Lecture 10

Misallocations in networks

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Baqaee and Farhi (2020)

Introduction

- Hsieh and Klenow imposed a lot of structure on the economy
  - very simple network structure of the economy
  - CES aggregator
- They are unrealistic
- We can dispense with most of them!
- This lecture: Baqaee and Farhi (2020) measuring changes in distortions in the U.S. and abroad over last 20 years

Plan

1. Competitive network model
2. Distorted network model
3. Baqaee-Farhi application

1. Competitive network model

Starting point

- N producers
  - prices $p_1, \dots, p_N$
- F factors supplied inelastically with quantities $\bar{L}_f$
  - prices $w_1, \dots, w_F$
- Arbitrary production network
- Final good sector that aggregates N producers into production

Producers problem

- Each firm is competitive and operates

$$\pi_i = \max_{y_i, \{x_{ij}\}_{j=1}^{N+F}} p_i y_i - \sum_{j=1}^{N} p_j x_{ij} - \sum_{f=1}^{F} w_f x_{if}$$

  s.t.

$$y_i = A_i F_i \left(\left\{ x_{ij} \right\}_{j=1}^{N+F} \right)$$

  where A_i is productivity and F is CRS

- F_i can depend on inputs of other firms in arbitrary way

Final goods and consumers

- Final good sector is competitive and solves

$$\Pi = \max_{Y, \{c_j\}_{j=1}^N} Y - \sum_{j=1}^N p_j c_j$$

s.t.

$$Y = \mathcal{D}\left(c_1, ..., c_N\right)$$

where $\mathcal{D}$ is CRS.

- Consumers solve

$$\max_C U\left(C\right)$$

s.t.

$$C = \sum_{f=1}^F w_f \bar{L}_f$$

Competitive eqm

- CE is C, Y, $\left\{c_j, y_j\right\}_{j=1}^{N}$, $\left\{x_{ij}\right\}_{i=1\ldots N, j=1\ldots N+F}$ $\{p_i\}_{i=1}^{N}$, $\{w_f\}_{f=1}^{F}$ such that
- Consumers, final good sector, all firms solve their problems
- Markets clear

$$C = Y,$$

$$y_i = c_i + \sum_{j=1}^{N} x_{ji} \text{ for all } i = 1, \ldots, N$$

$$\bar{L}_f = \sum_{j=1}^{N} x_{jf} \text{ for all } f = 1, \ldots, F$$

Social planner's problem

$$\max_{C,\{c_i,y_i,x_{ij}\}} U(C)$$

s.t.

$$\begin{aligned} C &= \mathcal{D}(c_1, ..., c_N), \\ y_i &= A_i F_i\left(\{x_{ij}\}_{j=1}^{N+F}\right) \text{ for all } i, \end{aligned}$$

$$y_i = c_i + \sum_{j=1}^{N} x_{ji} \text{ for all } i = 1, ..., N,$$

$$\bar{L}_f = \sum_{j=1}^{N} x_{jf} \text{ for all } f = 1, ..., F.$$

Excercise

Verify that eqm in the simple network economy is efficient

Remarks

1. F_i is CRS
   - decreasing returns to scale can be modelled as CRS function with firm-specific factors
2. A is Hicks-neutral: $F_i(A_i, ...) = A_i F_i(...)$
   - non-neutral productivity shock of input j to producer i is equivalent to a fictitious producer buying input j and selling to i with linear technology subject to Hicks-neutral shock
   - since N is arbitrary, just redefines what firms/networks are
3. No need for final good sector if we assume that consumers have *homothetic* utility $U(c_1, ..., c_N)$

WANT operator

- The GDP in this economy is Y
- We will think of $\{F_i\}$ as fixed but $\{A_i\}_{i=1}^{N}$ and $\{\bar{L}_f\}_{f=1}^{F}$ as potentially changing.
- We WANT to find a way to decompose ΔY into contributions of ΔA_i and $\Delta \bar{L}_f$.
    - a-priori seems difficult question: a shock to firm i will affect prices and demands on firm i, which in turn will affect prices and demands of its supplies, and so on along the production network.

Accounting identities

- Our two sets of feasibilities in quantities

$$y_i = c_i + \sum_{j=1}^{N} x_{ji}, \quad \bar{L}_f = \sum_{j=1}^{N} x_{jf}$$

- Re-write them in \$ shares

$$\underbrace{\frac{p_i y_i}{Y}}_{\equiv \lambda_i} = \underbrace{\frac{p_i c_i}{Y}}_{\equiv b_i} + \sum_j \underbrace{\frac{p_i x_{ji}}{p_j y_j}}_{\equiv \Omega_{ij}} \underbrace{\frac{p_j y_j}{Y}}_{\equiv \lambda_j}$$

$$\underbrace{\frac{w_f \bar{L}_f}{\mathrm{Y}}}_{\equiv \eta_f} = \sum_{i=1}^{N} \underbrace{\frac{p_i y_i}{\mathrm{Y}}}_{=\lambda_i} \underbrace{\frac{w_f x_{if}}{p_i y_i}}_{\equiv \Phi_{if}}$$

- Matrix form

$$\lambda' = b' \left(I - \Omega \right)^{-1}, \tag{AI 1}$$

$$\eta' = \lambda' \Phi. \tag{AI 2}$$

Accounting identities

- Matrix form
$$\lambda' = b' \left(I - \Omega\right)^{-1}, \quad \eta' = \lambda' \Phi.$$
- All objects are directly observable
  - λ_i : share of sales of firm i to GDP
  - b_i : share of consumption expenditures on good i to GDP
  - η_f : share of factor f in GDP
  - Ω : input-output matrix
  - Φ : factor expenditure matrix
- Note that
$$\begin{aligned} \sum_i b_i &= 1 \text{ always} \\ \sum_i \lambda_i &\geq 1, \text{ with } > \text{ if network is nontrivial} \end{aligned}$$

Efficient economies

Theorem

(Hulten) In competitive network economy,

$$\frac{\partial \ln Y}{\partial \ln A_k} = \lambda_k, \ \frac{\partial \ln Y}{\partial \ln \bar{L}_f} = \eta_f$$

- Remarkable result: effect of A_k and $\bar{L}_f$ on Y is summarized by λ_k and η_f regardless of details of network structure, assumptions on elasticities, etc
- Easy way to decompose sources of GDP growth

$$\Delta \ln Y = \underbrace{\lambda' \left(\Delta \ln A\right)}_{\text{Solow residual}} + \eta' \left(\Delta \ln \bar{L}\right).$$

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$$\Delta \ln Y = \underbrace{\lambda' \left(\Delta \ln A\right)}_{\text{Solow residual}} + \eta' \left(\Delta \ln \bar{L}\right).$$

- Even more remarkable: similar result holds for inefficient economies as well (see below and Baqaee and Farhi)

Next steps

- I will only prove that $\frac{\partial \ln Y}{\partial \ln A_k} = \lambda_k$
    - the other part is proved identically
- Step 1: envelope theorem for final good sector
- Step 2: envelope theorem for producers
- Step 3: combine them and the two accounting identities

Step 1

Lemma

In competitive network economy,

$$b' \frac{\partial \ln p}{\partial \ln A_k} = 0 \textit{ for all } k$$

Proof of step 1

- Problem of the final goods firm

$$\Pi = \max_{\{c_j\}_{j=1}^N} \mathcal{D}\left(c_1, ..., c_N\right) - \sum_{j=1}^N p_j c_j$$

- Apply the Envelope theorem

$$\frac{\partial \Pi}{\partial \ln A_k} = -\sum_{j=1}^N c_j \frac{\partial p_j}{\partial \ln A_k} = -\sum_{j=1}^N p_j c_j \frac{\partial \ln p_j}{\partial \ln A_k}$$

- Profits of final sector is always zero. Divide by previous eqn by Y to get

$$0 = \sum_{j=1}^N \underbrace{\frac{p_j c_j}{Y}}_{=b_j} \frac{\partial \ln p_j}{\partial \ln A_k} = b' \frac{\partial \ln p}{\partial \ln A_k}$$

Step 2

Lemma

In competitive network economy

$$\frac{\partial \ln p}{\partial \ln A_k} = (I - \Omega)^{-1} \left(\Phi \frac{\partial \ln w}{\partial \ln A_k} - e_k \right)$$

where e_k is the k-th basis vector.

Proof of step 2

- Problem of the producer i

$$\pi_i = \max_{\{x_{ij}\}_{j=1}^{N+F}} p_i A_i F_i \left(\{x_{ij}\}_{j=1}^{N+F} \right) - \sum_{j=1}^{N} p_j x_{ij} - \sum_{f=1}^{F} w_f x_{if}$$

- Apply the Envelope theorem

$$\frac{\partial \pi_i}{\partial \ln A_k} = \iota_{i=k} p_i y_i + p_i y_i \frac{\partial \ln p_i}{\partial \ln A_k} - \sum_{j=1}^{N} p_j x_{ij} \frac{\partial \ln p_j}{\partial \ln A_k} - \sum_{f=1}^{F} w_f x_{if} \frac{\partial \ln w_f}{\partial \ln A_k}$$

- Profits are zero. Divide previous equation by $p_i y_i$ to get

$$\frac{\partial \ln p_i}{\partial \ln A_k} = \sum_{j=1}^{N} \underbrace{\frac{p_j x_{ij}}{p_i y_i}}_{=\Omega_{ij}} \frac{\partial \ln p_j}{\partial \ln A_k} + \sum_{f=1}^{F} \underbrace{\frac{w_f x_{if}}{p_i y_i}}_{=\Phi_{if}} \frac{\partial \ln w_f}{\partial \ln A_k} - \iota_{i=k}$$

- Matrix form:

$$\frac{\partial \ln p}{\partial \ln A_k} = (I - \Omega)^{-1} \left(\Phi \frac{\partial \ln w}{\partial \ln A_k} - e_k \right)$$

Corollary

In competitive network economy

$$\eta' \frac{\partial \ln w}{\partial \ln A_k} = \lambda_k$$

Proof.

Sequentially apply Lemma 1, Lemma 2, AI1, AI2:

$$\begin{aligned} 0 &= b' \frac{\partial \ln p}{\partial \ln A_k} = b' \left(I - \Omega\right)^{-1} \left(\Phi \frac{\partial \ln w}{\partial \ln A_k} - e_k\right) \\ &= \lambda' \left(\Phi \frac{\partial \ln w}{\partial \ln A_k} - e_k\right) = \eta' \frac{\partial \ln w}{\partial \ln A_k} - \lambda_k \end{aligned}$$

□

Final step

- Now we are ready to prove the main result,

$$\frac{\partial \ln Y}{\partial \ln A_k} = \lambda_k$$

- Definition of GDP from income side (consumer budget constraint)

$$Y = \sum_{f=1}^{F} w_f \bar{L}_f.$$

- Differentiate w.r.t. $\ln A_k$:

$$\frac{\partial \ln Y}{\partial \ln A_k} Y = \sum_{f=1}^{F} w_f \bar{L}_f \frac{\partial \ln w_f}{\partial \ln A_k}$$

or in matrix form

$$\frac{\partial \ln Y}{\partial \ln A_k} = \eta' \frac{\partial \ln w}{\partial \ln A_k} \underbrace{=}_{\text{from corr.}} \lambda_k$$

Some observations I

- We defined Ω in terms of revenues,

$$\Omega_{ij} := \frac{p_j x_{ij}}{p_i y_i}.$$

- We could have define $\widetilde{\Omega}$ in terms of costs,

$$\widetilde{\Omega}_{ij} := \frac{p_j x_{ij}}{\sum_{k=1}^{N+F} p_k x_{ik}}.$$

- In our simple equilibrium firms make zero profits, so

$$p_i y_i = \sum_{k=1}^{N+F} p_k x_{ik}$$

  and, therefore,

$$\Omega = \widetilde{\Omega}$$

- Similarly, we could have defined $\tilde{\lambda}$ and $\tilde{\eta}$ relative to costs rather than revenues, with zero profits implying $\tilde{\lambda} = \lambda$, $\tilde{\eta} = \eta$

Some observations II

- We have

$$1 = \sum_{f=1}^{F} \frac{w_f \bar{L}_f}{Y} = \sum_{f=1}^{F} \eta_f$$

- Therefore,

$$0 = \sum_{f=1}^{F} \eta_f \frac{\partial \ln \eta_f}{\partial \ln A_k} = \eta' \frac{\partial \ln \eta}{\partial \ln A_k}$$

- Therefore, we could have stated our theorem as follows

Theorem

In competitive network economy,

$$\begin{aligned} \frac{\partial \ln Y}{\partial \ln A_k} &= \tilde{\lambda}_k - \tilde{\eta}' \frac{\partial \ln \eta}{\partial \ln A_k} \\ \frac{\partial \ln Y}{\partial \ln \bar{L}_f} &= \tilde{\eta}_f. \end{aligned}$$

2. Distorted network economy

Distorted economy

- Suppose we now have economy with arbitrary distortions
  - monopoly power
  - wedges/taxes/etc
- These distortions can be summarized as mark ups μ_i over marginal costs
  - CRS implies that it is also mark up over average cost, and so

$$\mu_i = \frac{p_i y_i}{\sum_{k=1}^{N+F} p_k x_{ik}}$$

- Baqaee-Farhi follow almost the same steps as we did but for inefficient economy
  - distorted/monopolistic firms still want to minimize cost (remember Lecture 8), so they envelope cost-minimization terms
  - firms may earn profits/rents, need to keep track of those (another accounting identity)

Inefficient economies

Theorem

(Baqaee-Farhi) In distorted network economy

$$\begin{aligned}
\frac{\partial \ln Y}{\partial \ln A_k} &= \tilde{\lambda}_k - \tilde{\eta}' \frac{\partial \ln \eta}{\partial \ln A_k}, \\
\frac{\partial \ln Y}{\partial \ln \bar{L}_f} &= \tilde{\eta}_f, \\
\frac{\partial \ln Y}{\partial \ln \mu_k} &= -\tilde{\lambda}_k - \tilde{\eta}' \frac{\partial \ln \eta}{\partial \ln \mu_k}.
\end{aligned}$$

- If it no longer the case that $\eta = \tilde{\eta}$ or $\lambda = \tilde{\lambda}$, but they can also be constructed from the data using costs rather than revenues.
  - costs and revenues give also mark ups μ

Implications

- Can derive from there GDP decomposition

$$\Delta \ln Y = \underbrace{\tilde{\lambda}' \left(\Delta \ln A\right)}_{\text{technology}} + \underbrace{\tilde{\eta}' \left(\Delta \ln \bar{L}\right)}_{\text{factor supply}} \underbrace{-\tilde{\lambda}' \left(\Delta \ln \mu\right) - \tilde{\eta}' \left(\Delta \ln \eta\right)}_{\text{misallocation}}$$

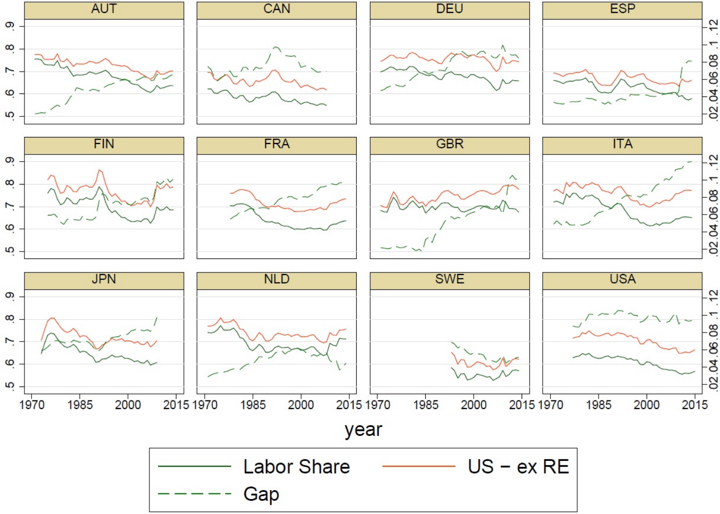
- See also in the paper second order expansions to get general version of Hsieh-Klenow measures of distortions in TFP

3. Application

Application

- Two stylized facts about U.S. in last 20 years

Labor share declined



Source: Gutierrez (2017)

Markups increased

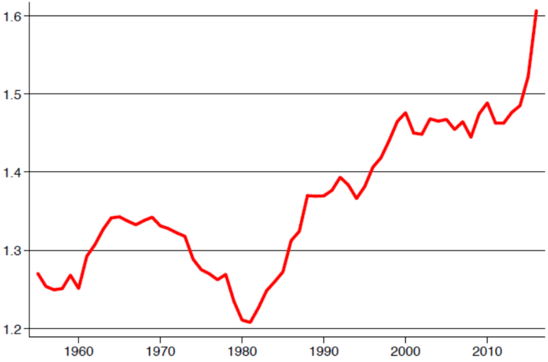


Figure 1: Average Markups. Output elasticities θ_{st} from estimated production function are time-varying and sector-specific (2 digit). Average is revenue weighted. Evolution 1955-2016.

Source: De Loecker, Eeckhout, Unger (2019)

Question

What does this mean about productivity and misallocation in the U.S. economy?

Key observation

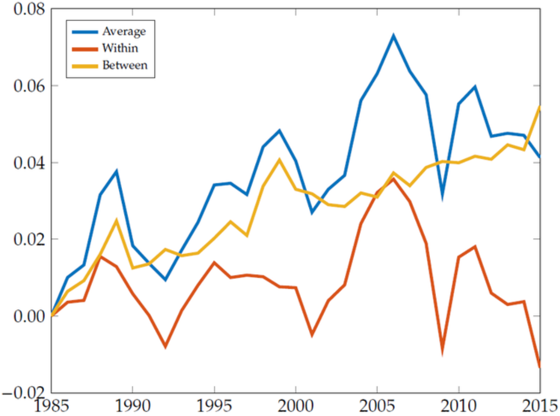


Figure 3: Decomposition of the increase in the average markup into a between and a within effect, using the user-cost approach markup data. All the changes are cumulated over time.

Source: Baqaee and Farhi (2019)

Decomposition of TFP growth

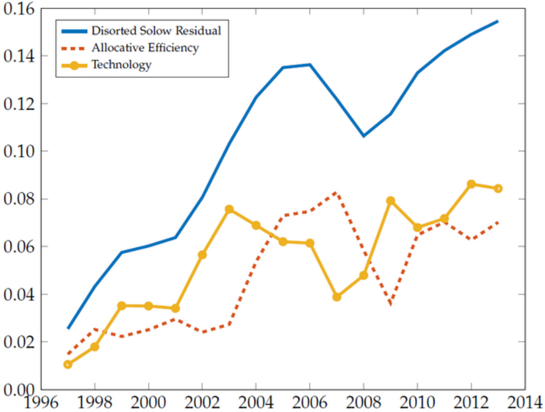


Figure 4: Cumulative decomposition of changes in aggregate TFP (distortion-adjusted Solow residual) into pure changes in technology and changes in allocative efficiency along the lines of equation (7), with markups obtained from the user-cost approach.

Source: Baqaee and Farhi (2019)

Discussion

- Half of GDP growth comes from improvement in allocative efficiency
- Intuition: firm mark-ups did not change but more resources flow to firms with higher markups
  - higher markup imply that firms use too little resources
  - increase in their size is a good thing
- Intuitively:
  - Amazon has low marginal costs and high markups (also, huge fixed costs!)
  - drives out of business high marginal cost mom-and-pop stores