

HW 6.1 Key

1. A \$100 par value bond with 6.5% annual coupons and maturing at par in 6 years sells at a price to yield 4%. Determine the modified duration of the bond. [9.c-h #02]

[A] 5.01 B) 4.96 C) 5.06 D) 5.11 E) 5.16 $i = 4\%$

$$MacD = \frac{6.5v + 6.5(2)v^2 + 6.5(3)v^3 + \dots + 6.5(6)v^6 + 100(6)v^6}{6.5v + 6.5v^2 + 6.5v^3 + \dots + 6.5v^6 + 100v^6}$$

$$\frac{6.5(Ia)_{\overline{6}|} + 600v^6}{6.5a_{\overline{6}|} + 100v^6} = 5.2124$$

$$ModD = v MacD = \boxed{5.0119}$$

2. On December 31, 2013, your company estimates that it will pay a total of \$10,000,000 in 2014 and subsequent years on health insurance claims that were incurred in 2013 and prior years. It expects the payout pattern to be as follows:

Year	2013	2014	2015	2016	2017
Cumulative Payout Pattern	0	50%	60%	75%	100%
		50%	10%	15%	25%

Determine the duration of the claim payments assuming that they are paid at the end of each calendar year and that the effective rate of interest is 8%. [9.c-h #06]

[A] 2.028 B) 1.866 C) 1.906 D) 1.947 E) 1.987 $i = 8\%$

$$MacD = \frac{0.5v + 0.1(2)v^2 + 0.15(3)v^3 + 0.25(4)v^4}{0.5v + 0.1v^2 + 0.15v^3 + 0.25v^4} = \boxed{2.0277}$$

3. A perpetuity-immediate with level payments has a duration of 25.25 years at an effective rate of interest of i . Determine i . [9.c-h #08]

[A] 4.12% B) 4.06% C) 4.19% D) 4.25% E) 4.31%

$$25.25 = \frac{v + 2v^2 + 3v^3 + 4v^4 + \dots}{v + v^2 + v^3 + v^4 + \dots} \quad \frac{(Ia)_{\infty}}{a_{\infty}}$$

$$25.25 = \frac{\frac{1}{i} + \frac{1}{i^2}}{\frac{1}{i}} \quad 1 + \frac{1}{i} \quad i = \boxed{0.041237}$$

4. An annuity-immediate has level annual payments for n years. The average time of the payments using the method of equated time is 8 years. Determine the duration of the payments if the effective rate of interest is 4.5%. [9.c-h #09]

[A] 7.18 B) 6.61 C) 6.9 D) 7.47 E) 7.76

$$i = 4.5\%$$

$$8 = \frac{1+2+3+\dots+n}{n} = \frac{1}{n} \left[\frac{n(n+1)}{2} \right] \rightarrow n = 15$$

$$MacD = \frac{v + 2v^2 + 3v^3 + \dots + 15v^{15}}{v + v^2 + v^3 + \dots + v^{15}} = \frac{(Ia)_{\overline{15}|}}{a_{\overline{15}|}} = \boxed{7.1843}$$

5. At an annual effective interest rate of 6%, the following assets have the same duration:

1. A 7-year bond paying annual coupons of $r\%$ and redeemable at par
2. A 13-year annuity with level annual payments.

$$i = 0.06$$

Find r . [09c_A1]

[A] 3.97% B) 3.79% C) 3.85% D) 3.91% E) 4.03%

$$\begin{array}{c} \text{(Bond)} \\ \frac{rv + 2rv^2 + 3rv^3 + \dots + 7rv^7 + 7v^7}{rv + rv^2 + rv^3 + \dots + rv^7 + v^7} = \frac{\text{(Annuity)}}{\frac{v + 2v^2 + 3v^3 + \dots + 13v^{13}}{v + v^2 + v^3 + \dots + v^{13}}} \end{array}$$

$$\frac{r(Ia)_{\overline{7}|} + 7v^7}{ra_{\overline{7}|} + v^7} = \frac{(Ia)_{\overline{13}|}}{a_{\overline{13}|}}$$

$$\frac{21.0321r + 4.6554}{5.5824r + 0.6651} = 6.1920$$

$$21.0321r + 4.6554 = 34.5660r + 4.1180$$

$$r = \boxed{3.9706\%}$$