Reserving

Stat 346 - Short-term Actuarial Math

Important Ratios

$$\text{Average Frequency} = \frac{\text{Number of Claims}}{\text{Exposure}}$$

- **Number of Claims:** The total number of claims reported in a given period.
- **Exposure:** The measure of risk, often represented in terms of policy years, vehicle years, or sum insured, providing a basis for comparing different risk units.

Understanding Exposure

We know the following about Scott and Joey.

- Scott has 2 losses
- Joey has 3 losses

Who has the higher likelihood of loss?

Understanding Exposure

The rest of the story

- Scott has 2 losses in the last 2 weeks
- Joey has 3 losses over the last 2 years.

Who has the higher likelihood of loss?

Understanding Exposure

Exposure is a basic unit of measure for risk.

Total Expected Loss = Expected Loss per Exposure $\times$ Exposures

Time is a common exposure level but there could even be better ones

- Discuss car claims in terms of claim per mile driven
- Disability claims for a business per employee, or even by employee hour worked.

Important Ratios

$$\text{Average Severity} = \frac{\text{Losses}}{\text{Number of Claims}}$$

$$\text{Pure Premium} = \frac{\text{Losses}}{\text{Exposure}} = \text{Frequency} \times \text{Severity}$$

$$\text{Loss Ratio} = \frac{\text{Pure Premium}}{\text{Actual Premium}}$$

Loss ratio is a measure of profitability. If it is low then profits are higher, but there are other things to consider.

Claim Related Expenses

ALAE (Allocated Loss Adjustment Expenses):

- Expenses assignable to a particular claim, including legal costs and expert witness fees.

ULAE (Unallocated Loss Adjustment Expenses):

- Expenses not easily allocated to a specific claim, such as payroll, rent, and computer expenses for the claims department.

DCC (Defense and Cost Containment):

- Expenses related to defense litigation and medical cost containment, whether provided internally or externally.

A&O (Adjusting and Other):

- Includes all claims adjusting expenses.

Examples of Claim Related Expenses

ALAE Examples:

- Payment to a law firm for defending a claim.
- Fees for an expert witness in a court case.

ULAE Examples:

- Salaries of the claims department staff.
- Office rent and utilities for the claims processing center.

DCC Examples:

- Costs associated with legal defense strategies.
- Expenses for medical reviews to contain claim costs.

A&O Examples:

- Costs for claims investigation teams.
- Expenses for claims adjustment software.

Key Dates in an Insurance Claim

Understanding the timeline of an insurance claim is crucial for both insurers and insureds. Here are the key dates involved:

- **Accident Date/Occurrence Date:** The date the loss occurred.
- **Report Date:** The date the insured reports the claim to the insurer.
- **Claim Create Date:** The date the claim handler enters the claim information into the insurer's data systems.
- **Transaction Date:** The date a financial transaction is made on a claim.
- **Settlement Date/Closed Date:** The date the final payment is sent to the insured for a claim, and the case reserve is set to 0.
- **Reopened Date:** The date when a claim that had been closed is reopened for further investigation or additional payments.
- **Policy Effective Date:** The date when the insurance policy goes into effect, marking the beginning of the coverage period.
- **Policy Expiration Date:** The date the policy is no longer effective, marking the end of the coverage period.

Understanding Payments for a Claim

In managing insurance claims, several financial measures are crucial:

Ultimate: The total losses that will eventually be paid out for claims.

Paid Losses: Payments already made to any party for a claim.

Case Reserves: Money set aside for total claim payments to be made.

Incurred: The sum of paid losses and case reserves. Represented as Incurred = Paid + Case Reserves.

IBNR (Incurred But Not Reported): Losses that have been incurred but not yet reported to the insurer.

Total reserves should equal Ultimate − Paid Losses, the sum of case reserves and IBNR, providing a comprehensive financial overview of the insurer's liability for claims.

Delving into IBNR: IBNER and IBNYR

The IBNR reserve can be further classified into two distinct types to better understand and manage latent claim liabilities:

IBNER (Incurred But Not Enough Reported): Represents the additional costs expected for claims that have been reported but are underestimated in the current reserves.

IBNYR (Incurred But Not Yet Reported): Refers to claims that have occurred but have not yet been reported to the insurer at all.

One important job of an actuary is to estimate reserves, because these two values are important to know. i.e. IBNR gives actuaries jobs.

Claim Terminology

Practice

1. IBNR = $1000
   Ultimate Losses = $1500
   What is the incurred loss?

2. Incurred Loss = $500
   Ultimate Loss = $1000
   What is the IBNR?

3. Reserve = $1000
   IBNR = $500
   What is the case outstanding?

Practice *- Answers*

1. IBNR = $1000
   Ultimate Losses = $1500
   What is the incurred loss? *1500 – 1000 = $500*

2. Incurred Loss = $500
   Ultimate Loss = $1000
   What is the IBNR? *1000 – 500 = $500*

3. Reserve = $1000
   IBNR = $500
   What is the case outstanding?
   1000 – 500 = $500

What are Reserves?

Practice

- Problem 1
  - *Paid Loss = 500*
  - *Ultimate Loss = 1250*
  - *IBNR = 250*
  - *What are the case reserves (case o/s)?*
- Problem 2
  - *Reserve = 500*
  - *IBNR = 100*
  - *Paid Loss = 1000*
  - *What are the case reserves (case o/s)?*
- Problem 3
  - *Incurred Loss = 500*
  - *Reserve = 500*
  - *Case Reserves (case o/s) = 300*
  - *What is the ultimate loss?*

Practice - *Answers*

- Problem 1
  - *Paid Loss = 500*
  - *Ultimate Loss = 1250*
  - *IBNR = 250*
  - *What are the case reserves (case o/s)?* *1250 – 500 – 250 = 500*
- Problem 2
  - *Reserve = 500*
  - *IBNR = 100*
  - *Paid Loss = 1000*
  - *What are the case reserves (case o/s)?* *500 – 100 = 400*
- Problem 3
  - *Incurred Loss = 500*
  - *Reserve = 500*
  - *Case Reserves (case o/s) = 300*
  - *What is the ultimate loss?* *500 + (500 – 300) = 700*

Practice

Date	Transaction
4/1/2017	Case Reserve of $500K Established
5/1/2017	A $10K partial payment is made
6/1/2017	Case Reserve is increased by another $50K
7/1/2017	A $300K partial payment is made

1. What is the case reserve on the claim as of the latest transaction?

2. What is the incurred loss on the claim as of the latest transaction?

Practice - *Answers*

Date	Transaction
4/1/2017	Case Reserve of $500K Established
5/1/2017	A $10K partial payment is made
6/1/2017	Case Reserve is increased by another $50K
7/1/2017	A $300K partial payment is made

1. What is the case reserve on the claim as of the latest transaction?

   500 – 10 + 50 – 300 = $240K

2. What is the incurred loss on the claim as of the latest transaction?

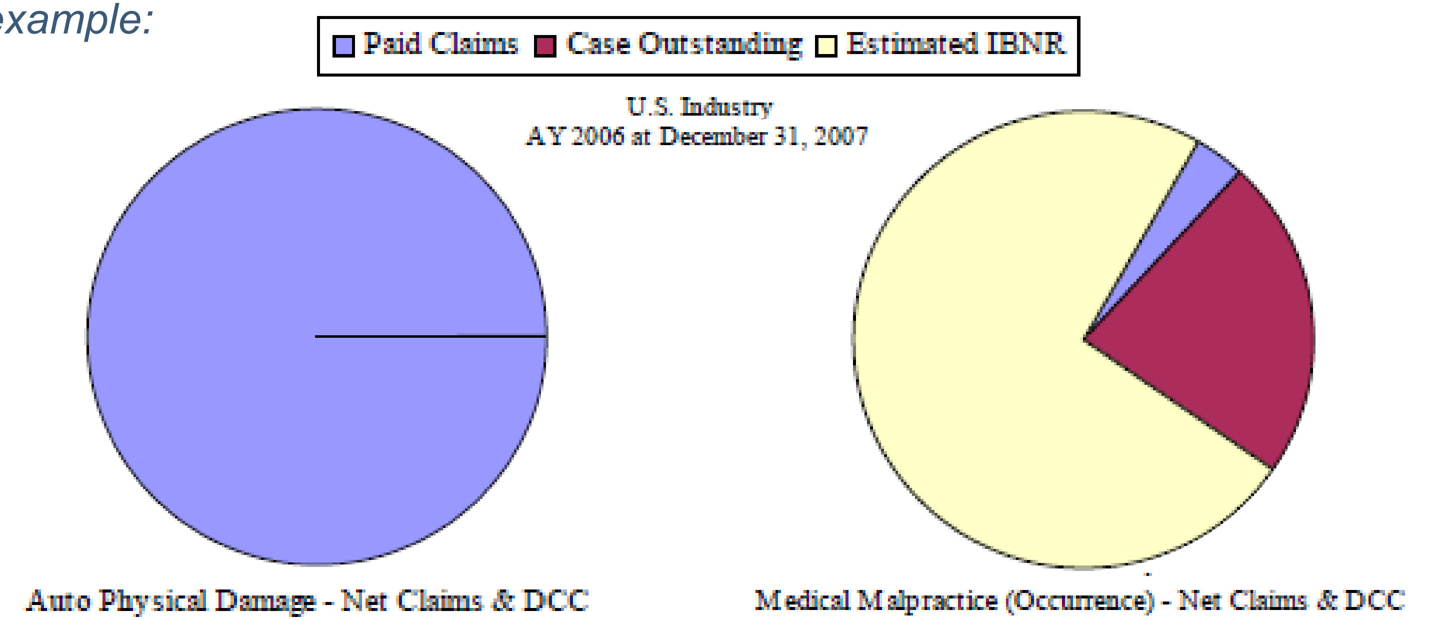
   240 + 10 + 300 = $550K

The Reserve Call

Why Is Loss Reserving Difficult?

- Loss Reserves are an estimate of what will happen in the future – a future that for some lines of business extends decades

For example:



- Also…Implicit in any loss reserve estimate are assumptions regarding:
  - *Inflation*
  - *Technology*
  - *Longevity*
  - *Company operations*
  - *Legal and judicial environment*
  - *Regulatory climate*

Data Aggregation

Calendar Year

- Transactional Data
- Used in financial statements
- Used for aggregation of exposures and diagnostic testing with accident year claim data
- Advantages:
  - *No future development*
  - *Readily available*
- Disadvantages:
  - *Cannot address development*

Example:

Calendar year 2015 reported claims =

2015 paid claims + (2015 ending case o/s - 2015 beginning case o/s)

Data Aggregation

Accident Year

- Date of occurrence data
- Accident year, quarter, or month
- Which exposures are best used with accident year claims?
- Advantages:
  - *Easy to group and easy to understand*
  - *Shorter time to fully develop than policy year*
  - *Numerous industry benchmarks*
  - *Useful for economic changes or major claim events*
- Disadvantages:
  - *Potential mismatch between claims and exposures*

Data Aggregation

Development triangle

Table that shows changes in value of various cohorts over time

- Use to estimate many different values
  - *reported claims*
  - *paid claims*
  - *claim-related expenses*
  - *reported claim counts*
- One of most common tools that actuaries use to organize data in order to identify and analyze patterns in historical data

Development - change in value for the cohort over time

- Actuaries interested in the *typical* development
- Look at the age (or maturity) of the cohort
  - *Generally measure in terms of time from start of cohort e.g. AY 2008 eval'd at 12/31/2008 is at 12 months*
- Development can be positive or negative

Data Aggregation

Triangle Structure *(P&C Customary Format)*

- Rows – experience period *(e.g. AY)*
  - *Can also be PY or RY*
- Columns – age or maturity *("as of" dates)*
- Diagonals – valuation date *(e.g. CY)*

Table 3 – Reported Claim Triangle

Accident Year	Reported Claims as of (months) 12	24	36	48
2005	1,500	2,420	2,720	3,020
2006	1,150	1,840	2,070	
2007	1,650	2,640		
2008	1,740			

AY

CY

Development Year (DY)

Data Aggregation

Practice

- *Using the triangles just created, construct an AYxDY cumulative reported (incurred) triangle*
- *Using the data below, construct an AYxDY cumulative reported (incurred) claim counts triangle*

Table 5 – Detailed Example – Claims Transaction Data

Claim ID	Accident Date	Report Date	2005 Transactions Total Payments	2005 Transactions Ending Case O/S	2006 Transactions Total Payments	2006 Transactions Ending Case O/S	2007 Transactions Total Payments	2007 Transactions Ending Case O/S	2008 Transactions Total Payments	2008 Transactions Ending Case O/S
1	Jan-5-05	Feb-1-05	400	200	220	0	0	0	0	0
2	May-4-05	May-15-05	200	300	200	0	0	0	0	0
3	Aug-20-05	Dec-15-05	0	400	200	200	300	0	0	0
4	Oct-28-05	May-15-06			0	1,000	0	1,200	300	1,200
5	Mar-3-06	Jul-1-06			260	190	190	0	0	0
6	Sep-18-06	Oct-2-06			200	500	0	500	230	270
7	Dec-1-06	Feb-15-07					270	420	0	650
8	Mar-1-07	Apr-1-07					200	200	200	0
9	Jun-15-07	Sep-9-07					460	390	0	390
10	Sep-30-07	Oct-20-07					0	400	400	400
11	Dec-12-07	Mar-10-08							60	530
12	Apr-12-08	Jun-18-08							400	200
13	May-28-08	Jul-23-08							300	300
14	Nov-12-08	Dec-5-08							0	540
15	Oct-15-08	Feb-2-09								

Practice - *Answer*

Construct an AYxDY cumulative reported (incurred) triangle:

Table 11 – Reported Claim Development Triangle

Accident Year	Reported Claims as of (months) 12	24	36	48
2005	1,500	2,420	2,720	3,020
2006	1,150	1,840	2,070	
2007	1,650	2,640		
2008	1,740			

- What was the IBNR for AY 2005 on 12/31/2006?
  (assume no development past 48 months)

Common Methods ~ *The Development Technique*

- Also known as the "*chain-ladder*" technique
- One of the most frequently used methods

Key Assumptions

- **Future claims' development is similar to prior years' development**

  Claims recorded to date will continue to develop in a similar manner in the future (Past is indicative of the future)
  Relative change in a given year's claims from one evaluation to next is similar to the relative change in prior years' claims at similar evaluation points

- **Implicitly assumes claims observed for an immature AY tell you something about claims yet to be observed**

  This is not assumed in the expected claims technique, BF, or Cape Cod

- **Other important assumptions**
  - *Consistent claims processing*
  - *Stable mix of claims types*
  - *Stable policy limits*
  - *Stable reinsurance retention limits throughout experience period*

Common Methods ~ *The Development Technique*

Mechanics

- Step 1 – Compile claims data in a development triangle
- Step 2 – Calculate age-to-age factors *(link ratios)*
- Step 3 – Calculate averages of the age-to-age factors
- Step 4 – Select loss development factors *(LDFs)*
- Step 5 – Select tail factor
- Step 6 – Calculate cumulative loss development factors *(CDFs)*
- Step 7 – Project ultimate claims

Common Methods ~ *The Development Technique*

Step 1 – Compile claims data in a development triangle (we did this yesterday…)

	Reported Claims as of *(cumulative)*				
Accident Year	12M	24M	36M	48M	60M
2014	3,575	4,400	5,500	6,000	6,200
2015	4,200	5,200	6,500	7,100	
2016	4,875	7,000	8,200		
2017	5,600	7,500			
2018	6,000				

Common Methods ~ *The Development Technique*

Step 2 – Calculate age-to-age factors (link ratios)

Reported Claims as of

Accident Year	12M	24M	36M	48M	60M
2014	3,575	4,400	5,500	6,000	6,200
2015	4,200	5,200	6,500	7,100	
2016	4,875	7,000	8,200		
2017	5,600	7,500			
2018	~~6,000~~				

$$\frac{4,400}{3,575}$$

$$\frac{5,200}{4,200}$$

$$\frac{5,500}{4,400}$$

Age to Age Factors

Accident Year	12-24	24-36	36-48	48-60	60-Ult
2014					
2015					
2016					
2017					

Step 3 – Calculate averages of the age-to-age factors

	Age to Age Factors				
Accident Year	12-24	24-36	36-48	48-60	60-Ult
2014	1.23	1.25	1.09	1.03	
2015	1.24	1.25	1.09		
2016	1.44	1.17			
2017	1.34				

Average	1.31	1.22	1.09	1.03	
Avr xHiLow	1.29	1.25	1.09	1.03	
Avr 2yr	1.39	1.21	1.09	1.03	

Common Methods ~ *The Development Technique*

Step 3 – Calculate averages of the age-to-age factors

- Simple Average – *arithmetic mean, which years to use*
- Medial Average – *xHiLo*
- Volume Weighted Average
  - *Use prior claims at earlier valuations as weights*
  - *Sum of claims at later valuation divided by sum of claims at earlier valuation*
  - *This is the quickest way using a calculator, but risky, as you could end up using a wild year (one where development is markedly different)*
- Geometric mean/average
- Note: Stability vs. Responsiveness
  - *Most recent experience most likely reflects the effect of latest changes in environment*
  - *Greater number of experience periods increases stability*

Common Methods ~ *The Development Technique*

	Reported Claims as of				
Accident Year	12M	24M	36M	48M	60M
2014	3,575	4,400	5,500	6,000	6,200
2015	4,200	5,200	6,500	7,100	
2016	4,875	7,000	8,200		
2017	5,600	7,500			
2018	6,000				

- Simple Average

$$12\text{to}24 = \left(\frac{4{,}400}{3{,}575} + \frac{5{,}200}{4{,}200} + \frac{7{,}000}{4{,}875} + \frac{7{,}500}{5{,}600}\right) / 4 = 1.31$$

- Volume Weighted Average

$$12\text{to}24 = \frac{(4{,}400 + 5{,}200 + 7{,}000 + 7{,}500)}{(3{,}575 + 4{,}200 + 4{,}875 + 5{,}600)} = 1.32$$

Common Methods ~ *The Development Technique*

	Age to Age Factors				
Accident Year	12-24	24-36	36-48	48-60	60-Ult
2014	1.23	1.25	1.09	1.03	
2015	1.24	1.25	1.09		
2016	1.44	1.17			
2017	1.34				

- Simple Average

  12to24 = (1.23 + 1.24 + 1.44 + 1.34) / 4 = 1.31

- Medial Average *(xHiLo)*

  12to24 = (~~1.23~~ + 1.24 + ~~1.44~~ + 1.34) / 2 = 1.29

- Geometric mean/average

  12to24 = (1.23 * 1.24 * 1.44 * 1.34) ^ $\frac{1}{4}$ = 1.31

Common Methods ~ *The Development Technique*

Step 4 – Select loss development factors (LDFs)

	Age to Age Factors				
Accident Year	12-24	24-36	36-48	48-60	60-Ult
2014	1.23	1.25	1.09	1.03	
2015	1.24	1.25	1.09		
2016	1.44	1.17			
2017	1.34				

Average	$\boxed{1.31}$	$\boxed{1.22}$	1.09	1.03
Avr xHiLow	1.29	1.25	1.09	$\boxed{1.03}$
Avr 2yr	1.39	1.21	$\boxed{1.09}$	1.03
Selected	1.31	1.22	1.09	1.03

Common Methods ~ *The Development Technique*

Step 4 – Select loss development factors (LDFs)

When selecting dev factors, review experience for following characteristics:

- Smooth progression of age-to-age factors across development periods - Ideally, pattern should steadily decrease with age
- Stability of age-to-age factors for the same development period
  Greatest variability in age-to-age factors at earlier ages
- Credibility of experience
  If limited due to claims volume, organizational changes or other factors, may be necessary to use benchmarks
- Changes in patterns
  Systematic patterns may suggest changes in internal operations or external environment
- Applicability of historical experience
  Based on qualitative info regarding changes in book or business and operations
- Also consider effect of external changes not in the data

Common Methods ~ *The Development Technique*

Step 5 – Select tail factor

	Age to Age Factors				
Accident Year	12-24	24-36	36-48	48-60	60-Ult
2014	1.23	1.25	1.09	1.03	
2015	1.24	1.25	1.09		
2016	1.44	1.17			
2017	1.34				
Average	1.31	1.22	1.09	1.03	
Avr xHiLow	1.29	1.25	1.09	1.03	
Avr 2yr	1.39	1.21	1.09	1.03	
Selected	1.31	1.22	1.09	1.03	60-Ult 1.01

Common Methods ~ *The Development Technique*

Step 5 – Select tail factor

- When data is available, should analyze development out to point where ceases
  - *Number of periods varies by line, jurisdiction, and data type*
- Sometimes most mature development period in data has factors greater than 1.0
  - *Actuary will need to determine tail factor*
- Tail factor can be difficult to select due to limited availability of relevant data
  - *Some med mal and WC claims take more than 15 years to reach final settlement*
- Tail factor is crucial because it affects unpaid estimate for all accident years
  - *Can create disproportionate leverage on the total estimated unpaid claims*
- Tail factor plays an important role in almost every technique to estimate unpaid claims

Several approaches

(1) Use industry benchmark development factors

(2) Fit a curve to selected or observed development factors and extrapolate

- *Exponential decay is a common assumption*

(3) For paid development when reported development is considered at ultimate, use reported-to-paid ratios at the latest observed paid development period

Common Methods ~ *The Development Technique*

Step 6 – Calculate cumulative loss development factors (CDFs)

Accident Year	12-24	24-36	36-48	48-60	60-Ult
2014	1.23	1.25	1.09	1.03	
2015	1.24	1.25	1.09		
2016	1.44	1.17			
2017	1.34				
Average	1.31	1.22	1.09	1.03	
Avr xHiLow	1.29	1.25	1.09	1.03	
Avr 2yr	1.39	1.21	1.09	1.03	
Selected	1.31	1.22	1.09	1.03	1.01
Age-to-Ult CDF					

*1.31 * 1.22 * 1.09 * 1.03 * 1.01*

*12-Ultimate = 12-24 * 24-36 * 36-48 * 48-60 * 60-Ult*

Common Methods ~ *The Development Technique*

Step 7 – Project ultimate claims

	Reported Claims as of *(cumulative)*					
Accident Year	12M	24M	36M	48M	60M	Projected Ultimates
2014	3,575	4,400	5,500	6,000	6,200	*6,262*
2015	4,200	5,200	6,500	7,100		*7,384*
2016	4,875	7,000	8,200			*9,266*
2017	5,600	7,500				*10,350*
2018	6,000					*10,860*

	12-Ult	24-Ult	36-Ult	48-Ult	60-Ult
Age-to-Ult CDF	1.81	1.38	1.13	1.04	1.01

*6,200 * 1.01 = 6,262*

*7,100 * 1.04 = 7,384*

Step 7 – Project ultimate claims

Accident Year	Reported Claims as of *(cumulative)* 12M	24M	36M	48M	60M	Projected Ultimates
2014	3,575	4,400	5,500	6,000	6,200	*6,262*
2015	4,200	5,200	6,500	7,100		*7,384*
2016	4,875	7,000	8,200			*9,266*
2017	5,600	7,500				*10,350*
2018	6,000					*10,860*

AY 2014 IBNR = *6,262 – 6,200 =* ***62***

AY 2015 IBNR = *7,384 – 7,100 =* ***284***

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AY 2018 IBNR = ***4,860***

Practice

- *Using the reported triangle below and the Development Method, calculate the total indicated reserve for each accident year given. Use a volume weighted average for LDF selections.*
  - *Assume a tail factor of 1.02*

Total Paid Losses	Accident Year	Reported Claims as of 12M	24M	36M	48M	60M
5,900	2014	3,575	4,400	5,500	6,000	6,200
6,100	2015	4,200	5,200	6,500	7,100	
7,800	2016	4,875	7,000	8,200		
6,800	2017	5,600	7,500			
4,200	2018	6,000				

Practice *- Answers*

- *Using the reported triangle below and the Development Method, calculate the total indicated reserve for each accident year given. Use a volume weighted average for LDF selections.*
  - *Assume a tail factor of 1.02*

Reserve = Ult - Paid

Total Paid Losses	Accident Year	Reported Claims as of 12M	24M	36M	48M	60M	*Ults*	*Rsrv*
5,900	2014	3,575	4,400	5,500	6,000	6,200	*6,324*	*424*
6,100	2015	4,200	5,200	6,500	7,100		*7,483*	*1,383*
7,800	2016	4,875	7,000	8,200			*9,438*	*1,638*
6,800	2017	5,600	7,500				*10,500*	*3,700*
4,200	2018	6,000					*11,100*	*6,900*

Vol Wtd-> 12-24 = (4,400+5,200+7,000+7,500) / (3.575+4,200+4,875 + 5,600) = 1.32

	12M	24M	36M	48M	60M
LDFs:	*1.321*	*1.217*	*1.092*	*1.033*	*1.02*
CDFs:	*1.850*	*1.400*	*1.151*	*1.054*	*1.02*

Common Methods ~ *The Development Technique*

"Completing the Square"

Accident Year	12M	24M	36M	48M	60M	Projected Ultimates
	Reported Claims as of *(cumulative)*					
2014	3,575	4,400	5,500	6,000	6,200	*6,262*
2015	4,200	5,200	6,500	7,100	*7,313*	*7,384*
2016	4,875	7,000	8,200	*8,938*	*9,206*	*9,266*
2017	5,600	7,500	*9,150*	*9,974*	*10,273*	*10,350*
2018	6,000	*7,860*	*9,589*	*10,452*	*10,766*	*10,860*

*6,000 * 1.31 = 7,860*

*7,860 * 1.22 = 9,589*

	12-24	24-36	36-48	48-60	60-Ult
Selected	1.31	1.22	1.09	1.03	1.01
	12-Ult	24-Ult	36-Ult	48-Ult	60-Ult
Age-to-Ult CDF	1.81	1.38	1.13	1.04	1.01

Common Methods ~ *The Development Technique*

Expected CY Development

	Reported Claims as of *(cumulative)*					
Accident Year	12M	24M	36M	48M	60M	Projected Ultimates
2014	3,575	4,400	5,500	6,000	6,200	*6,262*
2015	4,200	5,200	6,500	7,100	*7,313*	*7,384*
2016	4,875	7,000	8,200	*8,938*	*9,206*	*9,266*
2017	5,600	7,500	*9,150*	*9,974*	*10,273*	*10,350*
2018	6,000	*7,860*	*9,589*	*10,452*	*10,766*	*10,860*

Expected Development in CY 2019
(assuming AY 2014 hits Ultimate during CY 2019) =

7,860 – 6,000 + 9,150 – 7,500 + 8,938 – 8,200 + 7,313 – 7,100 + 6,262 – 6,200 = ***4,523***

Practice

- *Using the ultimates and LDFs given, complete the square for the below triangle (through 48months) AND calculate the 2009 CY expected development, assuming AY 2005 reaches ultimate during CY 2009:*

Table 11 – Reported Claim Development Triangle

Accident	Reported Claims as of (months)				
Year	12	24	36	48	Ults
2005	1,500	2,420	2,720	3,020	3,080
2006	1,150	1,840	2,070		2,339
2007	1,650	2,640			3,353
2008	1,740				3,532
LDFs:	*1.6*	*1.12*	*1.11*	*1.02*	
CDFs:	*2.03*	*1.27*	*1.13*	*1.02*	

Practice *- Answers*

- *Using the ultimates and LDFs given, complete the square for the below triangle (through 48months) AND calculate the 2009 CY expected development, assuming AY 2005 reaches ultimate during CY 2009:*

*2,640 * 1.12 = 2,957*

Table 11 – Reported Claim Development Triangle

Accident Year	Reported Claims as of (months) 12	24	36	48	*Ults*
2005	1,500	2,420	2,720	3,020	*3,080*
2006	1,150	1,840	2,070	*2,298*	*2,339*
2007	1,650	2,640	$\boxed{2{,}957}$	*3,282*	*3,353*
2008	1,740	*2,784*	*3,118*	*3,461*	*3,532*
LDFs:	*1.6*	*1.12*	*1.11*	*1.02*	
CDFs:	*2.03*	*1.27*	*1.13*	*1.02*	

2009 CY Expected Development =
(3,080 – 3,020) + (2,298 – 2,070) + (2,957 – 2,640) + (2,784 – 1,740) = 1,649

Review of Loss Ratio

The **Loss Ratio** is a fundamental concept in insurance, defined as the ratio of total losses paid out by an insurer to the premiums earned. Mathematically, it is represented as:

$$\text{Loss Ratio} = \frac{\text{Total Losses}}{\text{Earned Premium}} = \frac{\text{Loss per Unit of Exposure}}{\text{Premium per Exposure}}$$

It serves as a measure of an insurance company's profitability and efficiency in underwriting risks.

Loss Ratio Method (Expected Claims Method)

The Loss Ratio Method, or Expected Claims Method, estimates future claims based on the expected loss ratio and the amount of earned premium. It is expressed as:

$$\text{Ultimate Loss} = \text{Expected Loss Ratio} \times \text{Earned Premium}$$

This method is particularly useful when historical data is limited or not reflective of future expectations.

Example: Loss Ratio Method

Suppose an insurer expects a loss ratio of 60% for the current policyholders, with $10,000 in earned premiums for a given year. The estimated claims would be:

$$\text{Estimated Claims} = 0.60 \times 10,000 = \$6,000$$

Now suppose that we are three years down the road and only $5,000 has been paid. Reserves should then be set to $6,000 - 5,000 = 1,000$.

Loss Ratio Method Example

We are given the following yearly payments and. case reserves for accidents occurring in Year A

Year	Payments	Case Reserves
Year 1	\$56,000	\$32,000
Year 2	\$18,000	\$20,000
Year 3	\$10,000	\$12,000

We also know the loss ratio for accident year A was estimated using the folllowng values

- 80 units of exposures.
- 150 claims expected
- The average severity per claim is \$750
- The total earned premium is \$140,000

What is the current IBNR for accident year A after year A+2?

Bornhuetter-Ferguson Method

The Bornhuetter-Ferguson Method combines one major element from a claims triangle and one major element from the loss ratio method. The idea is that

$$\text{Expected Ultimate Loss} = \text{Expected Loss Ratio} \times \text{Earned Premium}$$

but also

$$\text{Expected Ultimate Loss} = \text{Paid Losses} * \prod_{i=1}^{\infty} f_i$$

where f_i is the i-th loss development factor. Reserves are then calculated using

$$\text{Reserves} = \text{Expected Ultimate Loss} - \text{Paid Losses}$$

$$\text{Reserves} = \text{Expected Loss Ratio} \times \text{Earned Premium} \times \left(1 - \frac{1}{f_{ult}}\right)$$

where $f_{ult} = \prod_{i=1}^{\infty} f_i$

Example: Bornhuetter-Ferguson Method

Consider \$3,000 in paid claims, an expected loss ratio of 50%, \$10,000 in earned premiums, and a loss development factor of 1.5. The ultimate claims would be:

$$\text{Expected Ultimate Loss} = 0.50 \times 10,000 = 5,000$$

Then reserved would be

$$\text{Reserves} = 5000 \left(1 - \frac{1}{1.5} \right) = 1666.67$$

what would the loss ratio method say?

Example: All Three Methods

You have chosen the following paid loss development factors to model the lower half of a claims paid triangle.

$1/0$	$2/1$	$3/2$	$4/3$	$\infty/4$
1.41	1.22	1.16	1.08	1.04

You are setting reserves for the annual report in calendar year 7. For accident year 6 you have paid-to-date claims of \$420,000. The earned premium calculated for accident year 6 is is \$1,000,000 and the expected loss ratio is 0.6. Determine the estimated loss reserve using the chain ladder. method the loss ratio method, and the Bornhuetter Ferguson method.