Solving LP

Saddle point of feasibility LP

$Ax \le 1$

$x \in X$

$$\underbrace{\min_{x \in X} \max_{p \in \Delta_m} p^T A x}_{\text{max violation}}$$

$$\partial f(x) = A^T \operatorname*{argmax}_{p \in \Delta_m} p^T A x$$

Smoothing

$$f_\mu(x) = \max_{p \in \Delta_m} p^T A x + \mu H(p)$$

$$H(p) = \sum_{i=1}^{m} p_i \log p_i$$

Lemma: $f(x) \le f_\mu(x) \le f(x) + \mu \log m$

Lemma: $\forall\, x, y \in \mathbb{R}^n$

$$\|\nabla f_\mu(x) - \nabla f_\mu(y)\|_1 \leq \frac{\|A(x-y)\|_\infty}{\mu}$$

$$\|\nabla \mathrm{smax}_\mu(z_1) - \nabla \mathrm{smax}_\mu(z_2)\|_1 \leq \frac{\|z_1 - z_2\|_\infty}{\mu}$$

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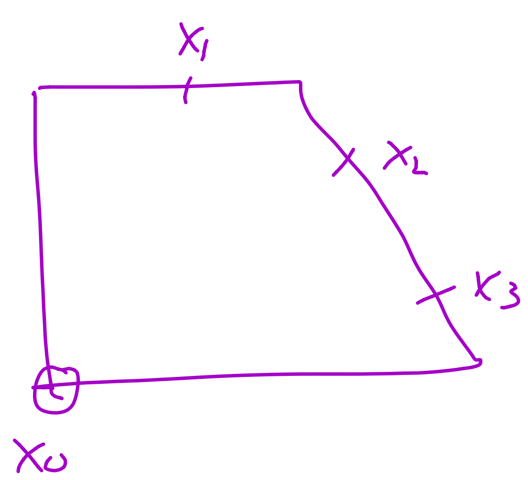

$$x_0^{(0)} = 0$$

$$x_0^{(1)} = x_0^{(0)} + \frac{x_1}{T}$$

$$x_0^{(2)} = x_0^{(0)} + \frac{x_1}{T} + \frac{x_2}{T}$$

$$x_1^{(T)} = x_0^{(0)} + \frac{1}{T}\sum_{i=1}^{T} x_i$$

$$\bar{x}_k = \frac{1}{T}\sum_{i=1}^{K} x_i$$

$$f_\mu(\bar{x}_{k+1}) - f_\mu(\bar{x}_k)$$

$$f_\mu\left(\bar{x}_k + \frac{x_{k+1}}{T}\right) - f_\mu(\bar{x}_k)$$

$$\le \frac{1}{T}\underbrace{\langle \nabla f(\bar{x}_k), x_{k+1}\rangle}_{\min\limits_{x\in X} \langle \nabla f(\bar{x}_k), x\rangle} + \frac{\|A - x_{k+1}\|_\infty^2}{2\mu T^2}$$

$$f_\mu(x) = \mu \log \sum_{i=1}^{m} e^{\left(\frac{Ax}{\mu}\right)_i}$$

$$\nabla_i f_\mu(x) = \frac{A^T e^{(Ax/\mu)_i}}{\sum^{n} (Ax)_{n/\ldots}}$$

$$\sum_{k=1} e^{\cdot} \quad /\mu$$

$$\min_{x \in X} \frac{\sum e^{\frac{(A\bar{x}_k)_i}{\mu}}}{\sum_{j=1}^{m} e^{\frac{(A\bar{x}_k)_{i'}}{\mu}}} (Ax)_i$$

$$= \begin{cases} \leq 1 & \text{continue} \\ > 1 & \text{infeasible / the softmax is the dual solution.} \end{cases}$$

$$f_\mu(\bar{x}_T) - f_\mu(0) \leq 1 + \frac{\sum_i^T \|Ax_i\|_\infty^2}{2\mu T^2}$$

Assume $\|Ax_i\|_\infty \leq \rho$ then

$$f(\bar{x}_T) \leq f_\mu(\bar{x}_T) \leq \mu \log m + 1 + \frac{T\rho^2}{2\mu T^2}$$

$$= \mu \log m + \frac{\rho^2}{}$$

$$2\mu T$$

Choose $\mu = \dfrac{\varepsilon}{2\log m}$

$$\leq \frac{\varepsilon}{2} + \frac{\rho^2 \log m}{\varepsilon T}$$

Choose $T \geq \dfrac{2\rho^2 \log m}{\varepsilon^2}$

So

$$\leq \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$$

Hence the running time is

$$\boxed{\frac{2\rho^2 \log m}{\varepsilon^2}}$$