Graphs and Matrix Algebra

- Reading: SAND, Chp 2
  SANDR, Chp 2/3

Connecting graph theory + matrix algebra: algebraic graph theory

Adjacency matrix $A_{ij} = \begin{cases} 1 & \{i,j\} \in E \\ 0 & \text{o.w.} \end{cases}$

A summarizes the network

- Degree $d_i = \sum_j A_{ij}$
- Walks A^r_{ij} = # of walks of length r between i and j
- Eigen-structure: G is regular iff the max degree is an eigenvalue of the matrix A

  (G regular $\Rightarrow$ all degrees are the same)

Rmk: For digraphs A is no longer symmetric

Laplacian: $L = D - A$ where $D = \text{diag}(A\mathbb{1})$

Properties: For $x \in \mathbb{R}^{N_v}$, vertex attributes, $x^T L x = \sum_{(i,j) \in E} (x_i - x_j)^2$

- Measures how "smooth" x is over the graph
- Eigen-analysis of L: $\boxed{\text{Chung-Graham has a book on this topic}}$
  - Connectivity, conductance, and graph structure

Graph Data Structures and Algorithms

- Adjacency matrices: $O(N_v^2)$
- Adjacency list: - $O(N_v + N_E)$
  - list of lists
- Edge list: - two columns (sender, reciever)

Rmk: For sparse networks, $N_E \simeq N_V$ so we get a full order reduction

for dense networks, $N_E \simeq N_V^2$ which is the same as the adjacency

Graph algorithms classification

- Directly $O(1)$ time ex: neighborhood, degree
- Answerable in poly time ex: shortest path, connected components
- Unanswerable in poly time ex: finding the maximum clique

Descriptive Statistics for Networks

Two main classes

- network mapping
- network characterization

Network Mapping: Production of a network-based visualization of a complex system.

Three stages (a) Data collection

(b) Network Graph construction

(c) visualization

Comments on (a): - think about 'relations' and 'elements'.

-