Scratch

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1 Chapter 3: Labor Demand

1.1 Terms

- q: Quantity of output produced by the firm
- E: Employee-hours (labor input)
- K: Capital input (machines, land, etc.)
- MP_E: Marginal product of labor
- MP_K: Marginal product of capital
- p: Price of output
- w: Wage rate (price of labor)
- r: Rental rate (price of capital)
- VMP_E: Value of marginal product of labor
- VAP_E: Value of average product of labor
- C: Cost of production

1.2 The Firm's Production Function

The firm's production function describes the technology that the firm uses to produce its output. We will assume for the moment that the firm only requires two inputs for its production: labor in the form of employee-hours (E) and capital (K), which includes land, machines, and other physical inputs. We will write the firm's production function as:

$$q = f(E, K)$$

1.2.1 Marginal Product and Average Product

Definition D.1: Marginal Product of Labor

The marginal product of labor (MP_E) is the additional output produced as a result of hiring one more employee-hour, holding the amount of capital constant.

Definition D.2: Marginal Product of Capital

The marginal product of capital (MP_K) is the additional output produced as a result of using one more unit of capital, holding the amount of labor constant.

Definition D.3: Average Product of Labor

The average product of labor (AP_E) is the average output produced per employee-hour, holding the amount of capital constant.

$$AP_E = \frac{q}{E}$$

Definition D.4: Law of Diminishing Returns

This is the assumption that the marginal product of labor eventually declines.

Figure 1 shows an example of how we would calculate the marginal product, average product, value of marginal product, and value of average product for labor.

TABLE 3-1

Calculating the Marginal and Average Product of Labor (Holding Capital Constant)

Note: The calculations for the value of marginal product and the value of average product assume that the price of the output is $2.

Number of Workers Employed	Output (Units)	Marginal Product (Units)	Average Product (Units)	Value of Marginal Product ($)	Value of Average Product ($)
0	0	—	—	—	—
1	11	11	11.0	22	22.0
2	27	16	13.5	32	27.0
3	47	20	15.7	40	31.3
4	66	19	16.5	38	33.0
5	83	17	16.6	34	33.2
6	98	15	16.3	30	32.7
7	111	13	15.9	26	31.7
8	122	11	15.3	22	30.5
9	131	9	14.6	18	29.1
10	138	7	13.8	14	27.6

Figure 1: Marginal Product Table

Figure 2 shows the relationship between the marginal product and average product curves.

FIGURE 3-1 **The Total Product, Marginal Product, and Average Product Curves**
(*a*) The total product curve gives the relationship between output and the number of workers hired by the firm, holding capital fixed. (*b*) The marginal product curve gives the output produced by each additional worker and the average product curve gives the output per worker.

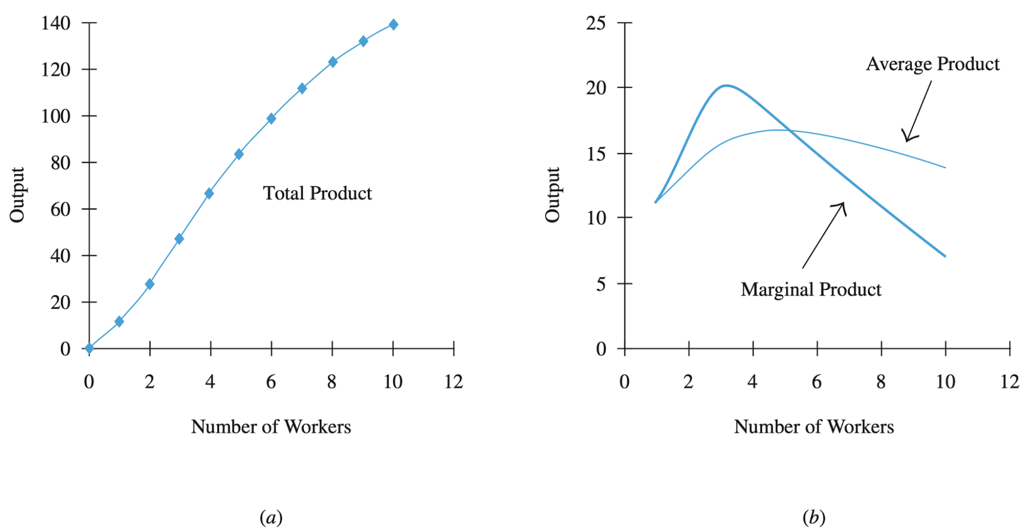


Figure 2: Marginal Product and Average Product Curves

1.2.2 Profit Maximization

The firm's profits are given by

$$\text{Profits} = pq - wE - rK$$

where

- p: Price of output
- w: Wage rate (price of labor)
- r: Rental rate (price of capital)

We assume that the firm is maximizing profits. We also, for the moment, take firms to be perfectly competitive, i.e., they cannot influence prices. Thus, given that these firms are price takers, they will maximize profits by choosing the optimal quantity of labor and capital.

Definition D.5: Perfectly Competitive Firm

We refer to a perfectly competitive firm as one that cannot influence prices.

1.3 The Short Run

Definition D.6: The Short Run

In our context, we define the short run to be a time horizon that is sufficiently short such that the firm cannot adjust its capital stock (K).

Definition D.7: Value of Marginal Product of Labor

The value of the marginal product of labor (VMP_E) is the additional revenue generated as a result of hiring one more employee-hour, holding the amount of capital constant.

$$VMP_E = p \cdot MP_E$$

Definition D.8: Value of Average Product of Labor

The value of the average product of labor (VAP_E) is the average revenue generated per employee-hour, holding the amount of capital constant.

$$VAP_E = p \cdot AP_E$$

1.3.1 How Many Workers Should the Firm Hire?

The firm will hire workers up to the point that the value of the marginal product of labor equals the wage rate and the value of the marginal product of labor is downward sloping. See Figure 3 for an example of VMP_E and VAP_E curves. In this figure, see the horizontal lines at 22 and 38. If the wage rate is 22, then the firm will hire 8 employees.[1] If the wage rate is 38, then the firm will hire 4 employees.

FIGURE 3-2 The Firm's Hiring Decision in the Short Run
A firm hires workers up to the point where the wage rate equals the value of marginal product of labor. If the wage is $22, the firm hires eight workers.

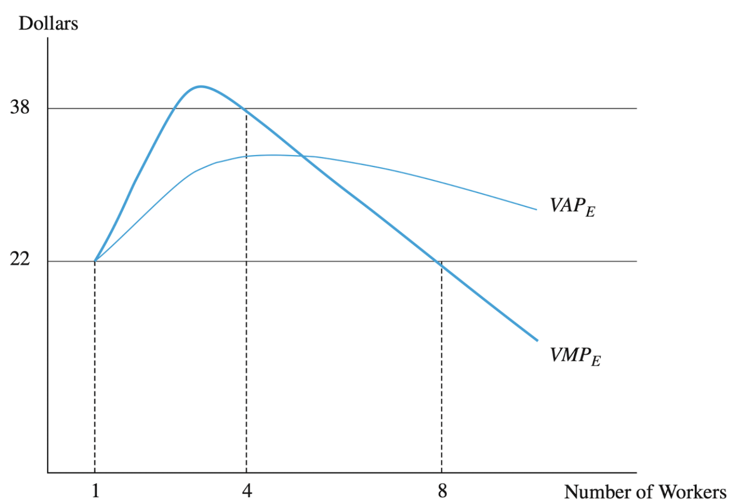


Figure 3: Short Run Hiring Decision

1.3.2 The Short-Run Labor Demand Curve for a Firm

With this setup, we can now derive the firm's short-run labor demand curve. In particular, the short-run labor-demand curve is given by the value of the marginal product of labor

[1] I think I'm kind of using employees and employee-hours as interchangeable concepts here.

curve. In particular, we can look at Figure 3, see at each value on the y-axis as a potential wage, draw a horizontal line to the point on the VMP_E curve that it intersects and is downward sloping, and then drop down to the x-axis to see how many employees the firm would hire at that wage. Doing this for all potential wages gives us the short-run labor demand curve, which is shown in Figure 4. Figure 4 also depicts what would happen if there was a shift in the VMP_E curve stemming from a rise in the price of the output. This would lead to more workers being hired, since it would raise the VMP_E curve to $VMP_{E'}$.

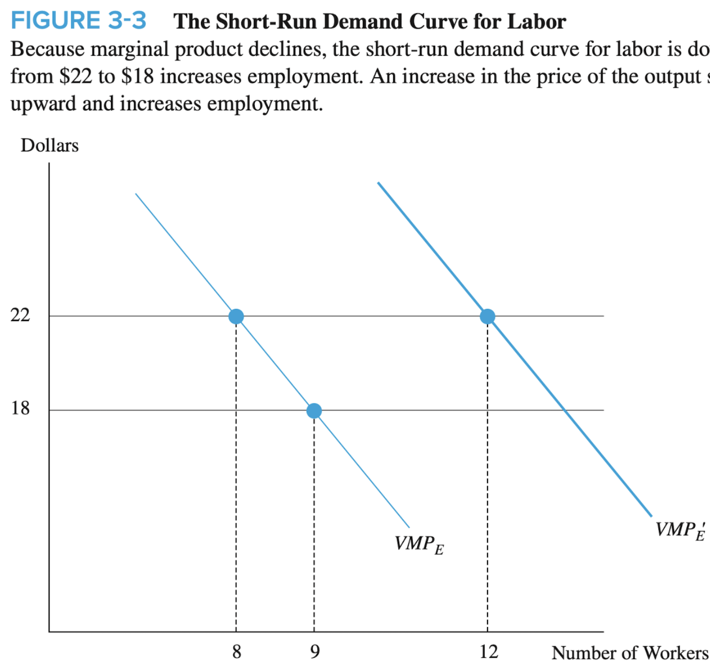


Figure 4: Short-Run Labor Demand Curve

Critical Point

It will be a recurring point to remember that the short-run labor demand curve for a firm is given by the VMP_E curve.

1.3.3 The Short-Run Labor Demand Curve in the Industry

The short-run labor demand curve in the industry is not simply the sum of the short-run labor demand curves of the individual firms, because, while one firm cannot influence prices, the industry as a whole can. If the industry increases production dramatically, the price of the output will fall, which lowers the VMP_E curve and hence demand. Thus, the short-run labor demand curve in the industry reflects this. See Figure 5 for an illustration.

FIGURE 3-4 **The Short-Run Demand Curve for the Industry**
Each firm in the industry hires 15 workers when the wage is \$20. If the wage falls to \$10, each firm hires 30 workers. If all firms expand, the output of the industry increases, reducing the price of the output and reducing the value of marginal product, so the labor demand curve of each individual firm shifts slightly to the left. At the lower price of \$10, each firm hires only 28 workers. The industry demand curve is not given by the horizontal sum of the firms' demand curves (*DD*), but takes into account the impact of the industry's expansion on output price (*TT*).

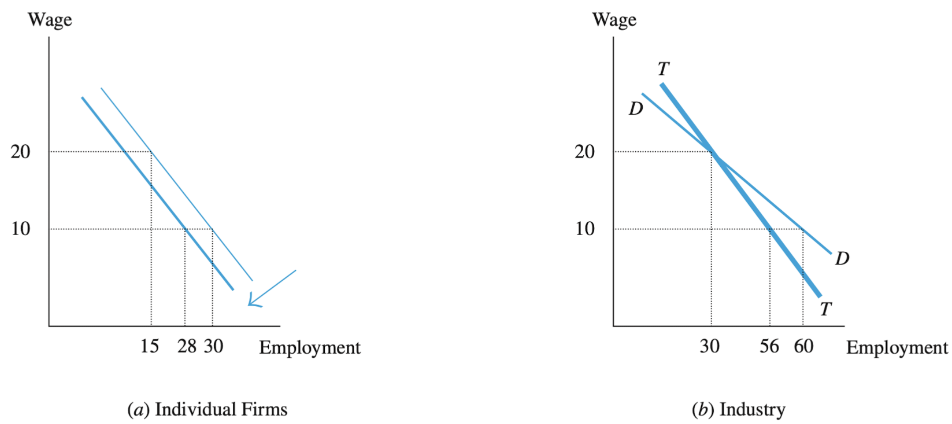


Figure 5: Short-Run Labor Demand Curve in the Industry

Definition D.9: Elasticity of Labor Demand

The elasticity of labor demand (δ_{SR}) is the percentage change in the quantity of labor demanded in the short run (E_{SR}) resulting from a percentage change in the wage rate.

$$\delta_{SR} = \frac{\Delta E_{SR}/E_{SR}}{\Delta w/w} = \frac{\Delta E_{SR}}{\Delta w} \cdot \frac{w}{E_{SR}}$$

1.3.4 An Alternative Interpretation of the Marginal Productivity Condition

An alternative way to express the point that the firm hires up to the point that the value of the marginal product of labor equals the wage rate is the following: The firm will hire up to the point that its marginal cost (MC)[2] equals its marginal revenue (MR). This interpretation is illustrated in Figure 6, where we show the marginal cost curve intersecting the marginal revenue curve (which in this case, is simply the price of the output, p). The optimal quantity of output is then given by q^*.

[2]That is, the marginal cost of producing another unit of the output.

FIGURE 3-5 The Firm's Output Decision
A profit-maximizing firm produces up to the point where the output price equals the marginal cost of production. This profit-maximizing condition is identical to the one stating that firms hire workers up to the point where the wage equals the value of marginal product.

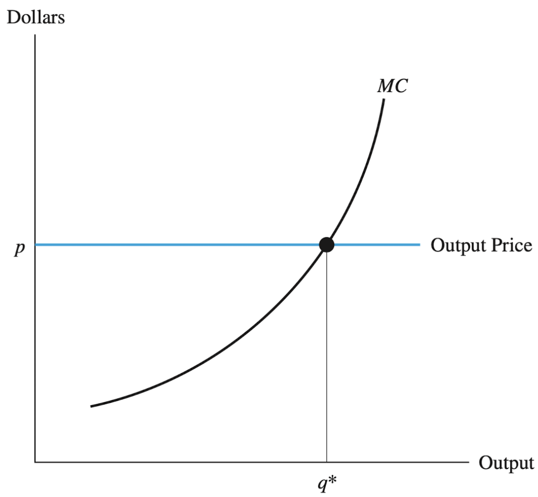


Figure 6: The Firm's Output Decision

We can note that the marginal cost of producing an extra unit of output is given by:

$$MC = w \times \frac{1}{MP_E}$$

That is, the wage scaled down by the marginal product of labor.[3]

Then, the profit-maximizing condition is that

$$w \times \frac{1}{MP_E} = p$$
$$\text{or } w = p \times MP_E = VMP_E$$

[3] You can think of this as saying something like: How much of a worker would you need to produce one more unit of output?

1.4 The Long Run

1.4.1 Isoquants

Definition D.10: Isoquant

An isoquant is a curve that captures the different combinations of labor and capital that produce the same level of output.

We demand the following properties of isoquants:

1. Isoquants must be downward sloping.
2. Isoquants do not intersect.
3. Higher isoquants are associated with higher levels of output.
4. Isoquants are convex to the origin.

See Figure 7 for an illustration of isoquants. Notice, as one would expect based on the definition, we are plotting the isoquants in K-E space.

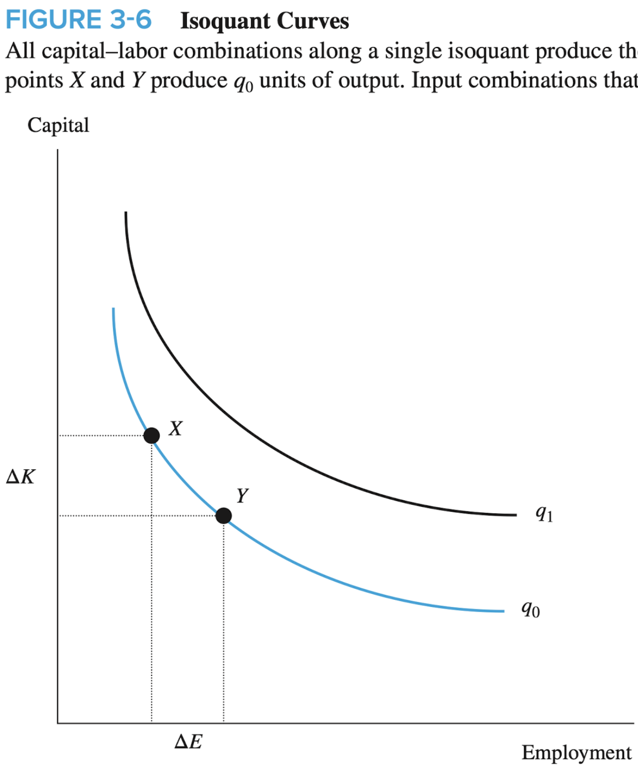


Figure 7: Isoquant Illustration

The slope of the isoquant is given by

$$\frac{\Delta K}{\Delta E} = -\frac{MP_E}{MP_K}$$

The LHS just captures the standard rise-over-run definition of slope. The RHS stems from the fact that, given the definition of an isoquant, it must be that:

$$(\Delta K)MP_K + (\Delta E)MP_E = 0$$

Thus, the slope of the isoquant is the negative ratio of the marginal products.

Definition D.11: Marginal Rate of Technical Substitution

The absolute value of the slope of the isoquant is referred to as the marginal rate of technical substitution (MRTS). That is, the MRTS is given by:

$$MRTS = \left|\frac{\Delta K}{\Delta E}\right| = \frac{MP_E}{MP_K}$$

Intuitively, this is telling us, if we increase labor by one unit, how much capital can we give up, while still keeping output constant.[a]

[a]The "one unit" framing isn't really right, since it's really about infinitesimal changes, but I'm being a little loose for the intuition.

1.4.2 Isocosts

The firm's cost of production is given by:

$$C = wE + rK$$

Definition D.12: Isocost Line

An isocost line is a curve that captures the different combinations of labor and capital that yield the same cost of production.

See Figure 8 for an illustration of isocost lines.

FIGURE 3-7 **Isocost Lines**

All capital–labor combinations along a single isocost curve are equally costly. Capital–labor combinations on a higher isocost curve are costlier. The slope of an isocost equals the ratio of input prices ($-w/r$).

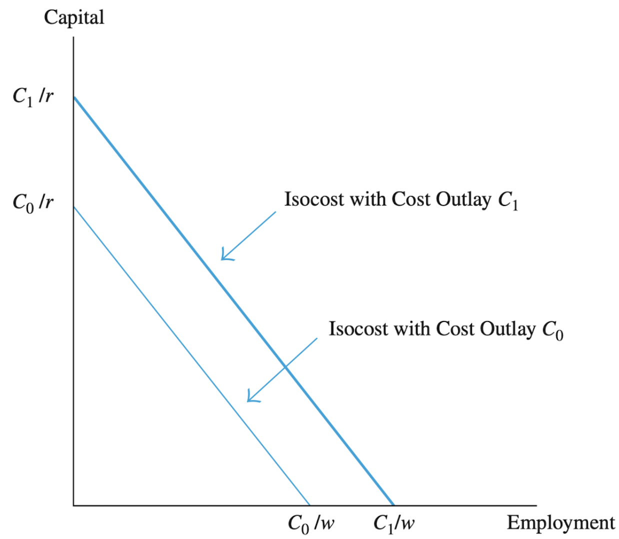


Figure 8: Isocost Lines

We can re-write the isocost line as:

$$K = \frac{C}{r} - \frac{w}{r}E$$

Thus, we can see that the slope of the isocost line is given by: $-\frac{w}{r}$. This is intuitive since it's telling us that the slope corresponds to telling us that an additional unit of labor costs $\frac{w}{r}$ units of capital.

1.4.3 Cost Minimization

Suppose that the firm is going to produce q_0 units of output. We can then ask: how can the firm produce q_0 at the lowest cost? The solution to this question is illustrated in Figure 9. The cost-minimizing solution is to produce at the point along the isocost line for q_0 which has a tangent isoquant.

FIGURE 3-8 The Firm's Optimal Combination of Inputs

A firm minimizes the cost of producing q_0 by using the capital–labor combination at point P, where the isoquant is tangent to the isocost. All other capital–labor combinations (such as those in points A and B) lie on a higher isocost.

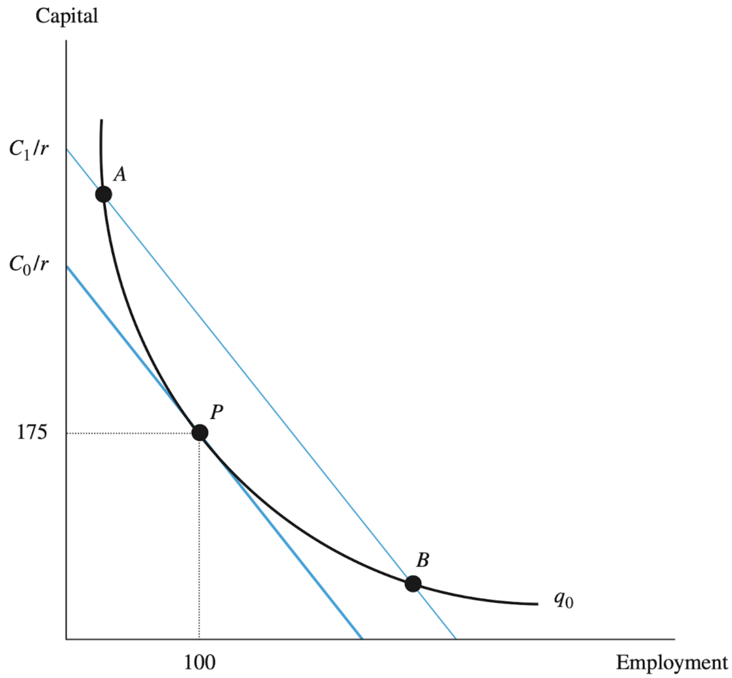


Figure 9: Cost Minimization

At the cost-minimizing solution, given the isoquant's tangency and the slope of each curve, we have:

$$\frac{MP_E}{MP_K} = \frac{w}{r}$$

That is the ratio of the marginal product of labor and capital equals the ratio of the cost of labor and capital.

The intuition may be more clearly seen by:

$$\frac{MP_E}{w} = \frac{MP_K}{r} \tag{1}$$

That is, the cost of the marginal product coming from labor and capital are equalized. It's logical that this is the optimal point, since if the cost of the marginal product from one was lower than the other, then the firm should re-allocate its spending toward that input.

1.4.3.1 Long-Run Profit Maximization

Note that the condition given by (1) is not the same as saying that firms maximize profit. This condition is only saying how firms minimize cost given a particular selection of output. Maximizing profit also requires choosing the optimal level of output.

Long-run profit maximization also necessitates that

$$w = p \times MP_E \quad \text{and} \quad r = p \times MP_K$$

which implies (1), but the reverse is not true.

Questions

Do we have a way of indicating the process of finding the profit-maximizing point on the isoquant-isocost diagram?

1.5 The Long-Run Demand Curve for Labor

Suppose that the wage for workers falls. How will firms respond in the long run? First, note that the worker's wage falling corresponds to a flattening of the isocost line (where the number of workers is on the x-axis).

Our first reaction to try to understand how the firms will re-allocate between capital and workers may be to draw Figure 10. However, this would be the wrong approach. In particular, this presupposes that the firm is going to hold costs fixed, but there is no reason to believe that this is the case.

FIGURE 3-9 The Impact of a Wage Reduction, Holding Constant Initial Cost Outlay at C_0

A wage reduction flattens the isocost curve. If the firm were to hold the initial cost outlay constant at C_0 dollars, the isocost would rotate around C_0 and the firm would move from point P to point R. A profit-maximizing firm, however, will not generally want to hold the cost outlay constant when the wage changes.

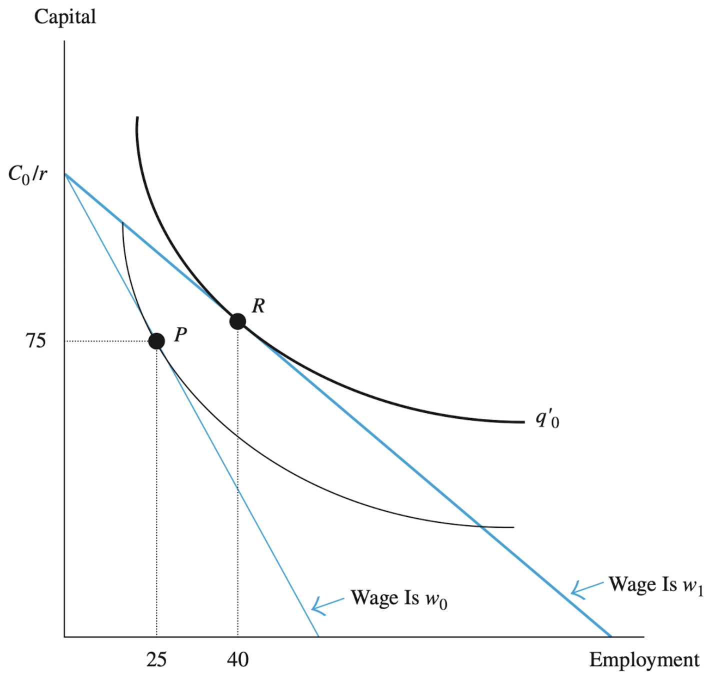


Figure 10: The Impact of a Wage Decrease if C_0 is Fixed

1.5.1 Will the Firm Expand if the Wage Falls?

Accommodating the reality that the cost may change, we can look to Figure 11 for an example of what might happen. Specifically, the fact that the marginal cost curve has shifted down encourages the firm to increase production. (See Panel (a).) Then, we can look at the relevant isoquant and isocost in Panel (b) to see what's happened to employment and capital usage. In this example, we display both as having increased. The important takeaway here, though, is primarily just that the firm is producing at a different level of cost.

FIGURE 3-10 **Impact of Wage Reduction on Output and Employment of a Profit-Maximizing Firm**
(*a*) A wage cut reduces the marginal cost of production and encourages the firm to expand (from producing 100 to 150 units). (*b*) The firm moves from point *P* to point *R*, increasing the number of workers hired from 25 to 50.

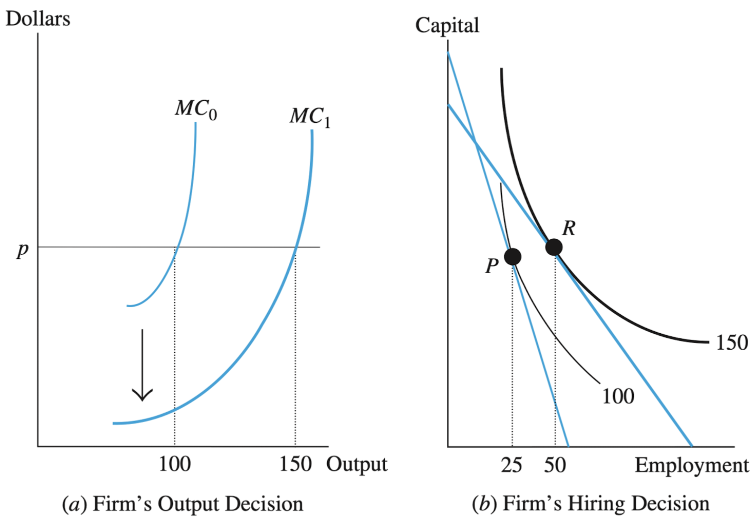


Figure 11: The Impact of a Wage Decrease if C_0 is Not Fixed

1.5.2 Substitution and Scale Effects

This section is analogous to our discussion of income and substitution effects for consumers. Here, we consider two effects that accompany a change in an input price for a firm: the scale effect and the substitution effect.

Definition D.13: Scale Effect

The scale effect refers to what happens to the firm's demands for its inputs as production increases, holding input prices constant.

Definition D.14: Substitution Effect

The substitution effect refers to what happens to the firm's demands for its inputs as input prices change, holding production constant.

In Figure 12, we illustrate these two effects in the context of a wage decrease. Prior to the wage change, the firm chooses employment and capital levels corresponding to P. If the production were shifted to the new level (150, in this example), holding input prices constant, the firm would choose the point Q, which corresponds to an increase in both capital and employment. This is the scale effect. The increase in both inputs holds as long as both are "normal inputs." However, if we now adjust the input prices to reflect the wage decrease, holding production constant at this new level, the firm would produce at point R. This reflects a decline in capital usage, relative to Q, and an increase in employment. This is the substitution effect. The directional effect of the substitution effect is not ambiguous.

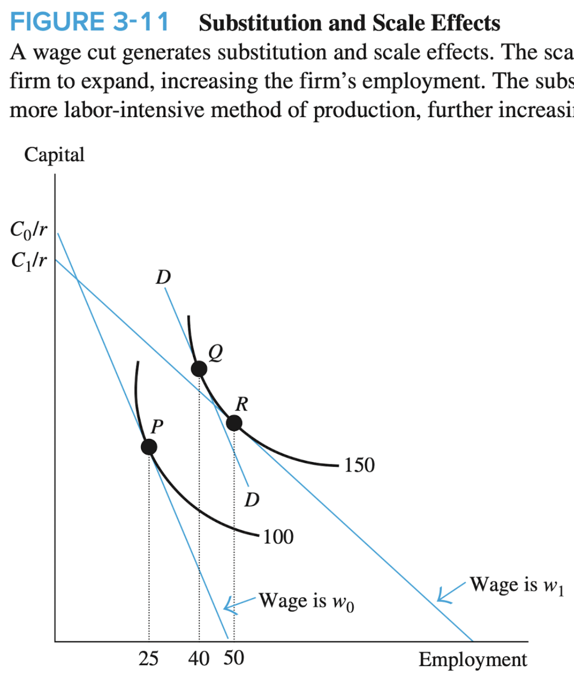


Figure 12: Substitution and Scale Effects

Definition D.15: Long-Run Elasticity of Labor Demand

The long-run elasticity of labor demand (δ_{LR}) is the percentage change in the quantity of labor demanded in the long run (E_{LR}) resulting from a percentage change in the wage rate.

$$\delta_{LR} = \frac{\Delta E_{LR}/E_{LR}}{\Delta w/w} = \frac{\Delta E_{LR}}{\Delta w} \cdot \frac{w}{E_{LR}}$$

In general, long-run labor demand will be more elastic than short-run labor demand. This dynamic is because in the long run, firms can adjust both capital and labor, so they can substitute between the two inputs.

FIGURE 3-12 **The Short- and Long-Run Labor Demand Curves**
In the long run, the firm can take full advantage of the economic opportunities introduced by a change in the wage. The long-run demand curve is more elastic than the short-run demand curve.

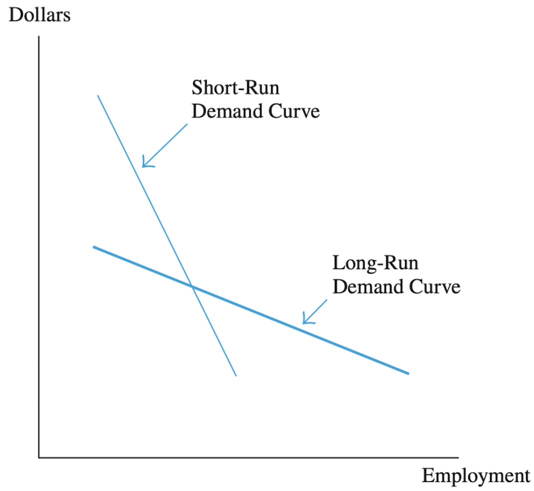


Figure 13: Long-Run vs Short-Run Elasticity of Labor Demand

1.5.3 Estimates of the Elasticity of Labor Demand

The author suggests that the short-run elasticity of labor demand is in the range of -0.4 and -0.5. That is, a 10% increase in wages leads to a 4-5% decrease in employment. He suggests that the long-run elasticity is estimated to be around -1%, i.e., a 10% increase in wages leads to a 10% decrease in employment.

1.6 The Elasticity of Substitution

"The size of the firm's substitution effect depends on the curvature of the isoquant."

Definition D.16: Perfect Substitute

If the isoquants are linear, then the inputs are perfect substitutes.

In the case of perfect substitutes, firms will simply use the cheapest input.

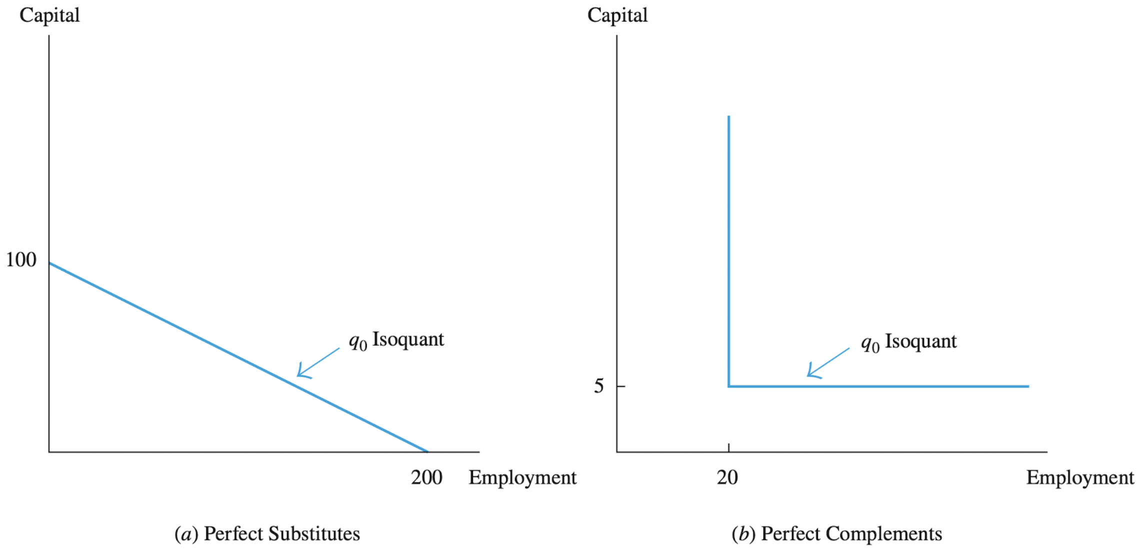
Definition D.17: Perfect Complement

If the isoquants are right angles, then the inputs are perfect complements.

In the case of perfect complements, firms will use inputs in fixed proportions.

See Figure 14 for an illustration of perfect substitutes and perfect complements.

FIGURE 3-13 **Isoquants When Inputs Are Either Perfect Substitutes or Perfect Complements**
Capital and labor are perfect substitutes if the isoquant is linear (so that two workers can always be substituted for one machine). The two inputs are perfect complements if the isoquant is right-angled. The firm then gets the same output when it hires 5 machines and 20 workers as when it hires 5 machines and 25 workers.



[5] The definition of perfect substitutes does *not* imply that the two inputs have to be exchanged on a one-to-one basis; that is, one machine hired for each worker laid off. Our definition only requires that the rate at which capital can be exchanged for labor is constant.

Figure 14: Perfect Substitutes and Perfect Complements

Definition D.18: Elasticity of Subsitution

The elasticity of substitution (σ) is the percentage change in the capital-labor ratio (K/L) resulting from a percentage change in the capital-labor price ratio (w/r).

$$\sigma = \frac{\text{Percent change in } K/L}{\text{Percent change in } w/r}$$

1.7 What Makes Labor Demand Elastic?

Marshall's rules of derived demand describe situations in which we would expect labor demand to be more elastic:

1. "Labor demand is more elastic the greater the elasticity of substitution."

2. "Labor demand is more elastic the greater the elasticity of demand for the output."

3. "Labor demand is more elastic the greater labor's share in total costs."

4. "The demand for labor is more elastic the greater the supply elasticity of other factors of production."

1.8 Factor Demand with Many Inputs

We now consider extending the theory we've developed so far to more inputs than simply labor and capital.

Suppose the firm's production function is instead given by:

$$q = f(x_1, x_2, x_3, \ldots, x_n)$$

where each x_i is a different input. We denote the marginal product of input i as MP_i, and the price of input i as w_i.

Under this setup, we still have the result that the ith input is purchased up to the point that:

$$w_i = p \times MP_i$$

Definition D.19: Cross-Elasticity of Factor Demand

As a measure of how the demand for one input responds to a change in the price of another input, we consider the cross-elasticity of factor demand:

$$\delta_{ij} = \frac{\%\Delta x_i}{\%\Delta w_j}$$

If the cross-elasticity is positive, then we say that the two inputs are substitutes. If the cross-elasticity is negative, then we say that the two inputs are complements.

Definition D.20: Capital-Skill Complementarity Hypothesis

The capital-skill complementarity hypothesis is the hypothesis that unskilled labor and capital are substitutes, while skilled labor and capital are complements.

1.9 Overview of Labor Market Equilibrium

We will cover it more fully in the next chapter, but now we will provide a brief overview of the idea of labor market equilibrium. The main idea is that the labor market occurs at the intersection of the labor supply and labor demand curves.

Figure 15 shows an example of labor market equilibrium at (E^*, w^*). At this point, in contrast to any other wage, the number of workers who want to work equals the number of workers that firms want to hire.

FIGURE 3-15 Wage and Employment Determination in a Competitive Market
In a competitive labor market, equilibrium is attained where supply equals demand. The equilibrium wage is w^* and E^* workers are employed.

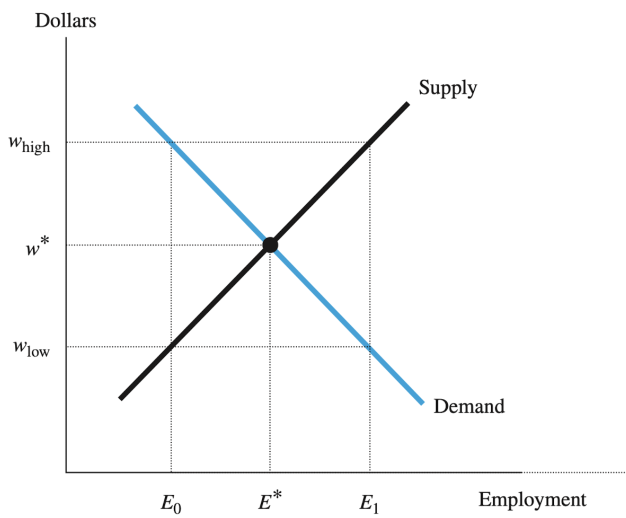


Figure 15: Labor Market Equilibrium Example

1.10 Rosie the Riveter as an Instrumental Variable

The author notes that much of modern labor economics research is focused on estimating labor supply and demand curves for various groups.

One method for doing so is to identify an instrumental variable (IV) that shifts only either supply or demand. This shift can then be used to trace out the other. The logic of this approach is visualized in Figure 16. Suppose the supply and demand curves start at S_0 and D_0, respectively. Then, consider three scenarios:

- S_0 moves to S_1, while D_0 stays fixed: Then the equilibrium moves from P to Q, and we can trace out the demand curve between these two points.

- D_0 moves to D_1, while S_0 stays fixed: Then the equilibrium moves from P to the unlabeled intersection of S_0 and D_1. Then, we can trace out the supply curve between these two points.

- S_0 moves to S_1 and D_0 moves to D_1: Then the equilibrium moves from P to R, and we can't say anything about either curve.

FIGURE 3-16 **Shifts in Supply and Demand Curves Generate Observed Wage and Employment Data**
The market is initially in equilibrium at P, and we observe wage w_0 and employment E_0. If only the supply curve shifts, we can observe w_1 and E_1, and the available data lets us to trace out the labor demand curve. But if both the supply and demand curves shift, we observe w_2 and E_2, and the available data trace out the curve ZZ, which does not provide any information about either the supply or the demand curve.

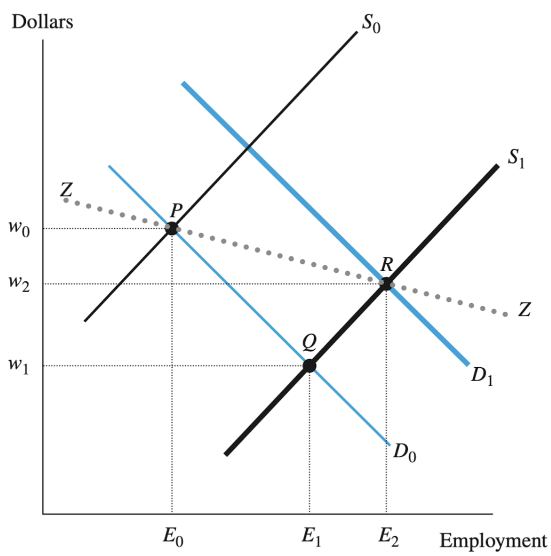


Figure 16: Instrumental Variable Approach

The author then describes one paper that uses this approach in the context of WWII, in which geographic differences in the mobilization rate of men was used as an instrument for female labor supply.

1.11 Policy Application: The Minimum Wage

We start by studying the effect of the minimum wage in a perfectly competitive labor market; we will discuss its effects under monospony in a future chapter. Figure 17 will be our reference point for this discussion. Prior to the minimum wage, the labor market is in equilibrium at (E^*, w^*). Then, suppose that a minimum wage is imposed at $\bar{w}$. Now, the labor supply is E_S, while the labor demand is $\bar{E}$. Thus, there are $E^* - \bar{E}$ fewer workers employed, and $E_S - E^*$ workers who want to work but cannot find jobs (i.e., who are "unemployed").

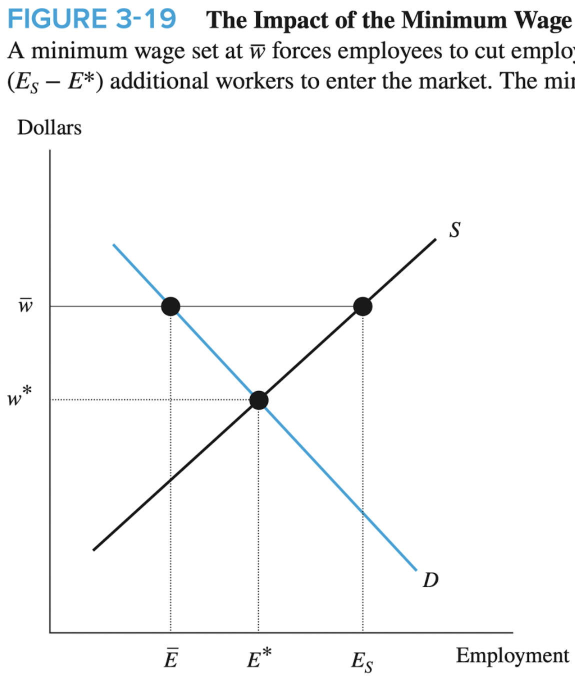


Figure 17: Minimum Wage Effects

The author then discusses compliance with minimum wage and how it relates to covered and uncovered sectors, but I won't discuss these here at the moment. The author then gives a lengthy discussion of the evidence and debates surrounding the effect of the minimum wage on employment. Again, I will not relay it all here, but it's available in the book for review.