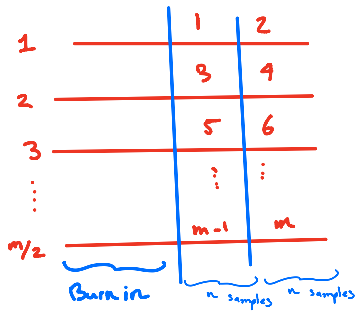
Convergence Criterion

Suppose we have $\psi^{(t)}$ samples



- MCMC samples are grouped in batches

$$\bar{\psi}_{\cdot j} = \frac{1}{n}\sum_{i=1}^{n}\psi_{ij}\ , \quad \bar{\psi} = \frac{1}{m}\sum_{j=1}^{m}\bar{\psi}_{\cdot j} \quad \text{unbiased for } \mathbb{E}[\psi|Y]$$

$$\widehat{Var}[\psi|Y] = \frac{1}{nm}\sum_{j=1}^{m}\Big[\underbrace{(\psi_{ij}-\bar{\psi}_{\cdot j})^2}_{\text{within}} + \underbrace{(\bar{\psi}_{\cdot j}-\bar{\psi})^2}_{\text{between}}\Big]$$

Unbiased variance estimate

$$\widehat{Var}^{+}[\psi|Y] = \frac{n-1}{n}\cdot\underbrace{\frac{1}{m}\sum_{j=1}^{m}\cdot\frac{1}{n-1}\sum_{i=1}^{n}(\psi_{ij}-\bar{\psi}_{\cdot j})^2}_{W} + \frac{1}{n}\cdot\underbrace{\frac{n}{m-1}\sum_{j=1}^{m}\big[(\bar{\psi}_{\cdot j}-\bar{\psi})^2\big]}_{B}$$

$$= \frac{n-1}{n}W + \frac{1}{n}B$$

Also estimates $Var[\psi|Y]$, $\mathbb{E}_{\psi|Y}[W] = \mathbb{E}_{\psi|Y}[B] = Var[\psi|Y]$

<u>Rmk:</u> $\widehat{Var}^{+}[\psi|Y] \longrightarrow Var(\psi|Y)$ as $n \longmapsto \infty$

Gelman & Rubin $\quad R = \sqrt{\dfrac{\widehat{Var}^{+}(\psi|Y)}{W}}$ <u>potential scale reduction</u>

$R = 1$ if chains are independent.

<u>Rmk:</u> Use R as a convergence criterion.

Rule of thumb: $\hat{R} < 1.1$

If ψ_{ij} are independant then $Var[\bar{\Psi}|Y] = \frac{1}{nm} Var(\psi|Y)$

As they are not,

$$\lim_{n \to \infty} nm \, Var[\bar{\Psi}|Y] = \left(1 + 2\sum_{t=1}^{\infty} \rho_t\right) Var(\psi|Y)$$

where $\rho_t = AutoCorr(t)$. Hence

$$Var[\bar{\Psi}|Y] = \frac{Var[\psi|Y]}{\boxed{n_{eff}}} \qquad n_{eff} = \frac{mn}{1 + 2\sum_{t=1}^{\infty} \rho_t}$$

Estimates: $\hat{n}_{eff} = \frac{nm}{1 + 2\sum_{t=1}^{T} \hat{\rho}_t}$ for large t.