Review

$$X(t+1) = \left(\frac{I+W}{2}\right) X(t)$$

$$\|p(t+1) - \pi\|_{D^{-1}}^2 = \|p(t) - \pi\|_D^2 \left(1 - \frac{\lambda_2}{2}\right)^2$$

If we look at the continuous version it's much more straightforward.

$$p(t) = e^{-tL} p(0)$$

$$\frac{d}{dt} \|p_t - \pi\|_{D^{-1}}^2$$

We can do this same proof with

$$e^{-tB} = \left(\frac{I+W}{2}\right)^t$$

$$B = -\log\left(\frac{I+W}{2}\right)$$

$$\|p_{t+1}-\pi\|^2_{D^{-1}} = \left\|\left(\frac{I+W}{2}\right)(p_t-\pi)\right\|^2_{D^{-1}}$$

$$= (p_t-\pi)^T \underbrace{\left(\frac{I+W}{2}\right)^T D^{-1} \left(\frac{I+W}{2}\right)}_{Z} (p_t-\pi)$$

$$= \frac{1}{4}\left(D^{-1} + D^{-1}AD^{-1} + D^{-1}AD^{-1} + D^{-1}AD^{-1}AD^{-1}\right)$$

$$= \frac{1}{4} D^{-1/2}\left(I + 2\, D^{-1/2}AD^{-1/2} + D^{-1/2}AD^{-1}AD^{-1/2}\right)D^{-1/2}$$

$$= D^{-1/2}\left(\frac{I}{4} + \frac{1}{2} D^{-1/2}AD^{-1/2} + \frac{\left(D^{-1/2}AD^{-1/2}\right)^2}{4}\right)D^{-1/2}$$

Recall $\mathcal{L} = I - D^{-1/2}AD^{-1/2}$ so

$$= D^{-1/2}\left(I - \frac{\mathcal{L}}{2}\right)^2 D^{-1/2}$$

Hence

$$\underbrace{(p_t - \pi)^T}_{\perp 1} D^{-1/2} \left(I - \frac{\mathcal{L}}{2}\right)^2 D^{-1/2} \underbrace{(p_t - \pi)}_{\perp 1}$$

$$\leq \max_{1 D^{-1/2} v = 0} v^T \left(I - \frac{\mathcal{L}}{2}\right) v$$

$$= \left(1 - \frac{\lambda_2}{2}\right)^2$$

$$L = \sum_E L_e = \sum_E x_e x_e^T$$

Want to find $\tilde{L}$ s.t.

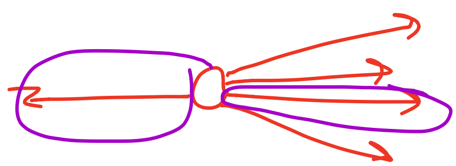
$$-\varepsilon L \preccurlyeq L - \tilde{L} \preccurlyeq \varepsilon L$$

$$-\varepsilon \preccurlyeq I - L^{-1/2} \tilde{L} L^{-1/2} \preccurlyeq \varepsilon$$

$$\sum \underbrace{L^{-1/2} x_e}_{u_e} \underbrace{x_e^T L^{-1/2}}_{u_e} = I$$

$$\sum_E u_e u_e^t = I$$

So we sparsify by choosing representative u_e.



$\|u_e\|^2 = x_e^T L x_e$ = effective resistance.

$$f_{ij} = \underline{v_i - v_j} = \underline{x_{ij}^T v}$$

$_{ij}$ $\quad r_{ij}$ $\quad \overline{}$ r_{ij}

$$x_{ij} = e_i - e_j$$

$$f = R^{-1} \underbrace{B}_{\text{incidence matrix.}} V$$

extra current.

$$i_{ext} = B^T f$$

$$i_{ext} = B^T R^{-1} B V$$

for $R = W^{-1}$ then

$$i_{ext} = LV$$

and the voltage is given by

$$V = L^{+} i_{ext}$$

and the effective resistence is given by

$$x_{ij}^T L^+ x_{ji}$$

Expect a different RW. on the final.