$$\mathcal{L}(\rho,\beta,\sigma^2 \mid y) = \prod_{j=1}^{J} (2\pi)^{-n_j/2} \, |R_j(\rho)|^{-1/2} \exp\left\{-\frac{1}{2}\left[(y_j - X_j\beta)^T \frac{R_j^{-1}(\rho)}{\sigma^2} (y_j - X_j\beta)\right]\right\}$$

$$\ell(\rho,\beta,\sigma^2 \mid y) = \sum_{j=1}^{J} \left\{ \frac{-n_j}{2} \log 2\pi - \frac{1}{2}\log|R_j(\rho)| - \frac{1}{2}\left[(y_j - X_j\beta)^T \frac{R_j^{-1}(\rho)}{\sigma^2}(y_j - X_j\beta)\right]\right\}$$

$$\frac{\partial \ell(\rho,\beta,\sigma^2 \mid y)}{\partial \rho} = \sum_{j=1}^{J} \frac{-1}{2}\frac{\partial}{\partial\rho}\log|R_j(\rho)| - \frac{1}{2}\frac{\partial}{\partial\rho}\left[(y_j - X_j\beta)^T \frac{R_j^{-1}(\rho)}{\sigma^2}(y_j - X_j\beta)\right]\}$$

$$\frac{\partial}{\partial\rho}\left[(n_j-1)\log(1-\rho) + \log(1+(n_j-1)\rho)\right] = -\frac{n_j-1}{1-\rho} + \frac{n_j-1}{1+(n_j-1)\rho}$$

$$\frac{\partial}{\partial\rho}\left[(y_j - X_j\beta)^T \left[\frac{\frac{1}{1-\rho}\left(I_n - \frac{\rho}{1+(n-1)\rho}1_{n_j}1_{n_j}^T\right)}{\sigma^2}\right](y_j - X_j\beta)\right]$$

$$= \frac{\partial}{\partial\rho}\left[\frac{1}{\sigma^2(1-\rho)} RSS_j(\rho)\right] + \frac{\partial}{\partial\rho}\left[\frac{-\rho}{\sigma^2(1-\rho)(1+(n-1)\rho)}\right]\left((y_j - X_j\beta)^T 1_{n_j}\right)\left((y_j - X_j\beta)^T 1_{n_j}\right)^T$$

$$= \frac{1}{\sigma^2(1-\rho)^2} RSS_j(\beta) + \frac{-\sigma^2(1-\rho)(1+(n_j-1)\rho) + \rho[-\sigma^2(1+(n_j-1)\rho) + \sigma^2(1-\rho)(n_j-1)]}{}$$