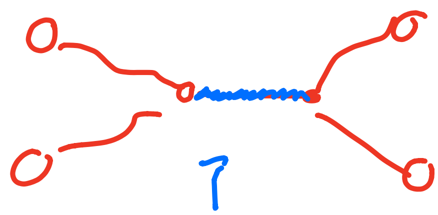
Traffic Matrix Estimation



Let z_{ij} be the volume of flow from $i \to j$.

Edge weights are combinations of these flows.

Let X_e be the flow on link e.

Goal: Use X to infer Z.

$X = BZ$ $\quad$ [—— B ——] $N_e \times$ # of origin-dest pairs

Large $p \gg n$ type problem.

Two solutions: — Gaussian Measurement Models

— Static / Dynamic approaches

Consider the model $X = B\mu + \varepsilon$, $\varepsilon \sim N(0, \sigma^2)$

— Often times need penalization / Bayesian approach

Consider the model

$$X \mid \mu \sim N(B\mu, \sigma^2 I)$$
$$\mu \mid \mu_0 \sim N(\mu_0, \tau^2 I)$$

Produces a Gaussian Post with

$$\mathbb{E}[\mu \mid X] = \mu^{(0)} + B^T (BB^T + \lambda I)^{-1} (x - B\mu^{(0)}) \qquad \lambda = \sigma^2 / \tau_0^2$$

$$Var(\mu \mid X = x) = \tau^2 [I - B^T (BB^T + \lambda I)^{-1} B]$$

<u>Maximization Entropy Regularization</u>

$$D(\mu \| \mu^0) = \sum_{ij} \frac{\mu_{ij}}{\mu_{++}} \log \frac{\mu_{ij}}{\mu_{ij}^{(0)}}$$ measures divergence of a certain dist.

Goal: $\min D(\mu \| \mu^{(0)})$ w.r.t. $(X - B\mu)^T (x - B\mu) \leq C$

equivalent to

$$\min_{\mu \geq 0} (X - B\mu)^T (x - B\mu) + \lambda D(\mu \| \mu^{(0)})$$

- All of these methods are static
  - ↳ want to do these in time

Kalman Filtering

$$\left.\mu^{(t+1)} = \Xi^{(t)} \mu^{(t)} + \eta^{(t)} \right]$$ structure added

$$X^{(t)} = B^{(t)} \mu^{(t)} + \varepsilon^{(t)}$$

Assume $\eta^{(t)} \sim f(0, \Psi^{(t)})$

$\varepsilon^{(t)} \sim f(0, \Sigma^{(t)})$

Blue Predictors

- $\hat{\mu}^{t+1|t}$ from $x^{(1)}, \ldots, x^{(t)}$

- update prediction to adjust for $B\hat{\mu}^{(t+1)} - x$ (observ)

- Steps can have the form from static penalization

- Σ, Ψ often assumed known

- Uses variants of EM.