# Theory of Income 2 - Pset 1

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## 1 Question 1

Wedges in agriculture

Consider a static, two sector economy, $A$ and $M$. There is a unit measure of agents, who have preferences $\ln C_A + \ln C_M$. Production technology in manufacturing is $Y_M = L_M$ and $Y_A = L_A^\alpha Z^{1-\alpha}$ where $L_M, L_A$ are labor supplied in $M$ and $A$, and $Z$ is land. Labor supply is inelastic, $L_A + L_M = 1$. Land is also inelastic, $Z = 1$.

Manufacturing technology is operated in the city, with perfectly competitive firms that pay wage $w$ to labor.

Agricultural technology is operated in the village. The village owns land in the communal property. That is, it divides land among all agents who live in the village (i.e. did not move into the city), and

villagers grow agricultural output, trade it for manufacturing goods, and consume.

Initially, all agents are born in the village, and choose whether to stay in the village or to move in the city.

(a) Set up social planner's problem and derive the key optimality condition that characterizes optimal intersectoral allocation of resources. [Hint: write the condition in terms of ratios between marginal utilities, marginal product of labor, etc. Lecture 7 can be helpful.]

(b) Define competitive equilibrium in this economy. [Hint: normalize price for manufacturing goods to 1. Villagers equally divide income from agricultural production. Also describe the relationship between villagers' utility and city workers' utility in equilibrium.]

(c) Show that this equilibrium is inefficient so that there is a wedge relative to the social planner's allocation. [Hint: what does the relationship you establish for villagers' and city works' utility imply about the consumption allocations in equilibrium?]

## 2 Question 2 Exercise 1

Slide 16 Exercise: Set up a competitive equilibrium version of this economy (i.e. intermediary firms act competitively) with inelastic supply.

(a) Show that this equilibrium is always efficient.

(b) Show that $P/W$ in competitive equilibrium differs from $P/W$ in monopolistic equilibrium even when labor is inelastic. How do you reconcline this with the fact that both equilibria are efficient in this case?

# 3 Question 2 Exercise 2

Slide 18 Exercise: Equilibrium consists of taxes $(\tau, T)$, prices $(\{P_i\}_i, W, P)$, allocations $(\{Y_i, \Pi_i, L_i\}_i, C, Y)$ such that:

- Consumers solve their problem taking prices and taxes as given.
- All firms and market clearing conditions as before.
- Government budget constraint holds.

$$T = \tau W L$$

Exercise (Walras' Law): Show that first two bullet points in the definition imply the third

# 4 Question 2 Exercise 3

Slide 19 Exercise 1: Show that equilibrium with taxes

$$\tau = -\frac{1}{\sigma - 1}$$

is efficient. Note: you need to specify the (unique) level of $T$ that is consistent with this $\tau$

# 5 Question 2 Exercise 4

Slide 19 Exercise 2:

- The optimum policy is to subsidize labor, $\tau < 0$
  - Monopolist under-employs labor, relative to efficient allocation.
  - The optimum policy subsidizes labor supply to offset this distortion.
  - Labor subsidies are financed with lump-sum taxes.
- Note that the optimal tax is independent of the elasticity of the labor supply

How do you reconcile this observation with the fact that monopolistic equilibrium is efficient when labor supply is inelastic?

# 6 Question 2 Exercise 5

Slide 20 Exercise

- Suppose we change the setup of Slide 3 by assuming the intermediate firm $i$ has productivity $A_i$.
- Keep the rest of the assumptions unchanged.
- Demand from final good firms is unchanged:

$$Y = \left(\frac{P}{P_i}\right)^{-\sigma} C_i$$

- Exercise: Show that the markup is constant in the model with heterogeneous firms:

$$P_i = \underbrace{\frac{\sigma}{\sigma - 1}}_{\text{markup}} \times \underbrace{\frac{W}{A_i}}_{\text{marginal cost of } i}$$

# 7 Question 2 Exercise 6

Slide 23 Exercise

Consider the cost function of the final good firm

$$\mathcal{C}(Y) = \min_{\{Y_i\}_i} \int P_i Y_i di$$

s.t.

$$Y = \left( \int Y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}.$$

- Let $\lambda$ be the Lagrange multiplier on this constraint.
- Exercise: Show that
  - Marginal cost satisfies $\mathcal{C}'(Y) = \lambda$ (hint: use the envelope theorem).
  - Average and marginal costs coincide, $\mathcal{C}'(Y) = \frac{\mathcal{C}(Y)}{Y}$ (hint: show that $\mathcal{C}$ is HD1).
  - If $P = \mathcal{C}'(Y)$, then final good producer profits are zero.

# 8 Question 2 Exercise 7

Slide 26 Exercise

- Show with elastic labor supply that this equilibrium is inefficient and that $\tau = -\frac{1}{\sigma-1}$ restores efficiency.
- Is this equilibrium efficient when labor supply is inelastic?