MA 575 HW 10

Dicussion Section 2: Monday 9:05

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Exercise 10.1

First we fit the random intercept model then fit the random slope/intercept model on the pig dataset.

```
#Read in data
pigs = read.csv(url("http://www.stat.tamu.edu/~sheather/book/docs/datasets/pigweights.csv"), he
library(nlme)

#fit random intercept model
ri_model = lme(weight~ weeknumber, data = pigs, random = ~1|pigid, method = "REML")
summary(ri_model)
```

```
## Linear mixed-effects model fit by REML
##  Data: pigs
##        AIC      BIC    logLik
##   2041.797 2058.052 -1016.898
##
## Random effects:
##  Formula: ~1 | pigid
##         (Intercept) Residual
## StdDev:    3.891253 2.096356
##
## Fixed effects: weight ~ weeknumber
##                 Value Std.Error  DF   t-value p-value
## (Intercept) 19.355613 0.6031390 383  32.09146       0
## weeknumber   6.209896 0.0390633 383 158.97012       0
##  Correlation:
##            (Intr)
## weeknumber -0.324
##
## Standardized Within-Group Residuals:
##         Min          Q1         Med          Q3         Max
## -3.73902210 -0.54562381  0.01835208  0.51221200  3.93133783
##
## Number of Observations: 432
## Number of Groups: 48
```

```
#fit random intercept and slope
ris_model = lme(weight ~ weeknumber, random=list(pigid = pdDiag(~ weeknumber)), data = pigs, me
summary(ris_model)
```

```
## Linear mixed-effects model fit by REML
##  Data: pigs
##        AIC      BIC    logLik
##   1751.029 1771.348 -870.5147
##
## Random effects:
##  Formula: ~weeknumber | pigid
##  Structure: Diagonal
##         (Intercept) weeknumber Residual
## StdDev:    2.630132  0.6135471  1.26443
##
## Fixed effects: weight ~ weeknumber
##                 Value Std.Error  DF  t-value p-value
## (Intercept) 19.355613 0.4021142 383 48.13462       0
## weeknumber   6.209896 0.0916386 383 67.76506       0
##  Correlation:
##            (Intr)
## weeknumber -0.075
##
## Standardized Within-Group Residuals:
##         Min          Q1         Med          Q3         Max
## -3.61354186 -0.54079895  0.01949449  0.54637821  3.01165383
##
## Number of Observations: 432
## Number of Groups: 48
```

(a)

From the output above, we see that the random intercept and slode model has $AIC = 1751.029$ and $logLik = -870.5147$ wich the random intercept model has $AIC = 2041.797$ and $logLik = -1016.898$. Therefor we see the random intercept model has higher AIC and lower log likelihood. This would suggest the second model is a better fit for the data.

(b)

While the AIC and log-likelihood suggest the random intercept and slope is a better fit to the data, the standard error for *weeknumber* is lower in the random intercept model than in the random slope and intercept model. This makes sense - as we assign more groupings to the data, the data to estimate each group slope reduces in power, so the overall estimate of the fixed effect is reduced.

Exercise 10.2

Suppose we have the random slopes model given by

$$y_{ij} = \beta_0 + \beta + 1t_{ij} + e^*_{ij}$$

For $e_{ij}^* = b_{0i} + b_{1i}t_{ij} + e_{ij}$ where $e_{ij} \overset{iid}{\sim} N(0, \sigma_e^2)$ are independent of $(b_{0i}, b_{1i})^T \overset{iid}{\sim} N(0, D)$ for the covariance matrix,

$$D = \begin{bmatrix} \sigma_0^2 & \sigma_{01}^2 \\ \sigma_{01}^2 & \sigma_1^2 \end{bmatrix}$$

Suppose that $j \neq j'$ then to calculate the correlation we first find the covariance below.

$$\begin{aligned}
Cov(e_{ij}^*, e_{ij'}^*) &= Cov(b_{0i} + b_{1i}t_{ij} + e_{ij}, b_{0i} + b_{1i}t_{ij'} + e_{ij'}) \\
&= Cov(b_{0i}, b_{0i}) + t_{ij'}Cov(b_{0i}, b_{1i}) + Cov(b_{0i}, e_{ij'}) + t_{ij}Cov(b_{1i}, b_{0i}) + t_{ij}t_{ij'}Cov(b_{1i}, b_{1i}) \\
&+ t_{ij}Cov(b_{1i}, e_{ij'}) + Cov(e_{ij}, b_{0i}) + t_{ij'}Cov(e_{ij}, b_{1i}) + Cov(e_{ij}, e_{ij'}) \\
&= \sigma_0^2 + t_{ij'}\sigma_{01}^2 + t_{ij}\sigma_{01}^2 + t_{ij}t_{ij'}\sigma_1^2 \\
&= \sigma_0^2 + \sigma_{01}^2(t_{ij'} + t_{ij}) + t_{ij}t_{ij'}\sigma_1^2
\end{aligned}$$

Using this same form for $j = j'$ we see that $Cov(e_{ij}^*, e_{ij'}^*) = Var(e_{ij}^*)$.

$$Var(e_{ij}) = t_{ij}^2\sigma_1^2 + 2t_{ij}\sigma_{01}^2 + (\sigma_0^2 + \sigma_e^2)$$

Using this we can write the correlation structure as

$$Corr(e_{ij}^*, e_{ij'}^*) = \frac{\sigma_0^2 + \sigma_{01}^2(t_{ij'} + t_{ij}) + t_{ij}t_{ij'}\sigma_1^2}{\sqrt{t_{ij}^2\sigma_1^2 + 2t_{ij}\sigma_{01}^2 + (\sigma_0^2 + \sigma_e^2)}\sqrt{t_{ij'}^2\sigma_1^2 + 2t_{ij'}\sigma_{01}^2 + (\sigma_0^2 + \sigma_e^2)}}$$