Recall for bootstrapping

$$X = (X_1, \dots, X_n) \overset{iid}{\sim} F$$

Want to estimate $\theta(F)$

<u>Idea</u>: plug-in estimator

$$\hat{\theta} = T(\hat{F}) \doteq g(X)$$

<u>Goal</u>: Estimate $\mathrm{Var}_F(\hat{\theta})$

good place to start is

$$\mathrm{Var}_F(\hat{\theta}) \simeq \mathrm{Var}_{\hat{F}}(\hat{\theta}) \simeq \widehat{\mathrm{Var}_{\mathrm{Boot}}}(\hat{\theta})$$

The "<u>bootstrap principle</u>" can be illustrated as the following

	CDF	Sample	Stat.
Real World	F	X	$\hat{\theta} = g(X)$
Boot Pop	$\hat{F}$	X^*	$\hat{\theta}^* = g(X^*)$

Types of Bootstraps

(0) Nonparametric: $\hat{F}(x) = \frac{1}{n} \sum_{i=1}^{n} I(x_i \le x)$

(i) Parametric $X_1, \dots, X_n \overset{iid}{\sim} F_\gamma$

then we get an estimate $\hat{\gamma}$

and bootstrap sample from $F_{\hat{\gamma}}$.

(ii) Bayesian

- put a "prior" on $\hat{F}$ to get $\hat{F}_{Bayes}$ and a posterior on X
- According to $\hat{F}$ each X_i gets mass $1/n$
- He instead puts random probabilities of sampling $X_i \sim p_i$

- Each iteration
  - $U_1, \dots, U_{n-1} \sim U(0,1)$
  - $U_{(1)}, \dots, U_{(n-1)}$
  - Define $U_{(0)} = 0$, $U_{(n)} = n$ then

    $p_i = U_{(i)} - U_{(i-1)}$

It can be shown $\mathbb{E}[p_i] = \frac{1}{n}$