HW 4.1 (c) Key

(This space for rent.)

1. You are given the following information on a bond:
   (i) Par value = 1000.
   (ii) Redemption value = 1000
   (iii) Coupon rate = 10%, convertible semiannually. → $r = 5\%$
   (iv) It is priced to yield 6%, convertible semiannually. → $i = 3\%$
   The bond has a term of n years. If the term of the bond is doubled, the price will increase by 80. Calculate the price of the n-year bond. [7.a-b #31]

   [A] 1090 B) 950 C) 990 D) 1040 E) 1140

(n-year) $P = 50 a_{\overline{n}|3\%} + 1000 v^n$

($2n$-year) $P + 80 = 50 a_{\overline{2n}|3\%} + 1000 v^{2n}$

$50 a_{\overline{n}|3\%} + 1000 v^n + 80 = 50 a_{\overline{2n}|3\%} + 1000 v^{2n}$ (×0.03)

$50(1 - v^n) + 30 v^n + 2.4 = 50(1 - v^{2n}) + 30 v^{2n}$

$20 v^{2n} - 20 v^n + 2.4 = 0 \rightarrow v^n = 0.8606$

$P = 50 \frac{1 - 0.8606}{0.03} + 1000(0.8606) = \boxed{1092.96}$

$r = 1.75$

2. Smith purchases a 1000 par value, 15 year, 3.5% bond with semiannual coupons. Smith pays a price of P_1. After 10 years, following coupon number 20, Smith sells the bond to Jones at a price of P_2. Jones retains the bond to maturity. The yield rate on Smith's investment is 6.5% convertible semiannually. The yield rate on Jone's investment is 5% convertible semiannually. What is P_1? [7.a-b #33]

A) At least \$740, but less than \$750
B) At least \$720, but less than \$730
C) At least \$730, but less than \$740
D) At least \$750, but less than \$760
E) At least \$760, but less than \$770

Smith: $P_1 = 17.5\, a_{\overline{20}|3.25\%} + P_2 (1.0325)^{-20} = \boxed{747.29}$

Jones: $P_2 = 17.5\, a_{\overline{10}|2.5\%} + 1000 (1.025)^{-10} = 934.36$

3. A 6% 100 bond matures in 11 years. It bears semiannual coupons and is purchased for 88.90 to yield $i^{(2)}$. A 9% 100 bond also matures in 11 years. It also bears semiannual coupons, but is purchased for 111.10 to yield $i^{(2)}$. What is $i^{(2)}$? [7.a-b #34]

A) At least 7.2%, but less than 7.6%
B) At least 6%, but less than 6.4%
C) At least 6.4%, but less than 6.8%
D) At least 6.8%, but less than 7.2%
E) At least 7.6%, but less than 8%

Bond 1: $88.90 = 3\, a_{\overline{22}|i} + 100 v^{22}$

Bond 2: $111.10 = 4.5\, a_{\overline{22}|i} + 100 v^{22}$

$22.2 = 1.5\, a_{\overline{22}|i} \rightarrow i = 3.7519\% \rightarrow i^{(2)} = \boxed{7.504\%}$

4. A 1000 par value bond with 5% coupons payable semiannually is purchased for 1088. The yield to the purchaser is 4%, convertible semiannually. If the same bond were redeemable at 125% of par, what price would have been paid to obtain the same yield? [7.a-b #37]

A) 1250 B) 1305 C) 1360 D) 1420 E) 1475

$1088 = 25\, a_{\overline{n}|2\%} + 1000 v^n \xrightarrow{\text{BA II}} n = 22$

$P = 25\, a_{\overline{n}|2\%} + 1250 v^n = \boxed{1250}$

5. Two 20-year bonds with 100 redemption values are each purchaed to yield an effective annual interest rate of 8%. The first bond bears annual g% coupons and is purchased at a premium of 19.64. The second bond bears annual $(g + 0.015)$% coupons. Find the purchase price of the second bond. [7.a-b #39]

A) 134 B) 128 C) 140 D) 146 E) 153

①: $119.64 = 100g\, a_{\overline{20}|8\%} + 100 v^{20}$

(Using TVM solver) PMT = $-10 \rightarrow g = 10\%$

②: $P = 100(0.1015)\, a_{\overline{20}|8\%} + 100 v^{20}$

$= \boxed{134.36}$