1. (a) Recall that our motivation for defining the leverage was to see the effect of each point (x_i, y_i) in our sample on a predicted value $\hat{y}_i$. Thus, expanding our estimate of $\hat{y}_i$ we have

$$\begin{aligned}
\hat{y}_i &= \hat{\beta}_0 + \hat{\beta}_1 x_i \\
&= \overline{y} - \hat{\beta}_1\overline{x} + \hat{\beta}_1 x_i \\
&= \overline{y} + \hat{\beta}(x_i - \overline{x}) \\
&= \overline{y} + \sum_{j=1}^{n} c_j y_j (x_i - \overline{x}) \\
&= \frac{1}{n}\sum_{j=1}^{n} y_j + (x_i - \overline{x})\sum_{j=1}^{n}\frac{(x_j - \overline{x})}{SXX}y_j \\
&= \frac{1}{n}\sum_{j=1}^{n} y_j + (x_i - \overline{x})\sum_{j=1}^{n}\frac{(x_j - \overline{x})}{SXX}y_j \\
&= \sum_{j=1}^{n}\left(\frac{1}{n} + \frac{(x_i - \overline{x})(x_j - \overline{x})}{SXX}\right)y_j
\end{aligned}$$

From here, we define $h_{ij} = \frac{1}{n} + \frac{(x_i-\overline{x})(x_j-\overline{x})}{SXX}$ and the leverage of point i as $h_{ii} = \frac{1}{n} + \frac{(x_i-\overline{x})^2}{SXX}$. This is the (marginal) effect of point y_i on the predicted value $\hat{y}_i$.

(b) High leverage points can be identified by looking at which points maximize h_{ii}. $\frac{1}{n}$ and SXX are constants in the data set, so we look for points that have large $(x_i - \overline{x})^2$. In a plot, we can find these by looking at those x values are that are far from the mean of x. That is, points with high leverage will be those that are absolutely "far" from the sample mean, $\overline{x}$.

(c) Suppose we are given a data set $\{(x_i, y_i) : i = 1, 2, \ldots, n\}$. Then, having $\overline{x}$ and SXX fixed, if there exists a point x_i with $h_{ii} = 1$ then

$$\begin{aligned}
1 &= \frac{1}{n} + \frac{(x_i - \overline{x})^2}{SXX} \\
\frac{n-1}{n}SXX &= (x_i - \overline{x})^2 \\
\sqrt{\frac{n-1}{n}SXX} &= x_i - \overline{x} \\
\sqrt{\frac{n-1}{n}SXX} + \overline{x} &= x_i
\end{aligned}$$