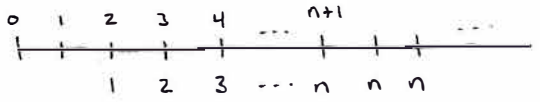
HW 2.6 (c) Key

1. A perpetuity costs 169.52 and makes annual payments at the end of the year. The perpetuity pays 1 at the end of year 2, 2 at the end of year 3, ... , n at the end of year $(n+1)$. After year $(n+1)$, the payments remain constant at n. The annual effective interest rate is 6.5%. Calculate n. [4.h-i #01]

A) 20 B) 21 C) 22 D) 23 E) 24



$i = 6.5\%$

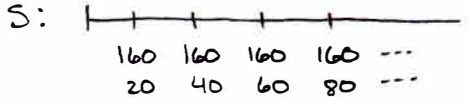
$$169.52 = \frac{\ddot{a}_{\overline{n}|}}{i} v \quad \rightarrow \quad 169.52 = \frac{a_{\overline{n}|}}{i} \quad \rightarrow \quad a_{\overline{n}|} = 11.0188$$

$\boxed{n = 20}$

2. Sandy purchases a perpetuity-immediate that makes annual payments. The first payment is 180, and each payment thereafter increases by 20. Danny purchases a perpetuity-due which makes annual payments of 500. Using the same effective interest rate, $i > 0$, the present value of both perpetuities are equal. Calculate i. [4.h-i #03]

A) 5.7% B) 5.2% C) 5.4% D) 6% E) 6.3%

S:



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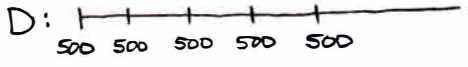


$$160 a_{\overline{\infty}|} + 20 (Ia)_{\overline{\infty}|} = 500 \ddot{a}_{\overline{\infty}|}$$

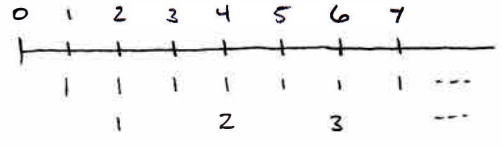
$$\frac{160}{i} + 20\left(\frac{1}{i} + \frac{1}{i^2}\right) = 500\left(1 + \frac{1}{i}\right)$$

$$160i + 20(i+1) = 500(i^2 + i)$$

$$500i^2 + 320i - 20 = 0 \qquad \boxed{i = 5.74\%}$$

3. A perpetuity has payments of 1, 2, 1, 3, 1, 4, 1, 5, ... Payments are made at the end of each year. You may assume an annual effective interest rate of 7%. Determine the present value of this perpetuity. [4.h-i #13]

A) At least 60, but less than 70
B) At least 20, but less than 30
C) At least 30, but less than 40
D) At least 40, but less than 50
E) At least 50, but less than 60



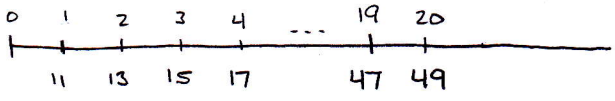
$i = 7\%$

$j = 14.49\%$ (2 year)

$$PV = a_{\overline{\infty}|i} + (Ia)_{\overline{\infty}|j} = \frac{1}{i} + \left(\frac{1}{j} + \frac{1}{j^2}\right) = \boxed{68.82}$$

4. A perpetuity immediate has annual payments of 11, 13, 15, 17, 19, ... The present value of payment number 19 equals the present value of payment number 20. Determine the present value of the perpetuity. [4.h-i #14]

A) At least 1,360, but less than 1,370
B) At least 1,350, but less than 1,360
C) At least 1,370, but less than 1,380
D) At least 1,380, but less than 1,390
E) At least 1,390, but less than 1,400



$47v^{19} = 49v^{20} \rightarrow 1+i = 1.04255 \rightarrow i = 0.04255$

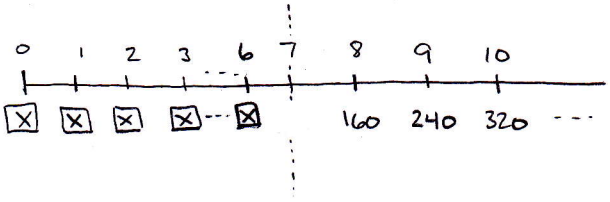
$PV = 9a_{\overline{\infty}|} + 2(Ia)_{\overline{\infty}|}$

$= 9\left(\frac{1}{i}\right) + 2\left(\frac{1}{i} + \frac{1}{i^2}\right)$

$= \boxed{1363}$

5. John will make 7 deposits of X each, at the beginning of each of the next 7 years, into a fund that earns a nominal rate of discount of 12% convertible semiannually. The fund must provide for a perpetuity of 160 beginning at the end of year 8 and increasing by 80 each year. Determine the smallest amount that X can be. [4.h-i #18]

A) At least 475, but less than 500
B) At least 400, but less than 425
C) At least 425, but less than 450
D) At least 450, but less than 475
E) At least 500, but less than 525



$d^{(2)} = 12\%$

$i = 13.1733\%$

$X\ddot{s}_{\overline{7}|} = 80a_{\overline{\infty}|} + 80(Ia)_{\overline{\infty}|}$

$X(1+i)\frac{(1+i)^7 - 1}{i} = 80\left(\frac{1}{i}\right) + 80\left(\frac{1}{i} + \frac{1}{i^2}\right)$

$\boxed{X = 492.00}$