3.9

$$\mathbb{P}(\sigma^2|y) \propto \mathbb{P}(y|\sigma^2)\mathbb{P}(\sigma^2) = \int \mathbb{P}(y|\mu,\sigma^2)\,\mathbb{P}(\mu|\sigma^2)d\mu\;\mathbb{P}(\sigma^2)$$

$$= \int \prod_{i=1}^{n} \mathbb{P}(y_i|\mu,\sigma^2)\,\mathbb{P}(\mu,\sigma^2)\,d\mu\;\;\mathbb{P}(\sigma^2)$$

$$\prod_{i=1}^{n} \mathbb{P}(y_i|\mu,\sigma^2) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n \exp\left(-\frac{1}{2\sigma^2}\left(\sum_{i=1}^{n}(y_i-\mu)^2\right)\right)$$

$$= \left(\frac{1}{2\pi\sigma^2}\right)^n \exp\left\{-\frac{1}{2\sigma^2}\left((n-1)\,s_y^2 + n(\bar{y}-\mu)^2\right)\right\}$$

$$= \left(\frac{1}{2\pi\sigma^2}\right)^n \exp\left\{-\frac{1}{2\sigma^2}\left((n-1)s_y^2\right)\right\} \exp\left\{\frac{-n}{2\sigma^2}(\bar{y}-\mu)^2\right\}$$

So

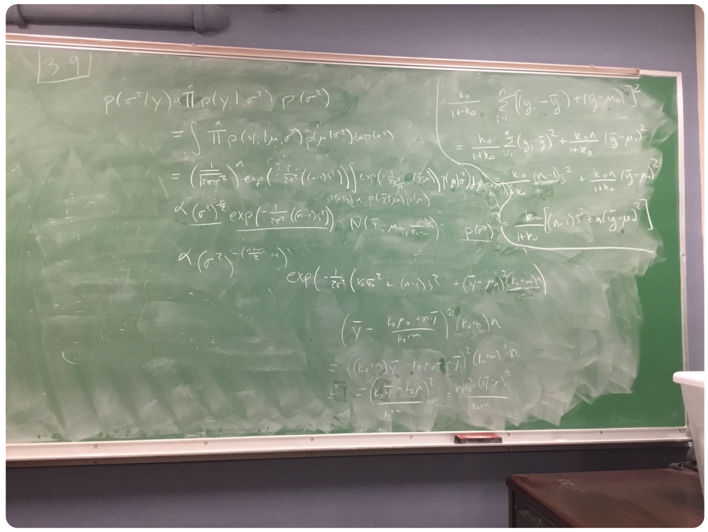
$$\int \prod_{i=1}^{n} \mathbb{P}(y_i|\mu,\sigma^2)\,p(\mu|\sigma^2)\,d\mu\;\;p(\sigma^2)$$

$$= \left(\frac{1}{2\pi\sigma^2}\right)^n \exp\left\{-\frac{1}{2\sigma^2}\left((n-1)s_y^2\right)\right\} \int \underbrace{\exp\left\{\frac{-1}{2\sigma^2/n}(\bar{y}-\mu)^2\right\}}_{\propto\,\mathbb{P}(\bar{y}|\mu,\sigma^2)} \underbrace{p(\mu|\sigma^2)}_{p(\mu|\sigma^2)}d\mu\;\;P(\sigma^2)$$

$$\Rightarrow \mathbb{P}(\mu|\bar{Y},\sigma^2) \propto \mathbb{P}(\bar{Y}|\mu,\sigma^2)\;\mathbb{P}(\mu|\sigma^2)$$

from part (a) we know

$\mu | y, \sigma^2 \sim N($



$\boxed{3.12}$

(a) Choose flat $p(\alpha,\beta) \propto 1$

(b) $\alpha = ?$, $\beta = -1$ if we know this

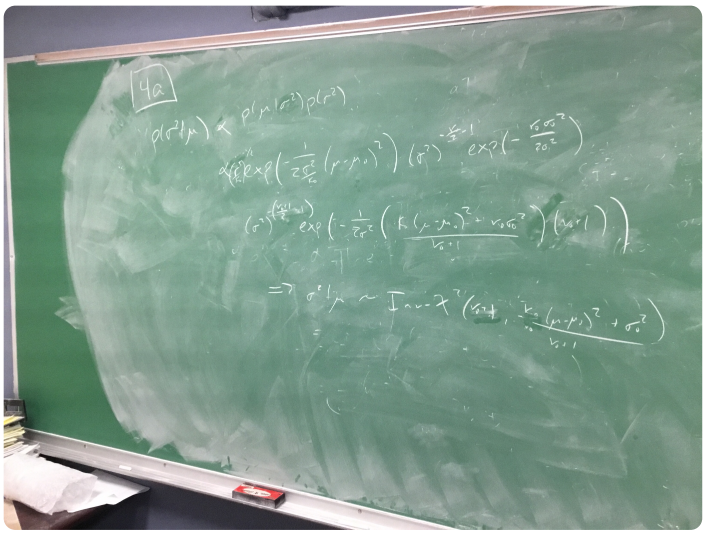
(c) $p(\alpha,\beta | y, t) \propto p(y|\alpha,\beta,t)\, p(\alpha,\beta)$

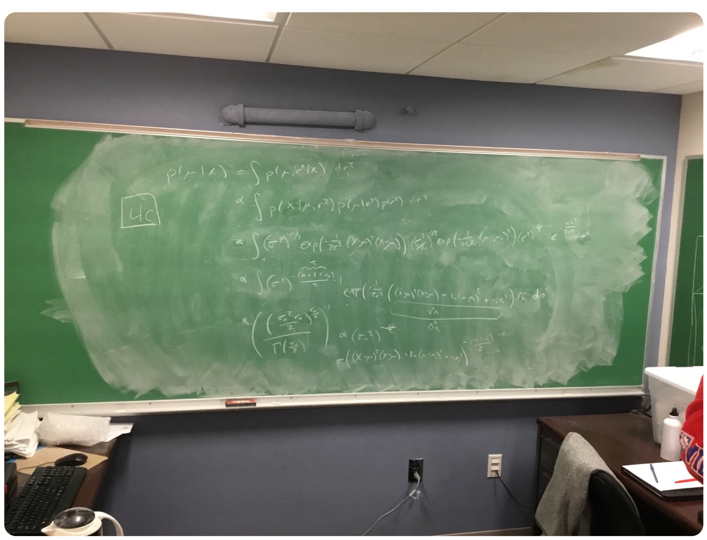
$$\propto p(\alpha,\beta) \prod_{i=1}^{n} p(y_i | \alpha \beta, t)$$

$$\propto 1 \cdot e^{n\alpha + \beta \sum t_i} \prod_{i=1}^{n} (\alpha + \beta t_i)^{y_i}$$

$$\propto \exp\left\{ n\alpha + \beta \sum t_i + \sum_{i=1}^{n} y_i \log(\alpha_i + \beta t_i) \right\}$$

4] (a)





BDA 3.12

$Y_i \sim \text{Poisson}(\lambda_i) \quad \lambda_i = \exp\{\alpha+\beta t_i\}$

$p(\alpha,\beta) \propto 1.$

$$p(\alpha,\beta \mid y) \propto \left(\prod_{i=1}^{n} e^{-\lambda} \lambda^{y_i}\right) p(\alpha,\beta)$$

$$\propto \prod_{i=1}^{n} \exp\left(-\exp(\alpha+\beta t_i)\right) \exp\left(y_i(\alpha+\beta t_i)\right)$$

$$\propto \exp\left\{-\sum_{i=1}^{n} \exp(\alpha+\beta t_i) + \alpha \sum y_i + \beta \sum y_i t_i\right\}.$$

$e^{\alpha} e^{\beta t_i}$