# Bias-Variance Tradeoffs in Joint Spectral Embeddings

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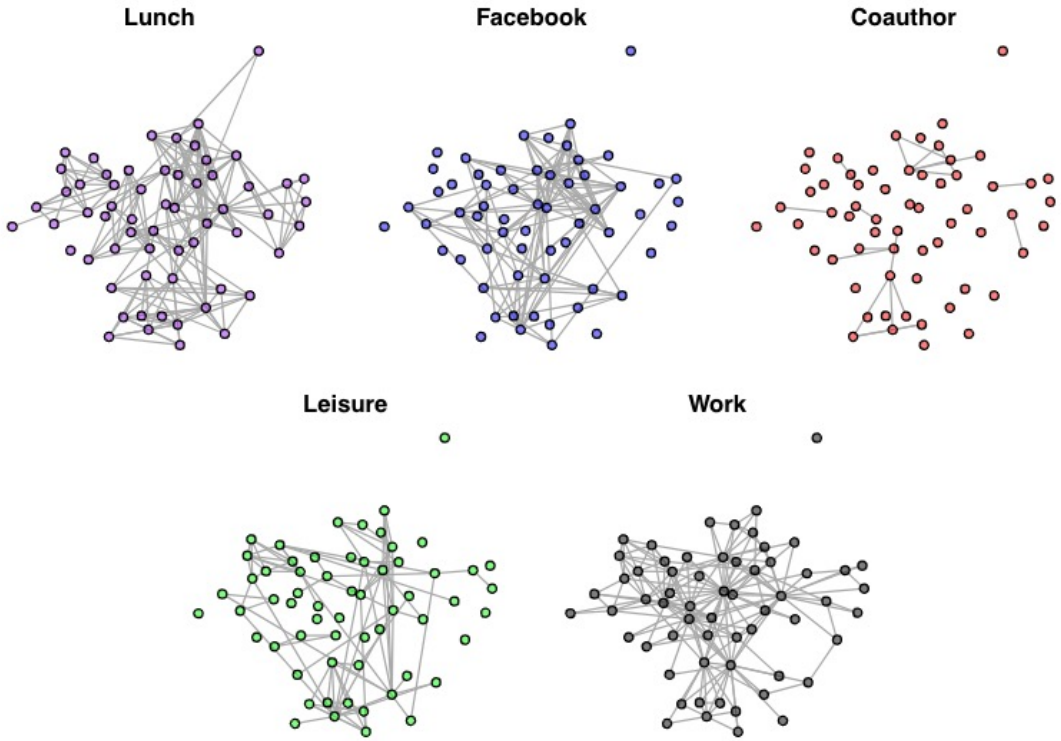
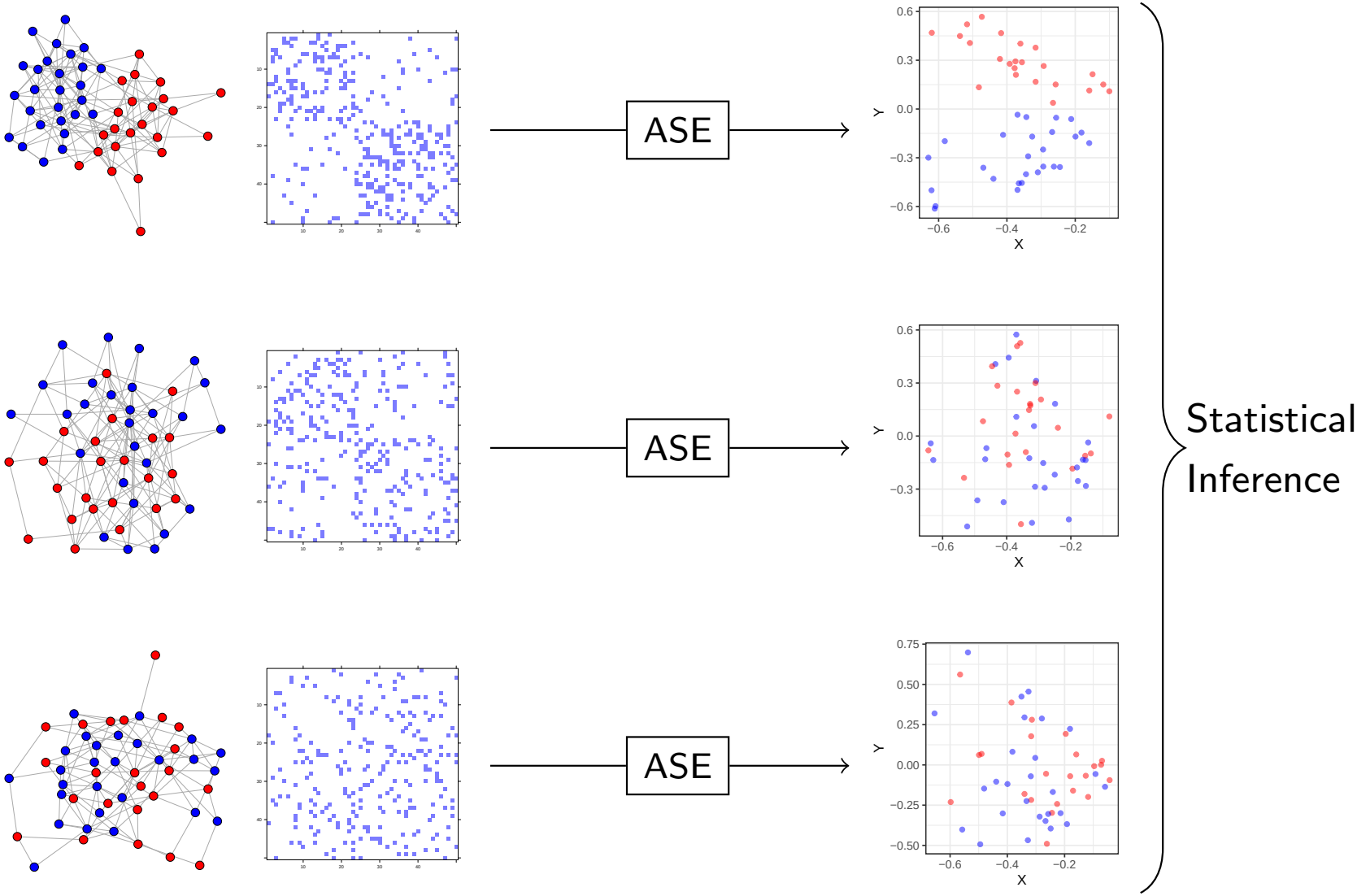
# Multiplex Networks

- Multiplex networks encode multiple relationships between entities as a collection of networks.

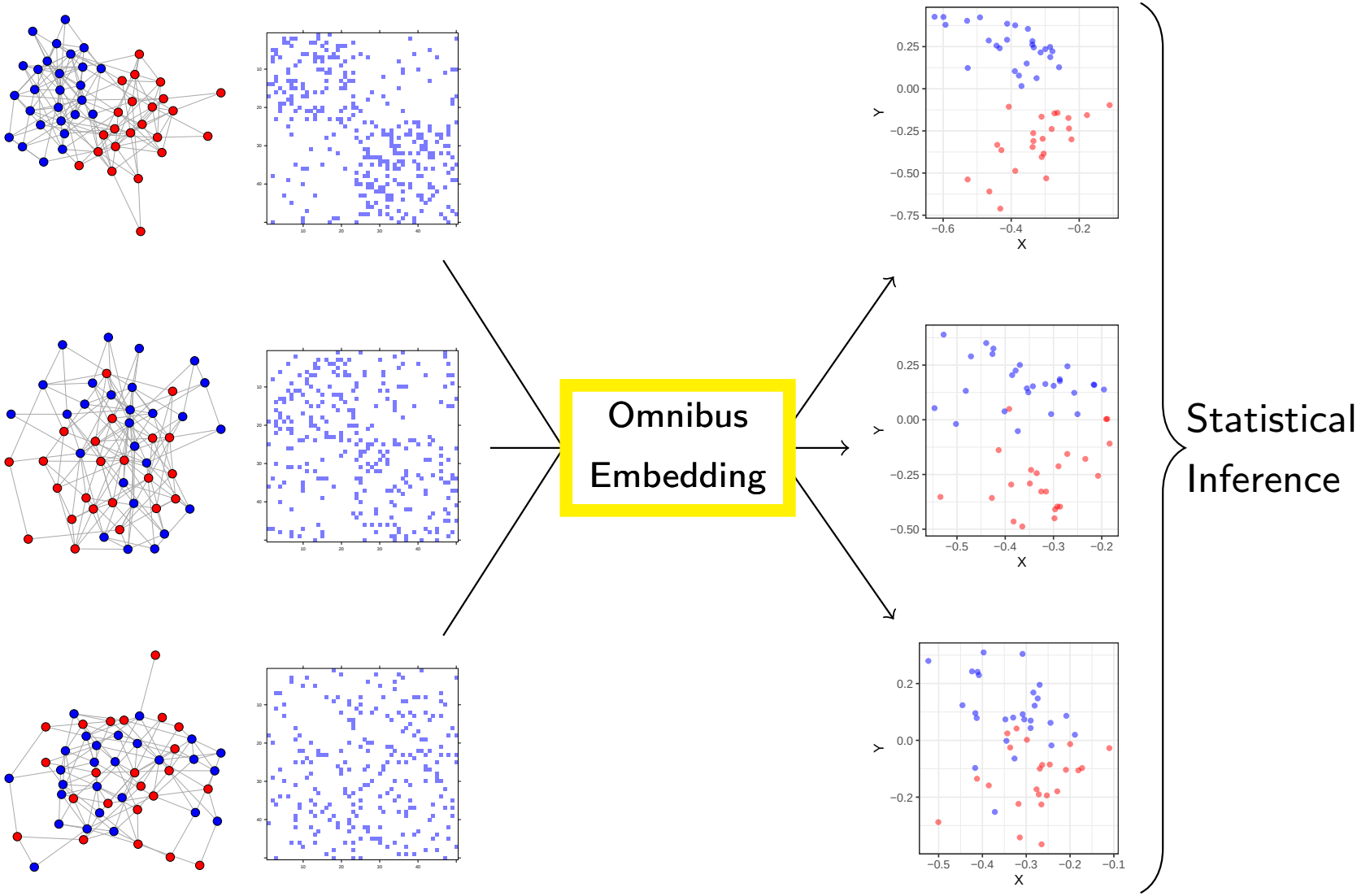



- Application areas; International Trade, Transportation Systems, Terrorist Groups, Neuroscience (Kivelä et al. 2014).

# Individual Spectral Embeddings



# Joint Spectral Embeddings



# Network Embedding Techniques

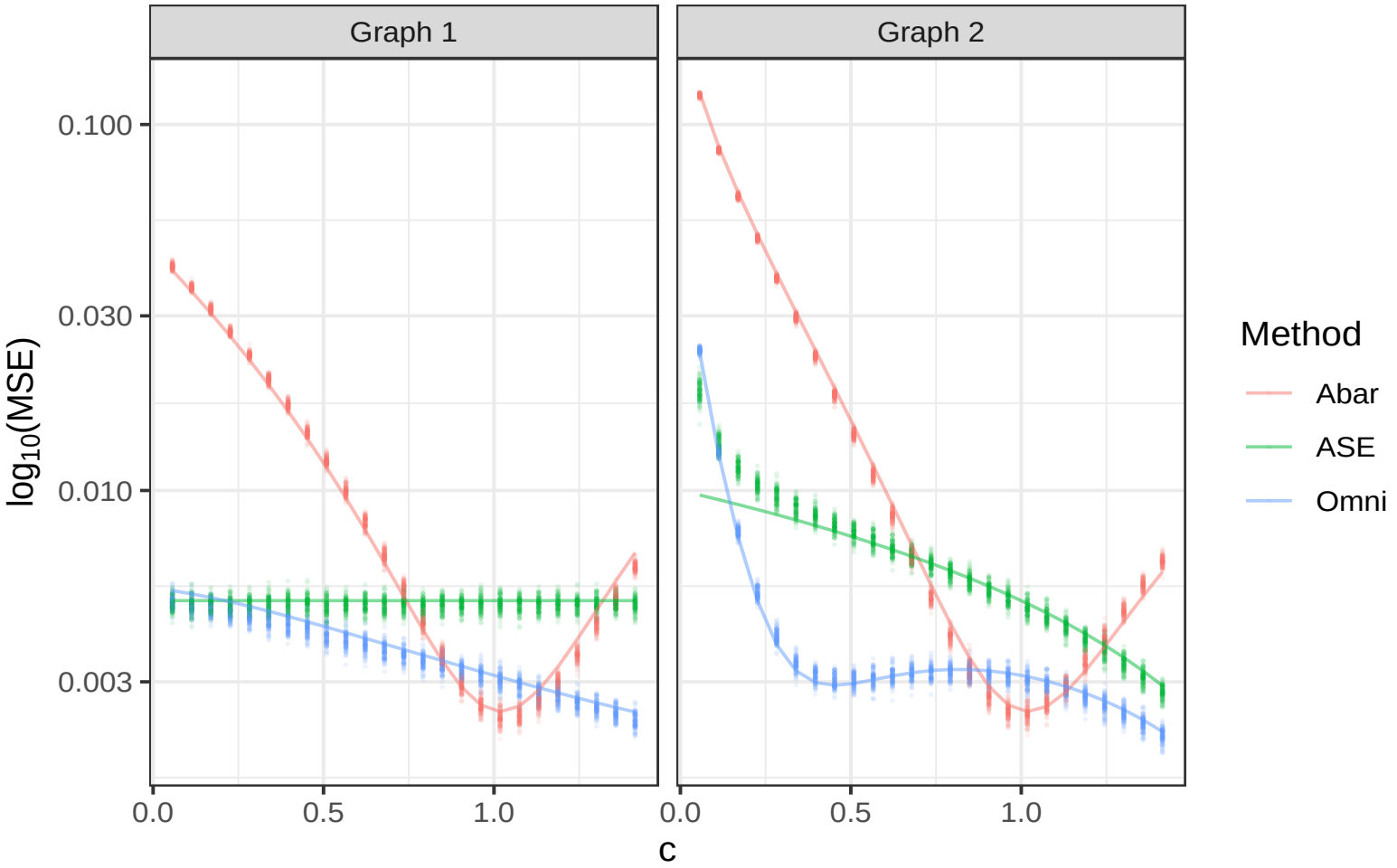
- ASE, Abar, Omnibus

# Estimation Task

- Introduce ESRDPG
- State Estimation task
- Plan to compare methods by MSE

# Mean Squared Error Comparison

- Suppose $\mathbf{A}^{(1)} \sim \text{ER}(p)$ and $\mathbf{A}^{(2)} \sim \text{ER}(c^2 p)$
- Under ESRDPG $\mathbf{X} = \sqrt{p}\mathbf{1}_n$, $\mathbf{C}^{(1)} = \mathbf{I}$, and $\mathbf{C}^{(2)} = c^2\mathbf{I}$



# Main Results

- Asympoptic Expansion
- Theorem Statements

# Pivotal Test Statistic

- Hypotheses, test statistic (both $W$ and $\hat{W}$), approximate asympoptic distribution

# Power Analysis

- Example, introduction of T statistic, power curves

# Conclusion & Future Work

- Introduced the *MCRDPG* probability model
- Highlighted an advantageous bias-variance tradeoff given by the Omnibus Embedding
- Established
  1. Bias of the Omnibus Estimator under the *MCRDPG*
  2. Uniform bound on the residual term at a $O(m^{3/2} \log nm/\sqrt{n})$ rate
- Highlighted second moment properties of the Omnibus Embedding

Questions?

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