

HW 3.1 (a) Key

1. Sally wishes to purchase a stereo system. She is offered the following payment options:

Option 1: \$0 down, \$479 in 1 year, \$248 in 2 years

Option 2: \$101.79 down, \$257.72 in 1 year, \$368 in 2 years

Determine the range of interest rate for which the present value of Option 1 is less than the present value of Option 2. [5.a-c #17]

- ☒ A) Less than 3.6% or more than 13.7% D) More than 4% but less than 15%
☐ B) More than 3.6% but less than 13.7% E) Less than 4.3% or more than 16.2%
☐ C) Less than 4% or more than 15%

$$479v + 248v^2 < 101.79 + 257.72v + 368v^2$$

$$479(1+i) + 248 < 101.79(1+i)^2 + 257.72(1+i) + 368$$

$$479i + 727 < 101.79i^2 + 461.3i + 727.51$$

$$101.79i^2 - 17.7i + 0.51 > 0$$

$$i < 3.65\% \quad i > 13.74\%$$



2. Bill deposits 500 into a fund for 22 years. The fund pays interest at the end of each 6-month period at a nominal rate of i convertible semiannually. The interest payments are reinvested in a separate fund earning interest at an annual effective rate of 7%. During the 22-year period, Bill earns an annual effective yield of 8.9%. Calculate i . [5.a-c #13]

- ☒ A) 11.08% B) 10.09% C) 10.42% D) 10.75% E) 11.42%

| | 0 | 1 | | | | 21 | 22 |
|---------------------|-----|------|------|------|------|------|------|
| Balance (B_t): | 500 | 500 | 500 | 500 | 500 | 500 | 500 |
| Interest (I_t): | | 500j | 500j | 500j | 500j | 500j | 500j |
| | | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| Payments (P_t): | | 500j | 500j | 500j | 500j | 500j | 500j |

$$r = 7\% \text{ (annual)}$$

$$k = 3.4408\% \text{ (semi-ann)}$$

$$500(1.089)^{22} = 500 + 500j S_{\overline{44}|k}$$

$$j = 5.5422\%$$

$$i = 11.08\%$$

3. John invests a total of 8,800. He purchases an annuity with payments of 400 at the beginning of each year for 22 years at an effective annual interest rate of 9%. As annuity payments are received, they are reinvested at an effective annual interest rate of 6%. The balance of the 8,800 is invested in a 22-year certificate of deposit with a nominal annual interest rate of 10%, compounded quarterly. Calculate the annual effective yield rate on the entire 8,800 investment over the 22-year period. [5.a-c #16]

☒ A) 9.08% B) 8.53% C) 8.81% D) 9.35% E) 9.62%

$$\text{Price of Annuity: } PV = 400 \ddot{a}_{\overline{22}|9\%} = 4116.90$$

$$\text{Amount invested in CD: } 4683.10$$

Annuity payments reinvested at 6%.

$$8800(1+i)^{22} = 4683.10(1.025)^{88} + 400 \ddot{s}_{\overline{22}|6\%}$$

$$i = 9.0789\%$$

4. 3000 is deposited into Fund X, which earns an annual effective rate of 4%. At the end of each year, the interest earned plus an additional 200 is withdrawn from the fund. At the end of year 15, the fund is depleted. The annual withdrawals of interest and principal are deposited into Fund Y, which earns an annual effective rate of 8%. Determine that accumulated value of Fund Y at the end of year 15. [5.a-c #01]

☒ A) 7,473 B) 5,680 C) 6,577 D) 8,370 E) 9,267

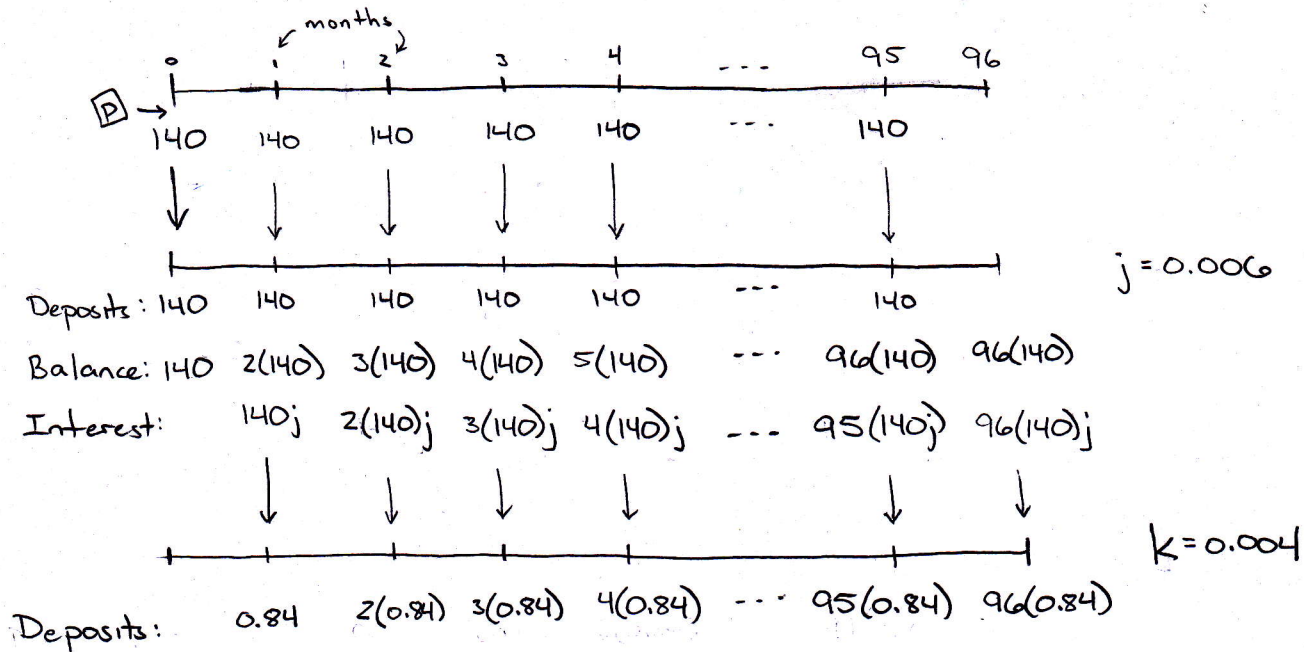
| | | | | | | | | |
|------------------------|------|-------------|-------------|-------------|-----|------------|-----------|-----------|
| | 0 | 1 | 2 | 3 | ... | 14 | 15 | |
| X: Balance (B_t): | 3000 | 2800 | 2600 | 2400 | ... | 200 | 0 | $i = 4\%$ |
| Interest (I_t): | | 120 | 112 | 104 | ... | 16 | 8 | |
| | | ↓ | ↓ | ↓ | | ↓ | ↓ | |
| Y: Deposits (R_t): | | 200 +120 | 200 +112 | 200 +104 | ... | 200 +16 | 200 +8 | $i = 8\%$ |

$$AV = 200 s_{\overline{15}|8\%} + 8 (Ds)_{\overline{15}|8\%}$$

$$= \boxed{7473.47}$$

5. An investor pays P for an annuity which provides payments of 140 at the beginning of each month for 8 years. These payments are invested at a nominal annual interest rate of 7.2% convertible monthly. Monthly interest payments are reinvested at a nominal annual interest rate of 4.8% convertible monthly. The annual yield rate over the 8-year period is 10% effective. Calculate P . [5.a-c #14]

(A) 8,350 B) 7,680 C) 7,850 D) 8,010 E) 8,180



$$P(1.1)^8 = 96(140) + 0.84(I\ddot{s})_{\overline{96}|k}$$

$$P(1.1)^8 = 17,896.69$$

$$P = \boxed{8348.94}$$