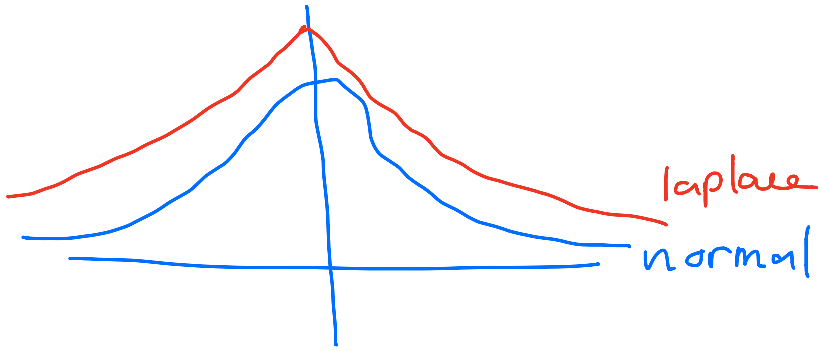
Ex: $X \sim N(0,1)$



Can we get a better envelop?

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

$$m g(x) = \frac{e^{1/2 - x}}{\sqrt{2\pi}} = \overbrace{\frac{e^{1/2}}{\sqrt{2\pi}}}^{m} \overbrace{e^{-x}}^{g(x)}$$

repeat

sample $X \sim \text{Exp}(1)$ $\quad U \sim U(0,1)$

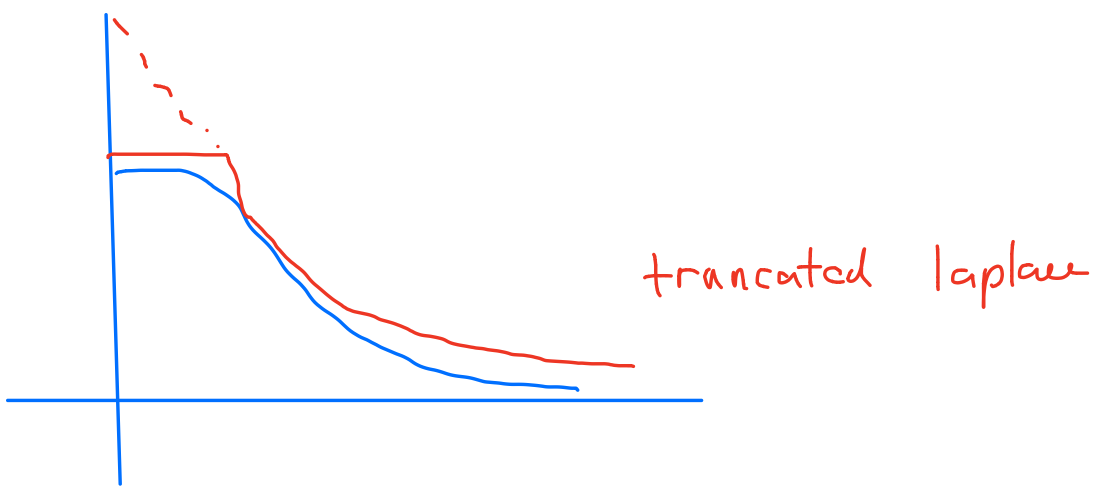
until $\quad U \leq \dfrac{f(x)}{m g(x)} = \dfrac{\frac{1}{\sqrt{2\pi}} e^{-x^2/2}}{\frac{1}{\sqrt{2\pi}} e^{1/2 - x}}$

$$= e^{-(x-1)^2/2}$$

Sample $S \sim \text{Bern}(1/2)$

$$X \leftarrow (2S-1)X$$

Improvement:



$$Mg(x) = \frac{1}{\sqrt{2\pi}} I\left(0 \le x \le \tfrac{1}{2}\right) + \frac{e^{1/2-x}}{\sqrt{2\pi}} I(x>1/2)$$

$$= \frac{1/2}{\sqrt{2\pi}} \underbrace{\frac{I(0 \le x \le 1/2)}{1/2}} + \frac{1}{\sqrt{2\pi}} \underbrace{e^{-(x-1/2)} I(x>1/2)}_{q_2 \sim \text{Exp}(1) + 1/2}$$

$g_1 \sim U(0, 1/2)$

$$= \left(\frac{0.5+1}{\sqrt{2\pi}}\right)\left[\frac{1/2}{1/2+1}\, g_1(x) + \frac{1}{0.5+1}\, g_2(x)\right]$$

$$= \frac{0.5+1}{\sqrt{2\pi}} \left\{ \underbrace{\lambda\, g_1(x) + (1-\lambda)\, g_2(x)}_{g(x)} \right\}$$

<u>repeat</u>

$B \sim U(0,1)$

If $B < \lambda$ then

sample $X \sim U(0, 1/2)$ $U \sim U(0,1)$

if $U \le \dfrac{f(x)}{M g(x)} = e^{-x^2/2}$ then <u>break</u>

else

Sample $X \sim \text{Exp}(1) + 0.5$ $\quad U \sim U(0,1)$

if $U \leq \frac{f(x)}{Mg(x)} = e^{-(x-1)^2/2}$ then break

until $S \sim \text{Bern}(1/2)$ then $X \leftarrow (2S-1)X$