Caliendo Paper Notes

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Contents

1 Terms

- n or i: Location index
- N: Number of locations
- j or k: Sector index
- J: Number of sectors
- θ^j: Sector-specific productivity dispersion parameter
- t: Time index
- L_0^{nj}: The mass of households in location n and sector j at time $t = 0$

- w_t^{nj}: Wage in location n and sector j at time t
- $U\left(C_t^{nj}\right)$: Utility function over baskets of final local goods
- C_t^{nj}: Basket of final local goods

$$C_t^{nj} = \prod_{k=1}^{J} \left(c_t^{nj,k}\right)^{\alpha^k}$$

- $c_t^{nj,k}$: Consumption of sector k good in market nj at time t.
- α^k: Final consumption share of sector k goods
  - $\sum_{k=1}^{J} \alpha^k = 1$
- P_t^n: Ideal price index in location n at time t

$$P_t^n = \prod_{k=1}^{J} \left(P_t^{nk}/\alpha^k\right)^{\alpha^k}$$

- P_t^{nk}: Price index of sector k goods for final consumption in location n at time t
- $b^n > 0$: Consumption obtained by non-employed individuals through home production[1]
- $C_t^{n0} = b^n$: Consumption in sector zero in location n at time t, which represents non-employment.
- $\beta \geq 0$: Discount factor
- $\tau^{nj,ik} \geq 0$: Labor relocation costs from market nj to ik
- $\boldsymbol{\epsilon}_t^{ik}$: Household-specific idiosyncratic shock for each choice of market.
  - $F(\boldsymbol{\epsilon})$: CDF of the idiosyncratic shock

$$F(\boldsymbol{\epsilon}) = \exp(-\exp(-\boldsymbol{\epsilon} - \bar{\gamma}))$$

  - $\bar{\gamma}$:

$$\bar{\gamma} \equiv \int_{-\infty}^{\infty} x \exp(-x - \exp(-x)) dx$$

- v_t^{nj}: The lifetime utility of a household currently in location n and sector j at time t, with the expectation taken over future realizations of the idiosyncratic shock.

$$\begin{aligned} \mathrm{v}_t^{nj} =& U\left(C_t^{nj}\right) + \max_{\{i,k\}_{i=1,k=0}^{N,J}} \left\{\beta E\left[\mathrm{v}_{t+1}^{ik}\right] - \tau^{nj,ik} + \nu \epsilon_t^{ik}\right\} \\ \text{s.t. } C_t^{nj} \equiv& \begin{cases} b^n & \text{if } j = 0 \\ w_t^{nj}/P_t^n & \text{otherwise} \end{cases} \end{aligned}$$

- ν: Parameter that scales the variance of the idiosyncratic shock

[1] Not totally sure if I'm understanding this term.

- $V_t^{nj} \equiv \mathbb{E}\left[\mathrm{v}_t^{nj}\right]$: Expected lifetime utility of a representative agent in location n and sector j at time t
- $\mu_t^{nj,ik}$: The fraction of households that relocate from market nj to ik at time t
- A_t^{nj}: Time-varying region-sector (market) productivity, common across varieties
- z^{nj}: Variety-specific market productivity
- q_t^{nj}: Expression for the quantity produced of an intermediate good

$$q_t^{nj} = z^{nj} \left(A_t^{nj} \left(h_t^{nj}\right)^{\xi^n} \left(l_t^{nj}\right)^{1-\xi^n} \right)^{\gamma^{nj}} \prod_{k=1}^{J} \left(M_t^{nj,nk}\right)^{\gamma^{nj,nk}}$$

- l_t^{nj}: Labor inputs in location n and sector j at time t
- h_t^{nj}: Structure inputs in location n and sector j at time t
- $M_t^{nj,nk}$: Material inputs from sector k demanded by a firm in sector j and region n at time t
- $\gamma^{nj} \geq 0$: Share of value-added in the production of sector j and region n
    - This comes from $\gamma^{nj} + \sum_{k=1}^{J} \gamma^{nj,nk} = 1$ and the Cobb-Douglas form, so the value-add share is the share that comes from the non-material inputs.
- $\gamma^{n,nk} \geq 0$: The share of materials from sector k in the production of sector j and region n.
- ξ^n: The share of structures in value-add.
- r_t^{nj}: The rental price of structures in region n and sector j at time t.
- x_t^{nj}: Unit price of an input bundle

$$x_t^{nj} = B^{nj} \left(\left(r_t^{nj}\right)^{\xi^n} \left(w_t^{nj}\right)^{1-\xi^n} \right)^{\gamma^{nj}} \prod_{k=1}^{J} \left(P_t^{nk}\right)^{\gamma^{nj,nk}}$$

2 The Model

2.1 Initial Setup

"In each region-sector combination, there is a competitive labor market. In each market, there is a continuum of perfectly competitive firms producing intermediate goods."

Firms have a Cobb-Douglas Constant-Returns-to-Scale (CRS) production function, which utilizes labor, "composite local factor that we refer to as structures, and materials from all sectors."

We assume that productivities are "distributed Fréchet with a sector-specific productivity dispersion parameter θ^j."

Time is discrete, $t = 0, 1, 2, \ldots$.

"Households are forward looking, have perfect foresight, and optimally decide where to move given some initial distribution of labor across locations and sectors. Households face costs to move across markets and experience an idiosyncratic shock that affects their moving decision."

2.2 Household Problem

At $t = 0$, there is a mass of households in location n and sector j, denoted by L_0^{nj}. Households are either *employed* or *non-employed*.

If employed in location n and sector j at time t, workers inelastically supply a unit of labor and receive wage w_t^{nj}.

Given income, the household decides how to allocate their consumption over final goods across sectors with a Cobb-Douglas aggregator. Preferences, $U\left(C_t^{nj}\right)$, are over baskets of final local goods:

$$C_t^{nj} = \prod_{k=1}^{J} \left(c_t^{nj,k}\right)^{\alpha^k}$$

"Households are forward-looking and discount the future at rate $\beta \geq 0$. Migration decisions are subject to sectoral and spatial mobility costs."

Assumption 1

Labor relocation costs $\tau^{nj,ik} \geq 0$ depend on the origin (nj) and destination (ik) and are time invariant, additive, and measured in terms of utility.

Households have idiosyncratic shocks ϵ_t^{ik} for each choice of market.

The timing for the household's problem is as follows:

1. Households observe the economic conditions in each market, as well as their own idiosyncratic shocks.
2. Returns
   - If they begin the period in the labor market, they work and receive the market wage.
   - If they are non-employed in a region, they receive home production.
3. Households choose whether to relocate.

Formally:

$$\begin{aligned} \mathrm{v}_t^{nj} =& U\left(C_t^{nj}\right) + \max_{\{i,k\}_{i=1,k=0}^{N,J}} \left\{\beta E\left[\mathrm{v}_{t+1}^{ik}\right] - \tau^{nj,ik} + \nu\epsilon_t^{ik}\right\} \\ &\text{s.t. } C_t^{nj} \equiv \begin{cases} b^n & \text{if } j = 0 \\ w_t^{nj}/P_t^n & \text{otherwise} \end{cases} \end{aligned}$$

v_t^{nj} is the lifetime utility of a household currently in location n and sector j at time t, with the expectation taken over future realizations of the idiosyncratic shock. ν is a parameter that scales the variance of the idiosyncratic shock.

Households choose to move to the labor market with the highest utility net of costs.

Assumption 2

The idiosyncratic shock ϵ is i.i.d. over time and distributed Type-I Extreme Value with zero mean.

Note that Assumption 2 is common in the literature[2] and is useful to allow "for simple aggregation of idiosyncratic decisions made by households."

Let

$$V_t^{nj} \equiv \mathbb{E}\left[\mathrm{v}_t^{nj}\right] \tag{1}$$

Then

$$V_t^{nj} = U\left(C_t^{nj}\right) + \nu \log\left(\sum_{i=1}^{N}\sum_{k=0}^{J} \exp\left(\beta V_{t+1}^{ik} - \tau^{nj,ik}\right)^{1/\nu}\right)$$

The derivation is given in subsection 3.1.

"V_t^{nj} can be interpreted as the expected lifetime utility of a household before the realization of the household preference shocks or, alternatively, as the average utility of households in that market."

From there we can derive the fraction of households that relocate from market nj to ik:

$$\mu_t^{nj,ik} = \frac{\exp\left(\beta V_{t+1}^{ik} - \tau^{nj,ik}\right)^{1/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J} \exp\left(\beta V_{t+1}^{mh} - \tau^{nj,mh}\right)^{1/\nu}} \tag{2}$$

To-do

Add derivation if desired.

All else equal (2) implies that markets with a higher lifetime utility (net of migration costs) will be the ones to attract more migrants. Moreover, it allows $\frac{1}{\nu}$ to be interpreted as a migration elasticity with respect to the expected utility of the destination market, i.e., $\beta V_{t+1}^{ik} - \tau^{nj,ik}$.

You can see the elasticity interpretation intuitively if you consider:

[2]Caliendo et al. (2019) refers readers to
Aguirregabiria, V., & Mira, P. (2010). Dynamic discrete choice structural models: A survey. *Journal of Econometrics, 156*(1), 38-67.

$$
\begin{aligned}
\mu_t^{nj,ik} &= \frac{\exp\left(\beta V_{t+1}^{ik} - \tau^{nj,ik}\right)^{1/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\exp\left(\beta V_{t+1}^{mh} - \tau^{nj,mh}\right)^{1/\nu}} \\
\Rightarrow \ln\mu_t^{nj,ik} &= \frac{1}{\nu}\left(\beta V_{t+1}^{ik} - \tau^{nj,ik} - \ln\sum_{m=1}^{N}\sum_{h=0}^{J}\exp\left(\beta V_{t+1}^{mh} - \tau^{nj,mh}\right)\right) \\
\Rightarrow \frac{\partial \ln\mu_t^{nj,ik}}{\partial(\beta V_{t+1}^{ik} - \tau^{nj,ik})} &\approx \frac{1}{\nu}
\end{aligned}
$$

(2) "conveys all the information needed to determine how the distribution of labor across markets evolves over time. In particular,"

$$
L_{t+1}^{nj} = \sum_{i=1}^{N}\sum_{k=0}^{J}\mu_t^{ik,nj}L_t^{ik} \tag{3}
$$

(3) characterizes the distribution of employment and non-employment across markets.

Note that, under the given timing assumptions, labor in period t is "fully determined by forward-looking decisions at period $t-1$."

2.3 Production

"Firms in each sector and region are able to produce many varieties of intermediate goods. "

Production requires labor, structures, and materials from all sectors.

"TFP of an intermediate good is composed of two terms, a time-varying sectoral-regional component $\left(A_t^{nj}\right)$, which is common to all varieties in a region and sector, and a variety-specific component $\left(z^{nj}\right)$."

2.3.1 Intermediate Good Production

2.3.1.1 Production Function

We assume that the production function is CRS and Cobb-Douglas.

The output for a producer of an intermediate good is given by:

$$
q_t^{nj} = z^{nj}\left(A_t^{nj}\left(h_t^{nj}\right)^{\xi^n}\left(l_t^{nj}\right)^{1-\xi^n}\right)^{\gamma^{nj}}\prod_{k=1}^{J}\left(M_t^{nj,nk}\right)^{\gamma^{nj,nk}}
$$

where

- l_t^{nj}: Labor inputs in location n and sector j at time t
- h_t^{nj}: Structure inputs in location n and sector j at time t
- $M_t^{nj,nk}$: Material inputs from sector k demanded by a firm in sector j and region n at time t
- $\gamma^{nj} \geq 0$: Share of value-added in the production of sector j and region n
  - This comes from $\gamma^{nj} + \sum_{k=1}^{J} \gamma^{nj,nk} = 1$ and the Cobb-Douglas form, so the value-add share is the share that comes from the non-material inputs.
- $\gamma^{n,nk} \geq 0$: The share of materials from sector k in the production of sector j and region n.
- ξ^n: The share of structures in value-add.

"Material inputs are goods from sector k produced in the same region n."

2.3.1.2 Unit Price of an Input Bundle

The unit price of an input bundle is given by:

$$x_t^{nj} = B^{nj} \left(\left(r_t^{nj} \right)^{\xi^n} \left(w_t^{nj} \right)^{1-\xi^n} \right)^{\gamma^{nj}} \prod_{k=1}^{J} \left(P_t^{nk} \right)^{\gamma^{nj,nk}}$$

3 Derivations

3.1 Derivation of V_t^{nj}

This is a derivation of Equation 2.2.

As a quick starting point, note that we know the value of $U\left(C_t^{nj}\right)$, so the uncertainty is over subsequent periods:

$$\begin{aligned}
V_t^{nj} \equiv & \mathbb{E}\left[\mathrm{v}_t^{nj}\right] \\
= & \mathbb{E}\left[U\left(C_t^{nj}\right) + \max_{\{i,k\}_{i=1,k=0}^{N,J}} \left\{\beta E\left[\mathrm{v}_{t+1}^{ik}\right] - \tau^{nj,ik} + \nu\epsilon_t^{ik}\right\}\right] \\
= & U\left(C_t^{nj}\right) + \mathbb{E}\left[\max_{\{i,k\}_{i=1,k=0}^{N,J}} \left\{\beta E\left[\mathrm{v}_{t+1}^{ik}\right] - \tau^{nj,ik} + \nu\epsilon_t^{ik}\right\}\right]
\end{aligned}$$

Denote the uncertain term as:

$$\Phi_t^{nj} \equiv \mathbb{E}\left[\max_{\{i,k\}_{i=1,k=0}^{N,J}} \left\{\beta E\left[\mathrm{v}_{t+1}^{ik}\right] - \tau^{nj,ik} + \nu\epsilon_t^{ik}\right\}\right]$$

Recall that we assumed that the idiosyncratic shock ϵ is i.i.d. over time and distributed Type-I Extreme Value with zero mean. Then, the CDF of ϵ is:

$$F(\epsilon) = \exp(-\exp(-\epsilon - \bar{\gamma}))$$

where

$$\bar{\gamma} \equiv \int_{-\infty}^{\infty} x \exp(-x - \exp(-x)) dx$$

is Euler's constant and

$$f(\epsilon) = \partial F / \partial \epsilon$$

Denote

$$\bar{\epsilon}_t^{ik,mh} \equiv \frac{\beta \left(V_{t+1}^{ik} - V_{t+1}^{mh} \right) - \left(\tau^{nj,ik} - \tau^{nj,mh} \right)}{\nu} \tag{4}$$

Then notice that:

$$
\begin{aligned}
&\Phi_t^{nj} \\
\equiv&\mathbb{E}\left[\max_{\{i,k\}_{i=1,k=0}^{N,J}}\left\{\beta E\left[\mathrm{v}_{t+1}^{ik}\right]-\tau^{nj,ik}+\nu\epsilon_t^{ik}\right\}\right] \\
=&\mathbb{E}\left[\max_{\{i,k\}_{i=1,k=0}^{N,J}}\left\{\beta V_{t+1}^{ik}-\tau^{nj,ik}+\nu\epsilon_t^{ik}\right\}\right] && \text{by (1)} \\
=&\mathbb{E}\left[\sum_{i=1}^{N}\sum_{k=0}^{J}\left(\beta V_{t+1}^{ik}-\tau^{nj,ik}+\nu\epsilon_t^{ik}\right)\mathbf{1}\left\{\underset{\{m,h\}_{m=1,h=0}^{N,J}}{\operatorname{argmax}}\left\{\beta V_{t+1}^{mh}-\tau^{nj,mh}+\nu\epsilon_t^{mh}\right\}=ik\right\}\right] \\
=&\mathbb{E}\left[\sum_{i=1}^{N}\sum_{k=0}^{J}\left(\beta V_{t+1}^{ik}-\tau^{nj,ik}+\nu\epsilon_t^{ik}\right)\prod_{mh\neq ik}\mathbf{1}\left\{\beta V_{t+1}^{mh}-\tau^{nj,mh}+\nu\epsilon_t^{mh}<\beta V_{t+1}^{ik}-\tau^{nj,ik}+\nu\epsilon_t^{ik}\right\}\right] \\
=&\sum_{i=1}^{N}\sum_{k=0}^{J}\mathbb{E}\left[\left(\beta V_{t+1}^{ik}-\tau^{nj,ik}+\nu\epsilon_t^{ik}\right)\prod_{mh\neq ik}\mathbf{1}\left\{\beta V_{t+1}^{mh}-\tau^{nj,mh}+\nu\epsilon_t^{mh}<\beta V_{t+1}^{ik}-\tau^{nj,ik}+\nu\epsilon_t^{ik}\right\}\right] \\
=&\sum_{i=1}^{N}\sum_{k=0}^{J}\mathbb{E}\left[\mathbb{E}\left[\left(\beta V_{t+1}^{ik}-\tau^{nj,ik}+\nu\epsilon_t^{ik}\right)\prod_{mh\neq ik}\mathbf{1}\left\{\beta V_{t+1}^{mh}-\tau^{nj,mh}+\nu\epsilon_t^{mh}<\beta V_{t+1}^{ik}-\tau^{nj,ik}+\nu\epsilon_t^{ik}\right\}\mid\epsilon_t^{ik}\right]\right] && \text{by LIE} \\
=&\sum_{i=1}^{N}\sum_{k=0}^{J}\mathbb{E}\left[\left(\beta V_{t+1}^{ik}-\tau^{nj,ik}+\nu\epsilon_t^{ik}\right)\prod_{mh\neq ik}\mathbb{E}\left[\mathbf{1}\left\{\beta V_{t+1}^{mh}-\tau^{nj,mh}+\nu\epsilon_t^{mh}<\beta V_{t+1}^{ik}-\tau^{nj,ik}+\nu\epsilon_t^{ik}\right\}\mid\epsilon_t^{ik}\right]\right] && \text{by iid} \\
=&\sum_{i=1}^{N}\sum_{k=0}^{J}\mathbb{E}\left[\left(\beta V_{t+1}^{ik}-\tau^{nj,ik}+\nu\epsilon_t^{ik}\right)\prod_{mh\neq ik}\Pr\left(\beta V_{t+1}^{mh}-\tau^{nj,mh}+\nu\epsilon_t^{mh}<\beta V_{t+1}^{ik}-\tau^{nj,ik}+\nu\epsilon_t^{ik}\mid\epsilon_t^{ik}\right)\right] && \text{binary variable property}
\end{aligned}
$$

Now, notice that

$$
\begin{aligned}
&\Pr\left(\beta V_{t+1}^{mh}-\tau^{nj,mh}+\nu\epsilon_t^{mh}<\beta V_{t+1}^{ik}-\tau^{nj,ik}+\nu\epsilon_t^{ik}\mid\epsilon_t^{ik}\right) \\
=&\Pr\left(\epsilon_t^{mh}<\epsilon_t^{ik}+\frac{\beta V_{t+1}^{ik}-\tau^{nj,ik}-\beta V_{t+1}^{mh}+\tau^{nj,mh}}{\nu}\mid\epsilon_t^{ik}\right) \\
=&\Pr\left(\epsilon_t^{mh}<\epsilon_t^{ik}+\bar{\epsilon}_t^{ik,mh}\mid\epsilon_t^{ik}\right) && \text{by (4)} \\
=&F\left(\epsilon_t^{ik}+\bar{\epsilon}_t^{ik,mh}\right)
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\Phi_t^{nj} &= \sum_{i=1}^{N}\sum_{k=0}^{J}\mathbb{E}\left[\left(\beta V_{t+1}^{ik} - \tau^{nj,ik} + \nu\epsilon_t^{ik}\right)\prod_{mh\neq ik} F\left(\epsilon_t^{ik} + \bar{\epsilon}_t^{ik,mh}\right)\right] \\
&= \sum_{i=1}^{N}\sum_{k=0}^{J}\int_{-\infty}^{\infty}\left(\beta V_{t+1}^{ik} - \tau^{nj,ik} + \nu\epsilon_t^{ik}\right) f\left(\epsilon_t^{ik}\right)\prod_{mh\neq ik} F\left(\bar{\epsilon}_t^{ik,mh} + \epsilon_t^{ik}\right) d\epsilon_t^{ik}
\end{aligned}
$$

From there, we can note that

$$
\begin{aligned}
f\left(\epsilon_t^{ik}\right) &= \exp(-\epsilon_t^{ik} - \bar{\gamma})\exp\left(-\exp\left(-\epsilon_t^{ik} - \bar{\gamma}\right)\right) \\
F\left(\bar{\epsilon}_t^{ik,mh} + \epsilon_t^{ik}\right) &= \exp\left[-\exp\left(-\left[\bar{\epsilon}_t^{ik,mh} + \epsilon_t^{ik} + \bar{\gamma}\right]\right)\right]
\end{aligned}
$$

Thus,

$$
\begin{aligned}
& f\left(\epsilon_t^{ik}\right)\prod_{mh\neq ik} F\left(\bar{\epsilon}_t^{ik,mh} + \epsilon_t^{ik}\right) \\
&= \exp(-\epsilon_t^{ik} - \bar{\gamma})\exp\left(-\exp\left(-\epsilon_t^{ik} - \bar{\gamma}\right)\right)\exp\left[-\sum_{(m,h)\neq(i,k)}\exp\left(-\left(\bar{\epsilon}_t^{ik,mh} + \epsilon_t^{ik} + \bar{\gamma}\right)\right)\right] \\
&= \exp(-\epsilon_t^{ik} - \bar{\gamma})\exp\left[-\exp\left(-\epsilon_t^{ik} - \bar{\gamma}\right) - \sum_{(m,h)\neq(i,k)}\exp\left(-\left(\bar{\epsilon}_t^{ik,mh} + \epsilon_t^{ik} + \bar{\gamma}\right)\right)\right] \\
&= \exp(-\epsilon_t^{ik} - \bar{\gamma})\exp\left[-\sum_{m=1}^{N}\sum_{h=0}^{J}\exp\left(-\bar{\epsilon}_t^{ik,mh} - \epsilon_t^{ik} - \bar{\gamma}\right)\right] \qquad \text{since } \bar{\epsilon}_t^{ik,ik} = 0 \\
&= \exp(-\epsilon_t^{ik} - \bar{\gamma})\exp\left[-\exp(-\epsilon_t^{ik} - \bar{\gamma})\sum_{m=1}^{N}\sum_{h=0}^{J}\exp\left(-\bar{\epsilon}_t^{ik,mh}\right)\right]
\end{aligned}
$$

Thus,

$$
\begin{aligned}
& \Phi_t^{nj} \\
&= \sum_{i=1}^{N}\sum_{k=0}^{J}\int_{-\infty}^{\infty}\left(\beta V_{t+1}^{ik} - \tau^{nj,ik} + \nu\epsilon_t^{ik}\right) f\left(\epsilon_t^{ik}\right)\prod_{mh\neq ik} F\left(\bar{\epsilon}_t^{ik,mh} + \epsilon_t^{ik}\right) d\epsilon_t^{ik} \\
&= \sum_{i=1}^{N}\sum_{k=0}^{J}\int_{-\infty}^{\infty}\left(\beta V_{t+1}^{ik} - \tau^{nj,ik} + \nu\epsilon_t^{ik}\right)\exp(-\epsilon_t^{ik} - \bar{\gamma})\exp\left[-\exp(-\epsilon_t^{ik} - \bar{\gamma})\sum_{m=1}^{N}\sum_{h=0}^{J}\exp\left(-\bar{\epsilon}_t^{ik,mh}\right)\right] d\epsilon_t^{ik}
\end{aligned}
$$

Denote:

$$\lambda_t^{ik} \equiv \log \sum_{m=1}^{N} \sum_{h=0}^{J} \exp\left(-\bar{\epsilon}_t^{ik,mh}\right)$$

$$\zeta_t^{ik} \equiv \epsilon_t^{ik} + \bar{\gamma}$$

Break

Putting this derivation on pause here. (The below lines are directly from the paper.)

Then through a change of variables, we have:

$$\Phi_t^{nj} = \sum_{i=1}^{N} \sum_{k=0}^{J} \int_{-\infty}^{\infty} \left(\beta V_{t+1}^{ik} - \tau^{njik} + \nu\left(\zeta_t^{ik} - \bar{\gamma}\right)\right) \exp\left(-\zeta_t^{ik} - \exp\left(-\left(\zeta_t^{ik} - \lambda_t^{ik}\right)\right)\right) d\zeta_t^{ik}$$

Consider an additional change of variables using:

$$\tilde{y}_t^{ik} = \zeta_t^{ik} - \lambda_t^{ik}$$

Then we obtain:

$$\Phi_t^{nj} = \sum_{i=1}^{N} \sum_{k=0}^{J} \exp\left(-\lambda_t^{ik}\right) \left(\left(\beta V_{t+1}^{ik} - \tau^{nj,ik} + \nu\left(\lambda_t^{ik} - \bar{\gamma}\right)\right) + \nu \int_{-\infty}^{\infty} \tilde{y}_t^{ik} \exp\left(-\tilde{y}_t^{ik} - \exp\left(-\tilde{y}_t^{ik}\right)\right) d\tilde{y}_t^{ik}\right)$$