Lecture 9

Misallocation within sectors

Mikhail Golosov

Hsieh and Klenow (2009)

Overview

- In Lecture 6 we saw that there is a lot of cross-country heterogeneity in TFP even within sectors
- We want to explore how misallocations may affect TFP
- Based on Hsieh and Klenow (2009)
- We use exactly the heterogeneous firm set up from Lecture 8
  - see Hseih and Klenow for extending model for more inputs (capital)
- Final good is a numerieire, $P = 1$

TFP of intermediate goods sector

- Value added of the intermediate good sector is

$$\int P_i Y_i di = Y$$

- We would measure sectoral TFP in the data from

$$Y = TFP \times L$$

- What is this TFP?

$$Y = \left(\int (A_i L_i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} = \underbrace{\left(\int \left[A_i \frac{L_i}{L} \right]^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}}_{=TFP} \times L$$

TFP of intermediate goods sector

- We showed in Lecture 8 that

$$L_i = const \times A_i^{\sigma-1} \text{ for all } i$$

- Therefore

$$\frac{L_i}{L} = \frac{L_i}{\int L_j dj} = \frac{A_i^{\sigma-1}}{\int A_j^{\sigma-1} dj}$$

- Plug and re-arrange

$$TFP = \left(\int A_i^{\sigma-1} di \right)^{\frac{1}{\sigma-1}}$$

Simplifying TFP

- Want to simplify

$$\ln TFP = \frac{1}{\sigma - 1} \ln \left(\int A_i^{\sigma-1} di \right)$$

- First of all, note that taking integrals is the same as taking averages or expectations

$$\begin{aligned} \ln TFP &= \frac{1}{\sigma - 1} \ln \mathbb{E} A_i^{\sigma-1} = \frac{1}{\sigma - 1} \ln \mathbb{E} \exp \left((\sigma - 1) \ln A_i \right) \\ &= \frac{1}{\sigma - 1} \ln \mathbb{E} \exp \left((\sigma - 1) \left(\bar{a} + a_i \right) \right) \end{aligned}$$

where

$$\bar{a} = \mathbb{E} \ln A_i, \quad a_i = \ln A_i - \mathbb{E} \ln A_i$$

- Re-arranged

$$\ln TFP = \bar{a} + \frac{1}{\sigma - 1} \ln \mathbb{E} \exp \left((\sigma - 1) a_i \right)$$

where $\mathbb{E} a_i = 0$.

- We want to approximate this to get more intuitive expressions

Approximations

Theorem

Suppose ε_i is a random variable with $\mathbb{E}\varepsilon_i = 0$. Then, to the second order approximation,

$$\ln \mathbb{E} \exp\left(\varepsilon_i\right) \approx \frac{1}{2} var\left(\varepsilon_i\right).$$

- Define function $F(x)$ for any scalar $x \geq 0$ as

$$F(x) = \ln \mathbb{E} \exp\left(x \cdot \varepsilon_i\right).$$

- Observe that Taylor expansions imply

$$\ln \mathbb{E} \exp\left(\varepsilon_i\right) = F(1) \approx F(0) + F'(0) + \frac{1}{2} F''(0)$$

- Compute directly

$$\begin{aligned}
F(0) &= \ln 1 = 0, \\
F'(0) &= \frac{\mathbb{E} \exp\left(0 \cdot \varepsilon_i\right) \varepsilon_i}{\mathbb{E} \exp\left(0 \cdot \varepsilon_i\right)} = \mathbb{E}\varepsilon_i = 0, \\
F''(0) &= \mathbb{E}\varepsilon_i^2 = var\left(\varepsilon_i\right).
\end{aligned}$$

Approximations

- Our formula is

$$\ln TFP = \bar{a} + \frac{1}{\sigma - 1} \ln \mathbb{E} \exp \left(\underbrace{(\sigma - 1)\, a_i}_{:=\varepsilon_i} \right)$$

- Apply this theorem

$$\ln TFP \approx \bar{a} + \frac{\sigma - 1}{2} var\,(a_i)$$

- What is the intuition for why TFP is increasing in σ?

Measures of efficiency

- Physical productivity

$$TFPQ_i \equiv \frac{Y_i}{L_i} = A_i$$

- Revenue productivity

$$TFPR_i \equiv \frac{P_i Y_i}{L_i} = P_i A_i$$

- Undistorted firm optimization implies

$$TFPR_i = TFPR_j \text{ for all } i, j$$

- Note that we can measure $TFPR_i$ directly by using data on firm revenue $P_i Y_i$ and employment L_i

Hsieh-Klenow

- Firm census data for firms in U.S., China, India for different industries and years
- HK have richer model with capital, but none of the insights are affected by that
  - but derivations are much longer

TPFR

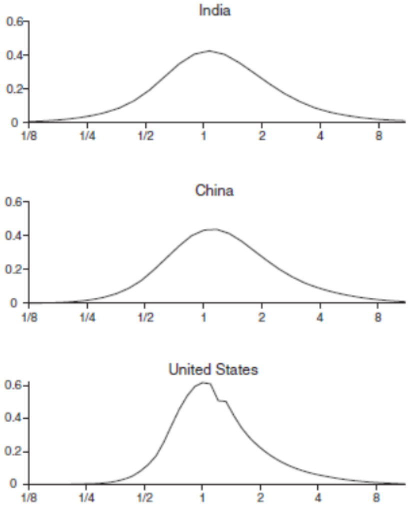


FIGURE II
Distribution of TFPR

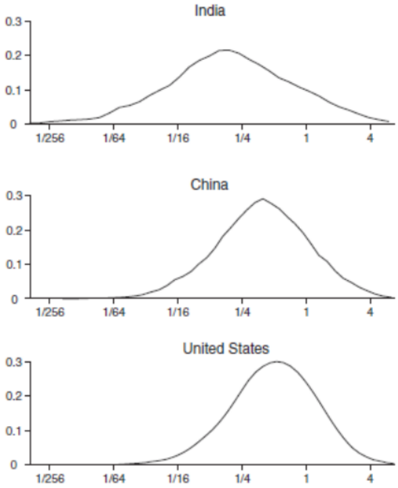
Discussion

- Dispersion of $\{TFPR_i\}_i$ is in all sectors but bigger in poorer countries
- What about phyical productivity? From $Y_i = YP_i^{-\sigma}$ we get

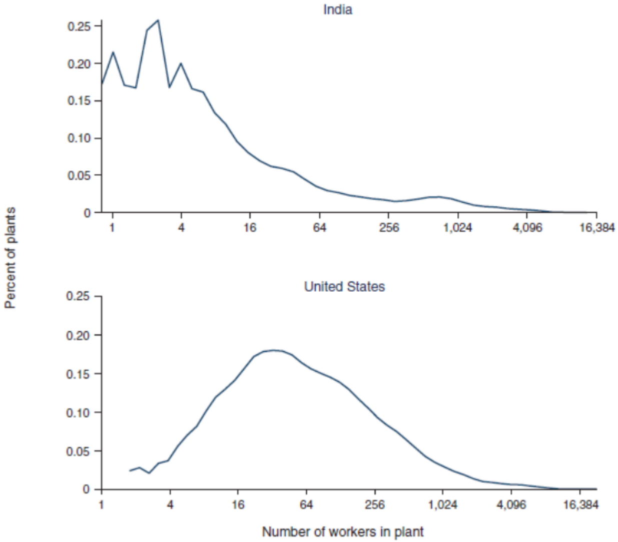
$$TFPQ_i = const \times \frac{(P_i Y_i)^{\frac{\sigma}{\sigma-1}}}{L_i}$$

- Make some assumptions on σ to compute dispersion of $\{TFPQ_i\}_i$

TPFQ


India
0.3
0.2
0.1
0
1/256
1/64
1/16
1/4
1
4
China
0.3
0.2
0.1
0
1/256
1/64
1/16
1/4
1
4
United States
0.3
0.2
0.1
0
1/256
1/64
1/16
1/4
1
4

Distribution of plan sizes



Distorted firm's optimization

- Something prevents firms from equalizing $\{TFPR_i\}_i \Longrightarrow$ want to study implications for TFP
- Let's do the same wedge-accounting excercise as in Lecture 7 $\Longrightarrow$ any allocation can be attained in the economy in which firm i faces distortion τ_i:

$$\max_{P_i, Y_i, L_i} P_i Y_i - (1+\tau_i) W L_i$$

s.t.

$$Y_i = Y P_i^{-\sigma}, \quad Y_i = A_i L_i$$

- Same steps as before imply that

$$TFPR_i = const \times (1+\tau_i)$$

Misallocation and TFP

- Going through the same algebra as in undistorted eqm, we find that $TFPR_i$ are no longer equalized for all i :

$$TFPR_i = 1 + \tau_i$$

- Compute sectoral TFP

$$TFP = \frac{\left(\int \left(1+\tau_i\right)^{1-\sigma} A_i^{\sigma-1} di \right)^{\frac{\sigma}{\sigma-1}}}{\int \left(1+\tau_i\right)^{-\sigma} A_i^{\sigma-1} di}$$

- Let's de-mean all variables

$$TFP^{dist} = \overline{A} \frac{\left(\int \left(\frac{1+\tau_i}{\overline{1+\tau}} \right)^{1-\sigma} \left(\frac{A_i}{\overline{A}} \right)^{\sigma-1} di \right)^{\frac{\sigma}{\sigma-1}}}{\int \left(\frac{1+\tau_i}{\overline{1+\tau}} \right)^{-\sigma} \left(\frac{A_i}{\overline{A}} \right)^{\sigma-1} di}$$

where $\ln \overline{1+\tau} := \mathbb{E} \ln \left(1+\tau_i\right)$.

- Let $t_i := \ln \left(1+\tau_i\right) - \mathbb{E} \ln \left(1+\tau_i\right)$.

Misallocation and TFP

- Apply our approximation theorem:

$$\ln TFP^{dist} \approx \bar{a} + \frac{1}{2}\sigma(\sigma-1)\, var\,(a_i - t_i) - \frac{1}{2}(\sigma-1)^2\, var\left(a_i - \frac{\sigma}{\sigma-1}t_i\right)$$

- Measured TFP
  - depends on variances and convarinace of distortions
  - does not depend on average distortions
- Suppose t_i are independent of a_i:

$$\ln TFP^{dist} \approx \underbrace{\bar{a} + \frac{\sigma-1}{2} var\,(a_i)}_{\text{undistorted TFP}} - \underbrace{\frac{\sigma}{2} var\,(t_i)}_{\text{effect of distortions}}$$

- Random distortions decrease measured TFP

Measuring distortions in the data

- TFPR and wedges can be measured directly in the data from the following FOCs
- Distortions from FOCs

$$1 + \tau_i = \frac{\sigma - 1}{\sigma} \times \frac{P_i Y_i}{W L_i} = \frac{\sigma - 1}{\sigma} \times \frac{\text{Revenue}_i}{\text{Wage bill}_i}$$

- Can measure their correlation with a_i (i.e., demeaned $\ln TFPQ_i$) to evaluate effect of misallocation

Output gains from removing misallocation

TABLE IV
TFP GAINS FROM EQUALIZING TFPR WITHIN INDUSTRIES

China	1998	2001	2005
%	115.1	95.8	86.6
India	1987	1991	1994
%	100.4	102.1	127.5
United States	1977	1987	1997
%	36.1	30.7	42.9

Output gains from removing misallocation

TABLE VI
TFP GAINS FROM EQUALIZING TFPR RELATIVE TO 1997 U.S. GAINS

China	1998	2001	2005
%	50.5	37.0	30.5
India	1987	1991	1994
%	40.2	41.4	59.2