Regression (X_i, Y_i), $Y_i = X_i^T \beta + e_i$

$\mathbb{E}[e_i | X_i] = 0$

$Var(e_i | X_i) = \sigma^2$

$Cov(e_i, e_j | X_i, X_j) = 0$

(i) Non parametric bootstrap

Bootstrap $\{(X_i, Y_i)\} \longrightarrow \{(X_i^*, Y_i^*)\}$

Assume that $(X_i, Y_i) \overset{iid}{\sim} F$

(ii) Parametric Bootstrap:

Take estimates $\hat{\beta}$, $\hat{\sigma}^2$, $X_i^* = X_i$,

$$Y_i^* = X_i^T \hat{\beta} + \hat{e}_i^*$$

$$\hat{e}_i^* \overset{iid}{\sim} N(0, \hat{\sigma}^2)$$

Make inferences on (x_i^*, y_i^*)

Assumptions: $y_i = x_i^T \beta + e_i$, $e_i | x_i \overset{iid}{\sim} N(0, \sigma^2)$

(iii) "Residual" Bootstrap

$\{ \hat{e}_i = y_i - \hat{y}_i \} \xrightarrow{\text{Boot}} \{ \hat{e}_i^* \}$

then $x_i^* = x_i$, $y_i^* = \hat{y}_i + \hat{e}_i^*$

assumption: $e_i | x_i \overset{iid}{\sim} F_e$

In all cases, obtain bootstrap samples $\hat{\beta}^*$ by regressing

$y^* \sim x^*$