Lecture 12
Basic New Keynesian model

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Introduction

- Simplest New Keynesian model
  - Dixit-Stigliz model from Lecture 8 in dynamic settings and *nominal rigidities*
- All prices $P, \{P_i\}_i$ are nominal (e.g., in \$), do not normalize them to 1
- Prices $\{P_i\}_i$ are sticky, it is costly for intermediate firms to change them

Rotemberg cost of price changes

- We consider simplest version of price adjustments, in the spirit of Rotemberg (1982)
  - not the most realistic, but to the first order of approximation we consider it will be equivalent to more realistic costs of price adjustments, e.g. ones proposed by Calvo or Taylor
- Suppose firm i had price $P_{i,t-1}$ in period t
- The cost of setting price $P_{i,t}$ in period t is

$$\Phi_{i,t} = \frac{\theta}{2} P_t Y_t \left(\frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2$$

  where $\theta \geq 0$ is parameter capturing how costly it is for firm i to change its price
- Costs are rebated lump-sum to consumers

Intermediate firms

- For now, all firms operate the same, fixed technology $A_{i,t} = A$ for all i, t:

$$Y_{i,t} = AL_{i,t}$$

  - later we might consider extensions when $A_{i,t}$ is stochastic
- Dividends

$$D_{i,t} = P_{i,t}Y_{i,t} - W_t L_{i,t} - \Phi_{i,t}$$

CB policy rule

- Central bank controls nominal interest rates I_t
- Let Q_t be a price of a nominal one period risk-free bond purchased at t. Then

$$I_t = 1/Q_t - 1$$

- For now, we let I_t to be an arbitrary stochastic process (aka "monetary policy")
  - equivalently, Q_t is some arbitrary stochastic process

Definition of eqm, I

Stochastic sequences $\{C_t, L_t, B_t, D_{i,t}, \Phi_{i,t}\}_{i,t}$, prices $\{P_t, P_{i,t}, W_t, Q_t, \Pi_t\}$ such that

- Consumers solve

$$\max_{\{C_t, L_t, B_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma}}{1-\gamma} - \frac{L_t^{1+\varphi}}{1+\varphi} \right]$$

  s.t. $B_{-1} = 0$ and

$$P_t C_t + Q_t B_t = W_t L_t + \int D_{i,t} di + \int \Phi_{i,t} di + B_{t-1}$$

- Final goods firms solve

$$\max_{\{Y_i\}_i, Y} P_t Y_t - \int P_i Y_i di$$

  s.t.

$$Y_t = \left(\int Y_{i,t}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}.$$

Definition of eqm, II

- Easiest to write problem of intermediate firms in recursive form:

$$V_t\left(P_{i,t-1}\right) = \max_{P_{i,t}, Y_{i,t}, L_{i,t}} P_{i,t} Y_{i,t} - W_t L_{i,t} - \Phi_{i,t} + Q_t \mathbb{E}_t V_{t+1}\left(P_{i,t}\right)$$

s.t.

$$Y_{i,t} = A L_{i,t}, \qquad Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\sigma} Y_t,$$

$$\Phi_{i,t} = \frac{\theta}{2} P_t Y_t \left(\frac{P_{i,t}}{P_{i,t-1}} - 1\right)^2.$$

- Dividends

$$D_{i,t} = P_{i,t} Y_{i,t} - W_t L_{i,t} - \Phi_{i,t}$$

Definition of eqm, III

- Monetary policy

$$Q_t \text{ is a given stochastic process}$$

- Market clearing conditions

$$C_t = Y_t, \ L_t = \int L_{i,t} di, \quad B_t = 0.$$

Consumer optimality

- Consumers intra-temporal FOC is the same as in Lectures 8:

$$C_t^{\gamma} L_t^{\varphi} = \frac{W_t}{P_t}$$

- The inter-temporal optimality conditions (make sure you can derive it!)

$$\begin{aligned} Q_t &= \beta \mathbb{E}_t \frac{P_t}{P_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \\ &= \beta \mathbb{E}_t \frac{1}{\Pi_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \end{aligned}$$

- All the interesting action is happening on the intermediate firm side

Intermediate firm optimality

- Firm problem

$$V_t\left(P_{i,t-1}\right) = \max_{P_{i,t}} P_{i,t}\left(\frac{P_{i,t}}{P_t}\right)^{-\sigma} Y_t - \frac{W_t}{A}\left(\frac{P_{i,t}}{P_t}\right)^{-\sigma} Y_t - \frac{\theta}{2} P_t Y_t \left(\frac{P_{i,t}}{P_{i,t-1}} - 1\right)^2 + Q_t \mathbb{E}_t V_{t+1}\left(P_{i,t}\right)$$

- FOC:

$$(1-\sigma)\left(\frac{P_{i,t}}{P_t}\right)^{-\sigma} Y_t + \sigma \frac{W_t}{A}\left(\frac{P_{i,t}}{P_t}\right)^{-\sigma} Y_t \frac{1}{P_{i,t}} - \theta P_t Y_t \left(\frac{P_{i,t}}{P_{i,t-1}} - 1\right)\frac{1}{P_{i,t-1}} + Q_t \mathbb{E}_t \frac{\partial}{\partial P_{i,t}} V_{t+1}\left(P_{i,t}\right) = 0$$

Intermediate firm optimality

- Firm problem

$$V_t\left(P_{i,t-1}\right)=\max_{P_{i,t}} P_{i,t}\left(\frac{P_{i,t}}{P_t}\right)^{-\sigma} Y_t-\frac{W_t}{A}\left(\frac{P_{i,t}}{P_t}\right)^{-\sigma} Y_t-\frac{\theta}{2} P_t Y_t\left(\frac{P_{i,t}}{P_{i,t-1}}-1\right)^2 + Q_t \mathbb{E}_t V_{t+1}\left(P_{i,t}\right)$$

- FOC:

$$(1-\sigma)\left(\frac{P_{i,t}}{P_t}\right)^{-\sigma} Y_t+\sigma \frac{W_t}{A}\left(\frac{P_{i,t}}{P_t}\right)^{-\sigma} Y_t \frac{1}{P_{i,t}} -\theta P_t Y_t\left(\frac{P_{i,t}}{P_{i,t-1}}-1\right) \frac{1}{P_{i,t-1}}+Q_t \mathbb{E}_t \frac{\partial}{\partial P_{i,t}} V_{t+1}\left(P_{i,t}\right)=0$$

- Envelope theorem:

$$\frac{\partial}{\partial P_{i,t}} V_{t+1}\left(P_{i,t}\right)=\theta P_{t+1} Y_{t+1}\left(\frac{P_{i,t+1}}{P_{i,t}}-1\right) \frac{P_{i,t+1}}{P_{i,t}^2}$$

Intermediate firm optimality, II

- All firms are identical, which implies that (see Lecture 8) that $P_{i,t} = P_t$ for all i
- This simplifies our previous equation to

$$(1-\sigma) + \sigma \frac{W_t / P_t}{A} - \theta \Pi_t \left(\Pi_t - 1\right) + \theta Q_t \mathbb{E}_t \left(\Pi_{t+1} - 1\right) \Pi_{t+1}^2 \frac{Y_{t+1}}{Y_t} \quad = \quad 0.$$

Equilibrium

- Combine previous equations and get rid of redundant variables
- $\{C_t, Q_t, L_t, W_t/P_t, \Pi_t\}_t$ are a competitive equilibrium if and only if they solve

$$(1-\sigma) + \sigma \frac{W_t/P_t}{A} - \theta \Pi_t (\Pi_t - 1) + \theta Q_t \mathbb{E}_t (\Pi_{t+1} - 1) \Pi_{t+1}^2 \frac{C_{t+1}}{C_t} = 0,$$

$$C_t^\gamma L_t^\varphi = \frac{W_t}{P_t},$$

$$C_t = AL_t,$$

$$Q_t = \beta \mathbb{E}_t \frac{1}{\Pi_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma},$$

$$\{Q_t\}_t \text{ is given}$$

Flexible price model

- **Exercise 1**: Suppose $\theta = 0$. Show that competitive equilibrium allocations $\{C_t, L_t\}_t$ and real wages $\{W_t/P_t\}_t$ are independent of monetary policy and coincide with our solution in Lecture 8
- Thus, a flexible price version of this economy works just like the model in Lecture 8

Sticky prices: zero inflation steady state

- With sticky prices, analysis is harder. Let's focus on steady state first
- We say that $\{C_t, L_t, W_t/P_t, L_t, Q_t, \Pi_t\}_t$ is *steady state* if it is constant and independent of t
- The *zero inflation* steady state, in which $\Pi_t = 1$, is a particularly convenient starting point.
- Observe that by setting $Q_t = \overline{Q} := \beta$ the central bank can attain such steady state:

$$\overline{Q} = \beta, \quad \overline{\Pi} = 1,$$

$$\overline{C}^{\gamma+\varphi} = \frac{\sigma - 1}{\sigma}, \ \overline{C} = A\overline{L}, \ \overline{W/P} = \frac{\sigma - 1}{\sigma}.$$

- I refer to vector $\left(\overline{C}, \overline{L}, \overline{W/P}, \overline{Q}, \overline{\Pi}\right)$ as the *zero inflation steady state*

Deterministic steady states

- Observe that real variables in the zero inflation steady state are the same as in our economy in Lecture 8 or flexible price model:
  - firms set their prices once and for all as a markup $\sigma/(\sigma-1)$ over marginal costs
  - no shocks $\Longrightarrow$ no need to ever change those prices
- **Exercise 2**: Suppose $\theta > 0$. Are there steady states that give higher welfare than flexible price equilibrium? If so, is the optimal inflation positive or negative? What is the intuition for this result? [Hint: is there a steady state in which allocations are efficient?]

A one time monetary shock

- Let's start with a simple thought experiment
  - economy is in the zero inflation steady state
  - the central bank introduces a one time cut $\Delta > 0$ in interest rates at $t = 0$
  - central bank reverts to its usual policy rule for $t > 0$
- Formally, monetary policy is

$$Q_0 = \overline{Q} + \Delta$$

$$Q_t = \overline{Q} \text{ for all } t > 0$$

A one time monetary shock: analysis

- Note that $\left(\overline{C}, \overline{L}, \overline{W/P}, \overline{Q}, \overline{\Pi}\right)$ still solve all our equations for $t > 0$
- Therefore, period zero allocations $C_0, L_0, W_0/P_0, \Pi_0$ solve

$$\beta + \Delta = \beta \left(\frac{\overline{C}}{C_0} \right)^{-\gamma}, \quad C_0 = AL_0, \quad C_0^{\gamma} L_0^{\varphi} = \frac{W_0}{P_0},$$

$$(1 - \sigma) + \sigma \frac{W_0}{P_0} - \theta \Pi_0 \left(\Pi_0 - 1 \right) = 0$$

- These four equations show main economics of NK models

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  - to satisfy demand, firm need to higher more labor, increases L_0
  - to incentive workers to supply more labor, real wages W_0/P_0 must increase
  - firms want to charge markups over their marginal costs W_0/P_0 so prices and inflation Π_0 must increase

Forward guidance

- This basic mechanism leads to some unexpected results
- Suppose that economy is in the zero inflation steady state and the central bank in period 0 suddenly announced a cut Δ in interest rates in period 1
- Formally, monetary policy is

$$Q_1 = \overline{Q} + \Delta$$
$$Q_t = \overline{Q} \text{ for all } t \neq 1$$

- **Exercise 3**: Show that response of output C_0 to this announcement is bigger than the response we found in the previous example
- Basic NK models imply that the further away in the future central banks promise rate cut, the stronger is the stimulus effect

Real shocks

- The demand channel through which interest rates affect economy also has unusual implications about responses to real shocks
- Suppose that economy is in the zero inflation steady state and there is a one-time drop in productivity (say, supply chain disruption):

$$A_0 = A - \Delta,$$
$$A_t = A \text{ for all } t > 0.$$

- **Exercise 4**: What are the response of output C_0 and inflation Π_0 to this shock, assuming that central bank keeps its nominal interest rate unchanged, $Q_t = \overline{Q}$ for all $t \geq 0$?