Name: HW 1.8(c) Key

1. If an investment of 1 will be tripled in 78.47 years at a constant force of interest δ, in how many years will an investment of 1 be doubled at a nominal rate of interest numerically equal to δ and convertible every four years? [1.b-f #18]

[A] At least 50, but less than 55
B) At least 35, but less than 40
C) At least 40, but less than 45
D) At least 45, but less than 50
E) At least 55, but less than 60

$e^{78.47\delta} = 3 \rightarrow \delta = 0.014$

$i^{(1/4)} = 0.014 \rightarrow j = 0.056$

$(1.056)^n = 2 \rightarrow n = 12.72$

$t = \boxed{50.88}$

2. Bruce deposits 240 into a bank account. His account is credited interest at a nominal rate of interest i convertible semiannually. At the same time, Peter deposits 240 into a separate account. Peter's account is credited interest at a force of interest δ. After 10.25 years, the value of each account is 696. Calculate $(i-\delta)$. [1.b-f #01]

[A] 0.27% B) 0.15% C) 0.19% D) 0.23% E) 0.32%

$240(1+i/2)^{20.5} = 696 \rightarrow i = 0.1066$

$240e^{10.25\delta} = 696 \rightarrow \delta = 0.1039$

$i - \delta = \boxed{0.0027}$

3. You are given a loan on which interest is charged over a 4-year period, as follows:
(i) an effective rate of discount of 11% for the first year;
(ii) a nominal rate of discount of 9% compounded every 2 years for the second year;
(iii) a nominal rate of interest of 12% compounded semiannually for the third year; and
(iv) a force of interest of 3% for the fourth year.

Calculate the annual effective rate of interest over the 4-year period. [1.b-f #13]

[A] 0.095 B) 0.100 C) 0.104 D) 0.109 E) 0.114

$(1+i)^4 = (1-0.11)^{-1}(1-0.18)^{-1/2}(1+0.06)^2 e^{0.03}$

$= (0.89)^{-1}(0.82)^{-1/2}(1.06)^2 e^{0.03}$

$i = \boxed{0.0948}$

4. Fund A accumulates at a force of interest $\frac{0.07}{1+0.07t}$ at time $t > 0$. $\rightarrow a(t) = 1+0.07t$

Fund B accumulates at a force of interest 0.04. $\rightarrow b(t) = e^{0.04t}$

You are given:

(i) The amount in Fund A at time zero is 2200.

(ii) The amount in Fund B at time zero is 800.

(iii) The amount in Fund C at any time $t > 0$ is equal to the sum of the amount in Fund A and the amount in Fund B. Fund C accumulates at a force of interest δ_t.

Calculate δ_2. [1.b-f #17]

A) 0.0559 B) 0.0587 C) 0.0615 D) 0.0643 E) 0.0671

A: $A_A(t) = 2200(1+0.07t)$

B: $A_B(t) = 800e^{0.04t}$

C: $A_C(t) = 2200(1+0.07t) + 800e^{0.04t}$

$A_C'(t) = 2200(0.07) + 32e^{0.04t}$

$A_C(2) = 3374.63$

$A_C'(2) = 188.67$

$\delta_t = \boxed{0.0559}$

5. Jill has 10,000 to invest at time $t = 0$, and two possible ways to invest it.

Investment A has a force of interest equal to $\delta_t = \frac{0.09}{1+0.09t}$ at time t. $\rightarrow a(t) = 1+0.09t$

Investment B provides a 6.84% effective annual interest rate.

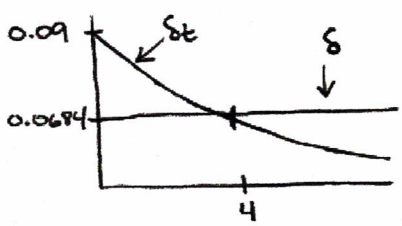
Jill can invest any portion of her principal in either investment A or B, and can transfer any portion of her money between the two investments at any time.

What is the maximum amount Jill can accumulate by time $t = 11$? [1.g #15]

A) At least 21,000, but less than 22,000
B) At least 20,000, but less than 21,000
C) At least 22,000, but less than 23,000
D) At least 23,000, but less than 24,000
E) At least 24,000, but less than 25,000

$i = 6.84\% \rightarrow \delta = \ln(1.0684) = 0.0661622$

$\delta_t = \delta \rightarrow \frac{0.09}{1+0.09t} = 0.0661622 \rightarrow t = 4$



$10{,}000\, a(4)(1.0684)^7 = 10{,}000\,(1.36)(1.0684)^7$

$= \boxed{\$21{,}611.06}$