1-a) size of vector $w = (d+1) \times 1$

size of vector $y = 1 \times n$

1-b) size of matrix $A = (d+1) \times n$

1-c) Letting $d+1 = n$

using induction,

let $n = 1, 2$ $\longrightarrow$ Let $n = k$

$\det(A_1) = |1| = 1$

$\det(A_2) = \begin{vmatrix} 1 & x_1 \\ 1 & x_2 \end{vmatrix} = x_2 - x_1$

$P(k): \det(A_k) = \prod_{1 \le i < j \le k} (x_j - x_i)$

Thus let $n = k+1$

when

$$A_k' = \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^k \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_k & x_k^2 & \cdots & x_k^k \\ 1 & x & x^2 & \cdots & x^k \end{bmatrix}$$

using Laplace transform

$$\det A_k' = 1C_{(k+1)1} + xC_{(k+1)2} + x^2C_{(k+1)3} + \cdots + x^kC_{(k+1)(k+1)}$$

using fundamental theorem of algebra/factor theorem

$$f(x) = C(x - x_1)(x - x_2) \cdots (x - x_k)$$

$$C_{(k+1)(k+1)} = (-1)^{(k+1)+(k+1)} \det \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{k-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{k-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_k & x_k^2 & \cdots & x_k^{k-1} \end{bmatrix} = 1 \cdot \prod_{1 \le i < j \le k} (x_j - x_i)$$

As $C = C_{(k+1)(k+1)}$

$$P(k+1) = \det(A_k') = \left[\prod_{1 \le i < j \le k} (x_j - x_i)\right]\left[(x - x_1)(x - x_2) \cdots (x - x_k)\right]$$

$\det A_{k+1}$

$$= \prod_{1 \le i < j \le k+1} (x_j - x_i)$$

Thus we can see if $P(k)$ is true $P(k+1)$ is also true, proven by induction.

1-d) In order for matrix to be non-zero, the matrix A has to be invertible, ~~columns and rows are non-dependent vectors in $\mathbb{R}^n$, and~~ ~~and~~ and the linear map represented by matrix is an isomorphism.

1-e) Assuming that the determinant of A is non-zero to solve $Aw=y$ with respect to w:

↳ $Aw=y$

$wA^{-1}A = A^{-1}y$ multiplication on both side

$w = A^{-1}y$ thus, this is the solution.

2) Since inverse doesn't work, we need to use pseudo-inverse to obtain solution for $Aw=y$:

↳ $Aw=y$

$A^{+}Aw \approx A^{+}y$

$Iw \approx A^{+}y$ Identity matrix AA^{+}

$w \approx A^{+}y$ we can use this to get solutions for linear equations

since $A^{+} = (A^{T}A)^{-1}A^{T}$

$= A^{-1}$, which is literally a substitute of inverse matrix, but suitable for non-square matrix.

---------------몇개가 잘 안 보여서 따로 적어둡니다 ㅠㅠ...

1- a)

size of vector w = (d+1) x 1

size of vector y = 1 x n

1 - b)

size of matrix A = (d+1) x n

1- d)

In order for matrix to be non-zero, the matrix A has to be invertible, and the linear map represented by matrix is an isomorphism