CHAPTER 18

SOLUTIONS TO PROBLEMS

18.1 With z_{t1} and z_{t2} now in the model, we should use one lag each as instrumental variables, $z_{t\text{-}1,1}$ and $z_{t\text{-}1,2}$. This gives one overidentifying restriction that can be tested.

18.3 For $\delta \neq \beta$, $y_t - \delta z_t = y_t - \beta z_t + (\beta - \delta)z_t$, which is an I(0) sequence ($y_t - \beta z_t$) plus an I(1) sequence. Since an I(1) sequence has a growing variance, it dominates the I(0) part, and the resulting sum is an I(1) sequence.

18.5 Following the hint, we have

$$y_t - y_{t\text{-}1} = \beta x_t - \beta x_{t\text{-}1} + \beta x_{t\text{-}1} - y_{t\text{-}1} + u_t$$

or

$$\Delta y_t = \beta \Delta x_t - (y_{t\text{-}1} - \beta x_{t\text{-}1}) + u_t.$$

Next, we plug in $\Delta x_t = \gamma \Delta x_{t\text{-}1} + v_t$ to get

$$\begin{aligned} \Delta y_t &= \beta(\gamma \Delta x_{t\text{-}1} + v_t) - (y_{t\text{-}1} - \beta x_{t\text{-}1}) + u_t \\ &= \beta\gamma \Delta x_{t\text{-}1} - (y_{t\text{-}1} - \beta x_{t\text{-}1}) + u_t + \beta v_t \\ &\equiv \gamma_1 \Delta x_{t\text{-}1} + \delta(y_{t\text{-}1} - \beta x_{t\text{-}1}) + e_t, \end{aligned}$$

where $\gamma_1 = \beta\gamma$, $\delta = -1$, and $e_t = u_t + \beta v_t$.

18.7 If $unem_t$ follows a stable AR(1) process, then this is the null model used to test for Granger causality: under the null that gM_t does not Granger cause $unem_t$, we can write

$$unem_t = \beta_0 + \beta_1 unem_{t\text{-}1} + u_t$$

$$\mathrm{E}(u_t | unem_{t\text{-}1}, gM_{t\text{-}1}, unem_{t\text{-}2}, gM_{t\text{-}2}, \ldots) = 0$$

and $|\beta_1| < 1$. Now, it is up to us to choose how many lags of gM to add to this equation. The simplest approach is to add $gM_{t\text{-}1}$ and to do a t test. But we could add a second or third lag (and probably not beyond this with annual data), and compute an F test for joint significance of all lags of gM_t.

18.9 Let $\hat{e}_{n+1}$ be the forecast error for forecasting y_{n+1}, and let $\hat{a}_{n+1}$ be the forecast error for forecasting Δy_{n+1}. By definition, $\hat{e}_{n+1} = y_{n+1} - \hat{f}_n = y_{n+1} - (\hat{g}_n + y_n) = (y_{n+1} - y_n) - \hat{g}_n = \Delta y_{n+1} - \hat{g}_n = \hat{a}_{n+1}$, where the last equality follows by definition of the forecasting error for Δy_{n+1}.

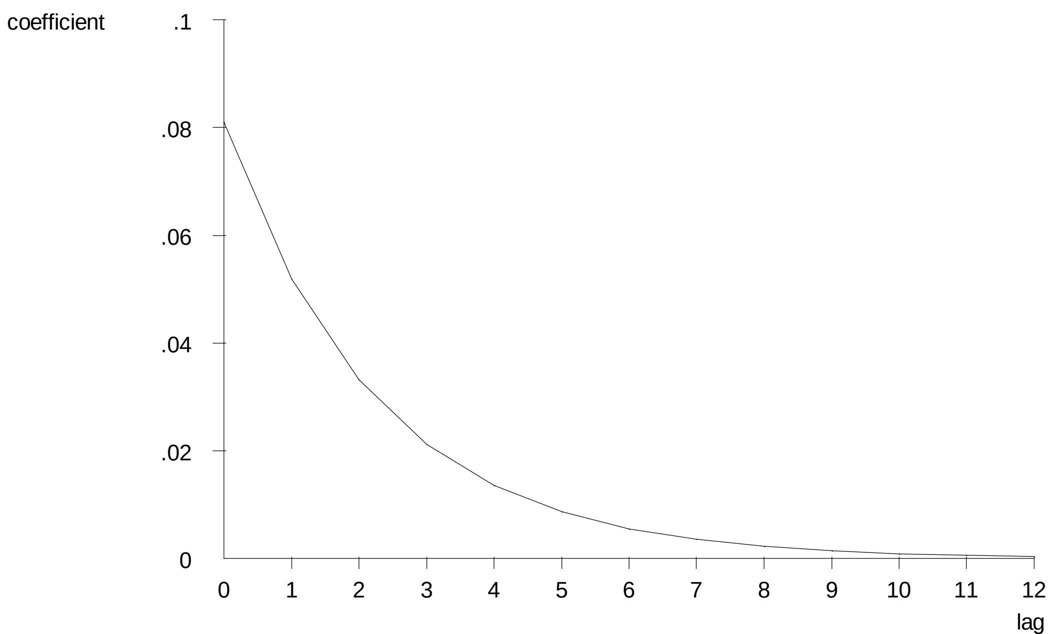
SOLUTIONS TO COMPUTER EXERCISES

C18.1 (i) The estimated GDL model is

$$\widehat{gprice} = \underset{(.0003)}{.0013} + \underset{(.031)}{.081}\, gwage + \underset{(.045)}{.640}\, gprice_{-1}$$

$$n = 284, \quad R^2 = .454.$$

The estimated impact propensity is .081 while the estimated LRP is .081/(1 – .640) = .225. The estimated lag distribution is graphed below.



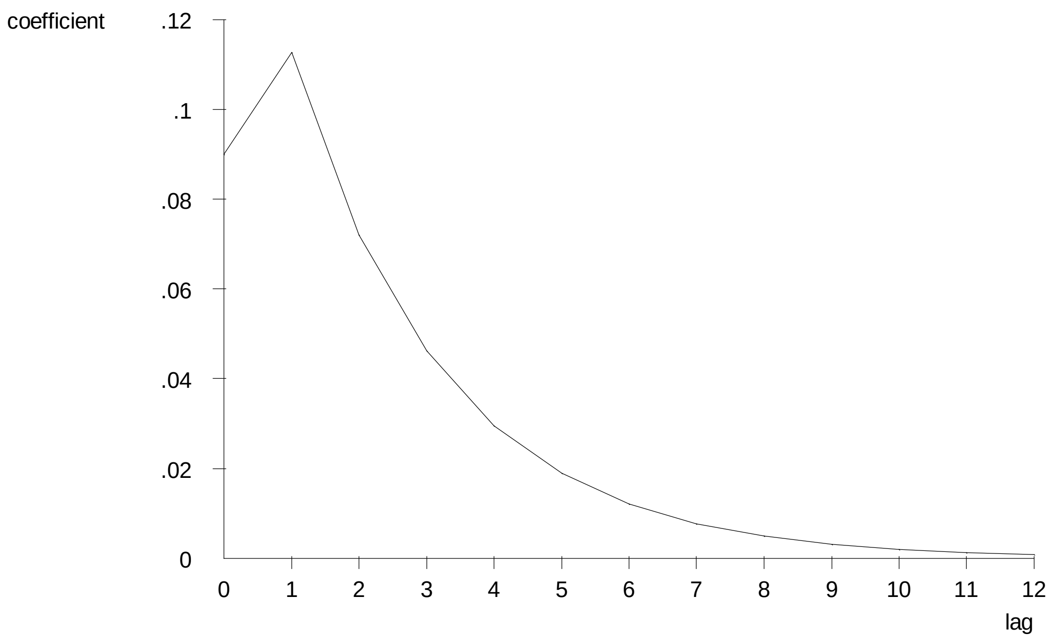
(ii) The IP for the FDL model estimated in Problem 11.5 was .119, which is substantially above the estimated IP for the GDL model. Further, the estimated LRP from GDL model is much lower than that for the FDL model, which we estimated as 1.172. Clearly we cannot think of the GDL model as a good approximation to the FDL model. One reason these are so different can be seen by comparing the estimated lag distributions (see below for the GDL model). With the FDL, the largest lag coefficient is at the ninth lag, which is impossible with the GDL model (where the largest impact is always at lag zero). It could also be that $\{u_t\}$ in equation (18.8) does not follow an AR(1) process with parameter ρ, which would cause the dynamic regression to produce inconsistent estimators of the lag coefficients.

(iii) When we estimate the RDL from equation (18.16) we obtain

$$\widehat{gprice} = .0011 + .090\, gwage + .619\, gprice_{-1} + .055\, gwage_{-1}$$
$$\quad\quad (.0003) \quad (.031) \quad\quad (.046) \quad\quad (.032)$$

$$n = 284, \quad R^2 = .460.$$

The coefficient on $gwage_{-1}$ is not especially significant but we include it in obtaining the estimated LRP. The estimated IP is .09 while the LRP is $(.090 + .055)/(1 - .619) \approx .381$. These are both slightly higher than what we obtained for the GDL, but the LRP is still well below what we obtained for the FDL in Problem 11.5. While this RDL model is more flexible than the GDL model, it imposes a maximum lag coefficient (in absolute value) at lag zero or one. For the estimates given above, the maximum effect is at the first lag. (See the estimated lag distribution below.) This is not consistent with the FDL estimates in Problem 11.5.



C18.3 (i) The estimated AR(3) model for $pcip_t$ is

$$\widehat{pcip_t} = 1.80 + .349\, pcip_{t-1} + .071\, pcip_{t-2} + .067\, pcip_{t-3}$$
$$\quad\quad (0.55) \quad (.043) \quad\quad (.045) \quad\quad (.043)$$

$$n = 554, \quad R^2 = .166, \quad \hat{\sigma} = 12.15.$$

When $pcip_{t-4}$ is added, its coefficient is .0043 with a t statistic of about .10.

(ii) In the model

$$pcip_t = \delta_0 + \alpha_1 pcip_{t-1} + \alpha_2 pcip_{t-2} + \alpha_3 pcip_{t-3} + \gamma_1 pcsp_{t-1} + \gamma_2 pcsp_{t-2} + \gamma_3 pcsp_{t-3} + u_t,$$

the null hypothesis is that *pcsp* does not Granger cause *pcip*. This is stated as H_0: $\gamma_1 = \gamma_2 = \gamma_3 = 0$. The F statistic for joint significance of the three lags of $pcsp_t$, with 3 and 547 *df*, is $F = 5.37$ and p-value = .0012. Therefore, we strongly reject H_0 and conclude that *pcsp* does Granger cause *pcip*.

(iii) When we add $\Delta i3_{t-1}$, $\Delta i3_{t-2}$, and $\Delta i3_{t-3}$ to the regression from part (ii), and now test the joint significance of $pcsp_{t-1}$, $pcsp_{t-2}$, and $pcsp_{t-3}$, the F statistic is 5.08. With 3 and 544 *df* in the F distribution, this gives p-value = .0018, and so *pcsp* Granger causes *pcip* even conditional on past $\Delta i3$.

C18.5 (i) The estimated equation is

$$\widehat{hy6}_t = \underset{(.028)}{.078} + \underset{(0.016)}{1.027}\, hy3_{t-1} - \underset{(0.038)}{1.021}\, \Delta hy3_t - \underset{(.037)}{.085}\, \Delta hy3_{t-1} - \underset{(.037)}{.104}\, \Delta hy3_{t-2}$$

$$n = 121, \quad R^2 = .982, \quad \hat{\sigma} = .123.$$

The t statistic for H_0: $\beta = 1$ is $(1.027 - 1)/.016 \approx 1.69$. We do not reject H_0: $\beta = 1$ at the 5% level against a two-sided alternative, although we would reject at the 10% level.

(ii) The estimated error correction model is

$$\widehat{hy6}_t = \underset{(.049)}{.070} + \underset{(.278)}{1.259}\, \Delta hy3_{t-1} - \underset{(.256)}{.816}\, (hy6_{t-1} - hy3_{t-2})$$

$$+ \underset{(.272)}{.283}\, \Delta hy3_{t-2} + \underset{(.256)}{.127}\, (hy6_{t-2} - hy3_{t-3})$$

$$n = 121, \quad R^2 = .795.$$

Neither of the added terms is individually significant. The F test for their joint significance gives $F = 1.35$, p-value = .264. Therefore, we would omit these terms and stick with the error correction model estimated in (18.39).

C18.7 (i) The estimated linear trend equation using the first 119 observations and excluding the last 12 months is

$$\widehat{chnimp}_t = \underset{(53.20)}{248.58} + \underset{(0.77)}{5.15}\, t$$

$$n = 119, \quad R^2 = .277, \quad \hat{\sigma} = 288.33.$$

The standard error of the regression is 288.33.

(ii) The estimated AR(1) model excluding the last 12 months is

$$\widehat{chnimp}_t = \underset{(54.71)}{329.18} + \underset{(.084)}{.416}\, chnimp_{t-1}$$

$$n = 118,\quad R^2 = .174,\quad \hat{\sigma} = 308.17.$$

Because $\hat{\sigma}$ is lower for the linear trend model, it provides the better in-sample fit.

(iii) Using the last 12 observations for one-step-ahead out-of-sample forecasting gives an RMSE and MAE for the linear trend equation of about 315.5 and 201.9, respectively. For the AR(1) model, the RMSE and MAE are about 388.6 and 246.1, respectively. In this case, the linear trend is the better forecasting model.

(iv) Using again the first 119 observations, the F statistic for joint significance of feb_t, mar_t, …, dec_t when added to the linear trend model is about 1.15 with p-value $\approx$ .328. (The df are 11 and 106.) So, there is no evidence that seasonality needs to be accounted for in forecasting *chnimp*.

C18.9 (i) Using the data up through 1989 gives

$$\hat{y}_t = \underset{(1{,}163.09)}{3{,}186.04} + \underset{(46.31)}{116.24}\, t + \underset{(.148)}{.630}\, y_{t-1}$$

$$n = 30,\quad R^2 = .994,\quad \hat{\sigma} = 223.95.$$

(Notice how high the R-squared is. However, it is meaningless as a goodness-of-fit measure because $\{y_t\}$ has a trend, and possibly a unit root.)

(ii) The forecast for 1990 (t = 32) is 3,186.04 + 116.24(32) + .630(17,804.09) $\approx$ 18,122.30, because y is \$17,804.09 in 1989. The actual value for real per capita disposable income was \$17,944.64, and so the forecast error is –\$177.66.

(iii) The MAE for the 1990s, using the model estimated in part (i), is about 371.76.

(iv) Without y_{t-1} in the equation, we obtain

$$\hat{y}_t = \underset{(103.38)}{8{,}143.11} + \underset{(5.64)}{311.26}\, t$$

$$n = 31,\quad R^2 = .991,\quad \hat{\sigma} = 280.87.$$

The MAE for the forecasts in the 1990s is about 718.26. This is much higher than for the model with y_{t-1}, so we should use the AR(1) model with a linear time trend.

C18.11 (i) For *lsp*500, the ADF statistic without a trend is $t = -.79$; with a trend, the t statistic is -2.20. These are both well above their respective 10% critical values. In addition, the estimated roots are quite close to one. For *lip*, the ADF statistic without a trend is -1.37 without a trend and -2.52 with a trend. Again, these are not close to rejecting even at the 10% levels, and the estimated roots are very close to one.

(ii) The simple regression of *lsp*500 on *lip* gives

$$\widehat{lsp500} = \underset{(.095)}{-2.402} + \underset{(.024)}{1.694}\, lip$$

$$n = 558,\ R^2 = .903.$$

The t statistic for *lip* is over 70, and the R-squared is over .90. These are hallmarks of spurious regressions.

(iii) Using the residuals $\hat{u}_t$ obtained in part (ii), the ADF statistic (with two lagged changes) is -1.57, and the estimated root is over .99. There is no evidence of cointegration. (The 10% critical value is -3.04.)

(iv) After adding a linear time trend to the regression from part (ii), the ADF statistic applied to the residuals is -1.88, and the estimated root is again about .99. Even with a time trend there is no evidence of cointegration.

(v) It appears that *lsp*500 and *lip* do not move together in the sense of cointegration, even if we allow them to have unrestricted linear time trends. The analysis does not point to a long-run equilibrium relationship.

C18.13 (i) The DF statistic is about -3.31, which is to the left of the 2.5% critical value (-3.12), and so, using this test, we can reject a unit root at the 2.5% level. (The estimated root is about .81.).

(ii) When two lagged changes are added to the regression in part (i), the t statistic becomes -1.50, and the root is larger (about .915). Now, there is little evidence against a unit root.

(iii) If we add a time trend to the regression in part (ii), the ADF statistic becomes -3.67, and the estimated root is about .57. The 2.5% critical value is -3.66, and so we are back to fairly convincingly rejecting a unit root.

(iv) The best characterization seems to be an I(0) process about a linear trend. In fact, a stable AR(3) about a linear trend is suggested by the regression in part (iii).

(v) For $prcfat_t$, the ADF statistic without a trend is –4.74 (estimated root = .62) and with a time trend the statistic is –5.29 (estimated root = .54). Here, the evidence is strongly in favor of an I(0) process whether or not we include a trend.

C18.15 (i) The usual DF test, obtained by regressing *curate* on a lag of *urate*, gives a very small coefficient, −.0063, and a *t* statistic, −.79, that is not close to being significant. Adding two lags of *curate* changes little: The coefficient on *urate*_1 becomes −.0086 with $t = -1.22$. There is very little evidence against the unit root hypothesis for *urate*. The coefficients on both lags of *curate* are positive and statistically very significant, but the outcome of the augmented DF test is essentially the same as the usual DF test.

(ii) For *vrate* the outcome is less clear cut. From the simple DF regression, the estimated ρ is about .925. The DF $t = -2.68$, is below the 10% critical value, –2.57, but above the 5% critical value, –2.86. When we use the ADF statistic the evidence against a unit root is even weaker with $t = -2.19$. Based on the ADF we cannot reject the null hypothesis of a unit root at even the 10% level and the two lags of *cvrate* are both very statistically significant, which justifies relying on the ADF statistic. Because the estimated value of ρ is pretty high, we operate under the assumption that *vrate* also has a unit root. (If we do not, the Beveridge Curve would make no sense, as it would be relating in I(1) variable, *urate*, to an I(0) variable, *vrate*.)

(iii) To implement the Engle-Granger test for cointegration, we regress *urate* on *vrate* and get the residuals, say, $\hat{u}$. We then run $\hat{u}$ through a standard DF test. The coefficient on $\hat{u}\_1$ is –.148 and the Engle-Granger *t* statistic is –3.23. This is below the 5% critical value, –3.04, but above the 5% critical value, –3.34. Thus, we reject the null of no cointegration at the 10% level but not the 5% level.

(iv) We obtain the leads-and-lags estimator of β by regressing $urate_t$ on $vrate_t$, $cvrate_t$, $cvrate_{t-1}$, and $cvrate_{t+1}$. We obtain $\hat{\beta} = -3.98$ (Newey-West se = .293). The 95% CI runs from −4.56 to −3.40. If we use the usual OLS standard error, the 95% CI is narrower because the standard error is smaller. The CI runs from −4.38 to −3.58, but we should not rely on this.

(v) When two lags are added to the EG regression, the *t* statistic falls dramatically in magnitude: $t = -1.05$. Now there is no evidence of cointegration, which is particularly troubling because the two lags have very statistically significant coefficients.

Given the evidence from the augmented EG statistic, it is clear that the claim that *urate* and *vrate* are cointegrated is not supported by the data. It seems that, from a modern time series perspective – at least during a recent period, and with monthly data – the Beveridge Curve may not be well defined.