HW 4.1 (e) Key

(This space for rent.)

1. An n-year 1000 par value bond with 7.1% annual coupons is purchased at a price to yield an annual effective rate of i. You are given:
   (i) If the annual coupon rate had been 9.3% instead of 7.1%, the price of the bond would have increased by 140.
   (ii) At the time of purchase, the present value of all the coupon payments is equal to the present value of the bond's redemption value of 1000.
   Calculate i. [7.a-b #10]

$\boxed{\text{A)}}$ 8.6% B) 8.4% C) 8.9% D) 9.1% E) 9.4%

$P = 71\, a_{\overline{n}|i} + 1000v^n$

(i) $\rightarrow$ $P + 140 = 93\, a_{\overline{n}|i} + 1000v^n$

$\rightarrow 140 = 22\, a_{\overline{n}|i} \rightarrow a_{\overline{n}|i} = 6.3636$

(ii) $\rightarrow$ $71\, a_{\overline{n}|i} = 1000v^n$

$\rightarrow 71(6.3636) = 1000\, v^n$

$v^n = 0.451818$

$a_{\overline{n}|i} = \frac{1-v^n}{i} \rightarrow 6.3636 = \frac{1-0.451818}{i}$

$\boxed{i = 0.0861}$

2. A 1000 bond with annual coupons is redeemable at par at the end of 20 years. At a purchase price of 744.73, the yield rate is i. The coupon rate is i - 0.026. Calculate i. [7.a-b#11]

A) 8% B) 7.4% C) 7.7% D) 8.3% E) 8.6%

Plugging in answers works well on this prob. The "table" function makes this go quickly.

$$744.73 = 1000(i-0.026)a_{\overline{20}|i} + 1000v^{20}$$

$$744.73 = (1000i - 26)a_{\overline{20}|i} + 1000v^{20}$$

$$744.73 = 1000(1-v^{20}) - 26a_{\overline{20}|i} + 1000v^{20}$$

$$744.73 = 1000 - 26a_{\overline{20}|i} \rightarrow 26a_{\overline{20}|i} = 255.27 \rightarrow \boxed{i = 8\%}$$

3. An n-year zero coupon bond with par value of 1000 was purchased for 460. An n-year 1000 par value bond with semiannual coupons of X was purchased for 680. A $3n$-year 1000 par value bond with semiannual coupons of X was purchased for P. All three bonds have the same yield rate. Calculate P. [7.a-b #12]

$\boxed{\text{A)}}$ 465 B) 458 C) 472 D) 479 E) 486

$$460 = 1000v^{2n} \rightarrow v^{2n} = 0.46$$

$$680 = Xa_{\overline{2n}|} + 1000v^{2n} \rightarrow Xa_{\overline{2n}|} = 220$$

$$P = Xa_{\overline{6n}|} + 1000v^{6n} \rightarrow Xa_{\overline{6n}|} = P - 97.336$$

$$\frac{P-97.336}{220} = \frac{a_{\overline{6n}|}}{a_{\overline{2n}|}} = 1+v^{2n}+v^{4n}$$

$$\frac{P-97.336}{220} = 1.6716$$

$$\boxed{P = 465}$$

4. Bart buys a 32-year bond with a par value of 1400 and annual coupons. The bond is redeemable at par. Bart pays 1923 for the bond, assuming an annual effective yield of i. The coupon rate on the bond is twice the yield rate. At the end of 8 years, Bart sells the bond for P, which produces the same annual effective yield rate of i to the new buyer. Calculate P. [7.a-b#14]

A) 1814 B) 1734 C) 1774 D) 1854 E) 1894

Original Terms: $1923 = 1400(2i)a_{\overline{32}|i} + 1400v^{32}$

$1923 = 2800(1-v^{32}) + 1400v^{32}$

$v^{32} = 0.6264 \rightarrow i = 1.4724\%$

Buyer: $P = 1400(2i)a_{\overline{24}|i} + 1400v^{24} = \boxed{1814.22}$

5. A 1000 par value 4-year bond has annual coupons of 20 for the first year, 40 for the second year, 55 for the third year, and 80 for the fourth year. The bond was purchased to yield a force of interest:

$$\delta_t = \frac{0.5t}{0.5t^2+1} \text{ for } t \geq 0.$$

Calculate the price of this bond. [7.a-b #19]

A) 423 B) 414 C) 432 D) 441 E) 451

$$\int_0^t \delta_r\,dr = \frac{1}{2}\int_0^t \frac{(2)0.5r}{0.05r^2+1}\,dr = \frac{1}{2}\ln(0.5r^2+1)\Big|_0^t = \frac{1}{2}\ln(0.5t^2+1)$$

$$a(t) = e^{\frac{1}{2}\ln(0.5t^2+1)} = \sqrt{0.5t^2+1}$$

$$P = \frac{20}{a(1)} + \frac{40}{a(2)} + \frac{55}{a(3)} + \frac{1080}{a(4)} = \boxed{422.88}$$