

Network Kriging

- End-to-End monitoring

↳ #paths $\propto O(N_v^2)$ infeasible

- Let \mathcal{P} be the set of all paths

Let $G_{ij} = \begin{cases} 1 & \text{if path } i \text{ traverses } j \\ 0 & \text{o.w.} \end{cases}$

Want to consider $y = Gx$ but only have access to some paths due to complexity.

$$\begin{matrix} \Rightarrow \\ \Rightarrow \\ \Rightarrow \\ \Rightarrow \end{matrix} \begin{bmatrix} | \\ | \\ | \\ | \end{bmatrix} = \begin{bmatrix} | \\ | \\ | \\ | \end{bmatrix} \begin{bmatrix} | \\ | \end{bmatrix}$$

Remark: Only need $O(N_e)$ to exactly recover y .

Q: Approximation error and statistical error when sampling less than $O(N_e)$

Path redundancy: key here. Measure that using SVD of G .

$$r_1 = \arg \max_{x \neq 0} \frac{x^T G^T G x}{x^T x} = \arg \max_{y(x)} y(x)^T y(x) = \text{path volume.}$$

Partial Characterization

$$B = G^T G \text{ non-decreasing diag}$$

$$\lambda(B) = \lambda_1 \geq \dots \geq \lambda_{N_e}$$

$$\frac{\lambda_k}{\lambda_1} \leq \frac{b_{kk}}{b_{11}} \dim(G)$$

- Betweenness centrality Controls decay
- Lots of optimal linear prediction stuff.

Path selection

- Minimise $\text{MSE}(a^T y_s)$
- Subset selection problem - NP complete
- Choose k paths such that they span the first k singular directions