Characterization of Networks

- Questions of interest can be phrased in terms of structural properties

Ex: Social dynamics ≃ patterns of edges

routes for movement of info ≃ shortest paths

importance of vertices ≃ centrality

groups / communities ≃ graph partitioning

Main contributors: SNA, math/CS, statistical physics

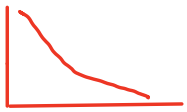
Def: Let f_d = prop. of vertices with degree d

Def: Vertex strength for a weighted graph is given by

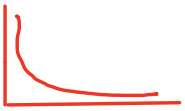
$$S_v = \sum_u A_{uv}$$

Rmk: Degree distributions fundamental objects

Good at discriminating between homogeneous/heterogeneous networks.



Exponential



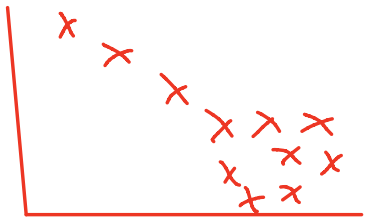
Power law

PL - several with low, with some high "hubbs"

Exp - more evenly dist.

Fitting Heterogeneous Degree Dist.

- Look at log-log distributions



- Difficult to fit this with a linear regression due to leverage
- Better (not good) estimators exist; Hill estimators
- Use as a descriptive tool

Centrality

- Questions related to importance of vertices
- No single definition or statistic

Def: Closeness : $Cen_j = \left(\sum_{i \neq j} \text{shortest path to } i \right)^{-1}$

betweenness: Counting # of shortest paths that go through j

eigenvector: Nodes importance dependent on importance of neighbors

$$c_i^e = \alpha \sum_{j \sim i} c_j^e \implies \frac{1}{\alpha} \vec{c} = A\vec{c}$$

page rank relates the RW to the eigen centrality

High order Centrality

- Grouping vertices then asking about things such as "Shortest path that goes through some/all of the vertices in the group"

Centrality in Di-Graphs

- Extensions are mostly straightforward

Kleinberg's HITS based on eigencentrality of $M_{hub} = AA^T$, $M_{auth} = A^TA$

Q: why do these include such things

Network Cohesion:

Q: · Are friends of friends, friends? (homophily)

- Similar vertices?
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