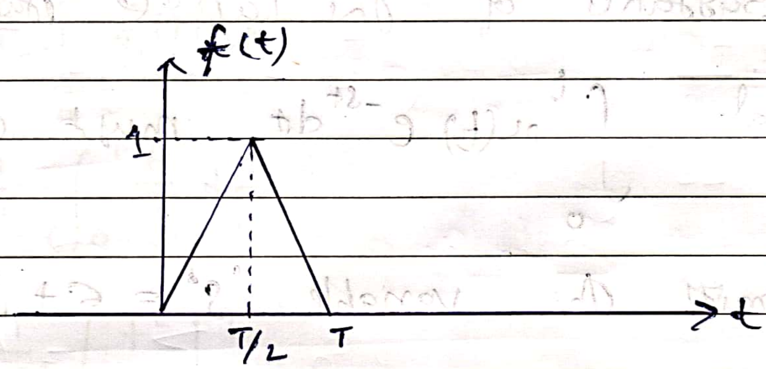
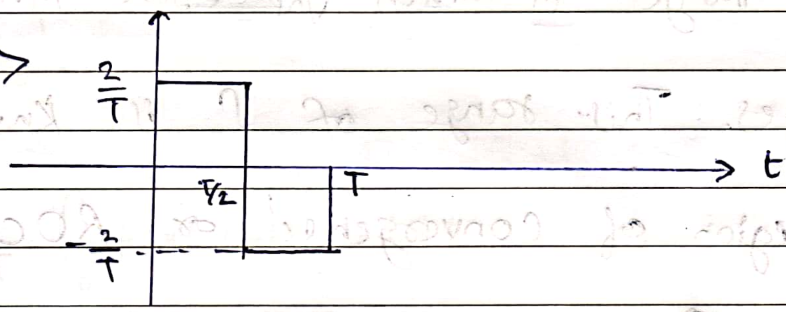
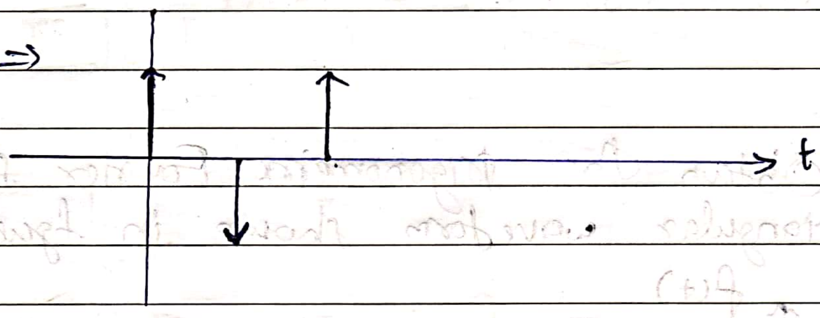
(d) Determine the Laplace transform of the triangular ~~phase~~ pulse shown in figure:



$\dfrac{d f(t)}{dt} \Rightarrow$



$\dfrac{d^2 f(t)}{dt} \Rightarrow$



$$\frac{d^2}{dt^2} = \frac{2}{T}\delta(t) - \frac{2}{T}\delta\left(t - \frac{T}{2}\right) + \frac{2}{T}\delta(t-T)$$

$$(j\omega)^2 F(\omega) = \frac{2}{T} - \frac{2}{T}e^{-j\omega T/2} + \frac{2}{T}e^{-j\omega T}$$

$$-\omega^2 F(\omega) = \frac{2}{T} - \frac{2}{T}e^{-j\omega T/2} + \frac{2}{T}e^{-j\omega T}$$

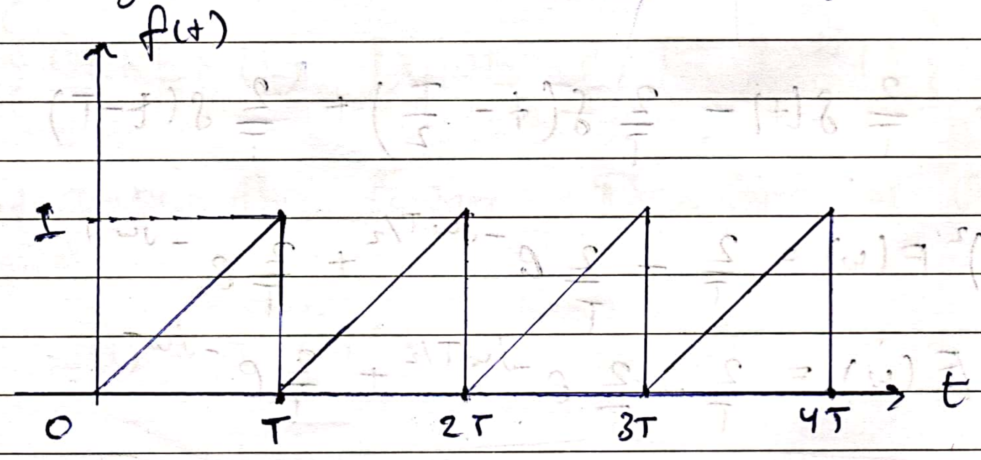
$$\boxed{F(\omega) = \frac{-2}{\omega^2 T}\left[1 - e^{-j\omega T/2} + e^{-j\omega T}\right]}$$

Q-5. (a) Define ROC in relation to Laplace Transform.

For existance of the laplace transform, the integral $\int_0^t x(t)\, e^{-st}\, dt$ must converge.

This limits the variable 's' = $\sigma + j\omega$' to a certain range in which the above integral converges. This range of s is known as the 'Region of convergence' or ROC for the Laplace Transform.

(b) Obtain the trigonometric Fourier series of the triangular waveform shown in figure



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$$f(t) = \left\{ \frac{t}{T} \; ; \quad 0 < t < T \right\}$$

$$a_0 = \frac{1}{T} \int_0^T f(t)\, dt$$

$$= \frac{1}{T} \int_0^T \frac{t}{T}\, dt$$

$$= \frac{1}{T^2} \left[\frac{t^2}{2} \right]_0^T$$

$$\boxed{a_0 = \frac{1}{2}}$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos n\omega_0 t\, dt$$

$$= \frac{2}{T} \int_0^T \frac{t}{T} \cos \frac{2n\pi t}{T}\, dt$$

$$= \frac{2}{T^2} \int_0^T t \cos \frac{2\pi n}{T} t\, dt$$

$$a_n = \frac{2}{T^2} \left[\frac{T^2}{4n^2\pi^2} - \frac{T^2}{4n^2\pi^2} \right] = \boxed{0}$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin n \frac{2\pi t}{T}\, dt$$

$$= \frac{2}{T} \int_0^T \frac{t}{T} \sin \frac{2\pi n}{T} t\, dt$$

$$= \frac{2}{T}\left[\frac{t\left(-\cos\frac{2n\pi t}{T}\right)}{\left(\frac{2n\pi}{T}\right)} + \frac{\sin\left(\frac{2\pi n t}{T}\right)}{\left(\frac{4n^2\pi^2}{T}\right)}\right]_0^T$$

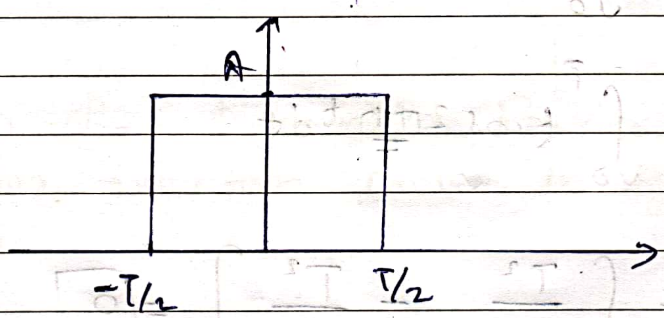
$$b_n = \frac{2}{T^2}\left[T^2 \frac{1}{2\pi n} + 0 - 0\right] = \boxed{\frac{1}{\pi n}}$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right)$$

$$\boxed{f(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{n\pi} \sin \frac{n 2\pi}{T} t}$$

(c) Obtain the fourier transform for the following functions:

(i) Gate function i.e., $f(t) = A$ for $-T/2 < t \leq T/2$



$$F(f(t)) = \int_{-T/2}^{T/2} A e^{-j\omega t}\, dt$$

$$= \frac{-A}{j\omega}\left[e^{-j\omega t}\right]_{-T/2}^{T/2}$$

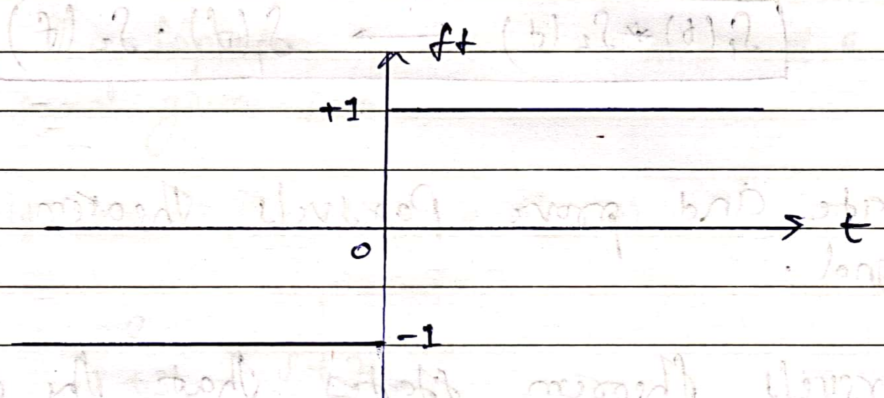
$$= \frac{-A}{j\omega}\left[e^{-j\omega T/2} - e^{j\omega T/2}\right]$$

$$= \frac{2A}{\omega} \sin\left(\frac{\omega T}{2}\right)$$

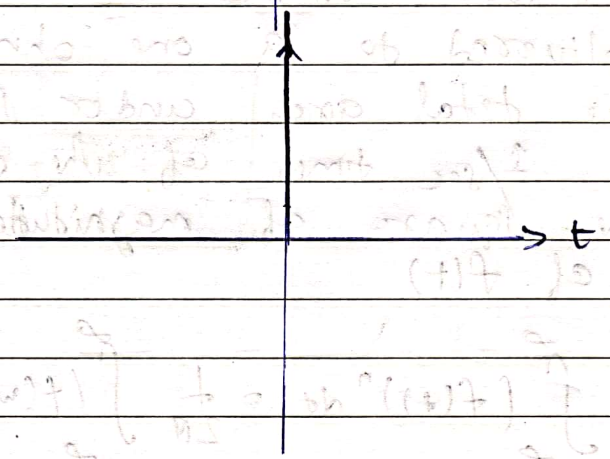
$$= AT \sin\frac{\omega T}{2}$$

$$= \boxed{AT \sin\left(\frac{\omega T}{2}\right)}$$

(ii) Signum function



$\frac{d f(t)}{dt} \Rightarrow$



$$(j\omega) F(\omega) = 2$$

$$\Rightarrow \boxed{F(\omega) = \frac{2}{j\omega}}$$

Q.8. (a) State convolution theorem of fourier transform

The convolution theorem of fourier transform states that the convolution of ~~the~~ two signals in time domain is equivalent to multiplication of the signals in frequency domain.

mathematically,

$$\boxed{S_1(t) * S_2(t) \longleftrightarrow S_1(f) \cdot S_2(f)}$$

(b) State and prove Parsvels theorem for energy signal.

Parsvels theorem states that the total energy delivered to a one ohm resistor equals the total area under the square of f(t) or 1/sec times of the total area under the square of magnitude of fourier transform of f(t)

$$W_{1\Omega} = \int_{-\infty}^{\infty} (f(t))^n \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 \, d\omega$$

$$W = \int_{-\infty}^{\infty} f(t) \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} \, d\omega \, dt$$

$$W = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-j(-\omega)t} \, dt \int_{-\infty}^{\infty} f(\omega) \, d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} F(-\omega) \cdot F(\omega) \, d\omega$$

$$\boxed{W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 \, d\omega}$$

(c) Determine the fourier - - - - - - - - - - - -

{ALREADY DONE in Q. 1. (b)}

(d) Find the Laplace transform ~~using~~ of a continuous time signal given below

$$x(t) = 2e^{-3t}u(t) - e^{-2t}u(t).$$

$$X(s) = \frac{2}{s+3} - \frac{1}{s+2}$$

$$\boxed{X(s) = \frac{s+1}{s^2+5s+6}}$$