Caliendo Paper Notes

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Contents

1 Terms

- n or i: Location index
- N: Number of locations
- j or k: Sector index
- J: Number of sectors
- θ^j: Sector-specific productivity dispersion parameter
- t: Time index
- L_0^{nj}: The mass of households in location n and sector j at time $t = 0$
- w_t^{nj}: Wage in location n and sector j at time t
- $U\left(C_t^{nj}\right)$: Utility function over baskets of final local goods
- C_t^{nj}: Basket of final local goods

$$C_t^{nj} = \prod_{k=1}^{J} \left(c_t^{nj,k}\right)^{\alpha^k}$$

- $c_t^{nj,k}$: Consumption of sector k good in market nj at time t.

- α^k: Final consumption share of sector k goods
  - $\sum_{k=1}^{J} \alpha^k = 1$
- P_t^n: Ideal price index in location n at time t

$$P_t^n = \prod_{k=1}^{J} \left(P_t^{nk}/\alpha^k\right)^{\alpha^k}$$

- P_t^{nk}: Price index of sector k goods for final consumption in location n at time t
- $b^n > 0$: Consumption obtained by non-employed individuals through home production[1]
- $C_t^{n0} = b^n$: Consumption in sector zero in location n at time t, which represents non-employment.
- $\beta \geq 0$: Discount factor
- $\tau^{nj,ik} \geq 0$: Labor relocation costs from market nj to ik
- $\boldsymbol{\epsilon}_t^{ik}$: Household-specific idiosyncratic shock for each choice of market.
- v_t^{nj}: The lifetime utility of a household currently in location n and sector j at time t, with the expectation taken over future realizations of the idiosyncratic shock.

$$\begin{aligned}\mathrm{v}_t^{nj} =& U\left(C_t^{nj}\right) + \max_{\{i,k\}_{i=1,k=0}^{N,J}} \left\{\beta E\left[\mathrm{v}_{t+1}^{ik}\right] - \tau^{nj,ik} + \nu\epsilon_t^{ik}\right\} \\ \text{s.t. } C_t^{nj} \equiv& \begin{cases} b^n & \text{if } j = 0 \\ w_t^{nj}/P_t^n & \text{otherwise} \end{cases}\end{aligned}$$

- ν: Parameter that scales the variance of the idiosyncratic shock

2 The Model

2.1 Initial Setup

"In each region-sector combination, there is a competitive labor market. In each market, there is a continuum of perfectly competitive firms producing intermediate goods."

Firms have a Cobb-Douglas Constant-Returns-to-Scale (CRS) production function, which utilizes labor, "composite local factor that we refer to as structures, and materials from all sectors."

We assume that productivities are "distributed Fréchet with a sector-specific productivity dispersion parameter θ^j."

Time is discrete, $t = 0, 1, 2, \ldots$.

"Households are forward looking, have perfect foresight, and optimally decide where to move given some initial distribution of labor across locations and sectors. Households face costs to move across markets and experience an idiosyncratic shock that affects their moving decision."

[1] Not totally sure if I'm understanding this term.

2.2 Household Problem

At $t = 0$, there is a mass of households in location n and sector j, denoted by L_0^{nj}. Households are either *employed* or *non-employed*.

If employed in location n and sector j at time t, workers inelastically supply a unit of labor and receive wage w_t^{nj}.

Given income, the household decides how to allocate their consumption over final goods across sectors with a Cobb-Douglas aggregator. Preferences, $U\left(C_t^{nj}\right)$, are over baskets of final local goods:

$$C_t^{nj} = \prod_{k=1}^{J} \left(c_t^{nj,k}\right)^{\alpha^k}$$

"Households are forward-looking and discount the future at rate $\beta \geq 0$. Migration decisions are subject to sectoral and spatial mobility costs."

Assumption 1

Labor relocation costs $\tau^{nj,ik} \geq 0$ depend on the origin (nj) and destination (ik) and are time invariant, additive, and measured in terms of utility.

Households have idiosyncratic shocks $\boldsymbol{\epsilon}_t^{ik}$ for each choice of market.

The timing for the household's problem is as follows:

1. Households observe the economic conditions in each market, as well as their own idiosyncratic shocks.
2. Returns
   - If they begin the period in the labor market, they work and receive the market wage.
   - If they are non-employed in a region, they receive home production.
3. Households choose whether to relocate.

Formally:

$$\begin{aligned} \mathrm{v}_t^{nj} =& U\left(C_t^{nj}\right) + \max_{\{i,k\}_{i=1,k=0}^{N,J}} \left\{\beta E\left[\mathrm{v}_{t+1}^{ik}\right] - \tau^{nj,ik} + \nu\epsilon_t^{ik}\right\} \\ \text{s.t. } C_t^{nj} \equiv& \begin{cases} b^n & \text{if } j = 0 \\ w_t^{nj}/P_t^n & \text{otherwise} \end{cases} \end{aligned}$$

v_t^{nj} is the lifetime utility of a household currently in location n and sector j at time t, with the expectation taken over future realizations of the idiosyncratic shock. ν is a parameter that scales the variance of the idiosyncratic shock.

Households choose to move to the labor market with the highest utility net of costs.

Assumption 2

The idiosyncratic shock ϵ is i.i.d. over time and distributed Type-I Extreme Value with zero mean.