Section 2-1: Reference Frames

- Every measurement must be made with respect to a **reference frame**. Usually, speed is relative to the Earth.

FIGURE 2–2 A person walks toward the front of a train at 5 km/h. The train is moving 80 km/h with respect to the ground, so the walking person's speed, relative to the ground, is 85 km/h.

- ***Specifically***, if a person walks towards the front of a train at **5 km/h** (with respect to the train floor) & the train is moving **80 km/h** with respect to the ground. The person's speed, relative to the ground is **85 km/h**.

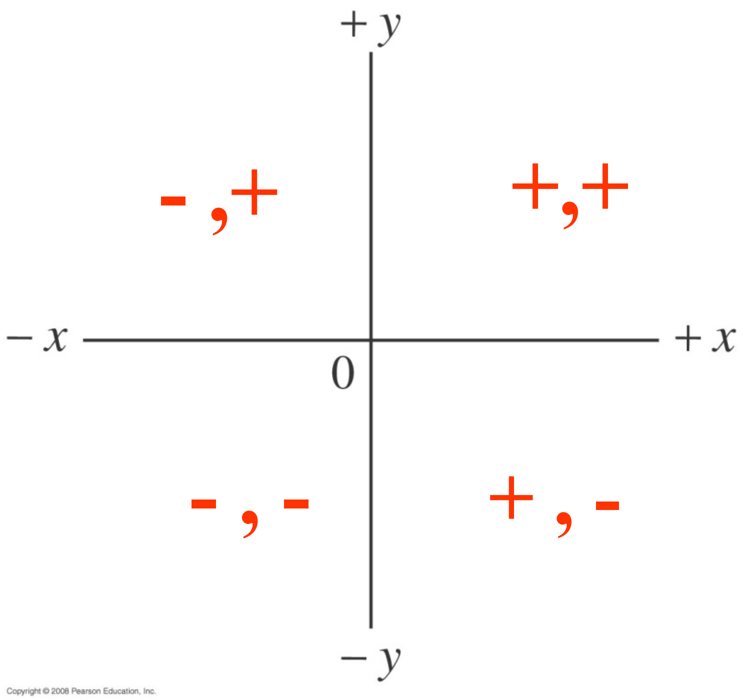
- When specifying speed, always specify the frame of reference unless its obvious (“with respect to the Earth”).
- Distances are also measured in a reference frame.

- When specifying speed or distance, we also need to specify **DIRECTION**.

Coordinate Axes

- Define a reference frame using a standard coordinate axes.
- 2 Dimensions **(x,y)**

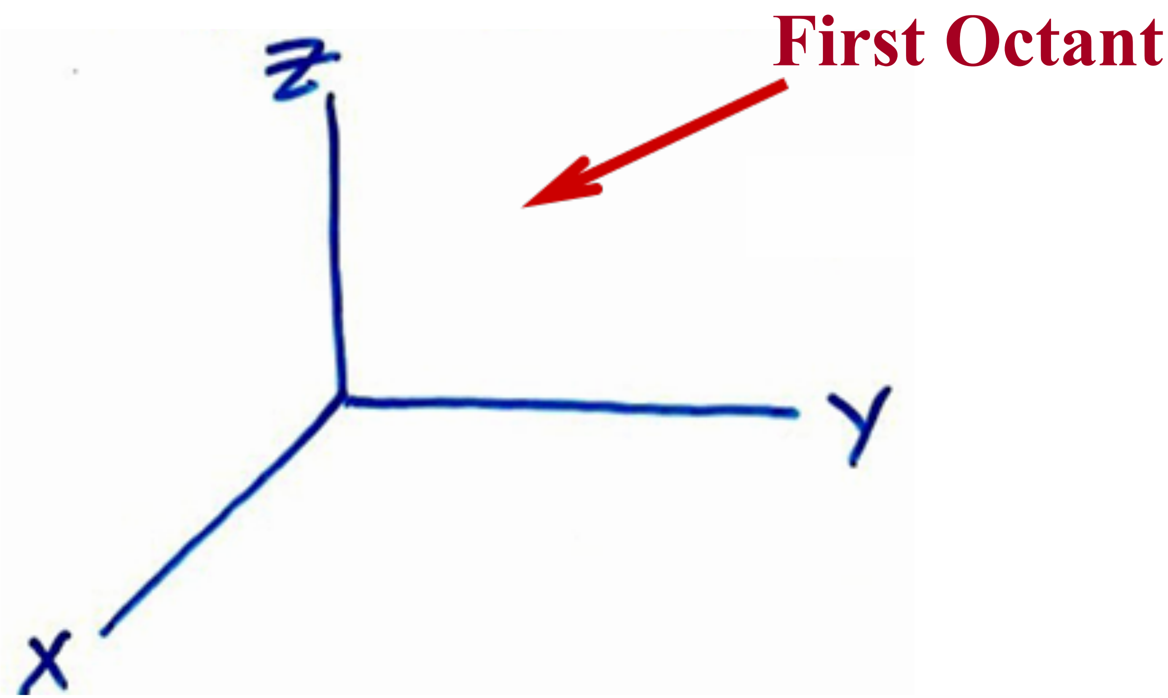
- Note, if its convenient, we could reverse + & - !



Standard set of xy coordinate axes

Coordinate Axes

- 3 Dimensions **(x,y,z)**

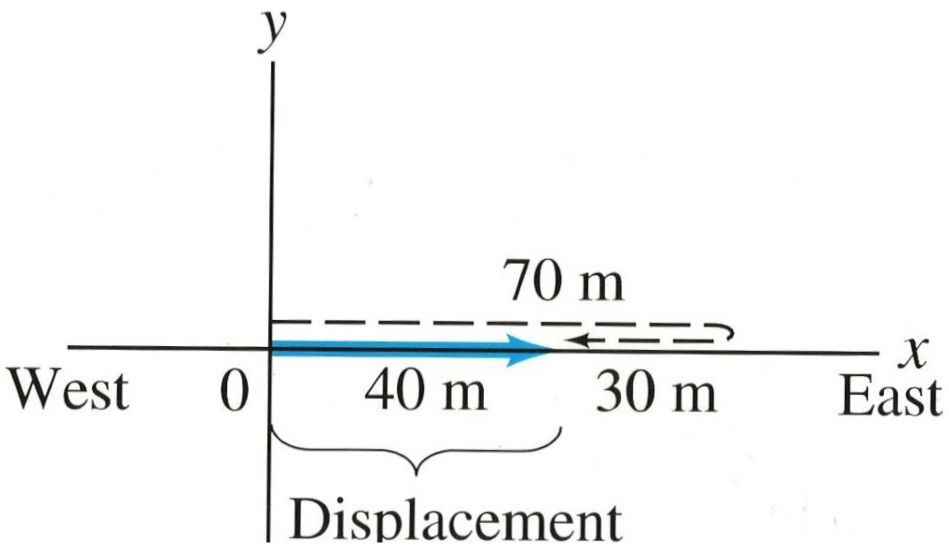


- Define direction using these.

Displacement & Distance

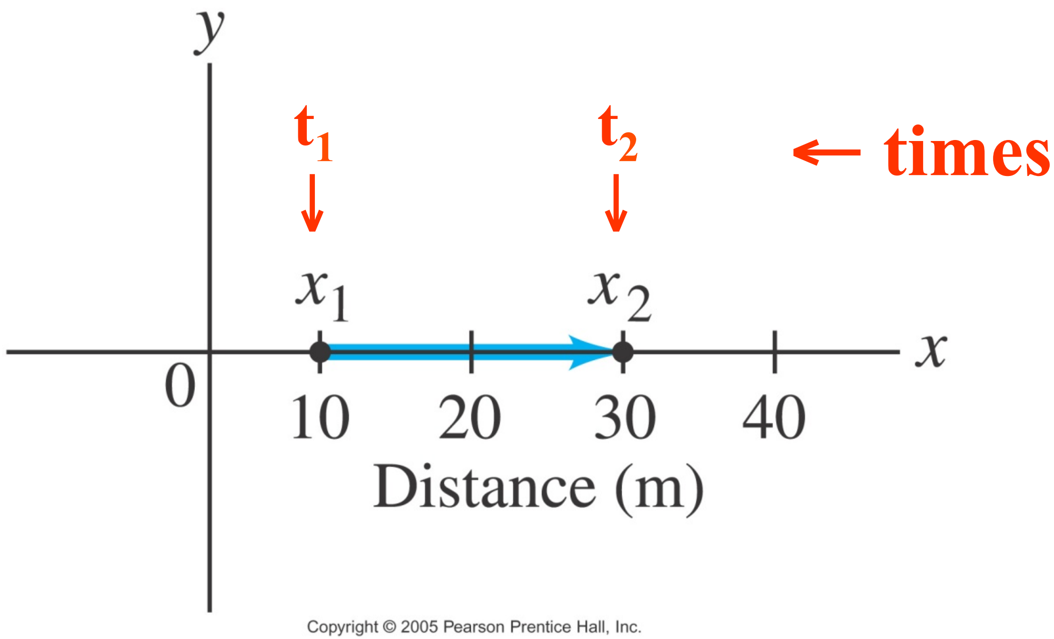
- Distance traveled by an object

  ≠ displacement of the object!



- Displacement = change in position of object.
- Displacement is a **vector** (magnitude & direction). Distance is a **scalar** (magnitude).
- Figure: distance = 100 m, displacement = 40 m East

Displacement



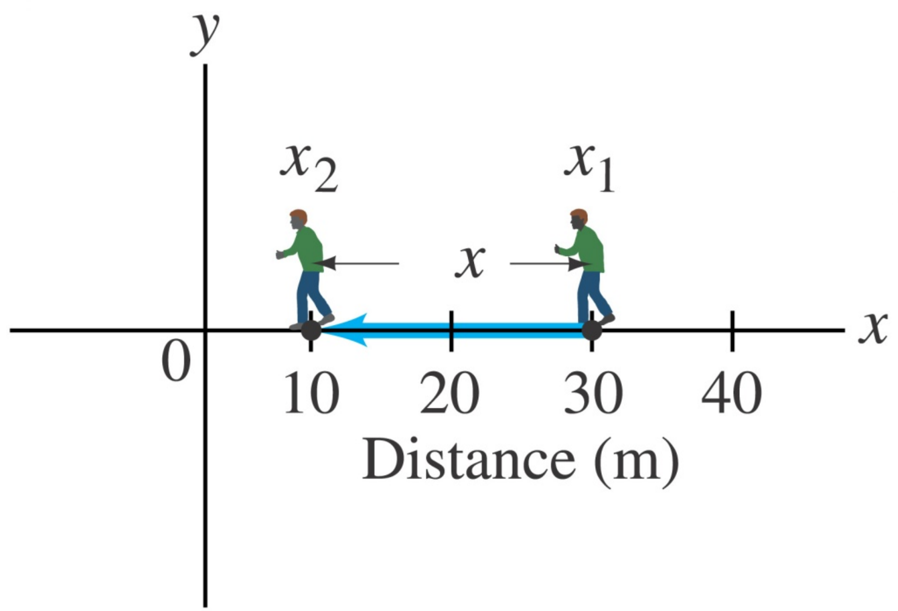
The arrow represents the displacement (in meters).

x_1 = 10 m, x_2 = 30 m

Displacement ≡ $\Delta x = x_2 - x_1$ = 20 m

Δ ≡ Greek letter "delta" meaning "change in"

FIGURE 2–6 For the displacement $\Delta x = x_2 - x_1 = 10.0\ \mathrm{m} - 30.0\ \mathrm{m}$, the displacement vector points to the left.



x_1 = 30 m, x_2 = 10 m

Displacement $\equiv \Delta x = x_2 - x_1 = -20$ m

Displacement is a **VECTOR**

Vectors and Scalars

- Many quantities in physics, like displacement, have a ***magnitude and a direction***. Such quantities are called ***VECTORS***.
  - Other quantities which are vectors: velocity, acceleration, force, momentum, ...
- Many quantities in physics, like distance, have a ***magnitude only***. Such quantities are called ***SCALARS***.
  - Other quantities which are scalars: speed, temperature, mass, volume, ...

- I usually denote vectors with arrows over the symbol.

$$\vec{v}$$

- In one dimension, we can drop the arrow and remember that a + sign means the vector points to right & a minus sign means the vector points to left.

Figure 1
The motion of a commuter train traveling along a straight route is an example of one-dimensional motion. Each train can move only forward and backward along the track.

Sect. 2-2: Average Velocity

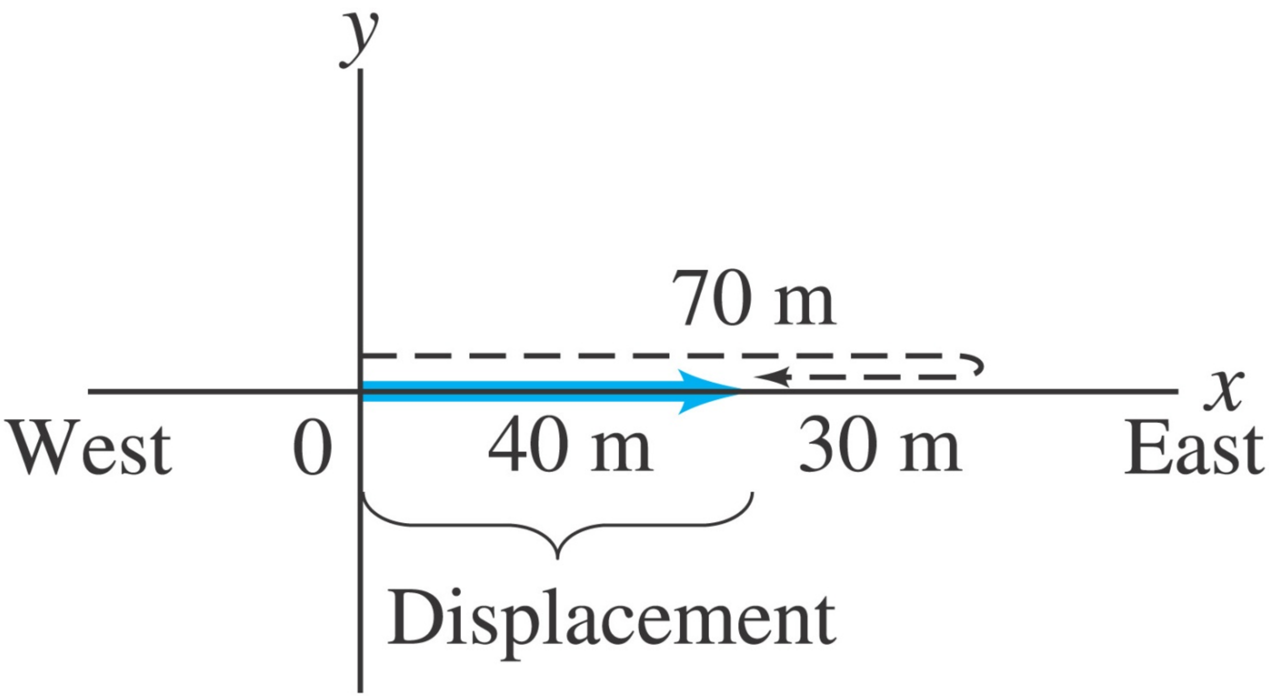
Scalar→ Average *Speed* ≡ (Distance traveled)/(Time taken)

Vector→ Average *Velocity* ≡ (Displacement)/(Time taken)

- **Velocity:** Both magnitude & direction describing how fast an object is moving. A ***VECTOR***. (Similar to displacement).
- **Speed:** Magnitude only describing how fast an object is moving. A ***SCALAR***. (Similar to distance).
- **Units: distance/time = m/s**

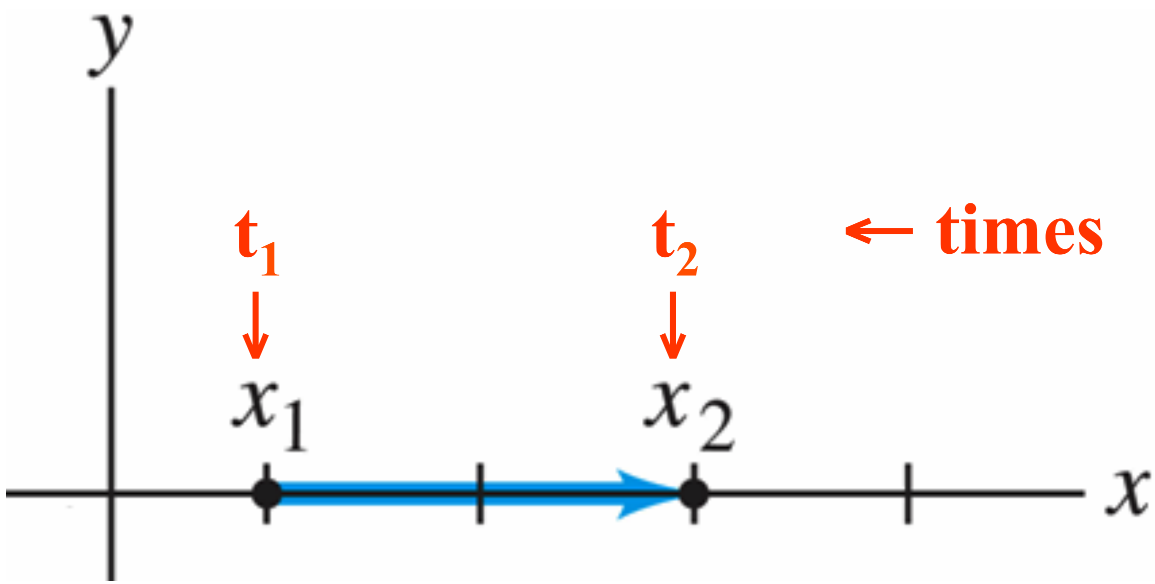
Average Velocity, Average Speed

- Displacement from before. Walk for 70 s.



- Average Speed = (100 m)/(70 s) = 1.4 m/s
- Average velocity = (40 m)/(70 s) = 0.57 m/s

- In general:



$\Delta x = x_2 - x_1$ = displacement

$\Delta t = t_2 - t_1$ = elapsed time

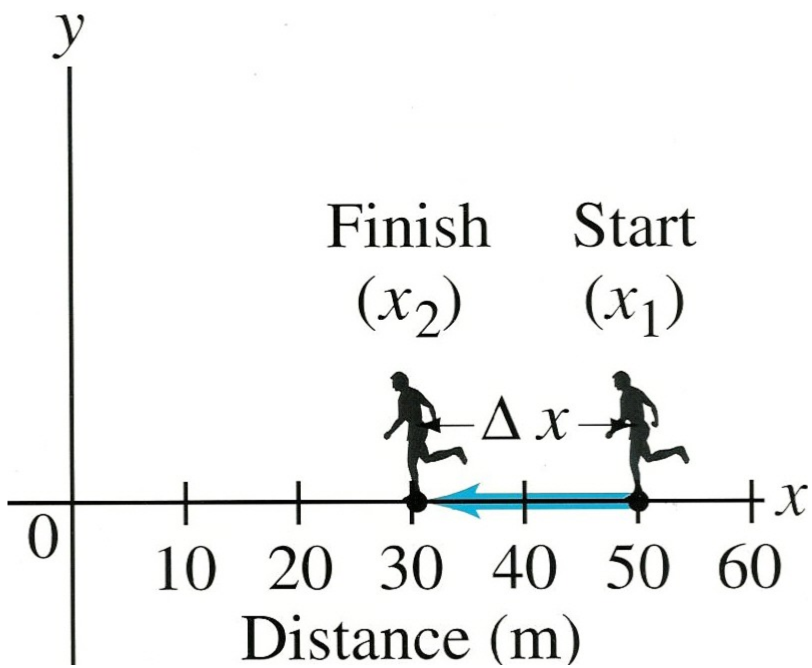
Average Velocity:

$$\bar{v} = \frac{\Delta x}{\Delta t} = (x_2 - x_1)/(t_2 - t_1)$$

Bar denotes average

Example 2-1

- Person runs from $\mathbf{x_1 = 50.0\ m}$ to $\mathbf{x_2 = 30.5\ m}$ in $\mathbf{\Delta t = 3.0\ s}$. $\mathbf{\Delta x = -19.5\ m}$



Average velocity = $\mathbf{\bar{v} = (\Delta x)/(\Delta t)}$

$\mathbf{= -(19.5\ m)/(3.0\ s) = -6.5\ m/s.}$ Negative sign indicates **DIRECTION**, (negative **x** direction)

Sect. 2-3: Instantaneous Velocity

- **Instantaneous velocity** ≡ velocity at any instant of time ≡ average velocity for an infinitesimally short time
- Mathematically, instantaneous velocity:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$\lim_{\Delta t \to 0}$ ratio $\frac{\Delta x}{\Delta t}$ considered as a whole for smaller & smaller **Δt**.

Mathematicians call this a derivative.

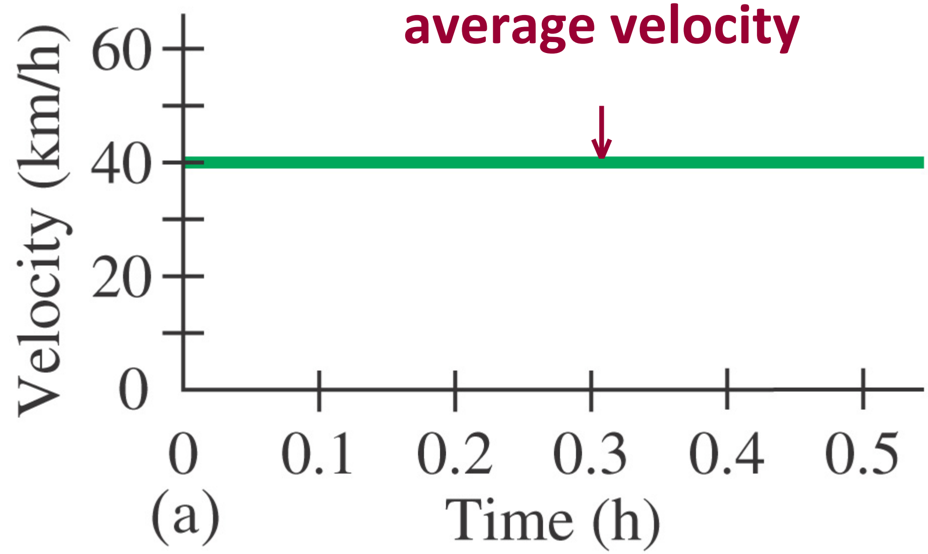
Do not set **Δt = 0** because **Δx = 0** then & **0/0** is undefined!

⇒ **Instantaneous velocity**

These graphs show

(a) constant velocity →

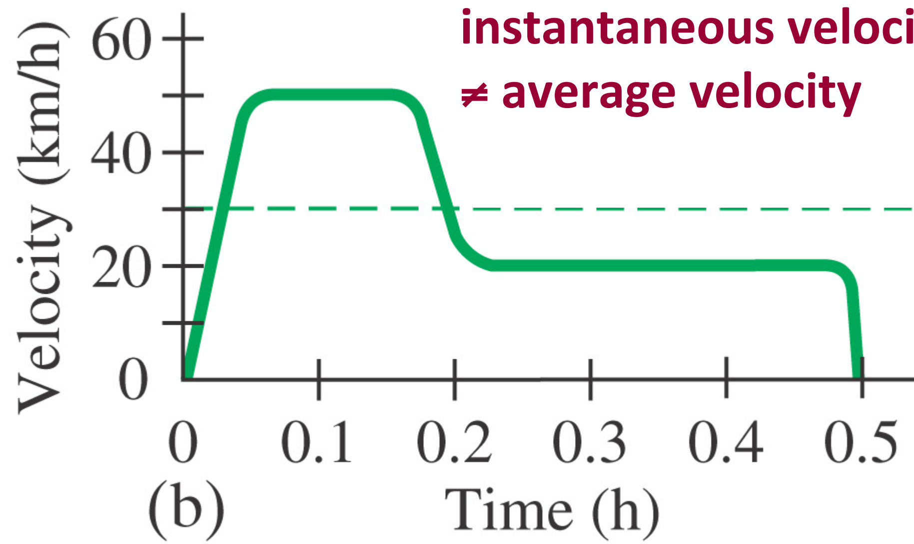
instantaneous velocity = average velocity



and

(b) varying velocity →

instantaneous velocity ≠ average velocity



The instantaneous velocity is the average velocity in the limit as the time interval becomes infinitesimally short.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

FIGURE 2–8 Car speedometer showing mi/h in white, and km/h in orange.

Ideally, a speedometer would measure instantaneous velocity; in fact, it measures average velocity, but over a very short time interval.

Sect. 2-4: Acceleration

- Velocity can change with time. An object with velocity that is changing with time is said to be ***accelerating.***
- Definition: **Average acceleration** = ratio of change in velocity to elapsed time.

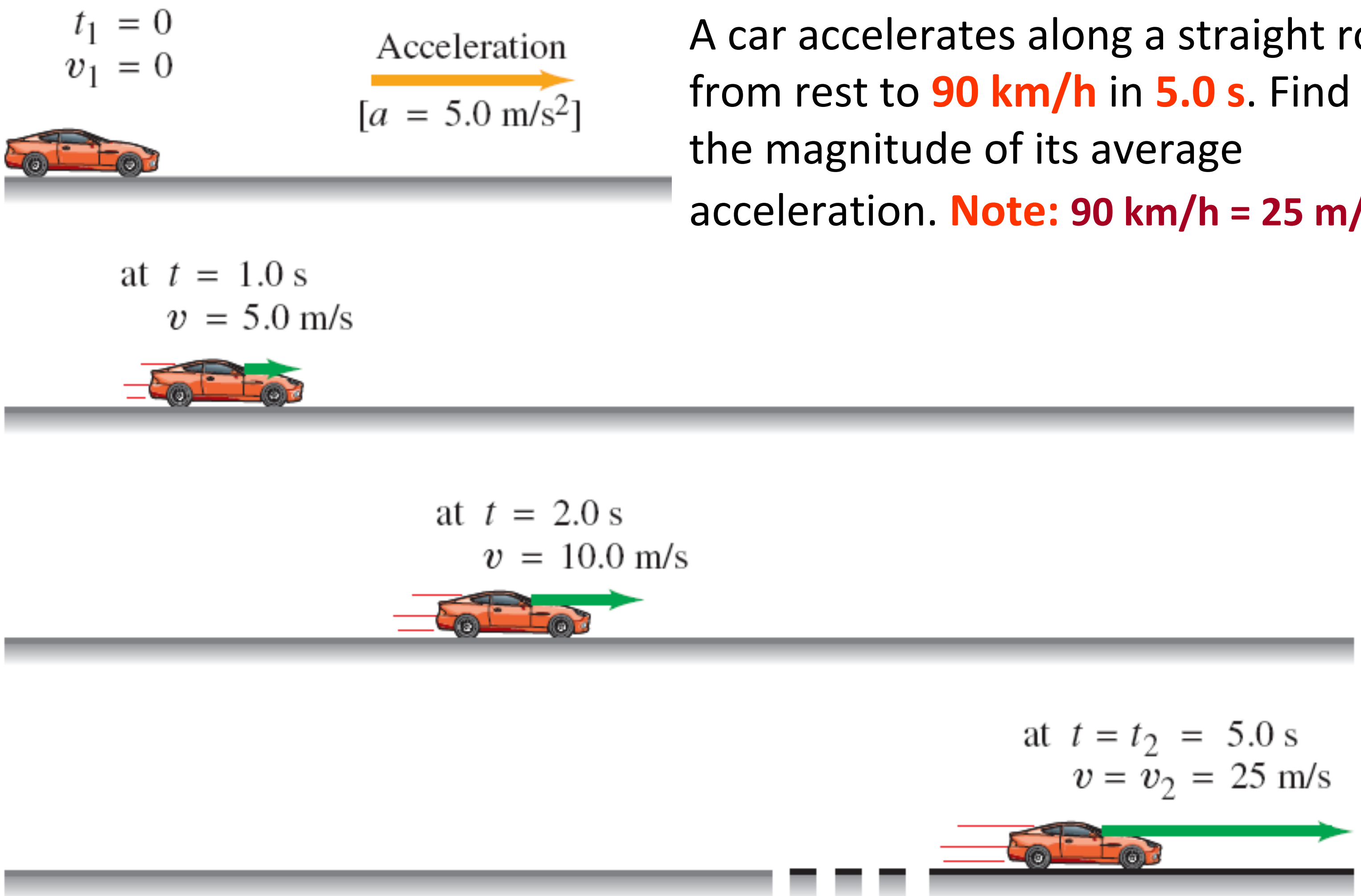
  $$\mathbf{a} \equiv \frac{\Delta v}{\Delta t} = (v_2 - v_1)/(t_2 - t_1)$$

  – Acceleration is a **vector**.
- **Instantaneous acceleration**

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

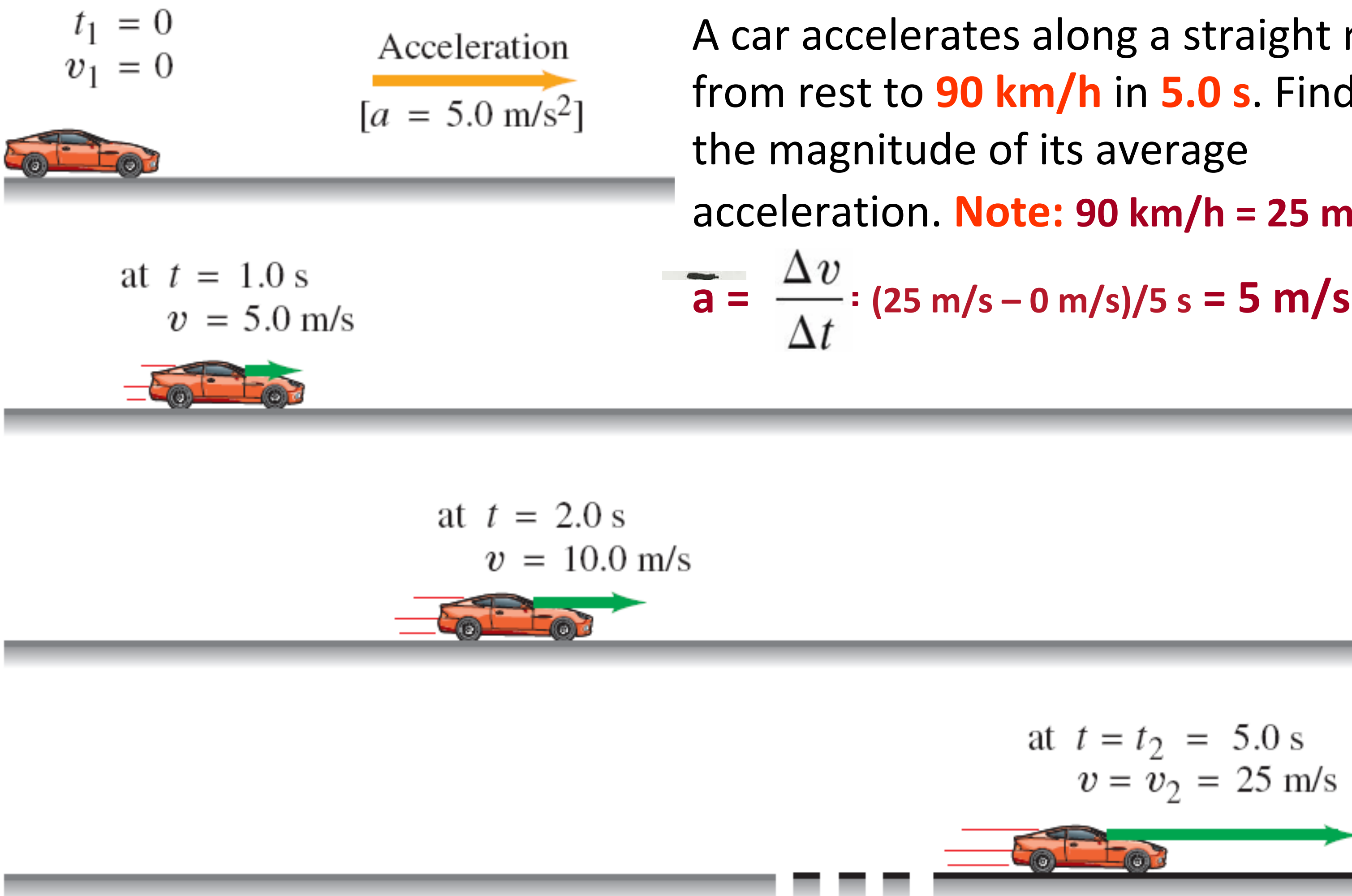
- **Units: velocity/time = distance/(time)^2 = m/s^2**

Example 2-4: Average Acceleration



A car accelerates along a straight road from rest to **90 km/h** in **5.0 s**. Find the magnitude of its average acceleration. **Note: 90 km/h = 25 m/s**

Example 2-4: Average Acceleration



A car accelerates along a straight road from rest to **90 km/h** in **5.0 s**. Find the magnitude of its average acceleration. **Note: 90 km/h = 25 m/s**

$$\mathbf{a} = \frac{\Delta v}{\Delta t} : (25\ \text{m/s} - 0\ \text{m/s})/5\ \text{s} = 5\ \text{m/s}^2$$

Conceptual Question

Velocity & Acceleration are both vectors.

Are the velocity and the acceleration always in the same direction?

Conceptual Question

Velocity & Acceleration are both vectors.

Are the velocity and the acceleration always in the same direction?

<u>NO!!</u>

If the object is ***<u>slowing down</u>***, the acceleration vector is in the opposite direction of the velocity vector!

Conceptual Question

Velocity & acceleration are both vectors.

Is it possible for an object to have a zero velocity and a non-zero acceleration?

Conceptual Question

Velocity & Acceleration are both vectors.

Is it possible for an object to have a zero acceleration and a non-zero velocity?

YES!!

If the object is ***moving at a constant velocity***, the acceleration vector is zero!

Conceptual Question

Velocity & acceleration are both vectors.

Is it possible for an object to have a zero velocity and a non-zero acceleration?

Conceptual Question

Velocity & acceleration are both vectors.

Is it possible for an object to have a zero velocity and a non-zero acceleration?

YES!!

If the object is ***instantaneously at rest*** **(v = 0)** ***but is either on the verge of starting to move or is turning around & changing direction***, the velocity is zero, but the acceleration is not!

Deceleration

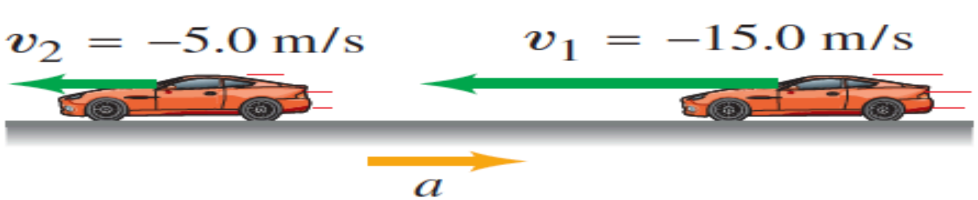
When an object is slowing down, we can say it is **decelerating**. But be careful: deceleration does not mean the negative acceleration.

ws down, as it comes in for landing in St. Maarten. Its acceleration is opposite in direction to its velocity. (credit: Steve Conry, Flickr)

FIGURE 2–12 The car of Example 2–6, now moving to the *left* and decelerating. The acceleration is $a = (v_2 - v_1)/\Delta t$, or

$$a = \frac{(-5.0\ \mathrm{m/s}) - (-15.0\ \mathrm{m/s})}{5.0\ \mathrm{s}}$$

$$= \frac{-5.0\ \mathrm{m/s} + 15.0\ \mathrm{m/s}}{5.0\ \mathrm{s}} = +2.0\ \mathrm{m/s^2}.$$



Motion with Constant Acceleration

- Many practical situations:
  - The magnitude of the **acceleration is *uniform*** (constant)
  - The motion is in a ***straight line***
  - *Free Fall*

- It's useful to derive some equations which apply ***in this case ONLY***.
  - The kinematic equations for uniform acceleration in one dimension.

Constant Acceleration

- Please Read on your own again!
- In the derivation, its useful to change notation slightly

$t_1 \equiv 0$ = time when the problem begins

$x_1 \equiv x_0$ = initial position (at $t_1 = 0$, often $x_0 = 0$)

$v_1 \equiv v_0$ = initial velocity (at $t_1 = 0$)

$t_2 \equiv t$ = time when we wish to know other quantities

$x_2 \equiv x$ = position at time t

$v_2 \equiv v$ = velocity at time t

$a \equiv$ acceleration = constant

(**average & instantaneous accelerations are equal**)

- Using these, **by definition** we have:
  - Average velocity:

    $\bar{v} = (x - x_0)/t \Rightarrow x = x_0 + \bar{v}t$ **(1)**
  - Acceleration (average = instantaneous):

    $a = (v - v_0)/t \Rightarrow v = v_0 + at$ **(2)**
  - Average velocity (another form):

    $\bar{v} = (½)(v + v_0)$ **(3)**

  This is because velocity increases at a uniform rate, thus the average velocity will be midway between initial and final velocities.

$$
\begin{aligned}
x &= x_0 + \bar{v}t \\
&= x_0 + \left(\frac{v_0 + v}{2}\right)t \\
&= x_0 + \left(\frac{v_0 + v_0 + at}{2}\right)t
\end{aligned}
$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2.$$

Eq (2): **$a = (v - v_0)/t \Rightarrow t = (v - v_0)/a$**

Eq (1): **$x = x_0 + \bar{v}t \quad \Rightarrow$**

$$x = x_0 + \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right) = x_0 + \frac{v^2 - v_0^2}{2a}.$$

$$v^2 = v_0^2 + 2a(x - x_0),$$

Constant Acceleration Equations

- Results (**one dimensional motion *only***!):

$$v = v_0 + at \qquad (1)$$

$$x = x_0 + v_0 t + (½)a t^2 \qquad (2)$$

$$v^2 = (v_0)^2 + 2a(x - x_0) \qquad (3)$$

$$\bar{v} = (½)(v + v_0) \qquad (4)$$

NOT VALID <u>*UNLESS*</u> a = CONSTANT!!!

Usually $x_0 = 0$. Sometimes $v_0 = 0$

Kinematic equations for constant acceleration (we'll use them a lot)

All we need for 1 dimensional constant-acceleration problems:

NOT VALID *UNLESS* a = CONSTANT!!!

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\bar{v} = \frac{v + v_0}{2}.$$

Physics and Equations

IMPORTANT!!!

- Even though these equations & their applications are important, Physics is ***not*** a collection of formulas to memorize & blindly apply!
- Physics is a set of ***PHYSICAL PRINCIPLES***.
- Blindly searching for the "equation which will work for this problem" can be ***DANGEROUS!!!!***

Problem Solving Strategies

1. **Read** the whole problem. Make sure you understand it. Read it again.
2. **Decide** on the objects under study & what the time interval is.
3. **Draw** a diagram & choose coordinate axes.
4. **Write** down the known (given) quantities, & the unknown ones needed.
5. **What *physics*** applies? Plan an approach to a solution.
6. **Which equations** relate known & unknown quantities? Are they valid in this situation? Solve ***algebraically*** for the unknown quantities, & check that your result is sensible (correct dimensions).
7. **Calculate the solution**, round it to appropriate number of significant figures.
8. **Look** at the result - is it reasonable? Does it agree with a rough estimate?
9. **Check** the units again.

<u>Bottom Line:</u>

<u>THINK!</u>

<u>DO NOT</u> **BLINDLY**

APPLY FORMULAS!!!!

Example 2-6: Runway Design

You' re designing an airport. A plane that will use this airport must reach a speed of **v_{min} = 100 km/h** (**27.8 m/s**) before takeoff. It can accelerate at **a = 2 m/s²**. **(a)** If the runway
is **x = 150 m** long, can this plane reach the speed of before it runs off the end of the runway? **(b)** If not, what is the minimum length required for the runway?

Solutions

(a) Use **Eq. (3)**:

$$v^2 = v_0^2 + 2a(x - x_0)$$
$$= 0 + 2(2.00\,\mathrm{m/s^2})(150\,\mathrm{m}) = 600\,\mathrm{m^2/s^2}$$
$$v = \sqrt{600\,\mathrm{m^2/s^2}} = 24.5\,\mathrm{m/s}.$$

Known	Wanted
$x_0 = 0$	v
$v_0 = 0$	
$x = 150\,\mathrm{m}$	
$a = 2.00\,\mathrm{m/s^2}$	

(b) Use **Eq. (3)** again with

$$(x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{(27.8\,\mathrm{m/s})^2 - 0}{2(2.00\,\mathrm{m/s^2})} = 193\,\mathrm{m}.$$

$$v = v_0 + at \quad \textbf{(1)}$$
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \quad \textbf{(2)}$$
$$v^2 = v_0^2 + 2a(x - x_0) \quad \textbf{(3)}$$
$$\bar{v} = \frac{v + v_0}{2}. \quad \textbf{(4)}$$

To be safe, make the runway

200 m long!