ASSIGNMENT 2

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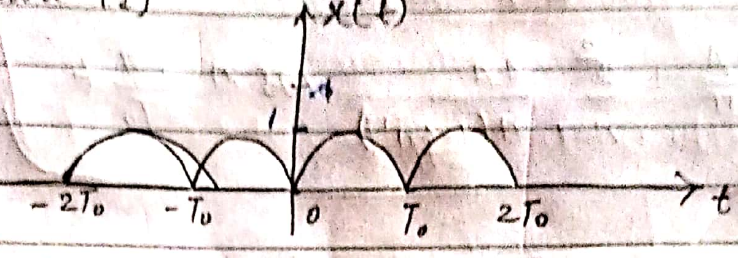
Q.1. (a). Find the trigonometric Fourier series representation for the Rectifier sine wave shown in figure (I)



$$f(t) = A \sin\left(\frac{\pi}{T_0}\right)t$$

$$a_0 = \frac{2}{T}\int_0^{T/2} f(t)\,dt$$

$$= \frac{2}{T_0}\int_0^{T_0/2} A \sin\left(\frac{\pi}{T_0}\right)t\,dt$$

$$= \frac{2A}{T_0}\int_0^{T_0/2} \sin\left(\frac{\pi}{T_0}\right)t\,dt$$

$$= \frac{2A}{\pi}\left[\cos\frac{\pi}{2} - 1\right]$$

$$= \frac{2A}{\pi}$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_0 t\,dt$$

$$= \frac{4}{T_0}\int_0^{T_0/2} A \sin\left(\frac{\pi}{T_0}\right)t\cos\left(\frac{n2\pi}{T_0}\right)t\,dt$$

$$= \frac{4A}{T_0}\int_0^{T_0/2}\left[\sin(1+2n)\frac{\pi}{T_0}t + \sin(1+2n)\frac{\pi}{T_0}t\right]dt$$

$$= \frac{-2A}{T_0}\left[\frac{\cos(1+2n)\frac{\pi}{T_0}t}{(1+2n)\frac{\pi}{T_0}} + \frac{\cos(1-2n)\frac{\pi}{T_0}t}{(1-2n)\frac{\pi}{T_0}}\right]_0^{T_0/2}$$

$$= -\frac{2A}{\pi}\left[\frac{1}{1+2n} + \frac{1}{1-2n}\right]$$

$$a_n = \frac{4A}{\pi(1-4n^2)}$$

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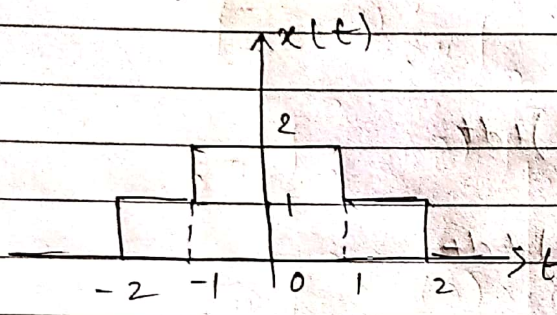
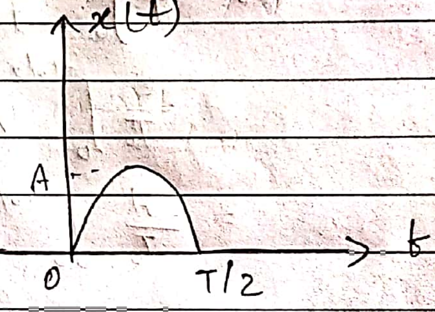
for even signal a_0, a_n, are present, $b_n = 0$

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n \omega_0 t)$$

$$\boxed{f(t) = \frac{2A}{\pi} + \sum_{n=1}^{\infty} \frac{4A}{n(1-4n^2)} \cos n \frac{2\pi}{T_0} t}$$

(b). Find the fourier transform of the signals shown in figure (II-a) and (II-b)





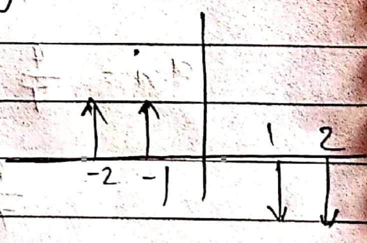
(II-a)

(II.a)

$$\frac{d}{dt} x(t) = \delta(t+2) + \delta(t+1) + \delta(t-1) + \delta(t-2)$$

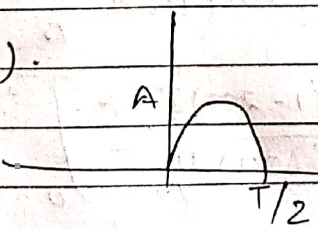
$$= e^{2j\omega} + e^{j\omega} + e^{-j\omega} + e^{-2j\omega}$$

$$(j\omega) X(\omega) = 2(\cos\omega + \cos 2\omega)$$



$$\boxed{X(\omega) = \frac{2}{j\omega} (\cos\omega + \cos 2\omega)}$$

(II-b).



$$A \sin\left(\frac{2\pi}{T}\right) t$$

$$= A\pi j \left[\delta\left(\omega + \frac{2\pi}{T}\right) - \delta\left(\omega - \frac{2\pi}{T}\right)\right]$$

Q- Explain the properties of Fourier transform and give the physical significance of each.

Properties of Fourier Transform:

(i) Linearity:

$$a_1 f_1(t) + a_2 f_2(t) \xrightarrow{F} a_1 F_1(\omega) + a_2 F_2(\omega)$$

(ii) Time Scaling:

$$f(at) \xrightarrow{F} \frac{1}{a} F\left(\frac{\omega}{a}\right)$$

(iii) Time shifting:

$$f(t-T_0) \xrightarrow{F} e^{-j\omega T_0} F(\omega)$$

(iv) Frequency shifting:

$$e^{j\omega_0 t} f(t) \xrightarrow{F} F(\omega-\omega_0)$$

(v) Time differentiation:

$$\frac{d^n}{dt^n} f(t) \longleftrightarrow (j\omega)^n F(\omega)$$

(vi) Frequency differentiation:

$$t^n f(t) = j^n \frac{d^n F(\omega)}{d\omega^n}$$

(vii) Convolution:

$$x_1(t) * x_2(t) = X_1(\omega) \cdot X_2(\omega)$$

(viii) Multiplication Properties

$$x_1(t) \cdot x_2(t) = \frac{1}{2\pi} [X_1(\omega) * X_2(\omega)]$$

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Q.3 (a)- State the multiplication theorem of Fourier transform

Statement: "Multiplication of two signals in time domain is equivalent to convolution of two ~~sing~~ signals in frequency domain."

$$\boxed{x_1(t)\cdot x_2(t) \longrightarrow \frac{1}{2\pi}\left[X_1(\omega) * X_2(\omega)\right]}$$

Proof:

$$\int_{-\infty}^{\infty} x_1(t)\, x_2(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} x_1(t) \frac{1}{2\pi}\int_{-\infty}^{\infty} X_2(\lambda)\, e^{j\lambda t}\, e^{-j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X_2(\lambda) \int_{-\infty}^{\infty} x_1(t)\, e^{-j(\omega-\lambda)t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X_2(\lambda)\, X_1(\omega-\lambda)\, d\omega$$

$$= \frac{1}{2\pi}\left[X_1(\omega) * X_2(\omega)\right]$$

$$\boxed{x_1(t)\cdot x_2(t) = \frac{1}{2\pi}\left[X_1(\omega) * X_2(\omega)\right]}$$

(b)- Solved in $I \rightarrow a$

(c)- Determine the Fourier Transform of the signal

$$x(t) = t\cos At$$

$$X(\omega) = (j\omega)\left[\frac{A}{B^2+A^2}\right]$$

$$= j(\omega)\frac{A}{(S^2+A^2)}$$

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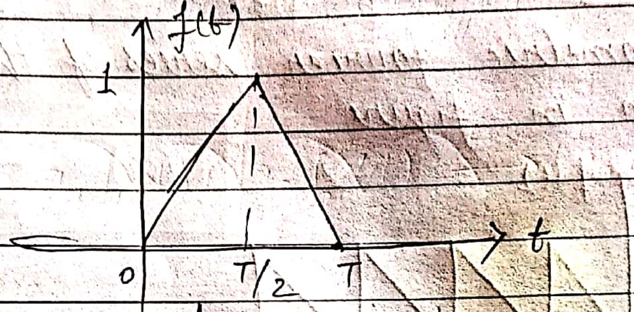
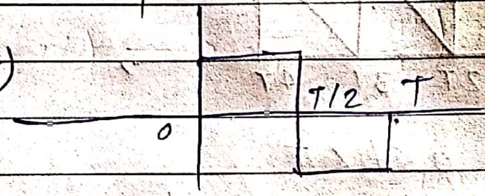
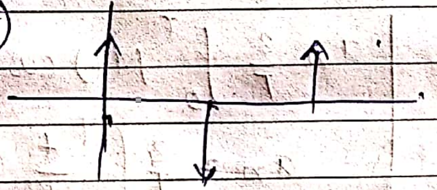
(d). Determine the Laplace transform of the triangular pulse shown in figure.



$\frac{d}{dt} f(t)$



$\frac{d^2}{dt^2} f(t)$



$$\frac{d^2}{dt^2} f(t) = \frac{2}{T}\delta(t) - \frac{2}{T}\delta\left(t - \frac{T}{2}\right) + \frac{2}{T}\delta(t - T)$$

$$(j\omega)^2 F(\omega) = \frac{2}{T} - \frac{2}{T} e^{-j\omega T/2} + \frac{2}{T} e^{-j\omega T}$$

$$-\omega^2 F(\omega) = \frac{2}{T} - \frac{2}{T} e^{-j\omega T/2} + \frac{2}{T} e^{-j\omega T}$$

$$\boxed{F(\omega) = -\frac{1}{\omega^2}\frac{2}{T}\left[1 - e^{-j\omega T/2} + e^{-j\omega T}\right]}$$

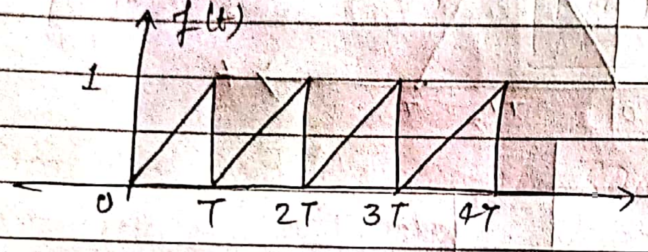
Q.5 (a) Define ROC in relation to Laplace Transform.

As we know that for the existence of the laplace transform the integral

$$X(s) = \int_0^t x(t)\, e^{-st}\, dt$$

must converge this limits the variable $s = \sigma + j\omega$ to apart of the complex plane known as ROC.

(b). Obtain trignometric fourier series of the triangular waveform shown in figure



$$f(t) = \frac{t}{T} \qquad 0<t<T$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$
$$= \frac{1}{T}\int_0^T \frac{t}{T}\,dt$$
$$= \frac{1}{T^2}\left(\frac{t^2}{2}\right]_0^T$$
$$\boxed{a_0 = \frac{1}{2}}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t\,dt$$
$$a_n = \frac{2}{T}\int_0^T \frac{t}{T}\cos n\frac{2\pi}{T}t\,dt$$
$$= \frac{2}{T^2}\int_0^T t\cos n\frac{2\pi}{T}t\,dt$$

$$a_n = \frac{2}{T^2}\left[\frac{t\sin\left(\frac{n2\pi}{T}\right)t}{n\left(\frac{2\pi}{T}\right)} + (1)\cdot\frac{\cos\left(n\frac{2\pi}{T}\right)t}{n^2\frac{4\pi^2}{T^2}}\right]_0^T$$

$$= \frac{2}{T^2}\left[\frac{T^2}{4\pi^2 n^2} - \frac{T^2}{n^2 4\pi^2}\right] = 0 \qquad \boxed{a_n = 0}$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\frac{2\pi}{T}t\,dt$$

$$= \frac{2}{T}\int_0^T \frac{t}{T}\sin\left(\frac{n2\pi t}{T}\right)dt$$

$$= \frac{2}{T^2}\int_0^T t \sin\left(\frac{2\pi n t}{T}\right) dt$$

$$= \frac{2}{T^2}\left[t\,\frac{\cos\frac{2\pi n t}{T}}{\frac{2\pi n}{T}} + (1)\,\frac{\sin\frac{2\pi n}{T}}{\frac{4\pi^2 n^2}{T^2}}\right]_0^T$$

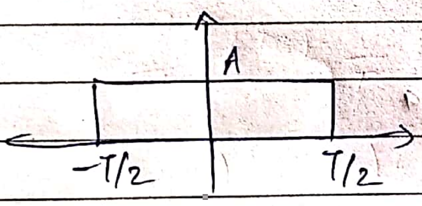
$$= \frac{2}{T^2}\left[-\frac{T^2}{2\pi n} + 0 - 0\right]$$

$$b_n = \frac{1}{\pi n}$$

$$f(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{\pi n} \sin \frac{n 2\pi t}{T}$$

(c)- Obtained the Fourier transform for the following function

(i)- Gate function i.e. $f(t) = A$ for $-T/2 \leq t \leq T/2$



$$= \int_{-T/2}^{T/2} A e^{-j\omega t} dt$$

$$= A\int_{-T/2}^{T/2} e^{-j\omega t} dt$$

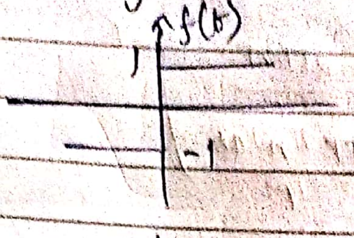
$$= -\frac{A}{j\omega}\left[e^{-j\omega t}\right]_{-T/2}^{T/2}$$

$$= -\frac{A}{j\omega}\left[e^{-j\omega T/2} - e^{+j\omega T/2}\right] = \frac{A}{j\omega}\left[e^{j\omega T/2} - e^{-j\omega T/2}\right]$$

$$= \frac{2A}{\omega}\sin\left(\frac{\omega T}{2}\right) = \frac{AT}{\omega T/2}\sin\frac{\omega T}{2}$$

$$\boxed{f(t) = AT\,\mathrm{sinc}\left(\frac{\omega T}{2}\right)}$$

(ii) Signum function i.e $sgn(t)$



$\frac{d}{dt} f(t) =$ 

$(j\omega)F(\omega) = 2 \Rightarrow \boxed{F(\omega) = \frac{2}{j\omega}}$

Q.7 (a) - Explain the significance of ROC in Laplace transform using an example.

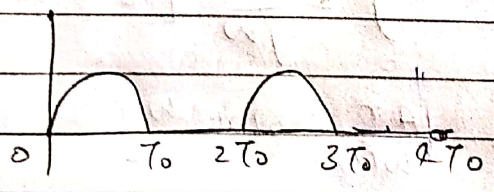
ROC is required in computing the laplace transform and the inverse Laplace transform. If ROC is not specified, the inverse Laplace transform is not unique

$$e^{at}u(t) \xrightarrow{L} \frac{1}{s-a} \quad s > a$$

$$-e^{-at}u(-t) \xrightarrow{L} \frac{1}{s+a} \quad s < -a$$

ROC depends on whether the signal is right sided ~~and~~ or left sided.

(b). Find the Trignometric Fourier series representation for a half wave rectified sine wave



$T = 2T_0 \Rightarrow \omega = \frac{2\pi}{2T_0}$

$\omega = \frac{\pi}{T_0}$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{2T_0}\int_0^{2T_0} A\sin\left(\frac{\pi}{T_0}t\right)dt$$

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$$= \frac{A}{2T_0}\int_0^{2T_0} \sin\left(\frac{\pi}{T_0}\right)t\,dt\Big] = -\frac{A}{2T_0}\left[\frac{\cos\left(\frac{\pi}{T_0}\right)t}{\frac{\pi}{T_0}}\right]_0^{T_0}$$

$$a_0 = -\frac{A}{2T_0}\frac{T_0}{\pi}\left[-1-1\right] = -\frac{A}{2\pi}(-2) = \frac{A}{\pi}$$

$$\boxed{a_0 = \frac{A}{\pi}}$$

$$a_n = \frac{2}{1}\int_0^1 f(t)\cos n\omega_0 t\,dt = \frac{2}{2T_0}\int_0^{T_0} A\sin\left(\frac{\pi}{T_0}\right)t\cos\frac{n\pi}{T_0}t\,dt$$

$$= \frac{A}{T_0}\int_0^{T_0}\sin\left(\frac{\pi}{T_0}\right)t\cos\left(\frac{n\pi}{T_0}\right)t\,dt$$

$$= \frac{A}{2T_0}\left[\int_0^{T_0}\left[\sin\left((1+n)\frac{\pi}{T_0}t\right) + \sin\left((1-n)\frac{\pi}{T_0}t\right)\right]dt\right]$$

$$= -\frac{A}{2T_0}\left[\frac{\cos(1+n)\frac{\pi}{T_0}t}{(1+n)\frac{\pi}{T_0}} + \frac{\cos(1-n)\frac{\pi}{T_0}t}{(1-n)\frac{\pi}{T_0}}\right]_0^{T_0}$$

$$= -\frac{A}{2T_0}\left[\left(\frac{-\cos n\pi}{(1+n)\pi/T_0} - \frac{\cos n\pi}{(1-n)\frac{\pi}{T_0}}\right) - \left(\frac{1}{(1+n)\pi/T_0} + \frac{1}{(1-n)\pi/T_0}\right)\right]$$

$$= -\frac{A}{2T_0}\left[-(-1)^n\left[\frac{1}{(1+n)\frac{\pi}{T_0}} + \frac{1}{(1-n)\frac{\pi}{T_0}}\right] - \left[\frac{1}{(1+n)\frac{\pi}{T_0}} + \frac{1}{(1-n)\frac{\pi}{T_0}}\right]\right]$$

$$= \frac{A\,T_0}{\pi\,2T_0}\left[\left((-1)^n+1\right)\left(\frac{1}{1+n} + \frac{1}{1-n}\right)\right] = \frac{A}{2\pi}\left[\left((-1)^n+1\right)\frac{2}{(1-n^2)}\right]$$

$$= \frac{A}{\pi}\left(\frac{(-1)^n+1}{1-n^2}\right)$$

$n =$ odd $= 0$

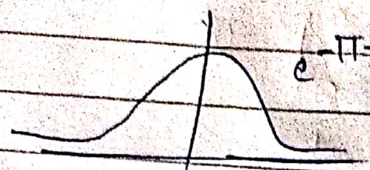
$n =$ eve $= \frac{2A}{\pi(1-n^2)}$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin\frac{n\pi}{T_0}t\,dt = \frac{2}{2T_0}\int_0^{T_0} A\sin\left(\frac{\pi}{T_0}\right)\sin\left(\frac{n\pi}{T_0}\right)t\,dt$$

$$= \frac{A}{2T_0}\int_0^{T_0}\left[\cos(1-n)\frac{\pi}{T_0}t - \cos(1+n)\frac{\pi}{T_0}t\,dt\right]$$

$$= \frac{A}{2T_0}\left[\frac{\sin(1-n)\frac{\pi}{T_0}t}{(1-n)\pi/T_0} - \frac{\sin(1+n)\frac{\pi}{T_0}t}{(1+n)\pi/T_0}\right]_0^{T_0}$$

(c) - show that a normalized Gaussian pulse is its own Fourier transform.



$e^{-\pi t^2} = \int_{-\infty}^{\infty} e^{-\pi t^2} e^{-j\omega t} dt$

$= \int_{-\infty}^{\infty} e^{-\left(\sqrt{\pi} t + \frac{j\omega}{2\sqrt{\pi}}\right)^2} e^{\frac{\omega^2}{4\pi}} dt$

$\sqrt{\pi} t + \frac{j\omega}{2\sqrt{\pi}} = p \qquad \sqrt{\pi}\, dt = dp$

$\frac{e^{-\omega/4\pi}}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-p^2} dp$

$\frac{e^{-\omega^2/4\pi}}{\sqrt{\pi}} \cdot 2\sqrt{\pi} = e^{\frac{4\pi^2 f^2}{4\pi}}$

$= e^{-\pi f}$

$e^{-\pi t^2} \xrightarrow{F} e^{-\pi f^2}$

(d) - Find the inverse Laplace Transform:

(i) - $x(s) = \frac{5s+13}{s(s^2+4s+13)}$

$\frac{5s+13}{s(s^2+4s+13)} = \frac{A}{s} + \frac{Bs+C}{s^2+4s+13}$

for $A\, s=0 \Rightarrow \boxed{A=1}$

for B & C $s=1$

$\frac{18}{18} = 1 + \frac{B+C}{18}$

$B+C = 0$

$s=2$

(ii) - $x(s) = \frac{1}{(s+a)^2}$

$\frac{1}{(s+a)^2} = \frac{A}{s+a} + \frac{B}{(s+a)^2}$

for B $s=-a$ $\quad B=1$

for $A = \frac{d}{ds}\left(\frac{1}{s+a}\right)(s+a)^2$

$A = 0$

$x(s) = \frac{1}{(s+a)^2}$

$\frac{L n}{(s+a)^{n+1}}$ is fourth transform of $te^{-at}u(t)$

so, $te^{-at}u(t)$

$\frac{28}{50} = \frac{1}{2} + \frac{2B+C}{25}$

$-\frac{2}{250} = \frac{2B+C}{25} \Rightarrow 2B+C = -1$

$B + C = 0 \qquad C = -B$

$B = -1 \qquad C = 1$

$= \frac{1}{s} + \frac{-s+1}{s^2+4s+13}$

$= \frac{1}{s} - \left[\frac{s+2-2}{(s+2)^2+3^2}\right] + \frac{1}{(s+2)^2+(3)^2}$

$= \frac{1}{s} - \frac{s+2}{(s+2)^2+3^2} + \frac{3}{(s+2)^2+3^2}$

$$\boxed{x(s) = u(t) - e^{-2t}\sin 3t\, u(t) + 3e^{-2t}\cos 3t\, u(t)}$$

Q.8.(a) State the convolution theorem of fourier transform

The convolution theorem of fourier transform states that the convolution of two signals in time domain is equal to multiplication of two signals in frequency domain.

(b). state and prove Parseval's theorem for energy signal

Parseval's theorem states that the total energy delivered to 1Ω resistor equals the total area under the square of $f(t)$ or $1/2\pi$ times of total area under the square of magnitude of fourier transform of $f(t)$.

$\cancel{W_{1\Omega}} = \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} \cancel{d\omega\, dt}$

~~=~~

$$W(2) = \int_{-\infty}^{\infty} [f(t)]^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$$

$$W = \int_{-\infty}^{\infty} f(t) \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega \, dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t) e^{-j(-\omega)t} dt \int_{-\infty}^{\infty} f(\omega) d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(-\omega) F(\omega) d\omega$$

$$\boxed{W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega}$$

(d). Find the Laplace transform of a continuous-time signal given below

$$x(t) = 2e^{-3t}u(t) - e^{-2t}u(t)$$

$$X(s) = \frac{2}{s+3} - \frac{1}{s+2}$$

$$= \frac{2s+4-s-3}{s^2+5s+6}$$

$$\boxed{X(s) = \frac{s+1}{s^2+5s+6}}$$