Problem Set 3

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Problem 1

a)

1,0,1,0

b)

$$
\begin{array}{ll}
\overline{z}(z+x)(y+v)(z+\overline{x}) & \\
(\overline{z}z+\overline{z}x)(y+v)(z+\overline{x}) & (\textbf{Distributive Law}) \\
(0+\overline{z}x)(y+v)(z+\overline{x}) & (\textbf{Complement Law}) \\
\overline{z}x(y+v)(z+\overline{x}) & (\textbf{Identity Law}) \\
(\overline{z}xz+\overline{z}x\overline{x})(y+v) & (\textbf{Distributive Law}) \\
(0\cdot x+0\cdot\overline{z})(y+v) & (\textbf{Complement Law}) \\
(0+0)(y+v) & (\textbf{Domination Law}) \\
0(y+v) & (\textbf{Idempotent Law}) \\
0 & (\textbf{Domination Law})
\end{array}
$$

c)

1,0,0,1

d)

$$
\begin{array}{ll}
(\overline{x}+\overline{y})\overline{(\overline{z}+v)}(y+x)(xy) & \\
\overline{(xy)}\overline{(\overline{z}+v)}(y+x)(xy) & (\textbf{DeMorgan's Law}) \\
0\cdot\overline{(\overline{z}+v)}(y+x) & (\textbf{Complement Law}) \\
0 & (\textbf{Domination Law})
\end{array}
$$

Problem 2

a)

Prove $\overline{A \cap \overline{B}} = \overline{A} \cup B$
We will start with $x \in (\overline{A \cap \overline{B}})$ and show that it is equivalent to $x \in (\overline{A} \cup B)$. By the definition of negation, $x \in (\overline{A \cap \overline{B}})$ is equivalent to $\neg(x \in (A \cap \overline{B}))$. We can then get $\neg((x \in A) \wedge \overline{(x \in B)})$ by the definition of intersect. Once again, by the definition of negation, we can get $\neg((x \in A) \wedge \neg(x \in B))$. Now, using De Morgan's law, we get $\neg(x \in A) \wedge \neg\neg(x \in B)$. With the double negation law, $\neg(x \in A) \wedge \neg\neg(x \in B)$ becomes $\neg(x \in A) \wedge (x \in B)$. We can apply the definition of negation to get $\overline{(x \in A)} \wedge (x \in B)$. Finally, by the definition of union, $\overline{(x \in A)} \wedge (x \in B)$ is equivalent to $x \in (\overline{A} \cup B)$, and therefore, $x \in (\overline{A \cap \overline{B}})$ is equivalent to $x \in (\overline{A} \cup B)$. $\square$

b)

Prove $A - (B \cap A) = A - B$ We will start with $x \in (A - (B \cap A))$ and show that it is equivalent to $x \in (A - B)$. Starting with the definition of difference, we show that $x \in (A - (B \cap A))$ is equivalent to $(x \in A) \wedge \neg(x \in (B \cap A))$. Then, by the definition of intersect, we get $(x \in A) \wedge \neg((x \in B) \wedge (x \in A))$. Using De Morgan's law, we get $(x \in A) \wedge (\neg(x \in B) \vee \neg(x \in A))$. Next, using the distributive law, we get $((x \in A) \wedge \neg(x \in B)) \vee ((x \in A) \wedge \neg(x \in A))$. With the complement law, we get $((x \in A) \wedge \neg(x \in B)) \vee F$, and with the idempotent law, we get $((x \in A) \wedge \neg(x \in B))$. Finally, by the definition of difference, $((x \in A) \wedge \neg(x \in B))$ is equivalent to $x \in (A - B)$. Therefore, $x \in (A - (B \cap A))$ is equivalent to $x \in (A - B)$. $\square$

Problem 3

a)

Let $A = \{1, 2, 4\}, B = \{2, 3, 4\}, C = \{1, 3, 4\}$. $(A \oplus B \oplus C) \cup (A \cap B \cap C) = \emptyset \cup \{4\} = \{4\}$, but $(A \cup B \cup C) = \{1, 2, 3, 4\}$. Therefore, the identity is false.

b)

Let $A = U, B = \emptyset$. $A \cap (\overline{A} \cup B)$ becomes $U \cap (\emptyset \cup \emptyset)$, which yields $\emptyset$. However, $A \cup (A \cap \overline{B})$ becomes $U \cup (U \cap U)$, which yields U. Therefore, the identity is false.