SDP

- More powerful than LPs for designing algorithms

  e.g. max cut

Maxcut Given G find $(S, \bar{S})$

s.t. $\dfrac{|E(S,\bar{S})|}{|E|}$ is maximized

Rmk: If G is bipartide,

$\text{maxcut}(G) = 1$

$$\text{relaxations} \left\{ \begin{array}{c} \text{LP} \\ \text{Spectral} \\ \text{SDPs} \end{array} \right\} \begin{array}{l} \text{no better} \\ \text{than} \\ 1/2 \text{ approx.} \end{array}$$

[illegible]

Trivial: Assign $v \in V$ to S

with prob $1/2$

$$\mathbb{E}(|E(S,\bar{S})|) = \sum_{e \in E} p(e \text{ cut})$$

$$= \frac{|E|}{2}$$

Linear Program

(mincut)

$$\min \frac{1}{2} \sum_{i,j \in E} \delta_{ij}$$

For all $\forall i \in V \quad x_i \in [-1, 1]$

$\bar{S} \quad S$

$$\forall ij \in E \quad \delta_{ij} \geq x_i - x_j$$

$$\delta_{ij} \geq x_j - x_i$$

better to look at mincut

$$\min \frac{1}{2} \sum |x_i + x_j| = \begin{cases} 0 & \text{cut} \\ 2 & \text{uncut} \end{cases}$$

with the relaxation

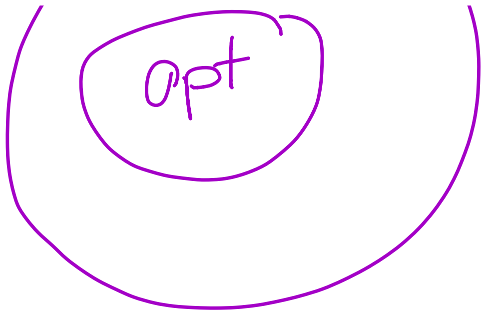
$$\min \frac{1}{2} \sum \delta_{ij}$$

s.t. $\delta_{ij} \geq x_i + x_j$

$\delta_{ij} \geq -x_i - x_j$

which we can reformulate as the max cut through

$$\max |E| - \frac{1}{2} \sum_{e} \delta_{ij}$$



≤ 1
LPmaxcut

Spectral Relaxation

$$\min \sum_E x_i x_j$$

$$\sum x_i^2 = |V| \qquad \text{ideally want } x_i = \pm 1 \ \forall i$$

or

$$\max \frac{1}{4} \sum (x_i - x_j)^2$$

$$\sum d_i x_i^2 = 2|E|$$

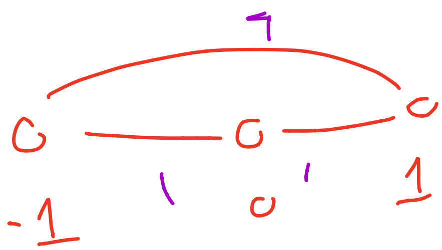
relaxation of Rayleigh Quotient

$$\max \frac{n}{4} \quad \frac{x^T L x}{x^T x}$$

$$= \frac{|E|}{2} \lambda_{max}(L)$$

Rmk: $\lambda_{max}(\mathcal{L}) = 2$ iff bipartite

Integrality Gap (for Spectral)



$$x^T D x = 2(1+0+1) = 4$$

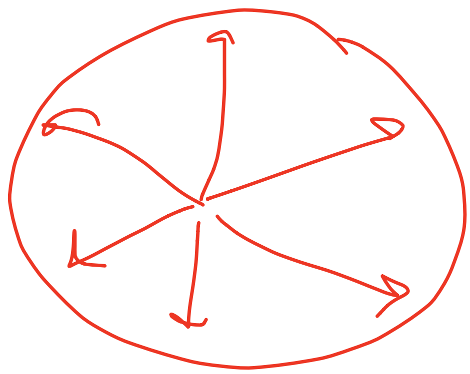
$$x^T L x = 6$$

Using the spectral version of the problem allows us to use

$$\max \frac{1}{4} \sum \|v_i - v_j\|^2$$

$$\text{s.t. } \|v_i\|^2 = 1$$

embedding in $\mathbb{R}^n$. Convex optimization over the sphere



Goemans-Williams shows that the integrability gap is 0.878.

<u>Rounding</u>: Choose a random hyperplane cut.