# APPENDIX C

## SOLUTIONS TO PROBLEMS

**C.1** (i) This is just a special case of what we covered in the text, with $n = 4$: $E(\bar{Y}) = \mu$ and $Var(\bar{Y}) = \sigma^2/4$.

(ii) $E(W) = E(Y_1)/8 + E(Y_2)/8 + E(Y_3)/4 + E(Y_4)/2 = \mu[(1/8) + (1/8) + (1/4) + (1/2)] = \mu(1 + 1 + 2 + 4)/8 = \mu$, which shows that $W$ is unbiased. Because the $Y_i$ are independent,

$$\begin{aligned} Var(W) &= Var(Y_1)/64 + Var(Y_2)/64 + Var(Y_3)/16 + Var(Y_4)/4 \\ &= \sigma^2[(1/64) + (1/64) + (4/64) + (16/64)] = \sigma^2(22/64) = \sigma^2(11/32). \end{aligned}$$

(iii) Because $11/32 > 8/32 = 1/4$, $Var(W) > Var(\bar{Y})$ for any $\sigma^2 > 0$, so $\bar{Y}$ is preferred to $W$ when each is unbiased.

**C.3** (i) $E(W_1) = [(n-1)/n]E(\bar{Y}) = [(n-1)/n]\mu$, and so $Bias(W_1) = [(n-1)/n]\mu - \mu = -\mu/n$. Similarly, $E(W_2) = E(\bar{Y})/2 = \mu/2$, and so $Bias(W_2) = \mu/2 - \mu = -\mu/2$. The bias in $W_1$ tends to zero as $n \rightarrow \infty$, while the bias in $W_2$ is $-\mu/2$ for all $n$. This is an important difference.

(ii) $plim(W_1) = plim[(n-1)/n] \cdot plim(\bar{Y}) = 1 \cdot \mu = \mu$ and $plim(W_2) = plim(\bar{Y})/2 = \mu/2$. Because $plim(W_1) = \mu$ and $plim(W_2) = \mu/2$, $W_1$ is consistent whereas $W_2$ is inconsistent.

(iii) $Var(W_1) = [(n-1)/n]^2 Var(\bar{Y}) = [(n-1)^2/n^3]\sigma^2$ and $Var(W_2) = Var(\bar{Y})/4 = \sigma^2/(4n)$.

(iv) Because $\bar{Y}$ is unbiased, its mean squared error is simply its variance. On the other hand, $MSE(W_1) = Var(W_1) + [Bias(W_1)]^2 = [(n-1)^2/n^3]\sigma^2 + \mu^2/n^2$. When $\mu = 0$, $MSE(W_1) = Var(W_1) = [(n-1)^2/n^3]\sigma^2 < \sigma^2/n = Var(\bar{Y})$ because $(n-1)/n < 1$. Therefore, $MSE(W_1)$ is smaller than $Var(\bar{Y})$ for $\mu$ close to zero. For large $n$, the difference between the two estimators is trivial.

**C.5** (i) While the expected value of the numerator of $G$ is $E(\bar{Y}) = \theta$, and the expected value of the denominator is $E(1 - \bar{Y}) = 1 - \theta$, the expected value of the ratio is not the ratio of the expected value.

(ii) By Property PLIM.2(iii), the plim of the ratio is the ratio of the plims (provided the plim of the denominator is not zero): $plim(G) = plim[\bar{Y}/(1-\bar{Y})] = plim(\bar{Y})/[1 - plim(\bar{Y})] = \theta/(1-\theta) = \gamma$.

**C.7** (i) The average increase in wage is $\bar{d} = .24$, or 24 cents. The sample standard deviation is about .451, and so, with $n = 15$, the standard error of $\bar{d}$ is $.451\sqrt{15} \approx .1164$. From Table G.2, the 97.5$^{th}$ percentile in the $t_{14}$ distribution is 2.145. So the 95% CI is $.24 \pm 2.145(.1164)$, or about –.010 to .490.

(ii) If $\mu = \mathrm{E}(D_i)$, then $\mathrm{H}_0$: $\mu = 0$. The alternative is that management's claim is true: $\mathrm{H}_1$: $\mu > 0$.

(iii) We have the mean and standard error from part (i): $t = .24/.1164 \approx 2.062$. The 5% critical value for a one-tailed test with $df = 14$ is 1.761, while the 1% critical value is 2.624. Therefore, $\mathrm{H}_0$ is rejected in favor of $\mathrm{H}_1$ at the 5% level but not the 1% level.

(iv) The $p$-value obtained from Stata is .029; this is half of the $p$-value for the two-sided alternative. (Econometrics packages, including Stata, report the $p$-value for the two-sided alternative.)

**C.9** (i) $X$ is distributed as Binomial(200,.65), and so $\mathrm{E}(X) = 200(.65) = 130$.

(ii) $\mathrm{Var}(X) = 200(.65)(1 - .65) = 45.5$, so $\mathrm{sd}(X) \approx 6.75$.

(iii) $\mathrm{P}(X \leq 115) = \mathrm{P}[(X - 130)/6.75 \leq (115 - 130)/6.75] \approx \mathrm{P}(Z \leq -2.22)$, where $Z$ is a standard normal random variable. From Table G.1, $\mathrm{P}(Z \leq -2.22) \approx .013$.

(iv) The evidence is pretty strong against the dictator's claim. If 65% of the voting population actually voted yes in the plebiscite, there is only about a 1.3% chance of obtaining 115 or fewer voters out of 200 who voted yes.

**C.11** Since $\bar{y} = .132$, $s = 1.27$, and $n = 400$, we have $\mathrm{se}(\bar{y}) = .064$. The $t$ statistic is $t = .132/.064 \approx 2.08$. The $p$-value is $.019 < .05$. Therefore, we conclude that the average change in GPAs is statistically greater than zero.