

# 1 Names and Symbols

A Block contains a set of factors, (design) a set of factors to cross with each other (crossing), and a set of constraints.

Let  $D$  be the set of factors in the design.

Let  $X$  be the set of factors in the crossing.

Let  $C$  be the set of constraints.

Let  $\overline{X}$  denote the factors in the design which are *not* in the crossing.

The number of factors/levels in  $X$  determines the length of the generated trial sequences, denoted  $l$ .

Lemmas:

1.  $X \subseteq D$
2.  $X \cup \overline{X} = D$
3.  $X \cap \overline{X} = \emptyset$

For a given factor  $f$ , let  $|f|$  denote the number of possible settings (levels) for  $f$ .

## 2 Counting Solutions

### 2.1 Simple Factors Only

This section is limited to designs with only factors that are simple, meaning not derived. (Not dependent on other factors)

So far, this all assumes simple factors, meaning windows with length and stride of 1. Let  $S$  be the set of all valid trial sequences for a given block. We wish to compute  $|S|$ .

When  $C \equiv \emptyset$ :

$$|S| = l! \prod_{f \in \overline{X}} |f|^l$$

Or simply:  $l!$  when  $\overline{X} \equiv \emptyset$ .

#### 2.1.1 Constraints

Only two constraints exist right now: **AtMostKInARow** and **ExactlyKInARow**. Exactly K is a special case of AtMostK too, so this should resolve to a single formula?

- **AtMostKInARow**( $k, f_v$ )
- **ExactlyKInARow**( $k, f_v$ )

Where  $k$  is the integer sequence limit for the level being constrained, and  $f_v$  denotes the level  $v$  in factor  $f$  being constrained.

Each constraint defines an equivalence relation on  $S$ , which in turn defines a set partition containing two subsets,  $A$  and  $B$ .  $A \subseteq S$ , while  $B \not\subseteq S$ .

Let  $Q$  be the set of all sets defined by constraints in  $C$  to not be in  $S$ . In otherwords:

$$\forall c \in C, c \rightarrow \begin{cases} A \mid A \subseteq S \\ B \mid B \not\subseteq S \end{cases}$$

Then:

$$Q = \{B_1, B_2, \dots, B_i\}$$

Where  $i = |C|$ .

Assuming that all  $B_i, B_n$  are disjoint:

$$|S| = l! \prod_{f \in \bar{X}} |f|^l - \sum_{q \in Q} |q|$$

But how can we count  $|B|$ ?

Because all factors/levels in  $X$  are being fully crossed, (this won't be true when specific crossings are excluded) we can compute the number of times  $v$  may appear in a sequence, denoted  $|v|$ .

$$|v| = \frac{l}{|f|}$$

For each sublength  $u$  in  $(k, |v|]$ , how many subsequences of length  $s$  can  $l$  contain? Denoted  $n$ . (If  $l = 4$  and  $u = 2$ , then  $n = 3$ )

It's more like this:

How many unique ways can you arrange  $|v|$  items in a sequence of  $l$  entries where there is at least one subsequence that is at least  $k$  consecutive elements.

With that,  $|B| = nl$ . (I think.) So:

$$|B| = \sum_{u \in L} ul$$

Where  $L$  is the range  $(k, |v|]$ .

But it will rarely be this simple, since multiple constraints will almost always overlap, right?