Project Presentations

- 10 minutes Presen, 5 mintes Q/A
  - Context
  - Goals
  - Summarize Progress
  - To Dos.
- 5 minutes
  - Attendance / participation accounted

Network Flows

- Flows - processes over networks
- Common Problems
  - Origin-Destination (OD) flow volume
  - Link volumes
  - OD and/or link flows
- Notation
  - G - directed
  - Routes over G by which flows move from origin to destination by routing matrix

$$(B_e)_{ij} = \begin{cases} 1 & e \text{ traversed in going from } i \to j \\ 0 & \text{o.w.} \end{cases}$$

- Flow volumes Z_{ij} from to $i \to j$, X_e volume on arc e

$$X = BZ$$

Gravity Models

- Let $\mathcal{I}$ = origins, $\mathcal{J}$ = destination
- Z_{ij} flows from $i \in \mathcal{I} \to j \in \mathcal{J}$.
- Specifying Z_{ij} as Poisson counts $\mathbb{E}(Z_{ij}) = h_O(i) h_D(j) h_S(i,j)$
- Netwons law analogy (classical)

$$\mathbb{E}[Z_{ij}] = \gamma \, \pi_{O,i} \pi_{D,i} \, d_{ij}^{-2}$$

γ — gravitational constant

- Standard: $h_O(i) = (\pi_{O,i})^{\alpha}$ $h_D(j) = (\pi_{D,j})^{\beta}$ $h_S(i,j) = \exp(\theta^T c_{ij})$

$$\log \mathbb{E}[Z_{ij}] = \alpha \log \pi_{O,i} + \beta \log \pi_{D,i} + \theta^T c_{ij}$$] invokes GLM estimation

ML:

$$\ell(\mu) = \sum_{\substack{i \in \mathcal{I} \\ j \in \mathcal{J}}} Z_{ij} \log \mu_{ij} - \mu_{ij} \qquad \Rightarrow \hat{\mu}_{ij} = \hat{\alpha}_i \hat{\beta}_j \exp(\hat{\theta}^T c_{ij})$$

satisfying certain ML constraints.

– Under mild conditions; well defined, $\hat{\theta}_k, \hat{\mu}_{ij}$ will be unique.

Traffic Matrix Estimation