Z Transform

By

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Z- Transform

- Z-transform takes sequences from the time domain to the frequency domain (also known as the complex plane or Z-domain or Z-plane), enabling us to explore various system properties and behaviors.

Applications of Z transform

1. **Analysis of Linear Discrete Signals**:
   1. The Z-transform allows us to analyze discrete signals in the frequency domain. It provides insights into stability, frequency response, and other characteristics.
2. **Pole-Zero Description of Discrete-Time Systems**:
   1. By applying the Z-transform, we can describe the behavior of discrete-time systems in terms of poles and zeros. This information is crucial for system analysis and design.
3. **Digital Filter Analysis**:
   1. Z-transform plays a key role in analyzing digital filters (both Finite Impulse Response - FIR and Infinite Impulse Response - IIR). It helps us understand their frequency response and stability.

Applications of Z transform

4. Impulse Response Estimation:

1. The Z-transform allows us to estimate the impulse response of a system from its transfer function. This is useful in practical applications such as signal processing and control systems.

5. Difference Equation Solutions:

1. When dealing with linear difference equations, the Z-transform provides an elegant way to find solutions. It relates the time-domain sequence to its Z-domain representation.

6. Transfer Function Determination:

1. We can obtain transfer functions for series and feedback systems using the Z-transform. These transfer functions are essential for system analysis and design.

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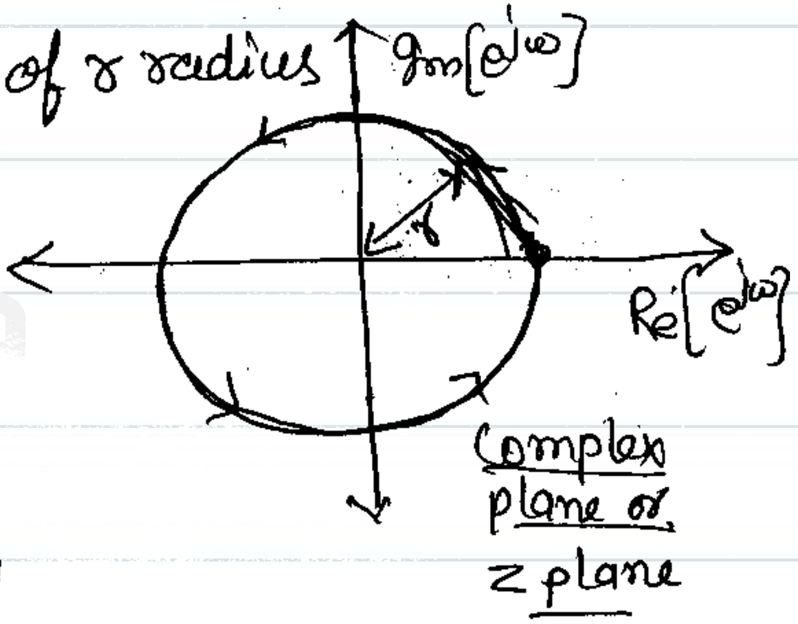
Z- Transform from DTFT

DTFT of signal is given by $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$ —— (1)

$$X(re^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, r e^{-j\omega n} \quad \text{—— (2)}$$

In the equation (2) the term $re^{j\omega}$ is a complex quantity which have r amplitude and rotates around a circle. of r radius. It represents a complex plane called. z-plane.



$$z = re^{j\omega}$$

therefore

$$\boxed{X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}}$$

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Bilateral and Unilateral Z-Transform

Case 1 :- $x(n)$ exists from $-\infty$ to ∞.

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$ double sided z transform.

Case:2 $x(n)$ exists only from 0 to ∞

$$X_{+}(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$ one sided z transfor (Causal z-transform)

Case 3 : If the sequence exists only from ~~$-\infty$ to 0~~ $n < 0$

the $$X_{-}(z) = \sum_{n=-\infty}^{-1} x(n) z^{-n}$$ Anticausal z-transform

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ROC The computation of $X(z)$ involves summation of infinite terms which are functions of z. hence it is possible that the infinite series may or may not give finite Value for certain values of z.

Therefore for every $X(z)$ there will be a set of values of z for which $X(z)$ can be computed. the set of such values is called Region of convergence (ROC) of $X(z)$

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ROC

Case(i) finite duration, right sided (causal) sequence.

Case (ii) finite duration, left sided (Anticausal) sequence.

Case (3) finite duration, double sided sequence

Case:(4) Infinite duration right sided sequence (causal)

Case (5) Infinite duration, left sided sequence. (Anticausal)

Case (6) Infinite, duration, double sided sequence. (Non causal)

Properties of ROC

(i) ROC of $X(z)$ consist of a ring or circle in the z plane centred around the origin.

(ii) ROC does not contain any pole.

(iii) ROC must be a connected region of values of z for which $X(z)$ is finite.

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Example 1 : Finite Duration, Right sided

$$x(n) = \{1, 2, 3, 4\}$$

$$x(0) = 1, \quad x(1) = 2, \quad x(2) = 3 \quad x(3) = 4$$

$$X(z) = \sum_{n=0}^{3} x(n) z^{-n}$$

$$= 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$$

$$= 1 + \frac{2}{z} + \frac{3}{z^2} + \frac{4}{z^3}$$

Example 1 $X(z) = 1 + \frac{2}{z} + \frac{3}{z^2} + \frac{4}{z^3}$

: ROC

when $z = 0$

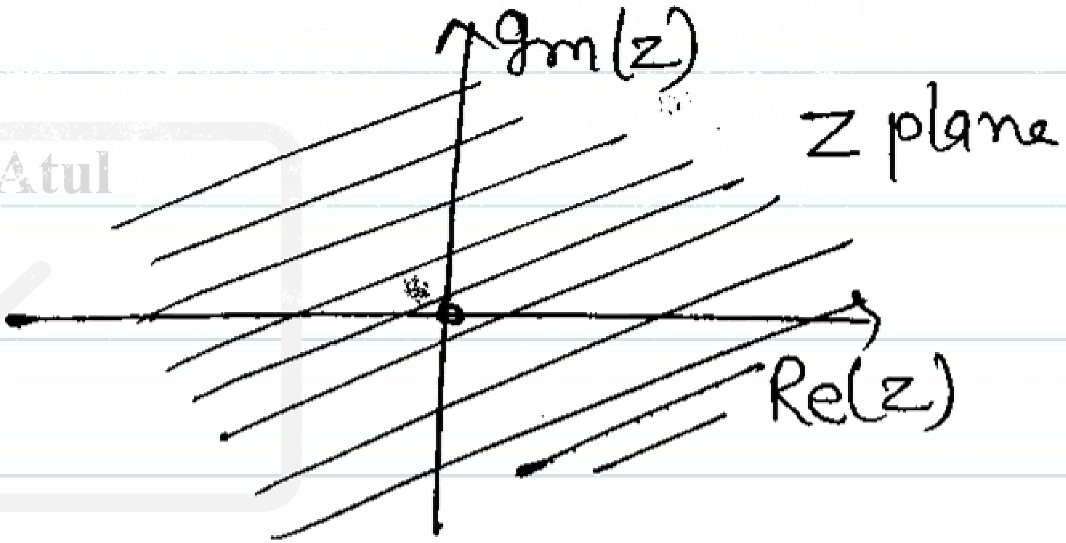
$X(z) = \infty$

if $z =$ constant.

$X(z) =$ finite.

if $z = \infty$

$X(z) = 1$ finite



Therefore $X(z)$ will be finite for all values of z except $z = 0$.

ROC is : Entire z plane except $z=0$

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Example 2 : Finite Duration , Left Sided

:- Determine Z transform :

$$x(n) = \{\underset{x(-3)}{6}, \underset{x(-2)}{4}, \underset{x(-1)}{5}, \underset{\substack{\uparrow \\ x(0)}}{3}\}$$

$x(-3) = 6$

$x(-2) = 4$

$x(-1) = 5$

$x(0) = 3$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-3}^{0} x(n) z^{-n}$$

$$= x(-3) z^{+3} + x(-2) z^{2} + x(-1) z^{1} + x(0)$$

$$X(z) = 6z^{3} + 4z^{2} + 5z + 3$$

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Example 2: ROC

$$X(z) = 6z^3 + 4z^2 + 5z + 3$$

if $z = 0$

$$X(z) = 3$$

if $z =$ Constant

$$X(z) = \text{constant (finite)}$$

if $z = \infty$

$$X(z) = \infty$$

therefore ROC of ~~the~~ $X(z)$ is ~~all~~ entire z plane. except $z = \infty$

ROC : Entire z plane except $z = \infty$

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Example 3 : Finite Duration, both sided

Determine Z transform and ROC of following signal $x(n)=\{4,3,2,\underset{\uparrow}{1},2,3,4\}$

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Example 3 : ROC

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Example 4 : Finite Duration

$x(n) = \delta(n)$ then determine z transform

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \delta(n) z^{-n}$$

$$= 0+0+1+0+0 \ldots = 1$$

ROC : Entire z plane

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Example 5 Right Sided, Infinite Duration Sequence

$x(n) = u(n)$

z plane

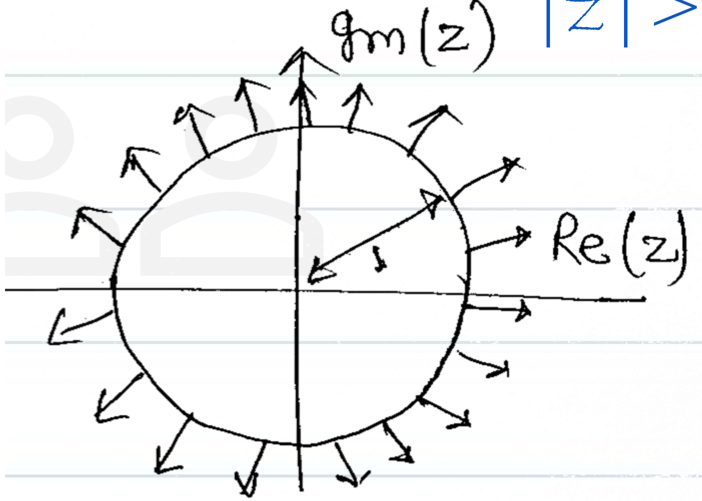
$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$X(z) = \sum_{n=0}^{\infty} u(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} (1z^{-1})^{n} = 1 + (z^{-1})^{1} + (z^{-1})^{2} + (z^{-1})^{3} + (z^{-1})^{4} + \dots$$

$$= \frac{1}{1-z^{-1}}$$

if $|z^{-1}| < 1$ therefore ROC is $|z| > 1$



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Example 5: Left Sided, Infinite Duration Sequence

Determine Z transform and ROC of following signal $x(n) = u(-n)$

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Example 5: Left Sided, Infinite Duration Sequence

Determine Z transform and ROC of following signal $x(n) = u(-n)$

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Example 5: Left Sided, Infinite Duration Sequence

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Assignment : Practice Problems

Find the z-transform and the ROC for the sequences $x[n]$ given below.

1. $x[n] = \{\underset{\uparrow}{2}, -1, 0, 3, 4\}$
2. $x[n] = \{1, -2, 3, -1, \underset{\uparrow}{2}\}$
3. $x[n] = \{5, 3, \underset{\uparrow}{-2}, 0, 4, -3\}$
4. $x[n] = \delta[n]$
5. $x[n] = u[n]$
6. $x[n] = u[-n]$
7. $x[n] = a^{-n}u[-n]$
8. $x[n] = a^{-n}u[-n-1]$
9. $x[n] = (-a)^{n}u[-n]$

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Thank you for your attention!