Two More Option Things

Put-Call Parity

Definition: Put-Call Parity establishes a relationship between the prices of a European call option and a European put option with the same strike price and expiration date.
Formula:

$$C - P = S - Ke^{-rT}$$

where:

- C = price of the call option
- P = price of the put option
- S = current stock price
- K = strike price
- r = risk-free interest rate
- T = time to maturity

Put-Call Parity Example

Given:

- Stock price $(S) = \$100$
- Strike price $(K) = \$100$
- Risk-free rate $(r) = 5\%$
- Time to maturity $(T) = 1$ year
- Call option price $(C) = \$10$

Find: Price of the put option (P)
Calculation:

$$P = C - S + Ke^{-rT} = 10 - 100 + 100e^{-0.05\times 1}$$

$$P = 10 - 100 + 95.12 = \$5.12$$

Put-Call Parity Practice

Practice Problem: Calculate the price of the put option given:

- Stock price $(S) = \$150$
- Strike price $(K) = \$155$
- Risk-free rate $(r) = 3\%$
- Time to maturity $(T) = 6$ months
- Call option price $(C) = \$8$

Delta Hedging

Definition: Delta hedging is an options strategy that aims to reduce, or hedge, the directional risk associated with price movements in the underlying asset by adjusting the position in the underlying asset and its options.
Delta (Δ):

$$\Delta = \frac{\partial C}{\partial S}$$

- For calls, Δ ranges from 0 to 1.
- For puts, Δ ranges from -1 to 0.

Black-Scholes Formula for Δ (Call Option):

$$\Delta_C = N(d_1)$$

where $d_1 = \frac{\log(S/K)+(r+0.5\sigma^2)T}{\sigma\sqrt{T}}$

Strategy:

- Short $\Delta_C \times$ Number of Options shares of the underlying stock.
- Shorting shares means selling shares you do not currently own, expecting to buy them back at a lower price.

Purpose: This strategy ensures that gains in the option's value due to increases in the underlying stock's price are offset by losses in the shorted stock position, leading to a less volatile overall investment position.

Black-Scholes Formula for Δ (Put Option):

$$\Delta_P = -N(-d_1) = N(d_1) - 1$$

where $d_1 = \frac{\log(S/K)+(r+0.5\sigma^2)T}{\sigma\sqrt{T}}$

Strategy:

- Buy $|\Delta_P| \times$ Number of Options shares of the underlying stock.
- Buying shares (longing) involves purchasing shares with the expectation that their value will increase.

Purpose: Longing shares in the context of put options hedging ensures that losses due to a decrease in the stock price (which increases the value of the put option) are offset by gains in the longed stock position, stabilizing the overall investment value.

Understanding Shorting and Longing in Hedging

Shorting Shares:

- Investors sell shares they do not own by borrowing them.
- The goal is to buy back the shares at a lower price and return them to the lender, pocketing the difference as profit.

Longing Shares:

- Investors purchase shares outright with the belief that the share price will increase.
- Profits are made when the shares are sold at a higher price than they were bought.

Role in Hedging: Both strategies are used to counterbalance the directional risk associated with holding options, aiming to neutralize the financial impact of significant price swings in the underlying asset.

Delta Hedging Example: Call

Given:

- Stock price $(S) = \$100$
- Delta of call option $(\Delta_C) = 0.6$
- Number of options $= 100$

Objective: Construct a delta-neutral portfolio

Action: Short 60 shares of the stock (since $100 \times 0.6 = 60$)

Delta Hedging Example: Put

Given:

- Stock price $(S) = \$100$
- Delta of put option $(\Delta_P) = -0.4$
- Number of options $= 100$

Objective: Construct a delta-neutral portfolio
Action: Buy 40 shares of the stock (since $100 \times -0.4 = -40$, and we negate the negative sign by buying)

Delta Hedging Practice

Practice Problems:

1. With a delta of 0.5 for a call option, how many shares should be shorted for a delta-neutral position if you own 150 options?
2. With a delta of -0.3 for a put option, how many shares should be bought for a delta-neutral position if you own 200 options?