Dynamic & Network Processes

- Dynamic Network – Indexed Processes
  - many complex systems are dynamic
  - Dynamics <u>of</u> the network + dynamics on the network
    - ↳ models for both and models jointly
- Processes on Network Graphs
  - dynamics a function of the network structure
  - Epidemic modeling
    - (a) Spread mechanism
    - (b) prediction
    - (c) control

SIR

- no network models
- $N + \underbrace{1}_{\text{infected}}$ population

Keep track of $(N_S(t), N_I(t), N_R(t))$ $\quad N+1 = N_S(t) + N_I(t) + N_R(t)$

susceptible $\quad$ infected $\quad$ removed

Model these in continuous time over discrete space.

Instantaneous Trans Prob.

$$\mathbb{P}(N_S(t+\delta t) = s-1,\ N_I(t+1) = i+1 \mid (s,i)) \simeq \beta s i \delta t$$

$$\mathbb{P}(N_S(t+\delta t) = s,\ N_I(t+1) = i-1 \mid (s,i)) \simeq \gamma i \delta t$$

$$\mathbb{P}(N_S(t+\delta t) = s,\ N_I(t+1) = i \mid (s,i)) \simeq 1 - \beta s i \delta t - \gamma i \delta t$$

model assumes homogeneous mixing – well mixed

Goal: model bivariate Stochastic Process $(N_S(t), N_I(t))$ (eventually with network structure)

Basic Reproduction # $R_0 = \frac{N\beta}{\gamma}$ = num. infected in early epidemic

Whittle: $R_0 \leq 1$ no chance of epidemic

$R_0 > 1$ epidemic with prob. $1 - \frac{1}{R_0}$

Branching Process analogue.

Introduce a network to get around homogenous mixing

Network modification to SIR

$$\mathbb{P}\left(X(t+\delta t)=x' \,\middle|\, X(t)=x\right) \propto \begin{cases} \beta m_i(x)\,\delta t & x_i=0 \quad x_i'=1 \\ \gamma\,\delta t & x_i=1 \quad x_i'=2 \\ 1-\beta m_i(x)\,\delta t-\gamma\,\delta t & \end{cases}$$

$m_i(x) = \#$ of neighbors

Under this model

$$R_0 = \frac{\beta}{\beta+\gamma}\left[\frac{\mathbb{E}[d^2]}{\mathbb{E}[d]} - 1\right]$$