Cs3340 Assignment 1

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1. a) Insertion sort's worst case performance is $\Theta(k^2)$ per k-element list. Therefore sorting all n/k lists of length k will take $\Theta(k^2 n/k) = \Theta(nk)$ worst-case time.

b) To merge the sublists in $\Theta(n\lg(n/k))$ time, we must merge the lists pairwise, then merge those lists pairwise, etc until there is only one list. Each level takes $\Theta(n)$ time and there are $\lg(n/k)$ levels. Therefore the merging takes $\Theta(n\lg(n/k))$ time.

c) $k = \Theta(\lg n)$.

$\Theta(nk+n\lg(n/k)) = \Theta(n\lg n+n\lg n-n\lg\lg n) = \Theta(2n\lg n-n\lg\lg n)$ which is in the order of $\Theta(n\lg n)$.

d) We should choose k so that it is the largest list length that is faster to sort using insertion sort than merge sort.

2.

A	B	O	o	Ω	ω	Θ
$\mathbf{Lg^2k}$	$\mathbf{n^{\epsilon}}$	Yes	Yes	No	No	No
$\mathbf{n^k}$	$\mathbf{c^n}$	Yes	Yes	No	No	No
$\mathbf{\sqrt{n}}$	$\mathbf{n^{\sin n}}$	No	No	No	No	No
$\mathbf{2^n}$	$\mathbf{2^{n/2}}$	No	No	Yes	Yes	No
$\mathbf{n^{\lg c}}$	$\mathbf{c^{\lg n}}$	Yes	No	Yes	No	Yes
lg(n!)	$\mathbf{\lg(n^n)}$	yes	No	Yes	No	Yes

3. a)

i) $T(n) = T(n/2)+c = \Theta(\lg n)$

ii) $T(n) = T(n/2)+cN$

$=2cN+T(n/4)$

$=3cN+T(n/8)$

$= \sum (i = 0 \rightarrow \lg n - 1) (2^i cN / 2^i)$

$=cN\lg n$

$=\Theta(n\lg n)$.

iii) $T(n)=T(n/2)+cn=\Theta(n)$.

b) i) $T(n) = 2T(n/2) + cn = \Theta(n\lg n)$

ii) $T(n) = 2T(n/2)+cn+2N=4N+cn+2c(n/2)+4T(n/4)$

$=8N+2cn+4c(n/4)+8T(n/8)$

$= \sum (cn+2^i N)$

$= \sum cn+N \sum 2^i$

$= cn \lg n + N ((2^{\lg n} – 1) / (2 – 1))$

$= cn \lg n + nN – N = O(nN)$

$= O(n^2)$

iii) $T(n) = 2T(n/2)+cn+2n/2$

$=2T(n/2)+(c+1)n$

$=\Theta(n\lg n)$.

5. a) $./asn1_a.sh [size] **b)** $./asn1_b.sh [size] **c)** $./asn1_c.sh [size] [k value]

d) Insertion sort's run time grows exponentially with input size, so an input of size 200,000,000 would take extremely long. The best value of K is 16 and at this length, insertion sort is faster than merge sort.