# Scratch

Dylan Baker

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## 1 Chapter 4: Labor Market Equilibrium

This chapter will study the notion of labor market equilibrium, which is the outcome of the interaction between labor supply, labor demand, and possibly external forces, such as government policies.

### Definition D.1: Invisible Hand Theorem

If markets are competitive, and workers and firms are free to enter and leave the market, then the equilibrium allocation of workers and wages will be efficient, in the sense that it maximizes the total gains that workers and firms obtain from trade with each other.

## 1.1 Equilibrium in a Single Labor Market

Figure 1 illustrates a competitive labor market with an equilibrium at  $(E^*, w^*)$ .

## FIGURE 4-1 Equilibrium in a Competitive Labor Market

The labor market is in equilibrium when supply equals demand;  $E^*$  workers are employed at a wage of  $w^*$ . The triangle P gives the producer surplus; the triangle Q gives the worker surplus. A competitive market maximizes the gains from trade, or the sum P + Q.

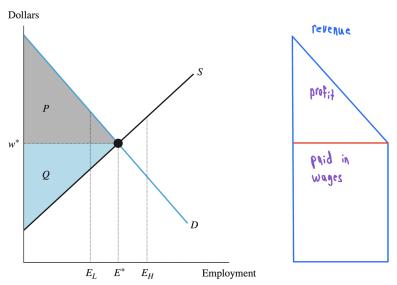


Figure 1: Labor Market Surplus

Since the labor demand curve corresponds to the value of the marginal product, we can see that the area under the labor demand curve up to any given point corresponds to the total revenue a firm receives from hiring that many workers. Thus, since the equilibrium is at  $(E^*, w^*)$ , the firm's total revenue is given by the area under the demand curve up to

 $E^*$ , which is drawn out in blue in Figure 1. Similarly, the area under the supply curve up to  $E^*$ , their profit, or producer surplus (denoted P in Figure 1), is given by the triangle above  $w^*$  and below the labor demand curve.

### Definition D.2: Producer Surplus

Producer surplus is the difference between the revenue a firm receives and the cost it incurs.

Similarly, workers are indifferent between working and not working along the labor supply curve. Thus, workers who are willing to work for less than  $w^*$  (which is everyone except the final workers hired) receive a surplus. This surplus is given by the triangle below  $w^*$  and above the labor supply curve, denoted Q in Figure 1.

The gains from trade are given by the sum of the producer and worker surplus: P + Q. The competitive market equilibrium maximizes the gains from trade. Such an outcome that maximizes gains from trade is said to be "efficient."

### Questions

It's somewhat challenging for me to think about how I should think about the connection between surplus and wellbeing. I understood surplus as a clear quantitative measure, but it seems like we would only care about it insofar as it connects to something more like the wellbeing of the parties involved. However, it's super unclear to me if we're trying to connect it back to that in any way.

## 1.2 Equilibrium across Labor Markets

So far, we have focused on a single labor market. Now, we turn to the case of multiple labor markets linked by migration. In Figure 2, we see two labor markets, N (north) and S (south). We start with the wages  $(w_n)$  in the north being higher than the wages  $(w_s)$  in the south. If workers are able to move freely, then workers from the South will move to the North, which will decrease supply in the South and increase supply in the North up to the point that wages are equalized across the two markets, at  $w^*$ . Note that this analysis relies on the idea that workers in each region are perfect substitutes for each other.

We find that this movement increases the total output across the markets. Specifically, the North market's output is originally the area under the demand curve up to A and after the move extends to B – the increase is characterized by the blue trapezoid. For the South, output reduces by the amount characterized by the blue trapezoid in its market. However, the increase in the North is larger than the decrease in the South. In particular, the growth in the North is large by the size of the triangle ABC.

#### FIGURE 4-2 Competitive Equilibrium in Two Labor Markets Linked by Migration

The wage in the northern region  $(w_N)$  exceeds the wage in the southern region  $(w_S)$ . Southern workers want to move north, shifting the southern supply curve to the left and the northern supply curve to the right. In the end, wages are equated across regions (at  $w^*$ ). The migration reduces the value of output in the South by the size of the shaded trapezoid in the southern labor market and increases the value in the North by the size of the larger shaded trapezoid in the northern labor market. Migration increases the value of aggregate output by the triangle ABC.

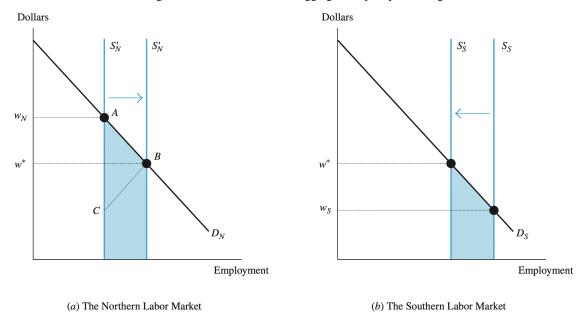


Figure 2: Equilibrium across Labor Markets

## 1.3 Policy Application: Payroll Taxes and Subsidies

### 1.3.1 Payroll Taxes Imposed on the Firm

The payroll tax is a tax imposed on the firm that is some fraction  $\tau$  of the wage paid to the worker.

Figure 3 illustrates the effect of a payroll tax. Here, we suppose there is a payroll tax of \$1 per hour. The initial equilibrium is at  $(E_0, w_0)$ , at point A. The payroll tax placed on the firm decreases the labor demand uniformly by the amount of the tax. Thus, the new equilibrium is at point B, where the employment level is now  $E_1$ , the wage received by the worker is  $w_1$ , and the effective wage paid by the firm is  $w_1 + 1$ .

### FIGURE 4-4 A Payroll Tax Imposed on Firms

A payroll tax of \$1 imposed on employers shifts down the demand curve (from  $D_0$  to  $D_1$ ). The tax cuts the wage that workers receive from  $w_0$  to  $w_1$  and increases the cost of hiring a worker from  $w_0$  to  $w_1 + 1$ .

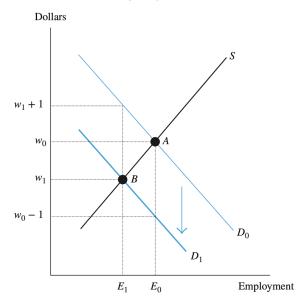


Figure 3: Payroll Tax

## 1.3.2 Payroll Taxes Imposed on the Worker

Figure 4 illustrates the effect of a payroll tax if it were imposed on the worker instead of the firm. Here, the effect is essentially the reverse in which the labor supply curve shifts up by the amount of the tax. Now, the equilibrium is at point B, where the employment level is  $E_1$ , the wage received by the worker is  $w_1$ , but the worker now pays a tax such that the effective wage received by the worker becomes  $w_1 - 1$ .

Whether the tax is imposed on the firm or the worker, the outcome is the same.

### FIGURE 4-5 A Payroll Tax Imposed on Workers

A payroll tax imposed on workers shifts the supply curve to the left (from  $S_0$  to  $S_1$ ). The payroll tax has the same impact on the equilibrium wage and employment regardless of who it is imposed on.

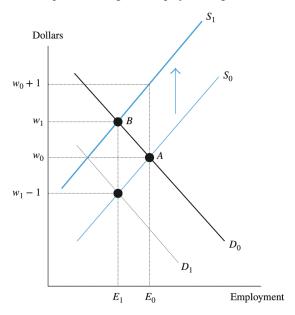


Figure 4: Payroll Tax Imposed on Workers

## 1.3.3 When Will the Payroll Tax Be Shifted Completely to Workers?

When the labor supply is perfectly inelastic, the entire burden of the payroll tax is shifted to the worker, as illustrated in Figure 5.

#### FIGURE 4-6 Inelastic Supply and a Payroll Tax Imposed on Firms

A payroll tax imposed on the firm is shifted completely to workers when the labor supply curve is perfectly inelastic. The wage is initially  $w_0$ . The \$1 payroll tax shifts the demand curve to  $D_1$ , and the wage falls to  $w_0 - 1$ .

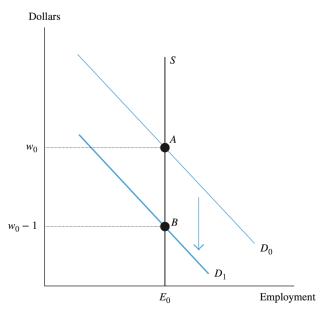


Figure 5: When Payroll Tax is Shifted Completely to Workers

## 1.3.4 Deadweight Loss

Figure 6 displays the reduction in the total surplus – aka the deadweight loss – that results from the imposition of a payroll tax. Recall that the initial equilibrium is at  $(E_0, w_0)$ , and the new equilibrium after the tax is at  $(E_1, w_{\text{NET}})$ , where the firm pays  $w_{\text{TOTAL}}$  and the worker receives  $w_{\text{NET}}$ . Now, the total surplus has been reduced by the DL triangle. The consumer surplus now becomes  $Q^*$ , rather than Q; the producer surplus now becomes  $P^*$ , rather than P; and the government collects tax revenue equal to the rectangle T.

If you look back at Figure 5, you can see that when the labor supply is perfectly inelastic, there is no deadweight loss from the imposition of the tax, and the producer surplus remains the same. All that changes is a transfer of consumer surplus to the government in the form of tax revenue. Specifically, the rectangle characterized by  $w_0AB(w_0-1)$  is transferred from consumer surplus to government revenue.

### FIGURE 4-7 Deadweight Loss of a Payroll Tax

(a) In a competitive equilibrium,  $E_0$  workers are hired at a wage of  $w_0$ . The triangle P gives the producer surplus and Q gives the worker surplus. The total gains from trade equal P + Q. (b) The payroll tax reduces employment to  $E_1$ ; raises the cost of hiring to  $w_{\text{TOTAL}}$ ; and reduces the worker's take-home pay to  $w_{\text{NET}}$ . The triangle  $P^*$  gives the producer surplus; the triangle  $Q^*$  gives the worker surplus; and the rectangle T gives the tax revenues. The net loss to society, or deadweight loss, is given by the triangle DL.

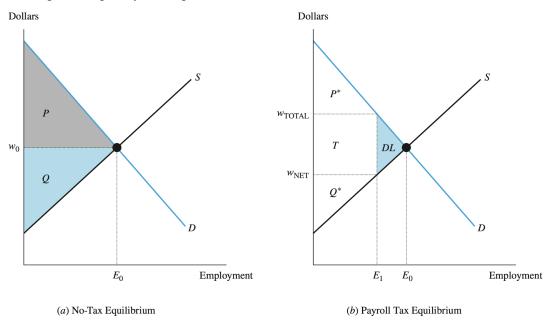


TABLE 4-1 Welfare Implications of a Payroll Tax

	No-Tax Equilibrium	Payroll Tax Equilibrium
Producer surplus	P	P*
Worker surplus	Q	Q*
Tax revenues	_	$\mathcal{T}$
Total gain from trade	P+Q	$P^* + Q^* + T$
Deadweight loss	_	DL

Figure 6: Deadweight Loss

## 1.3.5 Employment Subsidies

Figure 7 illustrates the effect of an employment subsidy of \$1. Effectively, this does the opposite of the payroll tax in terms of its effect on the demand curve; that is, the demand curve uniformly moves up by the size of the subsidy.

Under this setup, the equilibrium moves from A to B, where at B, the worker receives a wage of  $w_1$ , while the firm only has to pay  $w_1 - 1$ .

### FIGURE 4-8 The Impact of an Employment Subsidy

An employment subsidy of \$1 per worker hired shifts the demand curve from  $D_0$  to  $D_1$ , increasing employment. The wage that workers receive rises from  $w_0$  to  $w_1$ . The cost of hiring falls from  $w_0$  to  $w_1 - 1$ .

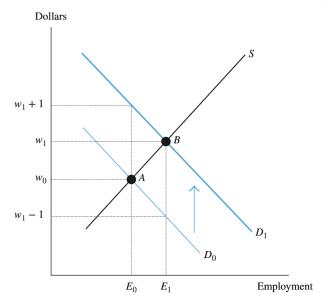


Figure 7: Employment Subsidy

The effect of the subsidy on employment will depend both on the labor supply elasticity and the labor demand elasticity. How it depends on the labor supply elasticity may be intuitive; if workers are very responsive to an increase in the wage, then more employment will be generated by the subsidy, and vice versa.

Figure 8 is a very quick demonstration that I draw of how the demand elasticity also impacts the effect on employment. For example, in the first panel, if demand is perfectly inelastic, then the subsidy naturally has no effect on employment. In the second panel, if demand is reasonably elastic, then the subsidy may have a more notable impact. The third panel shows the smaller effect when demand is reasonably inelastic but not perfectly inelastic.

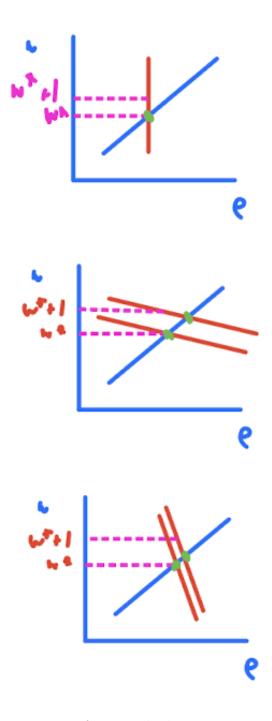


Figure 8: Impact of Demand Elasticity on Employment

## 1.4 Policy Application: Mandated Benefits

The government can assure every worker gets a certain benefit by mandating it. How does this affect equilibrium wages and employment?

The book uses the example of mandating a spinach pie for every worker at lunch.

Figure 9 depicts the effect of the mandatory benefit. Suppose the mandated benefit costs C dollars for the firm to provide, and the workers value it at B dollars. The demand curve shifts down uniformly by C, since they have to pay an extra C dollars per worker. At the same time, the supply curve shifts down uniformly by B, since they will get an extra B dollars of value. In the first case, where C > B, displayed in Panel (a), the shift in supply doesn't offset the shift in demand, so employment is lower than before the mandate, but not by as much as if a payroll tax had been implemented, since the payroll tax would've involved only the downward shift in demand, with no offsetting shift in supply. If C = B, as in Panel (b), then the shifts cancel out, and there is no effect on employment.

#### FIGURE 4-9 The Impact of a Mandated Benefit

(a) It costs firms C dollars to provide a mandated benefit, shifting the demand curve from  $D_0$  to  $D_1$ . Workers value the benefit only by B dollars, so the supply curve shifts down by less. Employment at the new equilibrium (point R) is higher than would have been the case if the firm had been assessed a payroll tax of C dollars (point Q), but lower than in a no-tax equilibrium (point P). (b) When the cost of providing the mandate equals the worker's valuation, the resulting equilibrium replicates the competitive no-tax equilibrium in terms of employment, total cost of hiring workers, and total compensation received by workers.

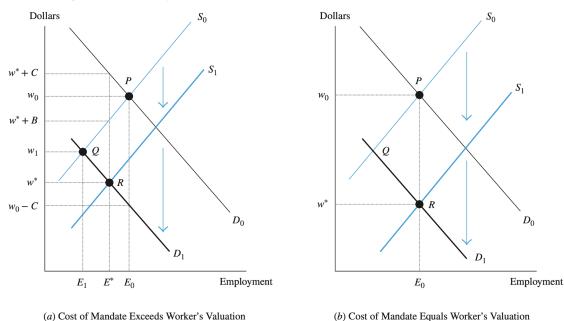


Figure 9: Effect of Mandated Benefits

The textbook highlights that the mandated benefit reduces the deadweight loss relative to a payroll tax when B > 0 and says that this means we prefer mandated benefits to payroll taxes.

## Questions

I am a bit confused by the last claim. It seems like if B>0 narrowly and C is much larger than B, then we have this scenarios where we basically burn a lot of money and don't even get tax revenue for it. This seems like the distortion triangle is smaller, which seems to be the author's point, but the fact that we've traded a lot of tax revenue for burned money seems like it would make the overall welfare effect negative. Am I thinking about this wrong?

## 1.5 The Labor Market Impact of Immigration

## 1.5.1 The Simplest Version

In the simplest model of analyzing the impact of immigration, we suppose that native and immigrant workers are perfect substitutes.

Panel (a) Figure 10 illustrates the impact of immigration under this assumption. Essentially, immigration constitutes a positive supply shock, which increases employment and reduces wages.

In contrast, if native and immigrant labor are complements, e.g., if the presence of immigrant workers permits native workers to specialize in tasks in which they are more adept, then this would correspond to a positive demand shock for native workers, as demonstrated in Panel (b) of Figure 10. In this case, both employment and wages of native workers increase.

#### FIGURE 4-10 The Short-Run Impact of Immigration

(a) Perfect substitutes. The two groups compete in the same labor market. Immigration shifts the supply curve to the right. The wage falls from  $w_0$  to  $w_1$  and total employment increases from  $N_0$  to  $E_1$ . The number of natives who work at the lower wage drops from  $N_0$  to  $N_1$ . (b) Complements. The two groups do not compete in the same labor market. Immigration makes natives more productive, shifting up the demand curve even though capital is fixed. Both the native wage and native employment increase.

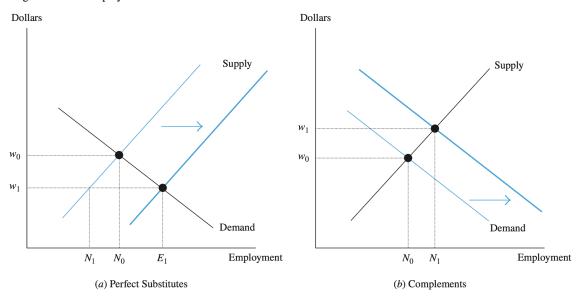


Figure 10: Impact of Immigration on Labor Market - Simplest Version

### 1.5.2 The Short Run and the Long Run

Suppose again that native and immigrant workers are perfect substitutes. What happens in the short run? As considered above, the supply curve shifts out, employment increases, and wages decrease. This decrease in worker wages and increase in workers will raise the returns to capital and increase profits. Thus, in the long run, firms will invest more in capital, as existing firms expand and new firms enter the market to take advantage of the cheap labor. The increase in the capital stock will then shift the labor demand curve to the right, attenuating the effects of the initial supply shock. However, by how much will the demand curve shift to the right in the long run? The answer depends on the production technology.

As an example, suppose we have a Cobb-Douglas production function:

$$q = AK^{\alpha}E^{1-\alpha}$$

where

• A: total factor productivity

- K: capital
- $\bullet$  E: labor
- $\alpha \in (0,1)$

This production function has constant returns to scale; that is, if we double both K and E, then output q will also double.

Recall that the profit-maximizing conditions in a competitive market require that the price of capital (r) and the price of labor (w) are equal to their respective value of their marginal products. Thus, the marginal productivity conditions for the Cobb-Douglas production function are:

$$r = MP_k p$$
 &  $MP_k = \alpha A K^{\alpha - 1} E^{1 - \alpha} \Rightarrow$   $r = \$1 \times \alpha A K^{\alpha - 1} E^{1 - \alpha}$   
 $w = MP_e p$  &  $MP_e = (1 - \alpha)AK^{\alpha}E^{-\alpha} \Rightarrow$   $w = \$1 \times (1 - \alpha)AK^{\alpha}E^{-\alpha}$ 

We can re-write this as:

$$r = \alpha A \left(\frac{K}{E}\right)^{\alpha - 1} = \alpha A \left(\frac{E}{K}\right)^{1 - \alpha}$$

$$w = (1 - \alpha) A \left(\frac{K}{E}\right)^{\alpha}$$
(1)

The marginal product of capital is increasing with the number of workers (and hence so is the value of the marginal product of capital). Thus, as the number of workers increases, the demand for capital increases. Similarly, the marginal product of labor is increasing with the amount of capital (and hence so is the value of the marginal product of labor). Thus, as the amount of capital increases, the demand for labor increases.

If we suppose the r is fixed, then (1) implies that the ratio of E/K is fixed. Thus, in this case, it must be that in the long run, the wage returns to its initial level. See Figure 11 for an illustration of this case.

#### FIGURE 4-11 Long-Run Impact of Immigration, Immigrants and Natives Are Perfect Substitutes

Immigration initially shifts out the supply curve, and the wage falls from  $w_0$  to  $w_1$ . Over time, capital expands as firms take advantage of the cheaper labor, shifting out the labor demand curve. If the aggregate production function has constant returns to scale, it must be the case that, after all capital adjustments have taken place, the wage returns to its initial level of  $w_1$ . In addition, the long-run level of native employment is exactly what it was prior to the immigrant influx.

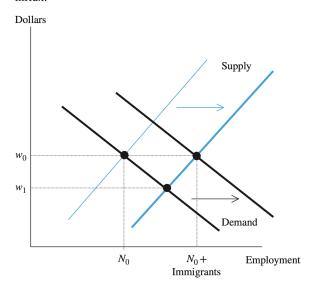


Figure 11: Long Run Effects of Immigration on Labor Market

### 1.5.3 The Immigration and Minimum Wage Debates

One interesting point is that the immigration and minimum wage debates are both focused on the labor demand elasticity. When we consider the results together, we should consider what certain pairs of results imply. For example, if we take the results (discussed in the textbook, but not in my notes) that immigration had no effect on employment in the Mariel natural experiment and that the minimum wage had no effect on employment in the NJ-Pennsylvania natural experiment, then this implies that the short-run labor demand elasticity is both perfectly elastic and perfectly inelastic, as shown in Figure 12. The author highlights the point that, though one can argue that these are simply different labor markets with different contexts, it would be troubling for the usefulness of the empirical evidence, since it would imply that the results in one setting tell you nothing about the dynamics in another setting.

### FIGURE 4-14 The Short-Run Labor Demand Curve Implied by Different Natural Experiments

(a) The short-run labor demand curve is perfectly elastic if the data from the Mariel natural experiment indicates that increased immigration did not affect the wage. (b) The short-run labor demand curve is perfectly inelastic if the data from the New Jersey–Pennsylvania minimum wage natural experiment indicates that the minimum wage does not affect employment.

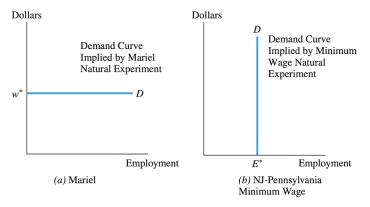


Figure 12: Labor Demand Elasticity in Immigration and Minimum Wage Debates

## 1.6 The Immigration Surplus

The author notes that although immigration may have negative effects on the wages of competing workers, it may generate gains for the aggregate economy. These gains come from the fact that the market wage equals the productivity of the last worker hired. Thus, all immigrants added to the economy, except for the last one, add more to the economy than they are paid. This is illustrated in Figure 13, where we take supply to be perfectly inelastic for the sake of simplicity. In this case, the immigration surplus is given by the triangle BCF. In fact, we can break up the segments further. Prior to the immigration supply shock, B reflects the equilibrium point, and the area ABN0 reflects the total output. After the immigration supply shock, the new equilibrium is at C, and the total output becomes ACM0.  $w_0BFw_1$  is switched from being accrued in wages to being accrued in profits for the firms, BCF is the new output generated that flows to firms, and CFMN is the new output generated that flows to the immigrant workers. Thus, immigration in this simple model corresponds to a loss in wages for workers, a greater increase in profits for firms (greater by the amount of the immigration surplus), and the introduction of new wages to the immigrants themselves. In general, the immigration surplus can be calculated as

Immigration surplus = 
$$\frac{1}{2} \times (w_0 - w_1) \times (M - N)$$

#### FIGURE 4-15 The Immigration Surplus

Prior to immigration, there are *N* native workers and national income is given by the trapezoid *ABN*0. Immigration increases labor supply to *M* workers and national income is given by the trapezoid *ACM*0. Immigrants are paid a total of *FCMN* dollars as salary. The immigration surplus gives the increase in national income that accrues to natives and is given by the area in the triangle *BCF*.

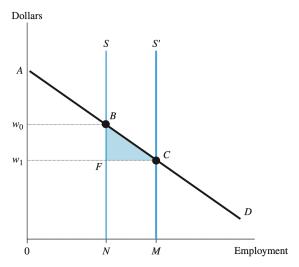


Figure 13: Immigration Surplus

## Questions

The book notes that, in the long run under constant returns to scale, the immigration surplus is zero. (GDP goes up but all the gains go to the immigrants.) I'm not sure if I totally understand this yet.

## 1.7 Policy Application: High-Skill Immigration

A supplementary theory introduces the notion that some immigrants, especially high-skill immigrants, may generate "human capital externalities." That is, immigrants may introduce new ideas, ways of thinking, etc. that make native workers more productive than they otherwise would've been. In this case, immigrants not only shift the supply curve, but also the demand curve, by shifting the value of the marginal product of labor.

Figure 14 illustrates this case. In this scenario, the increase to natives' income is given by the sum of the areas of BCD and ABEF. In this case, wages and employment can both rise.

### FIGURE 4-16 The Immigration Surplus in the Presence of Positive Externalities

There are N native workers. Immigration increases labor supply to M workers and the positive human capital externalities shift the demand curve to  $VMP'_L$ . The wage rises from  $w_0$  to  $w_1$ . Immigrants receive DCMN in wage payments. Native income increases by the sum of the trapezoid ABEF and the triangle BCD.

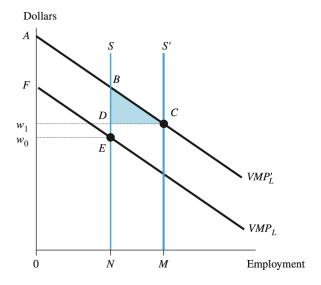


Figure 14: Immigration Surplus Amid Productivity Externalities

### Notes

Some diagnostic questions:

• Whose supply and demand curves are we looking at? When we look at a supply-demand diagram, we may be focusing on a particular group of workers, e.g., native workers in a particular skill group. This is important, as the effect of e.g., immigration or the minimum wage may be different for different groups of workers. For example, the impact of immigrants might increase the supply curve for one group of workers by being substitutes for them, but increase the demand curve for another group of workers by being complements for them.

### 1.8 The Cobweb Model