

A simple bound on  $L$

Lemma:  $L_G \preceq 2D$

Pf:  $x^T L_G x = \sum_{i,j \in E} (x_i - x_j)^2$

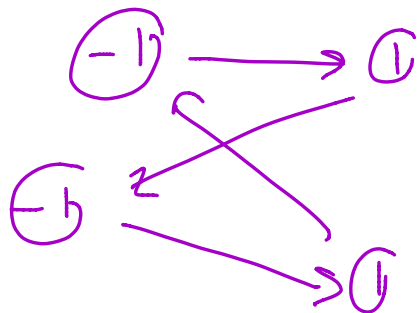
tight when  $x_i = -x_j$   $\leq \sum_{i,j} 2(x_i^2 + x_j^2)$

$$= 2 \sum_{i \in E} d_i x_i^2$$

$$= 2x^T D x$$

Lemma: If there exists  $x \neq 0$

$$x^T L_G x = 2x^T D x \iff x_i = -x_j$$



Bipartite graph.

For  $\lambda_2 = 0$  then  $G$  is disconnected

For  $\lambda_2 = 2 \iff G$  is bipartite.

Next time we will generalize for

(i) Robust clustering (ii) Robust approx  
to Maxcut