

## Problem Set 2

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### Problem 1.

$x = y = 1$  and  $w = z = 0$

**a)**

$$xyz\overline{w}$$

$$1 \wedge 1 \wedge \neg(0 \wedge 0)$$

$$1 \wedge 1$$

$$1$$

**b)**

$$x\overline{y} + z(\overline{w + z})$$

$$1 \wedge \neg 1 \vee 0 \wedge \neg(0 \vee 0)$$

$$1 \wedge 0 \vee 0 \wedge 1$$

$$0 \vee 0$$

**c)**

$$\overline{z}y\overline{x}(1 + w)$$

$$\neg 0 \wedge 1 \wedge \neg 1 \wedge (1 \vee 0)$$

$$1 \wedge 1 \wedge 0 \wedge 1$$

$$0$$

**d)**

$$xy\bar{z} + z\bar{w}$$

$$1 \wedge 1 \wedge \neg 0 \vee 0 \wedge \neg 0$$

$$1 \wedge 1 \wedge 1 \vee 0 \wedge 1$$

$$1 \vee 0$$

$$1$$

**e)**

$$\overline{(z + y)(w + x)}$$

$$\neg((0 \vee 1) \wedge (0 \vee 1))$$

$$\neg(1 \wedge 1)$$

$$\neg 1$$

$$0$$

**f)**

$$x\bar{y} + (\bar{x} + \bar{w} + \bar{y}\bar{z})$$

$$1 \wedge \neg 1 \vee \neg(\neg 1 \vee \neg 0 \vee \neg(1 \wedge 0))$$

$$1 \wedge 0 \vee \neg(0 \vee 1 \vee \neg(1 \wedge 0))$$

$$1 \wedge 0 \vee \neg(0 \vee 1 \vee 1)$$

$$1 \wedge 0 \vee 0$$

$$0 \vee 0$$

$$0$$

## Problem 2.

a)

Prove  $xy + x\bar{y} = x$

$$xy + x\bar{y}$$

$$x(y + \bar{y})$$

$$x \cdot 1$$

$$x$$

(Distributive Law)

(Complement Law)

(Domination Law)

b)

Prove  $x + xy = x$

$$x + xy$$

$$x$$

(Absorption Law)

c)

Prove  $x(\bar{y} + x) = x$

$$x(\bar{y} + x)$$

$$x\bar{y} + xx$$

$$x\bar{y} + x$$

$$x$$

(Distribution Law)

(Idempotent Law)

(Absorption Law)

d)

Prove  $\overline{xy + z\bar{x}} = (\bar{x} + \bar{y})(\bar{z} + x)$

$$\overline{xy + z\bar{x}}$$

$$\overline{xy} \cdot \overline{z\bar{x}}$$

$$(\bar{x} + \bar{y})(\bar{z} + \bar{\bar{x}})$$

$$(\bar{x} + \bar{y})(\bar{z} + x)$$

(DeMorgan's Law)

(DeMorgan's Law)

(Double Complement Law)

### Problem 3.

a)

$$x\bar{y}z + x\bar{z}$$

This formula is in DNF

$x$	$y$	$z$	$x\bar{y}z + x\bar{z}$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

CNF expression:

$$(x + y + z)(x + y + \bar{z})(x + \bar{y} + z)(x + \bar{y} + \bar{z})(\bar{x} + \bar{y} + \bar{z})$$

Simplified:

$$(x)(\bar{x} + \bar{y} + \bar{z})$$

b)

$$\bar{y}x\bar{z}w$$

This formula is in both CNF and DNF

c)

$$y(\bar{x} + z)(\bar{y} + \bar{z})(x + y + \bar{z})$$

This formula is in CNF

$x$	$y$	$z$	$x\bar{y}z + x\bar{z}$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

DNF expression:

$$\bar{x}y\bar{z}$$

d)

$$x + \overline{y} + z$$

This formula is in both CNF and DNF

## Problem 5.

$$(a_4 \oplus a_3 \oplus a_2 \oplus a_1 \oplus a_0) \oplus ((a_4 \vee a_3 \vee a_2 \vee a_1) \wedge \neg a_0)$$

## Problem 6.

a)

$$\begin{aligned}
& (F_O R_B) \oplus (F_O R_B + F_B R_O) \\
& ((F_O R_B) + (F_O R_B + F_B R_O)) \cdot \overline{((F_O R_B)(F_O R_B + F_B R_O))} & \text{(XOR Rewritten)} \\
& (F_O R_B + F_O R_B + F_B R_O) \cdot \overline{((F_O R_B)(F_O R_B + F_B R_O))} & \text{(Associative Law)} \\
& (F_O R_B + F_B R_O) \cdot \overline{((F_O R_B)(F_O R_B + F_B R_O))} & \text{(Idempotent Law)} \\
& (F_O R_B + F_B R_O) \cdot \overline{(F_O R_B)} + \overline{(F_O R_B + F_B R_O)} & \text{(DeMorgan's Law)} \\
& (F_O R_B + F_B R_O) \overline{(F_O R_B)} + (F_O R_B + F_B R_O) \overline{(F_O R_B + F_B R_O)} & \text{(Distributive Law)} \\
& (F_O R_B + F_B R_O) \overline{(F_O R_B)} + 0 & \text{(Complement Law)} \\
& (F_O R_B + F_B R_O) \overline{(F_O R_B)} & \text{(Identity Law)} \\
& \overline{(F_O R_B)}(F_O R_B) + \overline{(F_O R_B)}(F_B R_O) & \text{(Distributive Law)} \\
& 0 + \overline{(F_O R_B)}(F_B R_O) & \text{(Complement Law)} \\
& \overline{(F_O R_B)}(F_B R_O) & \text{(Identity Law)}
\end{aligned}$$

b)

The bank must be set on fire and the orphanage must be robbed. This is because the simplified version of The Riddler's final message says the orphanage being set on fire and the band being robbed cannot be true, and the bank being set on fire and the orphanage being robbed must be true.

c)

The Riddler's first message,  $F_O R_B$ , was a lie.

d)

It is possible that either the orphanage is set on fire or the bank is robbed, but not both. Therefore, Batman should send the fire department to the orphanage and Commissioner Gordon to the bank as well, just to be safe.

## Problem 8.

a)

$x \cdot y$  can be written as  $\overline{(x \rightarrow y) \rightarrow \bar{x}}$ , and since any function can be written using multiplication and complement,  $\rightarrow$  and complement must be functionally complete.

$x$	$y$	$x \rightarrow y$	$\bar{x}$	$(x \rightarrow y) \rightarrow \bar{x}$	$\overline{(x \rightarrow y) \rightarrow \bar{x}}$
0	0	1	1	1	0
0	1	1	1	1	0
1	0	0	0	1	0
1	1	1	0	0	1

b)

$\bar{x}$  can be written as  $x \oplus (x \rightarrow x)$ , and  $x \cdot y$  can be written as  $(x \rightarrow (x \oplus y)) \oplus (x \rightarrow x)$ . Since any function can be written using multiplication and complement,  $\rightarrow$  and  $\oplus$  must be functionally complete.

$x$	$y$	$x \oplus y$	$x \rightarrow (x \oplus y)$	$(x \rightarrow (x \oplus y)) \oplus (x \rightarrow x)$
0	0	0	1	0
0	1	1	1	0
1	0	1	1	0
1	1	0	0	1