Multilayer Network Inference

- Collections of network inference
  - ↳ two sample t-tests, ANOVA, anomaly detection
  - ↳ Classical approaches: sample means of networks
- Averages of networks – geometry, shape analysis, probability (CLT for Frechet means), then statistics such as Hotelling's T^2.
- Assume a weighted, undirected, simple, connected and $L_1, \ldots, L_n \overset{iid}{\sim} F$
- Construct a geometric over Laplacians that form a manifold with corners of dimension $\frac{d(d-1)}{2}$. and the manifold is a convex subset of an affine space in $\mathbb{R}^{d^2}$.

Basically mapping $W \longrightarrow \mathrm{vec}(W)_{1:\binom{d}{2}} \leftarrow$ (upper triangular)

- They prove a C.L.T.

$$\mathbb{E}_Q[L] = \underset{L \in \mathcal{L}_d}{\arg\min} \int \rho_F^2(L, \tilde{L})\, Q(d\tilde{L}) \qquad \hat{L}_n = \arg\min \frac{1}{n} \sum_{i=1}^{n} \rho_F^2(L, L_i)$$

in the labeled setting the CLT is given by

$$n^{1/2}\left(\phi(\hat{L}_n) - \phi(\Lambda)\right) \longrightarrow N(0, \Sigma)$$

where $\Lambda = \mathbb{E}_Q[L]$ and $\phi(\cdot)$ is the half vectorization.

Cor: $n\left(\phi(\hat{L}) - \phi(\Lambda_0)\right)^T \hat{\Sigma}^{-1}\left(\phi(\hat{L}) - \phi(\Lambda_0)\right) \xrightarrow{D} \chi^2_m$

$\hat{\Sigma}$ is sample covariance .... lots of issues here. Need sparse covariance estimation dua and Wolf, <u>Tony Cai</u> and collgues, sparse matrix estimation