CHAPTER 11

SOLUTIONS TO PROBLEMS

11.1 Because of covariance stationarity, $\gamma_0 = \text{Var}(x_t)$ does not depend on t, so $\text{sd}(x_{t+h}) = \sqrt{\gamma_0}$ for any $h \geq 0$. By definition, $\text{Corr}(x_t, x_{t+h}) = \text{Cov}(x_t, x_{t+h})/[\text{sd}(x_t) \cdot \text{sd}(x_{t+h})] = \gamma_h/(\sqrt{\gamma_0} \cdot \sqrt{\gamma_0}) = \gamma_h/\gamma_0$.

**11.3** (i) $\text{E}(y_t) = \text{E}(z + e_t) = \text{E}(z) + \text{E}(e_t) = 0$. $\text{Var}(y_t) = \text{Var}(z + e_t) = \text{Var}(z) + \text{Var}(e_t) + 2\text{Cov}(z, e_t) = \sigma_z^2 + \sigma_e^2 + 2 \cdot 0 = \sigma_z^2 + \sigma_e^2$. Neither of these depends on t.

(ii) We assume $h > 0$; when $h = 0$, we obtain $\text{Var}(y_t)$. Then $\text{Cov}(y_t, y_{t+h}) = \text{E}(y_t y_{t+h}) = \text{E}[(z + e_t)(z + e_{t+h})] = \text{E}(z^2) + \text{E}(ze_{t+h}) + \text{E}(e_t z) + \text{E}(e_t e_{t+h}) = \text{E}(z^2) = \sigma_z^2$ because $\{e_t\}$ is an uncorrelated sequence (it is an independent sequence and z is uncorrelated with e_t for all t. From part (i) we know that $\text{E}(y_t)$ and $\text{Var}(y_t)$ do not depend on t and we have shown that $\text{Cov}(y_t, y_{t+h})$ depends on neither t nor h. Therefore, $\{y_t\}$ is covariance stationary.

(iii) From Problem 11.1 and parts (i) and (ii), $\text{Corr}(y_t, y_{t+h}) = \text{Cov}(y_t, y_{t+h})/\text{Var}(y_t) = \sigma_z^2/(\sigma_z^2 + \sigma_e^2) > 0$.

(iv) No. The correlation between y_t and y_{t+h} is the same positive value obtained in part (iii) no matter how large is h. In other words, no matter how far apart y_t and y_{t+h} are, their correlation is always the same. Of course, the persistent correlation across time is due to the presence of the time-constant variable, z.

11.5 (i) The following graph gives the estimated lag distribution:
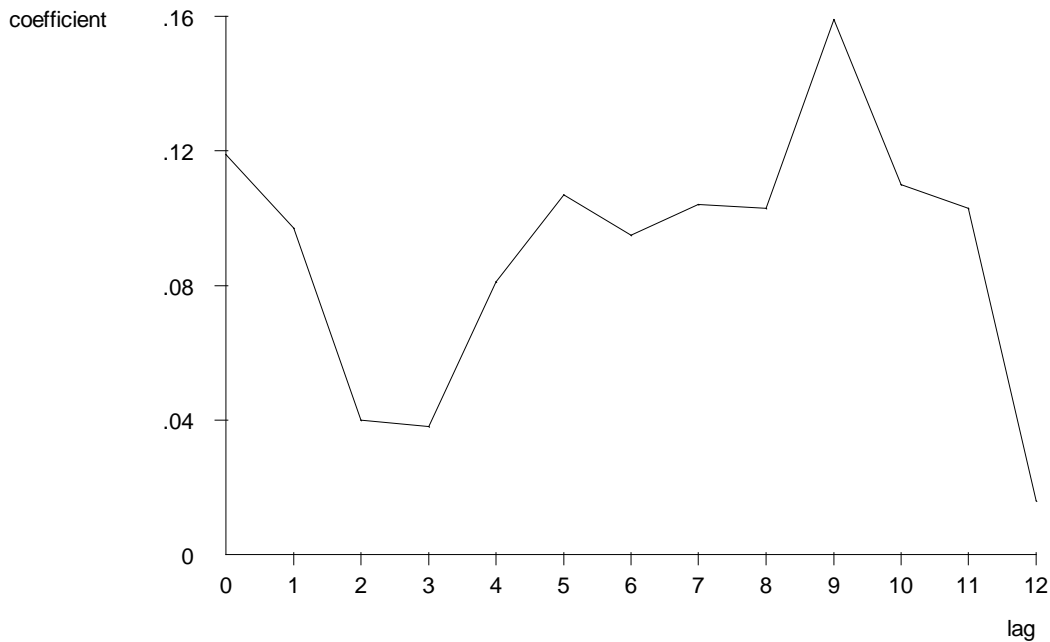
By some margin, the largest effect is at the ninth lag, which says that a temporary increase in wage inflation has its largest effect on price inflation nine months later. The smallest effect is at the twelfth lag, which hopefully indicates (but does not guarantee) that we have accounted for enough lags of *gwage* in the FLD model.

(ii) Lags two, three, and twelve have *t* statistics of less than two. The other lags are statistically significant at the 5% level against a two-sided alternative. (Assuming either that the CLM assumptions hold for exact tests or Assumptions TS.1′ through TS.5′ hold for asymptotic tests.)

(iii) The estimated LRP is just the sum of the lag coefficients from zero through twelve: 1.172. While this is greater than one, it is not much greater, and the difference from unity could be due to a sampling error.

(iv) The model underlying and the estimated equation can be written with intercept α_0 and lag coefficients $\delta_0, \delta_1, \ldots, \delta_{12}$. Denote the LRP by $\theta_0 = \delta_0 + \delta_1 + \ldots + \delta_{12}$. Now, we can write $\delta_0 = \theta_0 - \delta_1 - \delta_2 - \ldots - \delta_{12}$. If we plug this into the FDL model we obtain (with $y_t = gprice_t$ and $z_t = gwage_t$)

$$y_t = \alpha_0 + (\theta_0 - \delta_1 - \delta_2 - \ldots - \delta_{12})z_t + \delta_1 z_{t-1} + \delta_2 z_{t-2} + \ldots + \delta_{12} z_{t-12} + u_t$$

$$= \alpha_0 + \theta_0 z_t + \delta_1(z_{t-1} - z_t) + \delta_2(z_{t-2} - z_t) + \ldots + \delta_{12}(z_{t-12} - z_t) + u_t.$$

Therefore, we regress y_t on z_t, $(z_{t-1} - z_t)$, $(z_{t-2} - z_t)$, …, $(z_{t-12} - z_t)$ and obtain the coefficient and standard error on z_t as the estimated LRP and its standard error.

(v) We would add lags 13 through 18 of $gwage_t$ to the equation, which leaves $273 - 6 = 267$ observations. Now, we are estimating 20 parameters, so the *df* in the unrestricted model is $df_{ur} = 267$. Let R_{ur}^2 be the *R*-squared from this regression. To obtain the restricted *R*-squared, R_r^2, we need to reestimate the model reported in the problem but with the same 267 observations used to estimate the unrestricted model. Then $F = [(R_{ur}^2 - R_r^2)/(1 - R_{ur}^2)](247/6)$. We would find the critical value from the $F_{6,247}$ distribution.

11.7 (i) We plug the first equation into the second to get

$$y_t - y_{t-1} = \lambda(\gamma_0 + \gamma_1 x_t + e_t - y_{t-1}) + a_t,$$

and, rearranging,

$$y_t = \lambda\gamma_0 + (1 - \lambda)y_{t-1} + \lambda\gamma_1 x_t + a_t + \lambda e_t,$$

$$\equiv \beta_0 + \beta_1 y_{t-1} + \beta_2 x_t + u_t,$$

where $\beta_0 \equiv \lambda\gamma_0$, $\beta_1 \equiv (1 - \lambda)$, $\beta_2 \equiv \lambda\gamma_1$, and $u_t \equiv a_t + \lambda e_t$.

(ii) An OLS regression of y_t on y_{t-1} and x_t produces consistent, asymptotically normal estimators of the β_j. Under $E(e_t|x_t, y_{t-1}, x_{t-1}, …) = E(a_t|x_t, y_{t-1}, x_{t-1}, …) = 0$ it follows that $E(u_t|x_t, y_{t-1}, x_{t-1}, …) = 0$, which means that the model is dynamically complete [see equation (11.37)]. Therefore, the errors are serially uncorrelated. If the homoskedasticity assumption $Var(u_t|x_t, y_{t-1}) = \sigma^2$ holds, then the usual standard errors, *t* statistics and *F* statistics are asymptotically valid.

(iii) Because $\beta_1 = (1 - \lambda)$, if $\hat{\beta}_1 = .7$ then $\hat{\lambda} = .3$. Further, $\hat{\beta}_2 = \hat{\lambda}\hat{\gamma}_1$, or $\hat{\gamma}_1 = \hat{\beta}_2/\hat{\lambda} = .2/.3 \approx$ .67.

SOLUTIONS TO COMPUTER EXERCISES

C11.1 (i) The first order autocorrelation for $\log(invpc)$ is about .639. We first detrend $\log(invpc)$ by regressing on a linear time trend, $\hat{\rho}_1 \approx .485$. Especially after detrending there is little evidence of a unit root in $\log(invpc)$. For $\log(price)$, the first order autocorrelation is about .949, which is very high. After detrending, the first order autocorrelation drops to .822, but this is still pretty large. We cannot confidently rule out a unit root in $\log(price)$.

(ii) The estimated equation is

$$\widehat{\log(invpc_t)} \;=\; -.853 \;+\; 3.88\,\Delta\log(price_t) \;+\; .0080\,t$$
$$(.040) \quad (0.96) \qquad\qquad (.0016)$$

$$n = 41, \quad R^2 = .501.$$

The coefficient on $\Delta\log(price_t)$ implies that a one percentage point increase in the growth in price leads to a 3.88 percent increase in housing investment above its trend. [If $\Delta\log(price_t) = .01$ then $\Delta\widehat{\log(invpc_t)} = .0388$; we multiply both by 100 to convert the proportionate changes to percentage changes.]

(iii) If we first linearly detrend $\log(invpc_t)$ before regressing it on $\Delta\log(price_t)$ and the time trend, then $R^2 = .303$, which is substantially lower than that when we do not detrend. Thus, $\Delta\log(price_t)$ explains only about 30% of the variation in the trend of $\log(invpc_t)$.

(iv) The estimated equation is

$$\Delta\widehat{\log(invpc_t)} \;=\; .006 \;+\; 1.57\,\Delta\log(price_t) \;+\; .00004t$$
$$(.048) \quad (1.14) \qquad\qquad (.00190)$$

$$n = 41, \quad R^2 = .048.$$

The coefficient on $\Delta\log(price_t)$ has fallen substantially and is no longer significant at the 5% level against a positive one-sided alternative. The R-squared is much smaller; $\Delta\log(price_t)$ explains very little of the variation in $\Delta\log(invpc_t)$. Because differencing eliminates linear time trends, it is not surprising that the estimate on the trend is very small and very statistically insignificant.

C11.3 (i) The estimated equation is

$$\widehat{return_t} \;=\; .226 \;+\; .049\,return_{t-1} \;-\; .0097\,return_{t-1}^2$$
$$(.087) \quad (.039) \qquad\quad (.0070)$$

$$n = 689, \quad R^2 = .0063.$$

(ii) The null hypothesis is $H_0: \beta_1 = \beta_2 = 0$. Only if both parameters are zero does $E(return_t|return_{t-1})$ not depend on $return_{t-1}$. The F statistic is about 2.16 with p-value $\approx .116$. Therefore, we cannot reject H_0 at the 10% level.

(iii) When we put $return_{t-1}\cdot return_{t-2}$ in place of $return_{t-1}^2$, the null can still be stated as in part (ii): no past values of $return$, or any functions of them, should help us predict $return_t$. The R-squared is about .0052 and $F \approx 1.80$ with p-value $\approx .166$. Here, we do not reject H_0 at even the 15% level.

(iv) Predicting $return_t$ based on past returns does not appear promising. Even though the F statistic from part (ii) is almost significant at the 10% level, we have many observations. We cannot even explain 1% of the variation in $return_t$.

C11.5 (i) The estimated equation is

$$\widehat{\Delta gfr} = -1.27 - .035\,\Delta pe - .013\,\Delta pe_{-1} - .111\,\Delta pe_{-2} + .0079\,t$$
$$(1.05)\ \ (.027) \qquad (.028) \qquad\quad (.027) \qquad\qquad (.0242)$$

$$n = 69,\ \ R^2 = .234,\ \ \bar{R}^2 = .186.$$

The time trend coefficient is very insignificant, so it is not needed in the equation.

(ii) The estimated equation is

$$\widehat{\Delta gfr} = -.650 - .075\,\Delta pe - .051\,\Delta pe_{-1} + .088\,\Delta pe_{-2} + 4.84\,ww2 - 1.68\,pill$$
$$(.582)\ \ (.032) \qquad (.033) \qquad\quad (.028) \qquad\quad (2.83) \qquad (1.00)$$

$$n = 69,\ \ R^2 = .296,\ \ \bar{R}^2 = .240.$$

The F statistic for joint significance is $F = 2.82$ with p-value $\approx .067$. So, $ww2$ and $pill$ are not jointly significant at the 5% level, but they are jointly significant at the 10% level.

(iii) Including the time trend and the two dummy variables gives

$$\widehat{\Delta gfr} = -3.14 - .062\,\Delta pe - .039\,\Delta pe_{-1} + .095\,\Delta pe_{-2}$$
$$(1.15)\ \ \ (.032) \qquad (.032) \qquad\quad (.027)$$

$$+ .094\,t + 3.25\,ww2 - 4.89\,pill$$
$$(.038) \qquad (2.80) \qquad (1.62)$$

$$n = 69,\ \ R^2 = .359,\ \ \bar{R}^2 = .297.$$

The time trend coefficient is more than 10 times as large as when it appears by itself in part (i), and it is now statistically significant. The magnitude on the $pill$ coefficient has also gone way up, and its t statistic is greater than three in absolute value.

(iv) From the equation in part (iii), with the distributed lag coefficients rounded to three decimal places, the LRP is estimated to be $-.006$ and its t statistic is about $-.10$. So the LRP is practically and statistically equal to zero.

This example is a good illustration of how differencing variables before including them in a regression can lead to very different conclusions than a regression in levels.

C11.7 (i) If $E(gc_t|I_{t-1}) = E(gc_t)$ – that is, $E(gc_t|I_{t-1})$ does not depend on gc_{t-1}, then $\beta_1 = 0$ in $gc_t = \beta_0 + \beta_1 gc_{t-1} + u_t$. So, the null hypothesis is H_0: $\beta_1 = 0$ and the alternative is H_1: $\beta_1 \neq 0$. Estimating the simple regression using the data in CONSUMP.RAW gives

$$\widehat{gc_t} = .011 + .446\, gc_{t-1}$$
$$(.004) \quad (.156)$$
$$n = 35, \quad R^2 = .199.$$

The t statistic for $\hat{\beta}_1$ is about 2.86, and so we strongly reject the PIH. The coefficient on gc_{t-1} is also practically large, showing significant positive autocorrelation in consumption growth.

(ii) When gy_{t-1}, $i3_{t-1}$, and inf_{t-1} are added to the regression, the R-squared becomes about .304. The F test of joint significance of gy_{t-1}, $i3_{t-1}$, and inf_{t-1}, obtained using the Stata test command, is about 1.51 with p-value = .232. The p-values for the individual coefficients are .583, .507, and .416, respectively. Therefore, these three variables are neither jointly nor individually significant.

(iii) In the regression from part (ii), the p-value increases to .145 (from about .007) in the regression from part (i). This "watering down" occurs because gc_{t-1} is correlated with some of the variables gy_{t-1}, $i3_{t-1}$, and inf_{t-1}. We know that if we want to test four variables jointly it is not sufficient to test three of them jointly and then the fourth separately (based on its t statistic). So, we should not conclude the PIH is now supported by the data.

(iv) When we test all four lagged variables jointly, we get $F = 3.27$ with 4 and 30 df. The p-value is about .024, which means we reject the PIH at the 5% level. This is much more in line with part (i), even though the rejection of the PIH is now less strong (because the three lagged variables gy_{t-1}, $i3_{t-1}$, and inf_{t-1} seem to have no predictive power).

C11.9 (i) The first order autocorrelation for *prcfat* is .709, which is high but not necessarily a cause for concern. For *unem*, $\hat{\rho}_1 = .950$, which is a cause for concern in using *unem* as an explanatory variable in a regression.

(ii) If we use the first differences of *prcfat* and *unem*, but leave all other variables in their original form, we get the following:

$$\widehat{\Delta prcfat} = -.127 + \dots + .0068\, wkends + .0125\, \Delta unem$$
$$(.105) \qquad\qquad (.0072) \qquad\qquad (.0161)$$
$$- .0072\, spdlaw + .0008\, bltlaw$$
$$(.0238) \qquad\qquad (.0265)$$
$$n = 107, \quad R^2 = .344,$$

where the coefficient have been again suppressed on the time trend and seasonal dummies. This regression basically shows that the change in *prcfat* cannot be explained by the change in *unem* or any of the policy variables. It does have some seasonality, which is why the *R*-squared is .344.

(iii) This is an example of how estimation in first differences loses the interesting implications of the model estimated in levels. Of course, this is not to say the levels regression is valid. But, as it turns out, we can reject a unit root in *prcfat*, and so we can at least justify using it in level form; see Computer Exercise 18.13. Generally, the issue of whether to take first differences is very difficult, even for professional time series econometricians.

C11.11 (i) The estimated equation is

$$\widehat{pcrgdp}_t = \underset{(0.163)}{3.344} - \underset{(0.182)}{1.891}\,\Delta unem_t$$

$$n = 46,\ R^2 = .710.$$

Naturally, we do not get the exact estimates specified by the theory. Okun's Law is expected to hold, at best, on average. The estimates are not particularly far from their hypothesized values of 3 (intercept) and −2 (slope).

(ii) The *t* statistic for testing $H_0 : \beta_1 = -2$ is about .60, which gives a two-sided *p*-value of about .55. This is very little evidence against H_0; the null is not rejected at any reasonable significance level.

(iii) The *t* statistic for $H_0 : \beta_0 = 3$ is about 2.11, and the two-sided *p*-value is about .04. Therefore, the null is rejected at the 5% level, although it is not much stronger than that.

(iv) The joint test underlying Okun's Law gives $F = 2.41$. With (2,44) *df*, we get, roughly, *p*-value = .10. Therefore, Okun's Law passes at the 5% level, but only just at the 10% level.

C11.13 (i) Rounded to three decimal places, the correlation between *urate* and $urate_{t-1}$ is .996, which is very close to unity. This is indicative of a unit root process.

(ii) The correlation between *vrate* and $vrate_{t-1}$ is .945. While not as strong as that in part (i), it still suggests that *vrate* follows a unit root process.

(iii) The estimated equation is

$$\widehat{urate}_t = \underset{(0.592)}{17.119} - \underset{(0.207)}{3.741}\,vrate_t$$

$$n = 135,\ R^2 = .711.$$

The relationship is negative and the slope is very statistically significant with $|t| > 18$.

(iv) Unfortunately, because $urate_t$ and $vrate_t$ cannot be assumed to be weakly dependent, we cannot appeal to the usual asymptotic theory. And this is before we ask whether other assumptions, particularly the no serial correlation assumption in u_t, might be violated. Regressing an I(1) variable on another I(1) variable requires special treatment. It falls under the topic of "cointegration," which is introduced in Chapter 18.

(v) The equation estimated in first differences is

$$\widehat{\Delta urate_t} = 0.037 - 0.028 \, \Delta vrate_t$$
$$\phantom{\widehat{\Delta urate_t} = } (0.018) \quad (0.107)$$

$$n = 134, \quad R^2 = .0005.$$

This shows no relationship between the first differences of the variables. Until one understands something about the material in Chapter 18, we cannot know that the levels regression makes sense. What we do know is that the Beveridge curve in first differences provides no relationship, negative or otherwise, between unemployment rates and vacancies.