

Continuous RV

(i) F_x is continuous then X is continuous

(ii) If F_x is differentiable then

$$\frac{dF_x}{dx} = f \quad \text{density function}$$

$$\text{Discrete: } P(a \leq X \leq b) = \sum_{x=a}^b p(x)$$

$$\text{Cont: } P(a \leq X \leq b) = \int_a^b f(x) dx$$

$$\text{Def: } E(X) = \int x f(x) dx$$

$$\text{Var}(X) = E[(X - E(X))^2]$$

$$= \int (x - E(X))^2 p(x) dx$$

$$\int (f(x) - F(x)) f(x) dx$$

Ex: $f(x) = \frac{1}{b-a} I(a \leq x \leq b)$

$$F(x) = \frac{x-a}{b-a}$$

Ex: $f(x) = \lambda e^{-\lambda x} I(x \geq 0)$

$$X \sim \text{Exp}(\lambda)$$

$$F(x) = 1 - e^{-\lambda x}$$

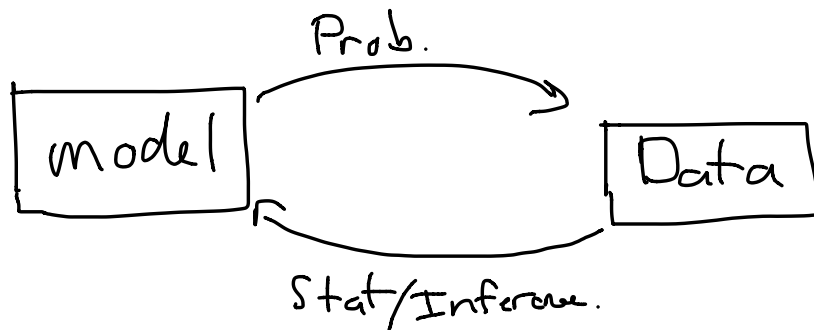
Ex: $f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$

$$X \sim N(\mu, \sigma^2)$$

CDF - Computed numerically

Rmk: Gaussian closed under linear operations

Statistics



- We'll be assuming a parametric setting in most of this course
- End with nonparametric inference.

Ex: $X_1, \dots, X_n \stackrel{iid}{\sim} \text{Bern}(p)$

* Parametric inference attempts to estimate parameters in a certain model.

$$\hat{g}(x_1, \dots, x_n) = \hat{\theta} \leftarrow \text{random}$$

So the distribution of $\hat{\theta}$ is the sampling

distribution

- $\hat{\theta}$ normally chosen to minimize some cost function

Ex: $\hat{p} = \frac{1}{n} \sum x_i \quad x_i \sim \text{Bern}(p)$

$$\hat{p} \xrightarrow{\text{as.}} p \quad \mathbb{E}(\hat{p}) = p$$

$$\frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} \xrightarrow{D} N(0, 1)$$

Confidence Intervals

Def: A confidence set is a stat.

$S(x_1, \dots, x_n)$ s.t. $S(D) \ni \theta$ with high probability.

$$\mathbb{P}(S(D) \ni \theta) = 1 - \alpha \quad (*)$$

Remark: $S(D)$ usually an interval

(*) usually want entirely specify $S(D)$

Ex: $X_1, \dots, X_n \sim \text{iid } N(\mu, \sigma^2)$
 $\underbrace{\sigma^2}_{\text{known}}$

$$\hat{\mu} = \frac{\sum X_i}{n} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

There always exists $z_{\alpha/2}$ s.t.

$$P(-z_{\alpha/2} \leq Z \leq z_{\alpha/2}) = 1 - \alpha$$

$$\Rightarrow P\left(-z_{\alpha/2} \leq \frac{\hat{\mu} - \mu}{\sigma/\sqrt{n}} \leq z_{\alpha/2}\right) = 1 - \alpha$$

$$P\left(\hat{\mu} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \hat{\mu} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha$$



Classical Confidence

Interview