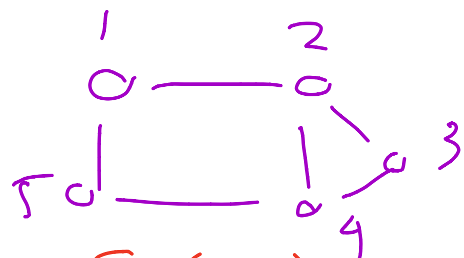
Last time we discussed the effect of conditioning and marginalizing over a set of graphical models.

If Gibbs



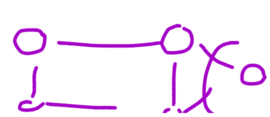
$$\mathbb{P}(x) = \frac{1}{Z}\prod_{c\in \mathcal{C}(G)} F_c(x_c)$$

we want $Z = \sum_{x} \prod_{c} F_c(x_c)$

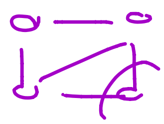
Fix a "good" order

x_3, x_1, x_2, x_5, x_1

$$Z = \sum_{x_1, x_2, x_4, x_5} F_{12}(\cdot)\, F_{15}(\cdot)\, F_{45}(\cdot) \underbrace{\sum_{x_3} F_{234}(\cdot)}_{f_{24}(x_2 x_4)}$$



$$= \sum_{x_1 x_2 x_5} F_{12} F_{15} \underbrace{\sum_{x_4} F_{45}(x_4, x_5) f_{24}}_{f_{25}(x_2, x_5)}$$



$$= \sum_{x_1 x_5} F_{12} \underbrace{\sum_{x_2} F_{12}(x_1, x_2) f_{25}(x_2, x_5)}_{f_{15}}$$

$$= \sum_{x_1} \underbrace{\sum_{x_5} F_{15}(x_1, x_5) \; f_{15}(x_1, x_5)}_{f_1}$$

$$= \sum_{x_1} f_1(x_1)$$

<u>Rmk</u>: The f functions are called "forward potentials".

If each $X_i \in S$ the a Brute force would take $O(|S|^5)$

but here we need $O(5|S|)$

<u>Rmk:</u> In general $O(|S|^n)$ versus $O(n|S|^k)$ k size of the largest marko blanket in the order.

What if we to estimate X

$$X^* = \operatorname*{argmax}_X \mathbb{P}(X)$$

$$\max_X \mathbb{P}(X) = \frac{1}{Z} \max_X \prod_c F_c(X_c)$$

$$= \max_{\substack{x_1 x_2 x_3 \\ x_4 x_5}} F_{12} F_{15} F_{45} F_{234}$$

We just need sub sums for max and keep track of max

$$\operatorname*{argmax}_{x_1,\dots,x_5} \mathbb{P}(X) = \operatorname*{argmax}_{x_1,\dots,x_5} F_{12}(\cdot) F_{15}(\cdot) F_{45}(\cdot) F_{234}$$

$$\max_{\substack{x_1 x_2 x_4 \\ x_5}} F_{12}(\cdot) F_{15}(\cdot) F_{45} \Big[\underbrace{\max_{x_3} F_{234}(x_2 x_3 x_4)}_{m_{24}} \Big]$$

$$b_3(x_2, x_4) = \operatorname*{argmax}_{x_3} F_{234}(\cdot)$$

$$= \max_{x_1 x_2 x_5} F_{12}(\cdot) F_{15}(\cdot) \max_{x_4} F_{45}(\cdot) m_{24}(\cdot)$$

$$\underbrace{\qquad\qquad\qquad}_{m_{25}}$$

$$b_4(x_2, x_4) = \arg\max_{x_4} F_{45}(\cdot)\, m_{24}(\cdot)$$

$$= \max_{x_1 x_5} F_{15}(\cdot) \underbrace{\max_{x_2} F_{12}\, m_{25}}_{m_{15}}$$

$$b_2(x_1 x_5) = \arg\max_{x_2} F_{12}(\cdot)\, m_{15}(\cdot)$$

$$= \max_{x_1} \underbrace{\max_{x_5} F_{15}\, m_{15}}_{m_1}$$

$$b_5(x_1) = \arg\max_{x_5} F_{15}\, m_{15}$$

$$= \max_{x_1} m_1(x_1) = \mathbb{P}(x^*)$$

then we "back track":

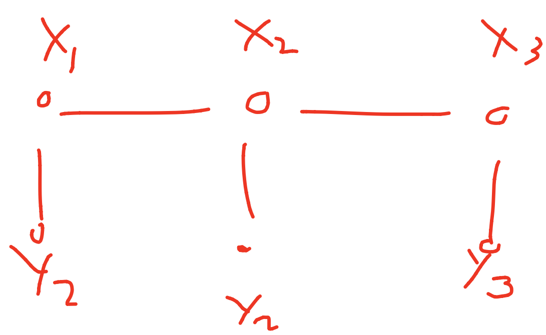
$$x_1^* = b_1, \; x_5^* = b_5(x_1^*)$$

$$x_2^* = b_2(x_1^*, x_5^*)$$

$$x_4^* = b_4(x_2^*, x_5^*)$$

$$x_3^* = b_3(x_2^*, x_4^*)$$

Ex: Hidden Markov Models



$$\mathbb{P}(Y) = \sum_{x} \mathbb{P}(Y|X)\,\mathbb{P}(X)$$

$$= \underbrace{\sum_{x_3} \underbrace{\sum_{x_2} \sum_{x_1} \underbrace{\mathbb{P}(x_1)\,\mathbb{P}(y_1|x_1)}_{f_1(x_1)}\,\mathbb{P}(x_2|x_1)\,\mathbb{P}(y_2|x_2)}_{f_2(x_2)}\,\mathbb{P}(x_3|x_2)\,\mathbb{P}(y_3|x_3)}_{f_3(x_3)}$$

$$f_1(x_1) = \mathbb{P}(x_1)\,\mathbb{P}(y_1|x_1)$$

$$f_i(x_i) = \sum_{x_{i-1}} \left[f_{i-1}(x_{i-1})\,\mathbb{P}(x_i|x_{i-1}) \right] \mathbb{P}(y_i|x_i)$$

$$=$$

$$\mathbb{P}(x_i,\, y_{1:i})$$