

Stat 346 Homework #3

1. For a commercial fire coverage

- In 2009, loss sizes follow a two-parameter Pareto distribution with parameters $\alpha=4$ and θ .
- In 2010, there is uniform inflation at rate r .
- The 65th percentile of loss size in 2010 equals the mean loss size in 2009.

Determine r . [.1107]

2. X follows a gamma distribution with parameters $\alpha=3.5$ and $\theta=100$. $Y = 1/X$
Evaluate $\text{Var}(Y)$ [$\frac{1}{93750}$]

3. X follows an exponential distribution with mean 50.
Determine the mean of X^4 . [150,000,000]

4. You are given the following:

- Losses in 1993 follow the density function

$$f(x) = 3x^{-4}, x \geq 1,$$

where x =losses in millions of dollars.

- Inflation of 10% impacts all claims uniformly from 1993 to 1994.

Determine the probability that losses in 1994 exceed 2.2 million. [.125]

5. Claim sizes in 2010 follow a lognormal distribution with parameters $\mu=4.5$ and $\sigma=2$. Claim sizes grow at 6% uniform inflation during 2011 and 2012.
Calculate $f(1000)$, the probability density function at 1000, of the claim size distribution in 2012.
[.000103]

6. Consider a random variable Z with the following moments:

- $E(Z) = 4.5$
- $E(Z^{0.5}) = 2$
- $E(Z^2) = 30$

Determine which distribution(s) from the following list best fit these moments:

- Lognormal distribution
- Exponential distribution
- Weibull distribution

7. For each of the following probability density functions, determine how many (integer) moments exist for the random variable:

a. $f(x) \propto e^{-2x}, \quad x > 0$

b. $f(y) = \frac{k}{y^3}, \quad y > 1$

c. $f(z) \propto z^{-4}e^{-3/z}, \quad z > 0$

8. Let X be a random variable following a Weibull distribution with parameters τ and θ . Prove that if you multiply X by a positive constant c , i.e., $Y = cX$, then Y follows a Weibull distribution with parameters τ and $c\theta$. Show the steps of your proof.