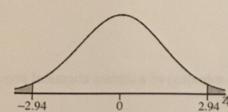
7.2.12. Determine the total area under the standard normal curve

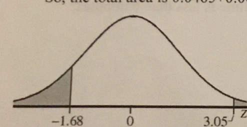
- **(a)** to the left of $z = -2.94$ or to the right of $z = 2.94$
- **(b)** to the left of $z = -1.68$ or to the right of $z = 3.05$
- **(c)** $z = -0.88$ or to the right of $z = 1.23$

Solution:

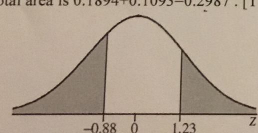
(a) The area to the left of $z = -2.94$ is 0.0016, and the area to the right of $z = 2.94$ is $1-0.9984=0.0016$. So, the total area is $0.0016+0.0016=0.0032$. [Tech: 0.0033]



(b) The area to the left of $z = -1.68$ is 0.0465, and the area to the right of $z = 3.05$ is $1-0.9989=0.0011$. So, the total area is $0.0465+0.0011=0.0476$.



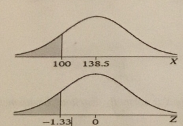
The area to the left of $z = -0.88$ is 0.1894, and the area to the right of $z = 1.23$ is $1-0.8907=0.1093$. So, the total area is $0.1894+0.1093=0.2987$. [Tech: 0.2988]



7.2.40. Wendy's Drive-Through Fast-food restaurants spend quite a bit of time studying the amount of time cars spend in their drive-throughs. Certainly, the faster the cars get through the drive-through, the more the opportunity for making money. *QSR Magazine* studied drive-through times for fast-food restaurants and found Wendy's had the best time, with a mean time spent in the drive-through of 138.5 seconds. Assuming drive-through times are normally distributed with a standard deviation of 29 seconds, answer the following.

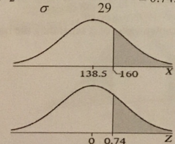
- **(a)** What is the probability that a randomly selected car will get through Wendy's drive-through in less than 100 seconds?
- **(b)** What is the probability that a randomly selected car will spend more than 160 seconds in Wendy's drive-through?
- **(c)** What proportion of cars spend between 2 and 3 minutes in Wendy's drive-through?
- **(d)** Would it be unusual for a car to spend more than 3 minutes in Wendy's drive-through? Why?

Solution: **(a)** $z = \frac{x-\mu}{\sigma} = \frac{100-138.5}{29} = -1.33.$



From Table V, the area to the left of $z = -1.33$ is 0.0918, so $P(X < 100) = 0.0918$. [Tech: 0.0922]

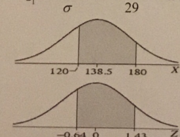
(b) $z = \frac{x-\mu}{\sigma} = \frac{160-138.5}{29} = 0.74.$



From Table V, the area to the left of $z = 0.74$ is 0.7704, so $P(X > 160) = 1 - 0.7704 = 0.2296$. [Tech: 0.2292]

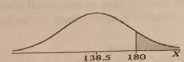
(c) Note that 2 minutes = 120 seconds and 3 minutes = 180 seconds.

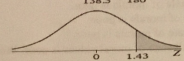
$z_1 = \frac{x_1-\mu}{\sigma} = \frac{120-138.5}{29} = -0.64$, $z_2 = \frac{x_2-\mu}{\sigma} = \frac{180-138.5}{29} = 1.43.$



From Table V, the area to the left of $z_1 = -0.64$ is 0.2611 and the area to the left of $z_2 = 1.43$ is 0.9236, so $P(120 \le X \le 180) = 0.9236 - 0.2611 = 0.6625$. [Tech: 0.6620]

(d) Note that 3 minutes = 180 seconds. $z = \frac{x-\mu}{\sigma} = \frac{180-138.5}{29} = 1.43.$





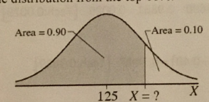
From Table V, the area to the left of $z = 1.43$ is 0.9236, so $P(X \geq 180) = 1 - 0.9236 = 0.0764$.
No, it would not be unusual for a car to spend more than 3 minutes (180 seconds) in Wendy's drive-through.
About 8 cars in 100 will spend more than 3 minutes in the drive-through.

7.2.48. Reading Rates The reading speed of sixth-grade students is approximately normal, with a mean speed of 125 words per minute and a standard deviation of 24 words per minute.

- **(a)** What is the reading speed of a sixth-grader whose reading speed is at the 90th percentile?
- **(b)** A school psychologist wants to determine reading rates for unusual students (both slow and fast). Determine the reading rates of the middle 95% of all sixth-grade students. What are the cutoff points for unusual readers?

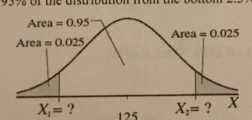
__Solution:__

(a) The figure below shows the normal curve with the unknown value of X separating the bottom 90% of the distribution from the top 10%.



From Table V, the area closest to 0.90 is 0.8997, which corresponds to the z-score 1.28.
So, the 90th percentile for the reading speed of sixth-grade students is $x = \mu + z\sigma = 125 + 1.28(24) \approx 156$ words per minute.

(b) The figure that follows shows the normal curve with the unknown values of X separating the middle 95% of the distribution from the bottom 2.5% and the top 2.5%.



From Table V, the area 0.0250 corresponds to the z-score -1.96.
Likewise, the area $0.0250 + .095 = 0.975$ corresponds to the z-score 1.96.
Now, $x_1 = \mu + z_1\sigma = 125 + (-1.96)(24) \approx 78$ and
$x_2 = \mu + z_2\sigma = 125 + 1.96(24) \approx 172$.
So, the cutoffs for unusual reading times are 78 and 172 words per minute.

7.4.22. Morality In a recent poll, the Gallup Organization found that 45% of adult Americans believe that the overall state of moral values in the United States is poor. Suppose a survey of a random sample of 500 adult Americans is conducted in which they are asked to disclose their feelings on the overall state of moral values in the United States. Use the normal approximation to the binomial to approximate the probability that

- **(a)** exactly 250 of those surveyed feel the state of morals is poor.
- **(b)** no more than 220 of those surveyed feel the state of morals is poor.
- **(c)** more than 250 of those surveyed feel the state of morals is poor.
- **(c)** between 220 and 250, inclusive, believe the state of morals is poor.
- **(e)** at least 260 adult Americans believe the overall state of moral values is poor. Would you find this result unusual? Why?

Solution:

From the parameters $n = 500$ and $p = 0.45$ we get $u_X = np = 500 \cdot 0.45 = 225$ and $\sigma_X = \sqrt{np(1-p)} = \sqrt{500 \cdot 0.45 \cdot (1-0.45)} = \sqrt{123.75} \approx 11.1243$. Note that $np(1-p) = 123.75 \geq 10$,

so the normal approximation to the binomial distribution can be used.

(a). $P(250) \approx P(249.5 \leq X \leq 250.5)$

$$= P\left(\frac{249.5-225}{\sqrt{123.75}} \leq Z \leq \frac{250.5-225}{\sqrt{123.75}}\right)$$

$$= P(2.20 \leq Z \leq 2.29) = 0.9890 - 0.9861 = 0.0029 \quad [\text{Tech:}0.0029]$$

(b). $P(X \leq 220) \approx P(X \leq 220.5)$

$$= P\left(Z \leq \frac{220.5-225}{\sqrt{123.75}}\right) = P(Z \leq -0.40) = 0.3446 \quad [\text{Tech:}0.3429]$$

(c). $P(X > 250) \approx P(X \geq 250.5)$

$$= P\left(Z \geq \frac{250.5-225}{\sqrt{123.75}}\right) = P(Z \geq 2.29)$$

$$= 1 - 0.9890 = 0.0110 \quad [\text{Tech:}0.0109]$$

(d). $P(219.5 \leq X \leq 250.5)$

$$= P\left(\frac{219.5-225}{\sqrt{123.75}} \leq Z \leq \frac{250.5-225}{\sqrt{123.75}}\right)$$

$$= P(-0.49 \leq Z \leq 2.29) = 0.9890 - 0.3121 = 0.6769 \quad [\text{Tech:}0.6785]$$

(e). $P(X \geq 260) \approx P(X > 259.5)$

$$= 1 - P(X \leq 259.5) = 1 - P\left(Z \leq \frac{259.5-225}{\sqrt{123.75}}\right)$$

$$= 1 - P(Z \leq 3.10) = 1 - 0.9990 = 0.0010 \quad [\text{Tech:}0.0010]$$

This is less than 0.05, so it would be unusual for at least 260 out of 500 adult Americans to indicate that they believe the overall state of moral values is poor.