

# Treelet Covariance Smoothers

## Better Estimation of Heritability

B. Draves<sup>1</sup>

<sup>1</sup>Department of Mathematics  
Lafayette College

Advisor: T. Gaugler

Lafayette College, 2016

# What is Treelets?

**An adaptive method for multi-scale representation and eigenanalysis of data where the variables can occur in any given order.<sup>1</sup>**

---

<sup>1</sup>Crossett, A., Lee, A. B., Klei, L., Devlin, B., and Roeder, K., Refining Genetically Inferred Relationships Using Treelet Covariance Smoothing, Annals of Applied Statistics, 7(2):669 690, 2013.

# What is Treelets?

**An adaptive method for multi-scale representation and eigenanalysis of data where the variables can occur in any given order.<sup>1</sup>**

What does *that* mean?

---

<sup>1</sup>Crossett, A., Lee, A. B., Klei, L., Devlin, B., and Roeder, K., Refining Genetically Inferred Relationships Using Treelet Covariance Smoothing, Annals of Applied Statistics, 7(2):669-690, 2013.

# What is Treelets? - My Take

Treelets is a method to systematically cluster correlated data in a way that yields information of the underlying structure while smoothing noisy samples.

# SNP Samples

- Each person has a specific genetic composition which can be described by a sample of single nucleotide polymorphisms (SNPs)

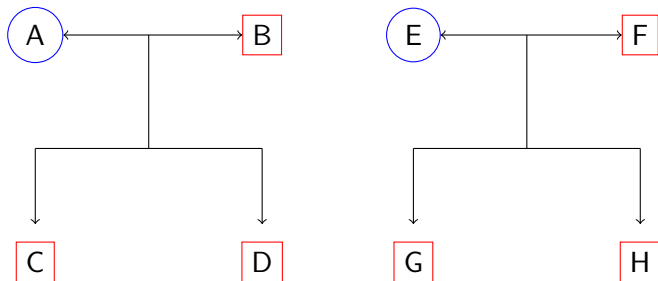
# SNP Samples

- Each person has a specific genetic composition which can be described by a sample of single nucleotide polymorphisms (SNPs)
- Idea: Use Treelets to refine an estimate of relatedness in a sample of individuals

# SNP Samples

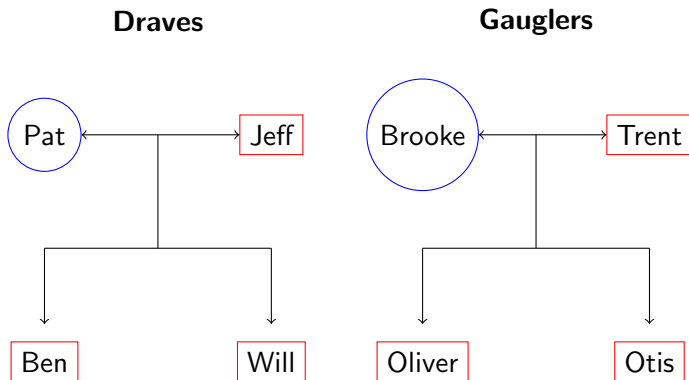
- Each person has a specific genetic composition which can be described by a sample of single nucleotide polymorphisms (SNPs)
- Idea: Use Treelets to refine an estimate of relatedness in a sample of individuals
- Use this information to estimate heritability of a phenotype in the population

# Pedigree Example





# Pedigree Example



# Estimating Relatedness - Theoretic

	Otis	Will	Trent	Pat	Jeff	Ben	Oliver	Brooke
Otis	1	0	$1/2$	0	0	0	$1/2$	$1/2$
Will	0	1	0	$1/2$	$1/2$	$1/2$	0	0
Trent	$1/2$	0	1	0	0	0	$1/2$	0
Pat	0	$1/2$	0	1	0	$1/2$	0	0
Jeff	0	$1/2$	0	0	1	$1/2$	0	0
Ben	0	$1/2$	0	$1/2$	$1/2$	1	0	0
Oliver	$1/2$	0	$1/2$	0	0	0	1	$1/2$
Brooke	$1/2$	0	0	0	0	0	$1/2$	1

# Estimating Relatedness - Theoretic

	Otis	Oliver	Trent	Brooke	Jeff	Ben	Will	Pat
Otis	1	1/2	1/2	1/2	0	0	0	0
Oliver	1/2	1	1/2	1/2	0	0	0	0
Trent	1/2	1/2	1	0	0	0	0	0
Brooke	1/2	1/2	0	1	0	0	0	0
Jeff	0	0	0	0	1	1/2	1/2	0
Ben	0	0	0	0	1/2	1	1/2	1/2
Will	0	0	0	0	1/2	1/2	1	1/2
Pat	0	0	0	0	0	1/2	1/2	1

# Estimating Relatedness - Sample

	Otis	Will	Trent	Pat	Jeff	Ben	Oliver	Brooke
Otis	0.82	0.31	0.52	0.04	0.39	0.15	0.51	0.44
Will	0.31	0.96	0.10	0.48	0.41	0.43	0.29	0.01
Trent	0.52	0.10	0.89	0.17	0.02	0.09	0.58	0.16
Pat	0.04	0.48	0.17	0.95	0.02	0.45	0.01	0.07
Jeff	0.39	0.41	0.02	0.02	0.83	0.54	0.05	0.13
Ben	0.41	0.43	0.09	0.45	0.54	0.96	0.03	0.04
Oliver	0.51	0.29	0.58	0.01	0.05	0.03	0.85	0.46
Brooke	0.44	0.01	0.16	0.07	0.13	0.04	0.46	0.79

# Treelets in Action

Otis

Oliver

Trent

Brooke

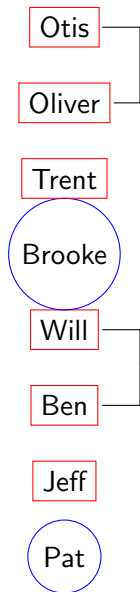
Will

Ben

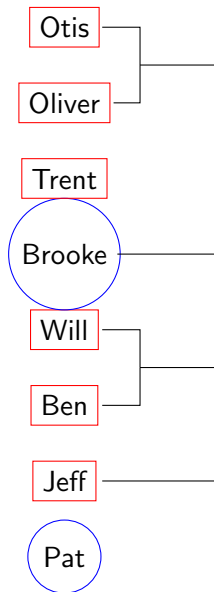
Jeff

Pat

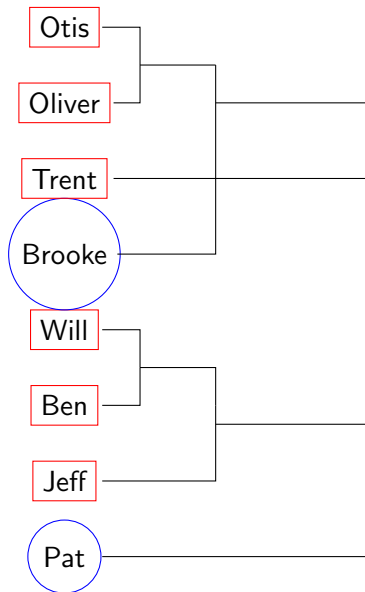
# Treelets in Action



# Treelets in Action

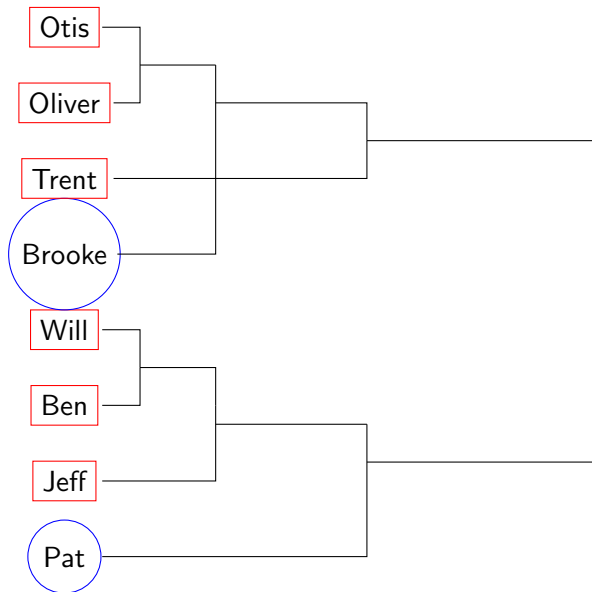


# Treelets in Action

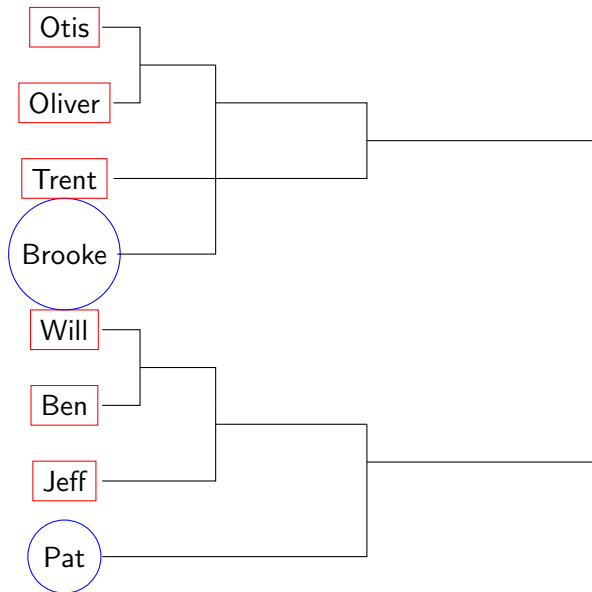




# Treelets in Action

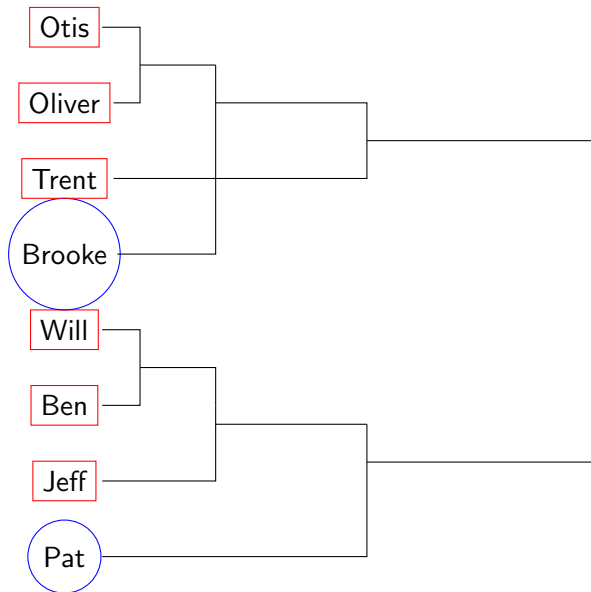


# Treelets in Action



1. Associated relationship matrix

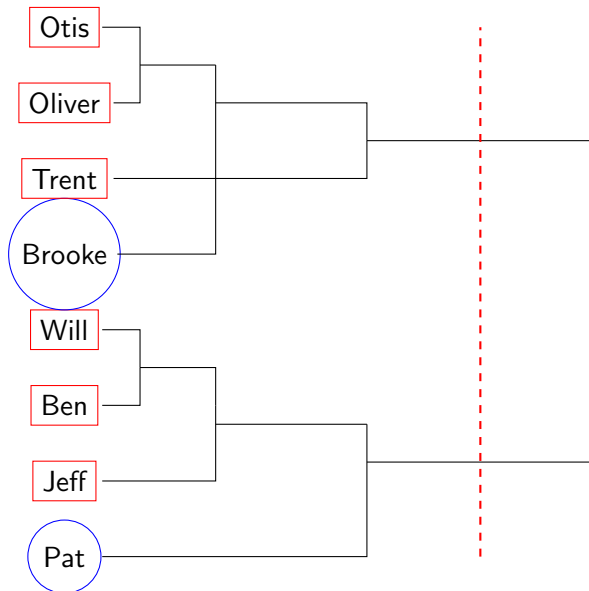
# Treelets in Action



1. Associated relationship matrix

2. Set any “insignificant” ( $< \lambda$ ) relationships to zero

# Treelets in Action



1. Associated relationship matrix

2. Set any “insignificant” ( $< \lambda$ ) relationships to zero

# Alternative Smoothing Methods

- How can we avoid clustering variables we expect are unrelated?

# Alternative Smoothing Methods

- How can we avoid clustering variables we expect are unrelated?
- Don't cluster variables we expect are unrelated

# Alternative Smoothing Methods

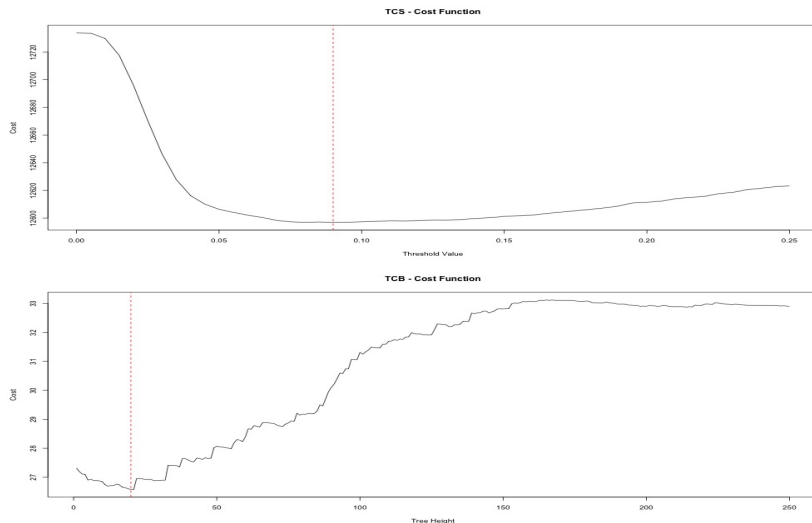
- How can we avoid clustering variables we expect are unrelated?
- Don't cluster variables we expect are unrelated
- Don't require Treelets to merge every variable

# Alternative Smoothing Methods

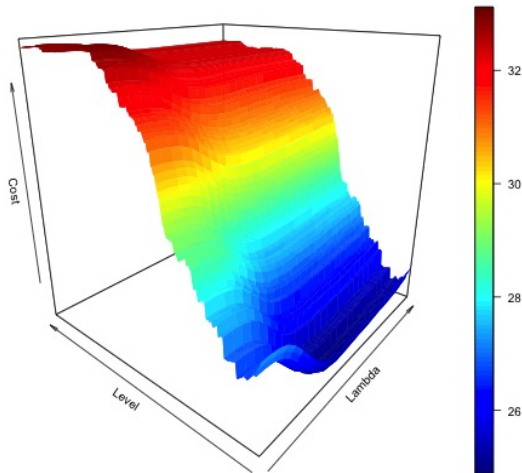
Method	Tree Height ( $\ell$ )	Threshold ( $\lambda$ )
Treelet Covariance Smoothing	Top	Variable
Treelet Covariance Blocking	Variable	Fixed
Treelet Covariance Blocked Smoothing	Variable	Variable



# Choosing Smoothing Parameters - TCS & TCB

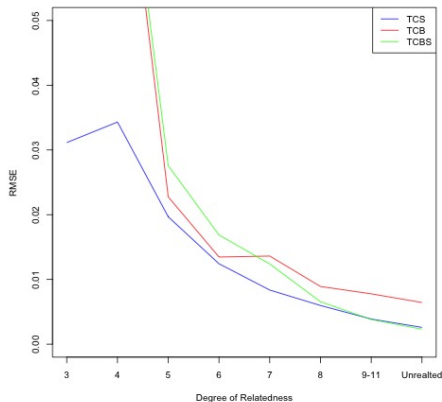


# Choosing Smoothing Parameters - TCBS

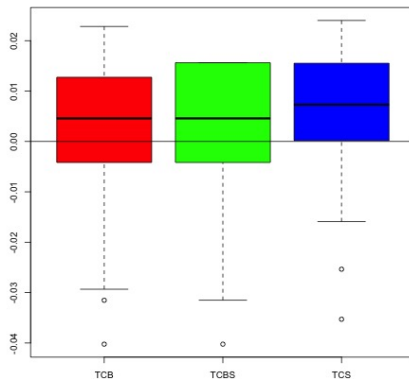


# Estimating Relatedness - Results

RMSE by Method



Degree Six Estimated Relatedness



# Current Work

- Using these refined estimations of relatedness, we estimate Body Mass Index (BMI) and Abdomen Visceral Fat Density (AVFD) in the Health Aging and Body Composition Study

# Current Work

- Using these refined estimations of relatedness, we estimate Body Mass Index (BMI) and Abdomen Visceral Fat Density (AVFD) in the Health Aging and Body Composition Study
- TCS has estimated the heritability of BMI close to the literature's accepted range and fails to do so for AVFD

# Current Work

- Using these refined estimations of relatedness, we estimate Body Mass Index (BMI) and Abdomen Visceral Fat Density (AVFD) in the Health Aging and Body Composition Study
- TCS has estimated the heritability of BMI close to the literature's accepted range and fails to do so for AVFD
- We look to show TCB or TCBS better estimates heritability of these traits

# Conclusions

- Treelets offers a new and useful methodology to the field of high - dimensional statistics

# Conclusions

- Treelets offers a new and useful methodology to the field of high - dimensional statistics
- TCB and TCBS look to improve this method by better utilizing the tree structure inherent in the algorithm



# Conclusions

- Treelets offers a new and useful methodology to the field of high - dimensional statistics
- TCB and TCBS look to improve this method by better utilizing the tree structure inherent in the algorithm
- TCB and TCBS can be applied to a variety of problems - specifically the estimation of relatedness and heritability

# Thanks for listening

Questions? Comments?