Lecture 2

Structural change: demand side

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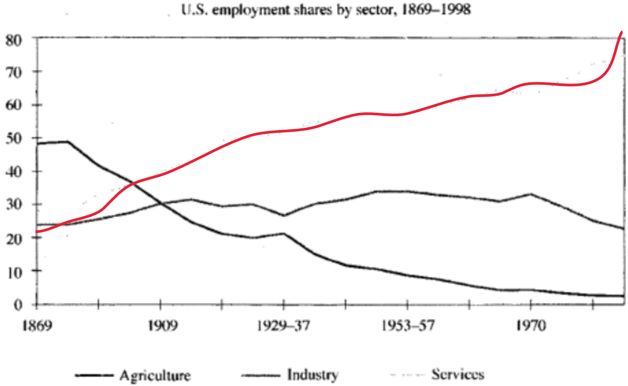
Introduction

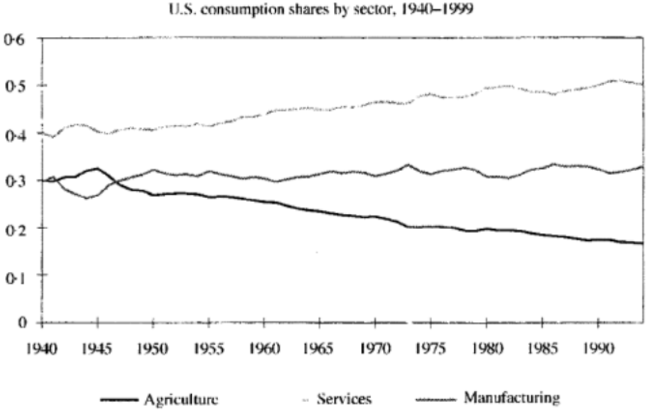
- Changes in composition of employment and production are important part of process of development.
  - Shift of employment and production from agriculture to manufacturing, and then from manufacturing to services.
- Emphasize how structural changes can be reconciled with balanced growth.

Kuznets facts I: employment shares



Sources: Historical Statistics of the United States, 1975 edition. Statistical Abstract of the United States, 1999.

Kuznets facts II: consumption shares



Source: Fconomic Report of the President (1990, 2000).

Kuznets facts III: summary

The Kuznets facts

	Share of total employment	Share of total consumption expenditures
Agriculture	declines	declines
Manufacturing	stable[5]	stable
Services	increases	increases

Non-Balanced Growth

- Major changes in structure of production in US economy over past 150 years
- Similar patterns present in all OECD economies
- Some less-developed economies still largely agricultural but trend towards smaller share of agriculture

Non-Balanced Growth

- How to reconcile broad structural changes emphasized by Kuznets with simultaneous constancy of aggregate variables emphasized by Kaldor?
- Two broad classes of explanations
  - preference-driven
  - technology-driven
- Both may be occurring at the same time

Non-Balanced Growth: The Demand Side

- Historically, demand side was emphasized more
- *Engel's law*: as a household's income increases, fraction that it spends on food (agricultural products) declines
  - fraction spent on services increases
- Kongsamut, Rebelo and Xie (AER, 2001)
  - can we incorporate this story into a neoclassical growth model?
  - get both Kaldor and Kuznets facts?

Demand side: key ingredients

- Preferences

$$\int_0^\infty \exp(-\rho t) \frac{c(t)^{1-\sigma} - 1}{1-\sigma} dt$$

with

$$\begin{aligned} c(t) &= \left(c^A(t) + \gamma^A\right)^{\eta^A} c^M(t)^{\eta^M} \left(c^S(t) + \gamma^S\right)^{\eta^S}, \\ \eta^i &> 0, \sum_{i \in \{A,M,S\}} \eta^i = 1, \\ \gamma^A &< 0, \gamma^S > 0 \end{aligned}$$

- Technology F is CRS with

$$\begin{aligned} Y^i(t) &= B^i F\left(K^i(t), X(t) L^i(t)\right), \\ \dot{X}(t)/X(t) &= g. \end{aligned}$$

with capital goods produced by sector M

- Inelastic labor supply ($\bar{L} = 1$)

Intuition for preferences

- Maximize preferences subject to budget constraint

$$\sum_{i \in \{A,M,S\}} p^i(t)\, c^i(t) + \dot{K}(t) = w(t) + (r(t) - \delta)\, K(t)$$

- Intratemporal conditions imply

$$p^i c^i = \eta^i \frac{c^{1-\sigma}}{\lambda} - p^i \gamma^i$$

Homothetic benchmark

- Suppose first $\gamma^A = \gamma^S = 0$:

$$\frac{p^i c^i}{p^j c^j} = \frac{\eta^i}{\eta^j}$$

- Preferences are *homothetic*: consumption shares are always constant as income grows
- Paremeter η^i captures consumption share of good i :

$$\eta^i = \frac{p^i c^i}{\sum_{j \in \{A,M,S\}} p^j c^j}$$

Nonhomothetic preferences

- More generally
$$\frac{p^i c^i}{p^M c^M} = \frac{\eta^i}{\eta^M} - \frac{p^i}{p^M}\frac{\gamma^i}{c^M}$$
- Holding prices fixed, $p^i c^i$ growths faster (slower) than $p^M c^M$ if $\gamma^i > 0$ (if $\gamma^i < 0$).
  - $\gamma^A < 0$: consumption share of A grows slower than M
  - $\gamma^S > 0$: consumption share of S grows faster than M
- Consistent with cross-sectional patterns in spending (Engel's curves)

Nonhomothetic preferences

- More generally

$$\frac{p^i c^i}{p^M c^M} = \frac{\eta^i}{\eta^M} - \frac{p^i}{p^M}\frac{\gamma^i}{c^M}$$

- Holding prices fixed, $p^i c^i$ growths faster (slower) than $p^M c^M$ if $\gamma^i > 0$ (if $\gamma^i < 0$).
  - $\gamma^A < 0$: consumption share of A grows slower than M
  - $\gamma^S > 0$: consumption share of S grows faster than M
- Consistent with cross-sectional patterns in spending (Engel's curves)
- The term $\frac{\gamma^i}{c^M} \to 0$ as $c^M \to \infty$
  - these preferences are become homothetic as economy grows
  - $\{\eta^i\}_i$ are long-run consumption shares

Technology

- Firms problem

$$\max p^i(t)\, Y^i(t) - w(t)\, L^i(t) - r(t)\, K^i(t)$$

s.t.

$$Y^i(t) = B^i F\left(K^i(t), X(t)\, L^i(t)\right)$$

Market clearing

- Market clearing for labor and capital:

$$K^A(t) + K^M(t) + K^S(t) = K(t),$$

$$L^A(t) + L^M(t) + L^S(t) = 1.$$

- Manufacturing good is used in production of investment good

$$\begin{aligned} I(t) + c^M(t) &= Y^M(t) \\ \dot{K}(t) &= I(t) - \delta K \end{aligned}$$

- Market clearing for agricultural and service goods:

$$c^A(t) = Y^A(t) \text{ and } c^S(t) = Y^S(t)$$

Competitive equilibrium

Definition

Given initial K_0, collection of prices and quantities, such that
(i) consumers choose their quantities optimally given prices
(ii) firms choose their quantities optimally given prices
(iii) all markets clear

Wlog, we normalize $p^M(t) \equiv 1$ for all t.

Optimality conditions for firms

- Capital
$$p^i(t)B^iF_K\left(K^i(t), X(t)\,L^i(t)\right) = r(t)$$
- Labor
$$p^i(t)B^iF_L\left(K^i(t), X(t)\,L^i(t)\right)X(t) = w(t)$$
- Almost all implications will be derived from these two conditions without using consumer side at all!

Reminder

Lemma: if $F\left(K,L\right)$ is HD1 then

$$F(K,L) = F_K\left(K,L\right)K + F_L\left(K,L\right)L \tag{*}$$

and $F_K\left(K,L\right)$, $F_L\left(K,L\right)$ are HD0.

Proof

- Since F is HD1

$$F(\lambda K, \lambda L) = \lambda F(K, L) \text{ for all } \lambda$$

- Differentiate w.r.t. λ and evaluate at $\lambda = 1$ to get (*)
- From (*) we have

$$F(\lambda K, \lambda L) = F_K(\lambda K, \lambda L)\,\lambda K + F_L(\lambda K, \lambda L)\,\lambda L$$

- Since F is HD1

$$F(K, L) = F_K(\lambda K, \lambda L)\,K + F_L(\lambda K, \lambda L)\,L$$

so F_K, F_L are HD0

Equalization of capital-labor ratios

- From firms' optimization in each sector

$$X(t)\,\frac{r(t)}{w(t)} = \frac{F_K\left(K^i(t), X(t)\,L^i(t)\right)}{F_L\left(K^i(t), X(t)\,L^i(t)\right)}$$

- Since F_K, F_L are HD0, the RHS is a function of $K^i(t)/X(t)\,L^i(t)$
- Since $r(t)/w(t)$ does not depend on i, there is some $k(t)$ s.t.

$$\frac{K^i(t)}{X(t)\,L^i(t)} = k(t) \text{ for all } i$$

Constant relative prices

- Also, from the first order conditions on firms

$$p^i(t)B^i F_K\left(K^i(t), X(t)\, L^i(t)\right) = r(t)$$

- From previous result,

$$F_K\left(K^i(t), X(t)\, L^i(t)\right) = F_K\left(K^j(t), X(t)\, L^j(t)\right) \ \ \forall i,j \in \{A, M, S\}$$

- Therefore (remember normalization $p^M(t) = 1$)

$$p^A(t) = \frac{B^M}{B^A}, \ \ p^S(t) = \frac{B^M}{B^S}$$

- In CE prices determined by technology, not preferences
  - same growth rate in all sectors $\Longleftrightarrow$ same relative prices

Optimality conditions for consumers

Excercise

Show that optimality conditions for consumers imply

$$\frac{1}{\sigma}(r-\delta-\rho)=\frac{\dot{c}^M(t)}{c^M(t)}=\frac{\dot{c}(t)}{c(t)},$$

$$\frac{\dot{c}^M}{c^M}=\frac{\dot{c}^A}{c^A+\gamma^A}=\frac{\dot{c}^S}{c^S+\gamma^S}.$$

Optimality conditions for consumers

Excercise

Show that optimality conditions for consumers imply

$$\frac{1}{\sigma}\left(r-\delta-\rho\right)=\frac{\dot{c}^{M}\left(t\right)}{c^{M}\left(t\right)}=\frac{\dot{c}\left(t\right)}{c\left(t\right)},$$

$$\frac{\dot{c}^{M}}{c^{M}}=\frac{\dot{c}^{A}}{c^{A}+\gamma^{A}}=\frac{\dot{c}^{S}}{c^{S}+\gamma^{S}}.$$

The second equation implies sectoral reallocation (since $\gamma^{A}<0$, $\gamma^{S}>0$)

$$\frac{\dot{c}^{A}}{c^{A}}<\frac{\dot{c}^{M}}{c^{M}}<\frac{\dot{c}^{S}}{c^{S}}$$

Structural change

- Since $c^A(t) = Y^A(t)$ and $c^S(t) = Y^S(t)$, we must have

$$\frac{\dot{Y}^A}{Y^A} < \frac{\dot{Y}^S}{Y^S}$$

- Hit Kuznets facts: reallocation of value added from A to S

Kaldor facts

- How does economy behaves in the aggregate?
  - when is it consistent with Kaldor facts?
- Want to aggregate it up

Aggregation

- Start with three feasibility conditions

$$
\begin{aligned}
c^A(t) &= B^A F\left(K^A(t), X(t) L^A(t)\right), \\
c^S(t) &= B^S F\left(K^S(t), X(t) L^S(t)\right), \\
c^M(t) + \dot{K}(t) &= B^M F\left(K^M(t), X(t) L^M(t)\right) - \delta K(t).
\end{aligned}
$$

- Apply HD1 Lemma

CGP growth rates

- Lemma about HD1:

$$\begin{aligned}
c^A(t) &= B^A \left\{ F_K^A(t)\, K^A(t) + F_L^A X(t) L^A(t) \right\}, \\
c^S(t) &= B^S \left\{ F_K^S(t)\, K^S(t) + F_L^S(t)\, X(t) L^S(t) \right\}, \\
c^M(t) + \dot{K}(t) &= B^M \left\{ F_K^M(t)\, K^M(t) + F_L^M(t)\, X(t) L^M(t) \right\} \\
&\quad -\delta K(t).
\end{aligned}$$

- Multiply by p^i, sum and firms optimality conditions

$$p^A c^A(t) + c^M(t) + p^S c^S(t) + \dot{K}(t) = r(t)\, K(t) + w(t)\, \bar{L} - \delta K(t)$$

- Want to get rid of $r(t)$ and $w(t)$

CGP growth rates

- Note that

$$\frac{K^i(t)}{X(t)\,L^i(t)} = k(t) \text{ for all } i$$

implies

$$\frac{K(t)}{X(t)\,\bar{L}} = k(t)$$

CGP growth rates

- Note that

$$\frac{K^i(t)}{X(t)\,L^i(t)} = k(t) \text{ for all } i$$

implies

$$\frac{K(t)}{X(t)\,\bar{L}} = k(t)$$

- Since F_K and F_L are HD0:

$$\begin{aligned} r(t) &= B^M F_K\left(K^M(t), X(t)\,L^M(t)\right) = B^M F_K(k(t), 1) \\ &= B^M F_K(K(t), X(t)\,\bar{L}) \end{aligned}$$

and same for $w(t)$

CGP growth rates

- Note that

$$\frac{K^i(t)}{X(t)\,L^i(t)} = k(t) \text{ for all } i$$

implies

$$\frac{K(t)}{X(t)\,\bar{L}} = k(t)$$

- Since F_K and F_L are HD0:

$$\begin{aligned} r(t) &= B^M F_K\left(K^M(t), X(t)\,L^M(t)\right) = B^M F_K(k(t), 1) \\ &= B^M F_K(K(t), X(t)\,\bar{L}) \end{aligned}$$

and same for $w(t)$

- Therefore, sum of feasibility constraints is

$$p^A c^A(t) + c^M(t) + p^S c^S(t) + \dot{K}(t) = B^M F(K(t), X(t)\,\bar{L}) - \delta K(t)$$

Kaldor facts

- Focus on *Constant growth path* (CGP): equilibrium with constant interest rates
  - assumption on K_0
- The plan:
  - necessary conditions: assume Constant Growth Path exist $\Longrightarrow$ what should be true?
  - sufficient conditions: verify (yourself) that under these conditions CGP exists and hits Kaldor facts

Constant capital-labor ratio

- If CGP exists, then $r(t) = r$ and therefore $k(t) = k$ so that all sectorial capital-labor ratios are constant

$$K^i(t) = k \cdot X(t) L^i(t)$$

- Sum up across i :

$$K(t) = k \cdot X(t) \bar{L}$$

- Therefore

$$\frac{\dot{K}(t)}{K(t)} = g.$$

- Aggregate feasibility:

$$p^A c^A(t) + c^M(t) + p^S c^S(t) = B^M F\left(K(t), X(t)\bar{L}\right) - (\delta + g) K(t)$$

- At what speed does each term grows?

Existence of Constant Growth Path

- Re-write this equation as

$$\begin{aligned} & p^A\left(c^A(t)+\gamma^A\right)+c^M(t)+p^S\left(c^S(t)+\gamma^S\right) \\ & -\left[p^A\gamma^A+p^S\gamma^S\right] \\ = \quad & B^M F(K(t),X(t)\bar{L})-(\delta+g)\,K(t) \end{aligned}$$

- The Excercise implies that $p^A\left(c^A(t)+\gamma^A\right), c^M(t), p^S\left(c^S(t)+\gamma^S\right)$ grow at the same rates
- Apply Uzawa's arguments: can have balanced growth only if

$$p^A\gamma^A+p^S\gamma^S=0$$

Structural change

Proposition In the above-described economy a CGP exists if and only if

$$\frac{\gamma^A}{B^A} + \frac{\gamma^S}{B^S} = 0.$$

In a CGP $k(t) = k$ for all t, and moreover

$$\frac{\dot{c}^A}{c^A} = g\frac{c^A + \gamma^A}{c^A}, \; \frac{\dot{c}^M}{c^M} = g, \; \frac{\dot{c}^S}{c^S} = g\frac{c^S + \gamma^S}{c^S}$$

- Growth rate in S starts high and asymptotes to g as $c^S \to \infty$
- Growth rate in A starts low and asymptotes to g as $c^A \to \infty$

Labor transition

- We have

$$c^i(t) = X(t)L^i(t)B^iF(k,1) \text{ for } i \in \{A, S\}$$

- This implies

$$\frac{\dot{c}^i}{c^i} = \frac{\dot{X}}{X} + \frac{\dot{L}^i}{L^i}$$

Labor transition

- We have

$$c^i(t) = X(t)L^i(t)B^iF(k,1) \text{ for } i \in \{A, S\}$$

- This implies

$$\frac{\dot{c}^i}{c^i} = \frac{\dot{X}}{X} + \frac{\dot{L}^i}{L^i}$$

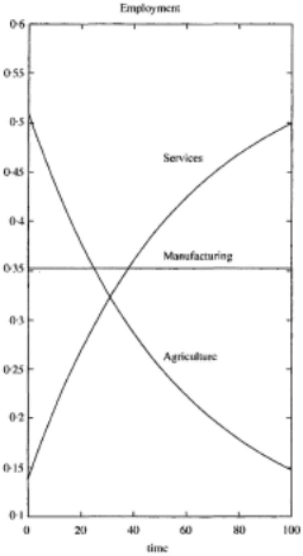
Excercise

Show that this + previous equations imply in CGP

$$\frac{\dot{L}^M}{L^M} = 0, \; \frac{\dot{L}^A}{L^A} < 0, \; \frac{\dot{L}^S}{L^S} > 0.$$

What are the limits $\lim_{t\to\infty} \frac{\dot{L}^A(t)}{L^A(t)}$ and $\lim_{t\to\infty} \frac{\dot{L}^S(t)}{L^S(t)}$?

Dynamics


Employment
Output Shares
Services
Manufacturing
Agriculture
time
0·6
0·55
0·5
0·45
0·4
0·35
0·3
0·25
0·2
0·15
0·1
0
20
40
60
80
100

Discussion

- The model is simultaneously consistent with balanced growth path facts of Kaldor and with structural change facts of Kuznets
- Condition necessary for a CGP is a “knife-edge” condition
  - but even when not satisfied, model may approximate structural change we observe