Lecture 8

Misallocation within sectors (Dixit-Stiglitz)

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Hsieh and Klenow (2009)

Summary

- Most of the differences in GDP is driven by differences in TFP, even on sectoral level
- Next: try to understand productivity differences within sectors

Monopolistic competition

- Perfect competition: all firms set price at marginal cost, most competitive firm gets the whole market
  - industry TFP = TFP of most efficient firm (assuming CRS)
- Not very realistic
- Monopolistic competition: firms have some monopoly power, charge a mark up over marginal cost
  - less productive firms operate in equilibrium
- Simplest monopolistic competition: all charge the same markup
  - based on work of Dixit and Stiglitz

Plan

1. Simple model of monopolistic competition
   1. eqm
   2. social planners problem
   3. mapping to data and aggregation
2. Distorted model of monopolistic competition
3. Hsieh-Klenow application

1a. Simple model of monopolistic competition: eqm

Basic set up

- Intermediate sector: measure one of firms. Firm $i \in [0,1]$ produces differentiated product Y_i with technology

$$Y_i = A_i K_i^{\alpha} L_i^{1-\alpha}$$

  - i is monopolist for i, charges price P_i

- Final sector: competitive with production function

$$Y = \left(\int Y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

  - throughout assume $\sigma > 1$, no eqm otherwise

- Consumer buys final good, have inelastic endowment of capital and labor $\bar{K}$ and $\bar{L}$, own firms and get their profits. Utility $U(C)$

Final sector

- Final sector solves (final good is a numeraire)

$$\max_{\{Y_i\}} Y - \int P_i Y_i di$$

s.t.

$$Y = \left(\int Y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

- FOC for i :

$$Y_i = Y \times P_i^{-\sigma}$$

- Downward demand curve for good i with constant elasticity σ

Definition of eqm, I

Prices $\{P_i\}_i$, r, w, allocations $\{Y_i, \Pi_i, K_i, L_i\}_i$, C, Y such that:

- Consumers own firms and get profits $\int \Pi_i di$, supply labor $\bar{L}$ and capital $\bar{K}$ inelastically at w and r and solve

$$\max_C U(C)$$

s.t.

$$C = w\bar{L} + r\bar{K} + \int \Pi_i di$$

- Final goods firms take prices $\{P_i\}$ as given and solve

$$\max_{\{Y_i\}_i, Y} Y - \int P_i Y_i di$$

s.t.

$$Y = \left(\int Y_i^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}},$$

which produces demand for good i as $Y_i(P_i) = YP_i^{-\sigma}$

Definition of eqm, II

- Intermediate firms take w and $Y_i(P_i)$ as given and solve

$$\Pi_i = \max_{P_i, Y_i, L_i, K_i} P_i Y_i - wL_i - RK_i$$

s.t.

$$\begin{aligned} Y_i &= YP_i^{-\sigma} \\ Y_i &= A_i K_i^{\alpha} L_i^{1-\alpha} \end{aligned}$$

- Market clearing conditions

$$C = Y, \int K_i di = \bar{K}, \int L_i di = \bar{L}$$

Remark

- We could have equivalently ditched the final good sector, and simply assume that consumers buy all the intermediate goods themselves, with preferences

$$\max_{C,\{C_i\}_i} U(C)$$

s.t.

$$C = \left(\int C_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}},$$

$$\int P_i C_i di = w\bar{L} + r\bar{K} + \int \Pi_i di$$

- It is the same setting, the math is just slightly more transparent with the final good sector formulation

Undistorted firm's problem

Excercise

Show that optimality for intermediate firms implies

$$P_i = \frac{\sigma}{\sigma - 1} \times \frac{1}{A_i} \left(\frac{R}{\alpha}\right)^{\alpha} \left(\frac{w}{1-\alpha}\right)^{1-\alpha},$$

$$\frac{K_i}{L_i} \text{ is the same for all } i$$

I will use symbols $\varsigma_1, \varsigma_2, ..$ to denote terms that are the same for all i

Excercise

Show that feasibilities then imply that

$$\frac{K_i}{L_i} = \frac{\bar{K}}{\bar{L}} \text{ for all } i$$

Solving eqm

- Previous results implies

$$P_i A_i = \varsigma_1$$

- Demand for good i is

$$Y_i = \varsigma_2 P_i^{-\sigma} = \varsigma_3 A_i^{\sigma}$$

- Previous results plus technology for firm i is

$$Y_i = \left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha} A_i L_i = \left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha-1} A_i K_i$$

which implies

$$L_i = \varsigma_L A_i^{\sigma-1}, \quad K_i = \varsigma_K A_i^{\sigma-1}$$

Eqm allocations

- Find constants from feasibilities:

$$\varsigma_L = \frac{\bar{L}}{\int A_i^{\sigma-1} di}, \ \varsigma_K = \frac{\bar{K}}{\int A_i^{\sigma-1} di}$$

and let

$$\varsigma_Y = \left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha} \varsigma_L$$

- Since we know ς_L, ς_K, ς_Y, we have solved for all eqm allocations:

$$L_i = \varsigma_L A_i^{\sigma-1}, \quad K_i = \varsigma_K A_i^{\sigma-1} \quad Y_i = \varsigma_Y A_i^{\sigma}$$

- More productive firms produce more output, use larger share of inputs
  - larger $\sigma \implies$ bigger fraction of market such firms have

Economics for the optimal price equation

- It is useful to understand the economics behind optimal price eqn

$$P_i = \frac{\sigma}{\sigma - 1} \times \frac{1}{A_i} \left(\frac{R}{\alpha} \right)^{\alpha} \left(\frac{w}{1-\alpha} \right)^{1-\alpha}$$

- Very important for understanding efficiency property of this eqm

Marginal cost

- Consider cost minimization

$$C(Y_i) = \min_{K_i, L_i} wL_i + RK_i$$

s.t.

$$Y_i = A_i K_i^{\alpha} L_i^{1-\alpha} \qquad (\lambda_i)$$

Marginal cost

- Consider cost minimization

$$C\left(Y_i\right) = \min_{K_i, L_i} wL_i + RK_i$$

s.t.

$$Y_i = A_i K_i^{\alpha} L_i^{1-\alpha} \qquad (\lambda_i)$$

Excercise

Show that optimality implies

$$\lambda_i = \frac{1}{A_i}\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}$$

Envelope theorem

Theorem

Consider maximization problem $V(\theta) = \max_{x \in \mathbb{R}^n} f(x;\theta)$ *where* θ *is a parameter. Let* x_θ^* *be a solution to this problem and* f_θ *be derivative of* f *w.r.t.* θ. *Then*

$$V'(\theta) = f_\theta\left(x_\theta^*;\theta\right).$$

- Our cost minimization in Lagrangian form:

$$C(Y_i) = \min_{K_i, L_i} wL_i + RK_i + \lambda_i\left[Y_i - A_i K_i^\alpha L_i^{1-\alpha}\right]$$

- Apply the envelope theorem:

$$C'(Y_i) = \lambda_i$$

- Therefore, λ_i is the marginal cost of firm i

Implication

- Easy to interpret optimal price

$$P_i = \underbrace{\frac{\sigma}{\sigma - 1}}_{\text{mark up} > 1} \times \underbrace{\frac{1}{A_i}\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}}_{\text{marginal cost, } \lambda_i}$$

- All firms charge the same mark up $\frac{\sigma}{\sigma-1}$ over marginal costs
    - perfect competition limit as $\sigma \to \infty$

Useful short-cut

- Marginal costs are independent of the choice of optimal prices
  - In many applications, the exact form of marginal costs is not very important, so people write the firm's problem as

$$\max_{P_i} P_i Y_i (P_i) - mc_i Y_i (P_i)$$

s.t. $Y_i (P_i) = Y P_i^{-\sigma}$

- This gives

$$\max_{P_i} Y P_i^{1-\sigma} - mc_i Y P_i^{-\sigma}$$

that we get right away the key implication

$$P_i = \frac{\sigma}{\sigma - 1} mc_i$$

1b. Simple model of monopolistic competition: social planner's problem

Social planner

Social planner chooses allocations subject to feasibilities:

$$\max_{Y,\{Y_i,K_i,L_i\}_i} U(Y)$$

s.t.

$$Y = \left(\int Y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}},$$

$$Y_i = A_i K_i^{\alpha} L_i^{1-\alpha},$$

$$\int K_i di = \bar{K}, \int L_i di = \bar{L}$$

Efficiency of eqm

Excercise

Show that SP optimum implies that

$$L_i = \varsigma_L A_i^{\sigma-1}, \quad K_i = \varsigma_K A_i^{\sigma-1} \quad Y_i = \varsigma_Y A_i^{\sigma}$$

where ς_L, ς_K, ς_Y are the ones we found in eqm

- Somewhat surprising, this implies that monopolistic eqm is **efficient** even though price $>$ marginal cost
- **Intuition**:
  - in our simple model, all factors are supplied inelastically
  - monopoly power lowers real wage and return to capital but cannot distort aggregate supply of $\bar{K}$ and $\bar{L}$
  - each firm has the same markup $\frac{\sigma}{\sigma-1}$: monopoly power does not distort allocation of $\bar{K}$ and $\bar{L}$ across firms

1c. Mapping to data and aggregation

Measures of efficiency

- Physical productivity

$$TFPQ_i \equiv \frac{Y_i}{K_i^\alpha L_i^{1-\alpha}} = A_i$$

- Revenue productivity

$$TFPR_i \equiv \frac{P_i Y_i}{K_i^\alpha L_i^{1-\alpha}} = P_i A_i$$

- Undistorted firm optimization implies

$$TFPR_i = TFPR_j \text{ for all } i, j$$

Measurements in the data

- Data for L_i (labor in some unit), K_i (book or market value of capital), $\mathcal{R}_i = P_i Y_i$ (revenues), $\mathcal{L}_i = wL_i$ (labor payments)
- From definition we can measure directly

$$TFPR_i \equiv \frac{\mathcal{R}_i}{K_i^{\alpha} L_i^{1-\alpha}}$$

- From $Y_i = YP_i^{-\sigma}$ we get

$$Y_i = Y^{\frac{1}{1-\sigma}} \left(P_i Y_i\right)^{\frac{\sigma}{\sigma-1}} = \varsigma_7 \mathcal{R}_i^{\frac{\sigma}{\sigma-1}}$$

- Therefore

$$TFPQ_i = \varsigma_7 \frac{\mathcal{R}_i^{\frac{\sigma}{\sigma-1}}}{K_i^{\alpha} L_i^{1-\alpha}}$$

Industry TFP

- Industry value added

$$\int P_i Y_i di = Y$$

- We would measure industry TFP as

$$Y = TFP \times \bar{K}^{\alpha} \times \bar{L}^{1-\alpha}$$

- What is TFP in terms of primitives?

$$\begin{aligned} Y &= \left(\int \left(A_i K_i^{\alpha} L_i^{1-\alpha} \right)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \\ &= \underbrace{\left(\int \left[A_i \left(\frac{K_i}{\bar{K}} \right)^{\alpha} \left(\frac{L_i}{\bar{L}} \right)^{1-\alpha} \right]^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}}_{=TFP} \times \bar{K}^{\alpha} \times \bar{L}^{1-\alpha} \end{aligned}$$

TFP in the undistorted economy

$$TFP = \left(\int \left[A_i \left(\frac{K_i}{\bar{K}} \right)^{\alpha} \left(\frac{L_i}{\bar{L}} \right)^{1-\alpha} \right]^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

- In eqm $L_i = \varsigma_L A_i^{\sigma-1}$, $K_i = \varsigma_K A_i^{\sigma-1}$, so that

$$\frac{L_i}{\bar{L}} = \frac{\varsigma_L A_i^{\sigma-1}}{\varsigma_L \int A_i^{\sigma-1} di} = \frac{A_i^{\sigma-1}}{\int A_i^{\sigma-1} di} = \frac{K_i}{\bar{K}}$$

- This gives

$$TFP = \left[\int A_i^{\sigma-1} di \right]^{\frac{1}{\sigma-1}}$$

Simplifying TFP

- Want to simplify

$$\ln TFP = \frac{1}{\sigma - 1} \ln \left[\int A_i^{\sigma-1} di \right]$$

- Wlog we can write

$$\ln A_i = \bar{a} + a_i \text{ with } \mathbb{E} a_i = 0.$$

- For any $x \geq 0$ define

$$F(x) = \frac{1}{\sigma - 1} \ln \left[\mathbb{E} \exp \left\{ (\sigma - 1)(\bar{a} + x a_i) \right\} \right]$$

- Note that

$$F(1) = \ln TFP$$

Approximations

- Use standard Taylor expansion

$$F(1) \approx F(0) + F'(0) + \frac{1}{2}F''(0)$$

- Compute explicitly

$$\ln TFP = F(1) \approx \mathbb{E}\ln A + \frac{\sigma - 1}{2} var(\ln A)$$

- What is the intuition for why TFP is increasing in σ?

2. Distorted monopolistic eqm

Distorted firm's optimization

- Suppose allocations are distorted with firm-specific wedges $\tau_{Y,i}, \tau_{K,i}$
- Actual firm problem

$$\max_{P_i, Y_i, L_i, K_i} \left(1 - \tau_{Y,i}\right) P_i Y_i - wL_i - \left(1 + \tau_{K,i}\right) RK_i$$

s.t.

$$\begin{aligned} Y_i &= YP_i^{-\sigma} \\ Y_i &= A_i K_i^{\alpha} L_i^{1-\alpha} \end{aligned}$$

Misallocation and TFP

- Going through the same algebra as in undistorted eqm, we find that $TFPR_i$ are no longer equalized for all i :

$$TFPR_i = \varsigma_8 \frac{\left(1 + \tau_{K,i}\right)^{\alpha}}{1 - \tau_{Y,i}}$$

- And sectoral TFP is

$$TFP^{dist} = \left[\int \left(A_i \frac{\overline{TFPR}}{TFPR_i} \right)^{\sigma - 1} di \right]^{\frac{1}{\sigma - 1}}$$

  where $\overline{TFPR}$ is certain average of $TFPR_i$
- Using approximations, it can be show that

$$\ln TFP^{dist} \approx \left[\underbrace{\mathbb{E} \ln A + \frac{\sigma - 1}{2} var\left(\ln A\right)}_{\text{undistorted TFP}} \right] - \underbrace{\frac{\sigma}{2} var\left(\ln TFPR\right)}_{\text{effect of distortions}}$$

Measuring distortions in the data

- TFPR and wedges can be measured directly in the data from the following FOCs

$$TFPR_i = \frac{\mathcal{R}_i}{K_i^{\alpha} L_i^{1-\alpha}}$$

- Also wedges from firm optimization

$$\begin{aligned} 1 + \tau_{K,i} &= \frac{\alpha}{1-\alpha} \frac{\mathcal{L}_i}{RK_i} \\ 1 - \tau_{Y,i} &= \frac{\sigma}{\sigma - 1} \frac{\mathcal{L}_i}{(1-\alpha)\, \mathcal{R}_i} \end{aligned}$$

3. Hsieh and Klenow application

Hsieh-Klenow

- Firm census data for firms in U.S., China, India for different industries and years

TPFR

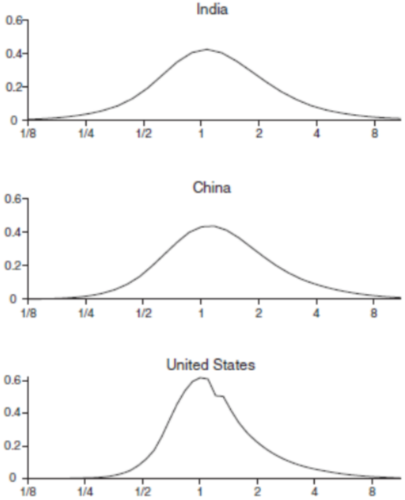
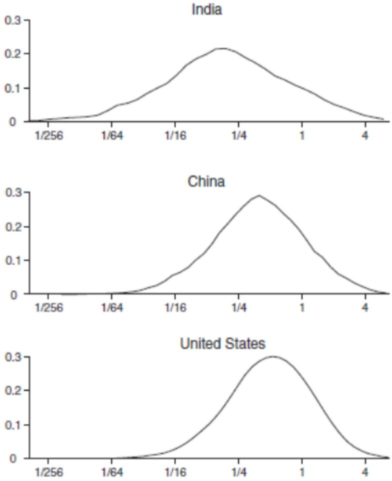
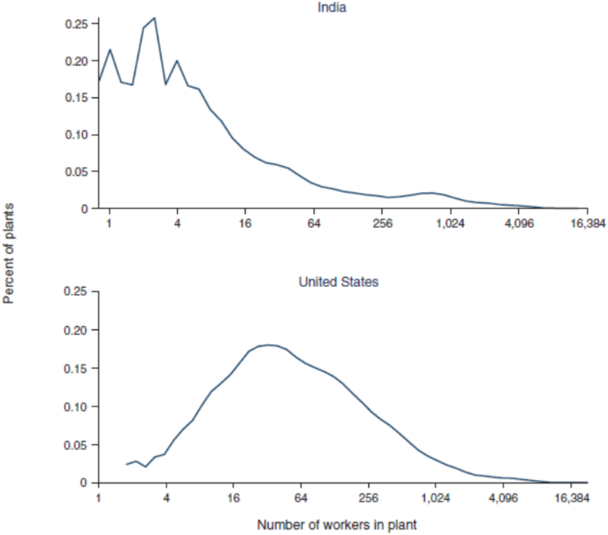


FIGURE II
Distribution of TFPR

TPFQ


India
0.3
0.2
0.1
0
1/256
1/64
1/16
1/4
1
4
China
0.3
0.2
0.1
0
1/256
1/64
1/16
1/4
1
4
United States
0.3
0.2
0.1
0
1/256
1/64
1/16
1/4
1
4

Distribution of plan sizes



Output loss due to misallocation

- We have

$$\frac{Y^{distorted}}{Y^{undistorted}} = \frac{TFP^{distorted} \times K^{\alpha} L^{1-\alpha}}{TFP^{undistorted} \times K^{\alpha} L^{1-\alpha}}$$
$$= \left[\int \left(\frac{A_i}{\bar{A}^{undistorted}} \frac{\overline{TFPR}}{TFPR_i} \right)^{\sigma-1} di \right]^{\frac{1}{\sigma-1}}$$

where

$$\bar{A}^{undistorted} = \left[\int A_i^{\sigma-1} di \right]^{\frac{1}{\sigma-1}}$$

Output gains from removing misallocation

TABLE IV
TFP GAINS FROM EQUALIZING TFPR WITHIN INDUSTRIES

China	1998	2001	2005
%	115.1	95.8	86.6
India	1987	1991	1994
%	100.4	102.1	127.5
United States	1977	1987	1997
%	36.1	30.7	42.9

Output gains from removing misallocation

TABLE VI

TFP Gains from Equalizing TFPR Relative to 1997 U.S. Gains

China	1998	2001	2005
%	50.5	37.0	30.5
India	1987	1991	1994
%	40.2	41.4	59.2