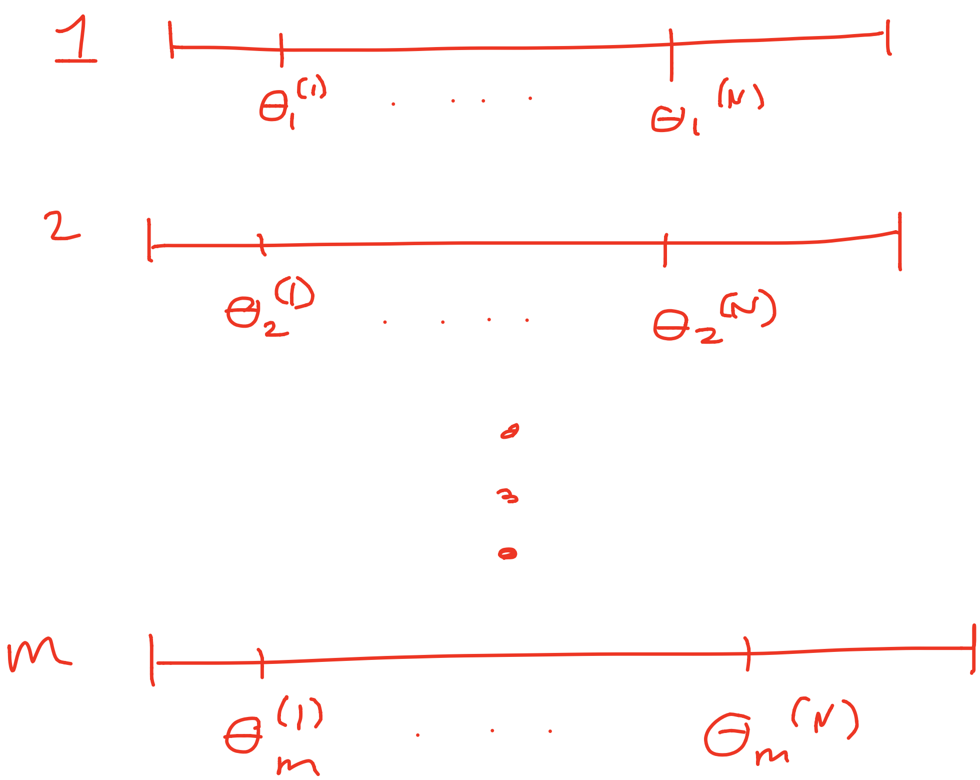
MCMC Convergence

Goal: infer θ

To assess convergence, sample parallel chains



Compute:

$$\bar{\theta}_j = \frac{1}{N} \sum_{t=1}^{N} \theta_j^{(t)}$$

$$s_j^2 = \frac{1}{N-1} \sum_{t=1}^{N} \left(\theta_j^{(t)} - \bar{\theta}_j\right)^2$$

$$B = \frac{N}{m-1} \sum_{j=1}^{m} (\bar{\theta}_j - \bar{\theta})^2$$

$$\bar{\theta} = \frac{1}{m} \sum_{j=1}^{m} \bar{\theta}_j$$

$$W = \frac{1}{m} \sum_{j=1}^{m} s_j^2$$

$B =$ between chain variance

$W =$ within chain variance

$$R = \frac{\frac{N-1}{N} W + \frac{1}{N} B}{W}$$

Scale reduction variance

If $B \simeq 0$ then $R \simeq 1$ and we see there is no inbetween chain variance.

Rule of thumb: $\hat{R} < 1.1$

Variance Estimation

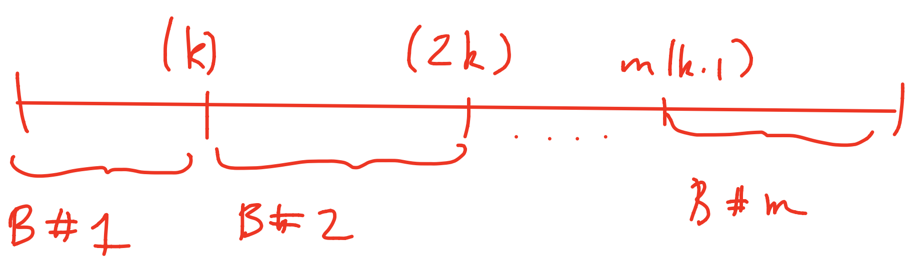
If $\theta^{(t)}$ independent, then

$$\hat{Var}(\bar{\theta}) = \frac{s_\theta^2}{N} = \frac{1}{N} \frac{1}{N-1} \sum_{i=1}^{n} (\theta^{(t)} - \bar{\theta})^2$$

but $\theta^{(t)}$ are autocorrelated.

Idea: "Batches"

Split chain into m batches of size k.



Compute $B_j = \frac{1}{k} \sum_{t=(j-1)k+1}^{jk} \theta^{(t)}$

and the following estimate is

$$\widehat{Var}_{Batch}(\bar{\theta}) = \frac{1}{m}\frac{1}{m-1}\sum_{j=1}^{m}(B_j - \bar{B})^2$$

Want to choose k & m such that autocorrelation is small.

Another Estimate

$$\widehat{Var}(\bar{\theta}) = \frac{S_\theta^2}{ESS(\theta)}$$

$$ESS(\theta) = \frac{N}{K(\theta)}$$

$$K(\theta) = 1 + \sum_{l=1}^{\infty} \rho_l(\theta)$$

l lagged autocorrelation

Rule of thumb:

$$\frac{ESS(\theta)}{N} = \frac{1}{K(\theta)} > 0.5$$

- Use ACF plot for diagnostics for MCMC convergence.