Ratemaking

Stat 346 - Short-term Actuarial Math

Setting Rates

- Setting rates involves determining the premium to charge policyholders.
- A common approach is to set rates equal to:

$$\text{Rate} = \text{Expected losses} + \text{Expenses} + \text{Profit}$$

- However, this approach can vary depending on the specific circumstances and regulatory requirements.

Permissible Loss Ratio

- The permissible loss ratio (PLR) is a key metric used in ratemaking.
- It is defined as:

$$\text{PLR} = 1 - \frac{\text{Expenses}}{\text{Premium}} - \frac{\text{Profit}}{\text{Premium}}$$

- PLR represents the proportion of premiums that can be used to cover losses.
- A higher PLR indicates a higher level of risk tolerance.

Alternate Approaches

- Rates can also be determined using alternate approaches:
  - Setting rates equal to losses divided by the permissible loss ratio:

$$\text{Rate} = \frac{\text{Losses}}{\text{PLR}}$$

  - Setting rates equal to:

$$\text{Rate} = \frac{\text{Losses} + F}{1 - V}$$

    where F represents fixed costs per exposure (e.g., salaries, overhead) and V represents costs that scale with premium (e.g., profit, contingencies).

Ratemaking Data

Policy Year *(Premium and Loss)*

- Considers all premium and loss transactions on policies that were written during a 12-month period, regardless of when the claim occurred or when it was reported, reserved or paid.
- Policy Year Premium and Exposures
  - *All premium and exposures earned on policies written during the year are considered part of that policy year's earned premium and earned exposures*
  - *Premium and exposures are not fixed until after the expiration date of all policies written during the year*
- Policy Year Paid Losses
  - *Payments made on those claims covered by policies written during the year*
- Policy Year Reported Losses
  - *Payments made plus case reserves only for those claims covered by policies written during the year*
- At the end of the policy year, losses change as additional claims occur, claims are paid or reserves are changed.

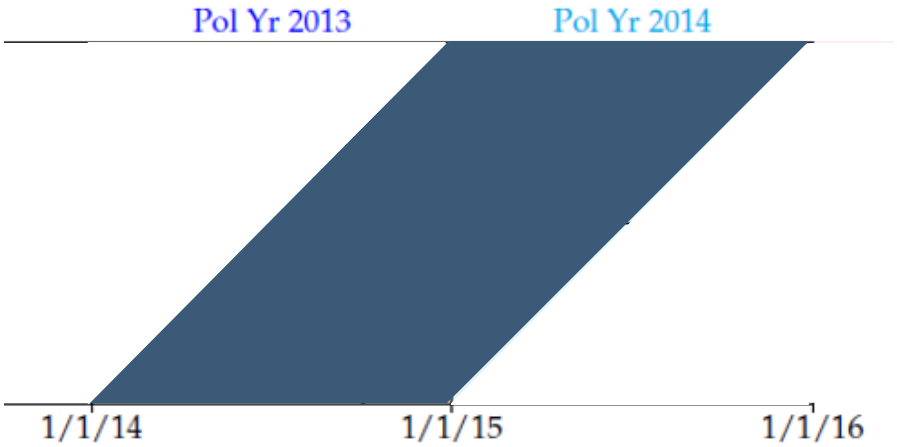
Ratemaking Data

Policy Year *(Premium and Loss)*

Advantage: Best match between losses and premium

- *Losses on policies written during the year are compared with premium earned on those same policies*

Disadvantage: Policy year exposures are not fully earned until after the end of the year, data takes longer to develop than both calendar and accident year.

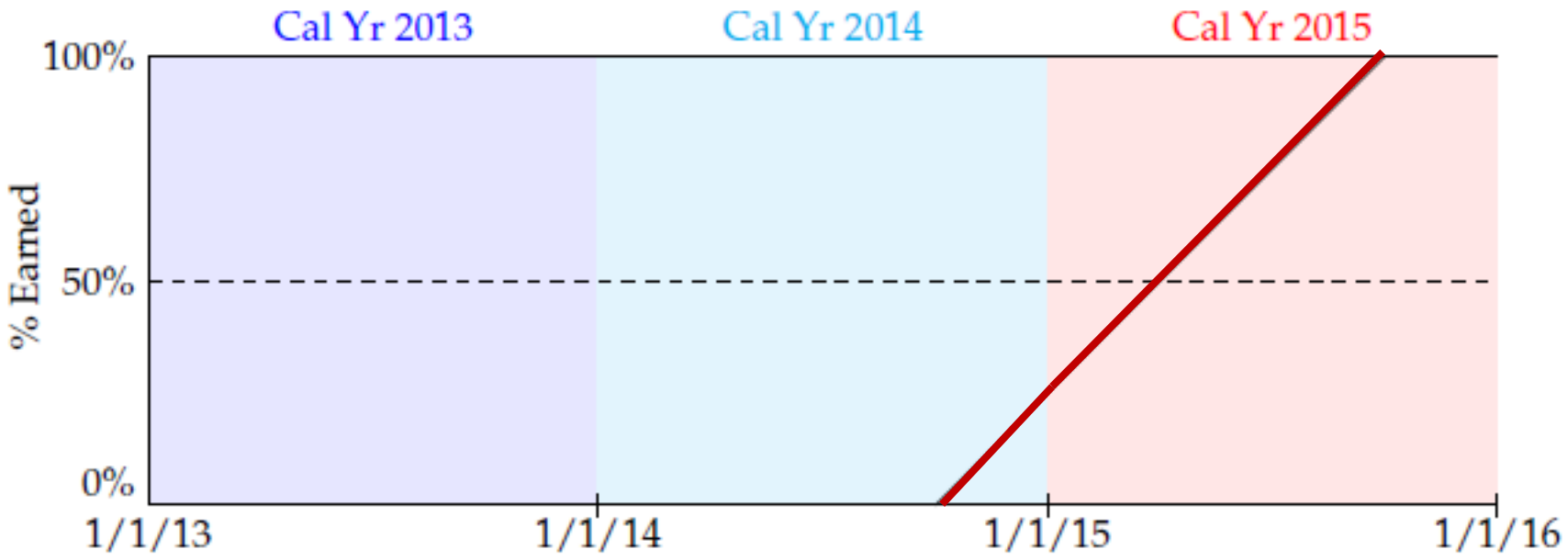


Earned Premium vs. Unearned Premium

- **Earned Premium**: Policyholder payments made in a given year.
- **Unearned Premium**: Payments from policies in that year but to be paid in later years.

Ratemaking Data

Calendar Year Earned Premium – Policy Effective 10/1/2014



EP in CY 2014 for this policy = WP * Proportion earned during 2014

= WP * (3/12) = WP * 0.25

EP in CY 2015 for this policy = WP – EP for CY 2014

Example Problem: One Year Policies

- Suppose a company has one year policies.
- For a given year, there are $24,000$ in written premiums.
- **Question**: How much is actually earned premium by the end of the year?

Example Problem Solution: One Year Policies

- Assuming policies are uniformly written over the year, earned premium is half of written premium.
- So, earned premium for the year is $12,000$.

Example Problem: Six Month Policies

- Now, let's consider six month policies.
- For the same year, with $24,000$ in written premiums.
- **Question**: How much is actually earned premium by the end of the year?

Example Problem Solution: Six Month Policies

- Policies written in the first half of the year contribute their entire premium to earned premium.
- For policies written in the second half, only half of the premium is earned.
- So, earned premium for the year is $12,000 + (12,000/2) = 18,000$.

Discussion

Given the following information answer the following questions. Assume 12-month policies.

Claim #	Policy Effective Date	Accident Date	Report Date	Transaction Date	Claim Status	Incremental Payment	Case Reserve as of Transaction Date
1	7/1/2013	9/9/2013	1/2/2014	1/7/2014	Open	$ -	$ 10,000
				6/8/2014	Open	$ 7,000	$ 3,000
				2/4/2015	Closed	$ 2,000	$ -
2	1/1/2014	3/3/2014	3/4/2014	3/5/2014	Open	$ 1,000	$ 5,000
				3/6/2014	Open	$ 2,000	$ 3,000
				3/10/2014	Closed	$ 1,000	$ -
3	4/1/2014	9/2/2014	10/15/2014	11/1/2014	Open	$ 5,000	$ 25,000
				2/2/2015	Open	$ 10,000	$ 15,000
				6/5/2015	Open	$ 10,000	$ 10,000

- *Calculate the calendar year 2014 incurred losses as of 12/31/2014*

  7+(3-0) + (1+2+1)+(0-0) + 5+(25-0) = $44,000
- *Calculate the policy year 2014 incurred losses as of 12/31/2014*

  (1+2+1)+0 + 5+25 = $34,000
- *Calculate the accident year 2014 incurred losses as of 12/31/2015*

  (1+2+1)+0 + (5+10+10)+10 = $39,000

Practice

Given the following claim activity on an annual policy effective 12/29/2006:

Claim Number	Transaction Date	Incremental Payment	Case Reserve as of Transaction Date	Transaction Description
1	December 31, 2006			Claim occurred
1	December 31, 2006		$1,000	Claim reported and reserve established
1	October 5, 2007		$10,000	Case reserve increased
1	July 5, 2008		$25,000	Case reserve increased
1	January 25, 2009	$30,000	$0	Settlement made, Payment made, Claim closed
2	April 1, 2007			Claim occurred
2	April 5, 2007		$25,000	Claim reported and reserve established
2	July 1, 2008		$0	Claim closed without payment

- Calculate 2008 CY incurred losses
- Calculate 2006 AY incurred losses evaluated as of 12/31/2007
- Calculate 2006 PY incurred losses evaluated as of 12/31/2007

Practice - *Answers*

Given the following claim activity on an annual policy effective 12/29/2006:

Claim Number	Transaction Date	Incremental Payment	Case Reserve as of Transaction Date	Transaction Description
1	December 31, 2006			Claim occurred
1	December 31, 2006		$1,000	Claim reported and reserve established
1	October 5, 2007		$10,000	Case reserve increased
1	July 5, 2008		$25,000	Case reserve increased
1	January 25, 2009	$30,000	$0	Settlement made, Payment made, Claim closed
2	April 1, 2007			Claim occurred
2	April 5, 2007		$25,000	Claim reported and reserve established
2	July 1, 2008		$0	Claim closed without payment

- Calculate 2008 CY incurred losses *0 + (25,000 – 10,000)+ (0 – 25,000) = -$10K*
- Calculate 2006 AY incurred losses evaluated as of 12/31/2007 *0 + 10K = $10K*
- Calculate 2006 PY incurred losses evaluated as of 12/31/2007 *0+10K+25K=$35K*

Practice

Find the following using the claim detail from the table below:

Note: Policy A was written on 7/1/2009 & Policy C was written on 7/1/2010

- *AY 2010 reported (incurred) claims @ 12/31/2010*
- *AY 2010 reported (incurred) claims @ 12/31/2011*
- *CY 2011 reported (incurred) claims*
- *PY 2009 reported (incurred) claims @ 12/31/2009*
- *PY 2010 reported (incurred) claims @ 12/31/2011*

Note that this is the Case Reserve as of the transaction date

Policy	Claim Number	Accident Date	Report Date	Transaction Date	Claim Status	Claim Chars	Loss Payment	Case Reserve
A	1	01/10/10	01/15/10	01/15/10	Open	…	$ -	$10,000
A	1	01/10/10	01/15/10	03/01/10	Open	…	$1,000	$9,000
A	1	01/10/10	01/15/10	05/01/10	Closed	…	$9,000	$ -
C	2	10/01/10	10/15/10	10/15/10	Open	…	$ -	$18,000
C	2	10/01/10	10/15/10	12/15/10	Open	…	$2,000	$17,000
C	2	10/01/10	10/15/10	03/01/11	Open	…	$7,000	$15,000
C	2	10/01/10	10/15/10	03/01/12	Closed	…	$15,000	$ -
C	3	02/01/11	02/15/11	02/15/11	Open	…	$ -	$15,000
C	3	02/01/11	02/15/11	12/01/11	Closed	…	$ -	$ -

Practice - *Answers*

Find the following using the claim detail from the table below:

Note: Policy A was written on 7/1/2009 & Policy C was written on 7/1/2010

- *AY 2010 reported (incurred) claims @ 12/31/2010* = *(1+9+2) + (0+17)* = ***$29K***
- *AY 2010 reported (incurred) claims @ 12/31/2011*
- *CY 2011 reported (incurred) claims*
- *PY 2009 reported (incurred) claims @ 12/31/2009*
- *PY 2010 reported (incurred) claims @ 12/31/2011*

Policy	Claim Number	Accident Date	Report Date	Transaction Date	Claim Status	Claim Chars	Loss Payment	Case Reserve
A	1	01/10/10	01/15/10	01/15/10	Open	…	$ -	$10,000
A	1	01/10/10	01/15/10	03/01/10	Open	…	$1,000	$9,000
A	1	01/10/10	01/15/10	05/01/10	Closed	…	$9,000	$ -
C	2	10/01/10	10/15/10	10/15/10	Open	…	$ -	$18,000
C	2	10/01/10	10/15/10	12/15/10	Open	…	$2,000	$17,000
C	2	10/01/10	10/15/10	03/01/11	Open	…	$7,000	$15,000
C	2	10/01/10	10/15/10	03/01/12	Closed	…	$15,000	$ -
C	3	02/01/11	02/15/11	02/15/11	Open	…	$ -	$15,000
C	3	02/01/11	02/15/11	12/01/11	Closed	…	$ -	$ -

Practice - *Answers*

Find the following using the claim detail from the table below:

Note: Policy A was written on 7/1/2009 & Policy C was written on 7/1/2010

- *AY 2010 reported (incurred) claims @ 12/31/2010*
- *AY 2010 reported (incurred) claims @ 12/31/2011* = *(1+9+2+7) + (0+15)* = ***$34K***
- *CY 2011 reported (incurred) claims*
- *PY 2009 reported (incurred) claims @ 12/31/2009*
- *PY 2010 reported (incurred) claims @ 12/31/2011*

Policy	Claim Number	Accident Date	Report Date	Transaction Date	Claim Status	Claim Chars	Loss Payment	Case Reserve
A	1	01/10/10	01/15/10	01/15/10	Open	…	$ -	$10,000
A	1	01/10/10	01/15/10	03/01/10	Open	…	$1,000	$9,000
A	1	01/10/10	01/15/10	05/01/10	Closed	…	$9,000	$ -
C	2	10/01/10	10/15/10	10/15/10	Open	…	$ -	$18,000
C	2	10/01/10	10/15/10	12/15/10	Open	…	$2,000	$17,000
C	2	10/01/10	10/15/10	03/01/11	Open	…	$7,000	$15,000
C	2	10/01/10	10/15/10	03/01/12	Closed	…	$15,000	$ -
C	3	02/01/11	02/15/11	02/15/11	Open	…	$ -	$15,000
C	3	02/01/11	02/15/11	12/01/11	Closed	…	$ -	$ -

Practice - *Answers*

Find the following using the claim detail from the table below:

Note: Policy A was written on 7/1/2009 & Policy C was written on 7/1/2010

- *AY 2010 reported (incurred) claims @ 12/31/2010*
- *AY 2010 reported (incurred) claims @ 12/31/2011*
- *CY 2011 reported (incurred) claims* = *(7+0+0) + (15-17 + 0-0)* = ***$5K***
- *PY 2009 reported (incurred) claims @ 12/31/2009*
- *PY 2010 reported (incurred) claims @ 12/31/2011*

Policy	Claim Number	Accident Date	Report Date	Transaction Date	Claim Status	Claim Chars	Loss Payment	Case Reserve
A	1	01/10/10	01/15/10	01/15/10	Open	…	$ -	$10,000
A	1	01/10/10	01/15/10	03/01/10	Open	…	$1,000	$9,000
A	1	01/10/10	01/15/10	05/01/10	Closed	…	$9,000	$ -
C	2	10/01/10	10/15/10	10/15/10	Open	…	$ -	$18,000
C	2	10/01/10	10/15/10	12/15/10	Open	…	$2,000	$17,000
C	2	10/01/10	10/15/10	03/01/11	Open	…	$7,000	$15,000
C	2	10/01/10	10/15/10	03/01/12	Closed	…	$15,000	$ -
C	3	02/01/11	02/15/11	02/15/11	Open	…	$ -	$15,000
C	3	02/01/11	02/15/11	12/01/11	Closed	…	$ -	$ -

Practice - *Answers*

Find the following using the claim detail from the table below:

Note: Policy A was written on 7/1/2009 & Policy C was written on 7/1/2010

- *AY 2010 reported (incurred) claims @ 12/31/2010*
- *AY 2010 reported (incurred) claims @ 12/31/2011*
- *CY 2011 reported (incurred) claims*
- *PY 2009 reported (incurred) claims @ 12/31/2009* = ***$0***
- *PY 2010 reported (incurred) claims @ 12/31/2011*

Policy	Claim Number	Accident Date	Report Date	Transaction Date	Claim Status	Claim Chars	Loss Payment	Case Reserve
A	1	01/10/10	01/15/10	01/15/10	Open	…	$ -	$10,000
A	1	01/10/10	01/15/10	03/01/10	Open	…	$1,000	$9,000
A	1	01/10/10	01/15/10	05/01/10	Closed	…	$9,000	$ -
C	2	10/01/10	10/15/10	10/15/10	Open	…	$ -	$18,000
C	2	10/01/10	10/15/10	12/15/10	Open	…	$2,000	$17,000
C	2	10/01/10	10/15/10	03/01/11	Open	…	$7,000	$15,000
C	2	10/01/10	10/15/10	03/01/12	Closed	…	$15,000	$ -
C	3	02/01/11	02/15/11	02/15/11	Open	…	$ -	$15,000
C	3	02/01/11	02/15/11	12/01/11	Closed	…	$ -	$ -

Practice - *Answers*

Find the following using the claim detail from the table below:

Note: Policy A was written on 7/1/2009 & Policy C was written on 7/1/2010

- *AY 2010 reported (incurred) claims @ 12/31/2010*
- *AY 2010 reported (incurred) claims @ 12/31/2011*
- *CY 2011 reported (incurred) claims*
- *PY 2009 reported (incurred) claims @ 12/31/2009*
- *PY 2010 reported (incurred) claims @ 12/31/2011* = *(2+7+0+0) + (15+0)* = ***$24K***

Policy	Claim Number	Accident Date	Report Date	Transaction Date	Claim Status	Claim Chars	Loss Payment	Case Reserve
A	1	01/10/10	01/15/10	01/15/10	Open	…	$ -	$10,000
A	1	01/10/10	01/15/10	03/01/10	Open	…	$1,000	$9,000
A	1	01/10/10	01/15/10	05/01/10	Closed	…	$9,000	$ -
C	2	10/01/10	10/15/10	10/15/10	Open	…	$ -	$18,000
C	2	10/01/10	10/15/10	12/15/10	Open	…	$2,000	$17,000
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C	3	02/01/11	02/15/11	02/15/11	Open	…	$ -	$15,000
C	3	02/01/11	02/15/11	12/01/11	Closed		$ -	$ -

Premium Trends

- Premiums often increase over time.
- This increase is typically exponential growth.
- One way to estimate the growth trend from data is using log-linear regression.

Projected Loss Cost

- Projected loss cost can be calculated using the formula:

$$\text{Projected Loss Cost} = \text{Experienced Loss Cost} \times \exp(\delta t)$$

- Where δ represents rate of growth and t is the time period for projection.

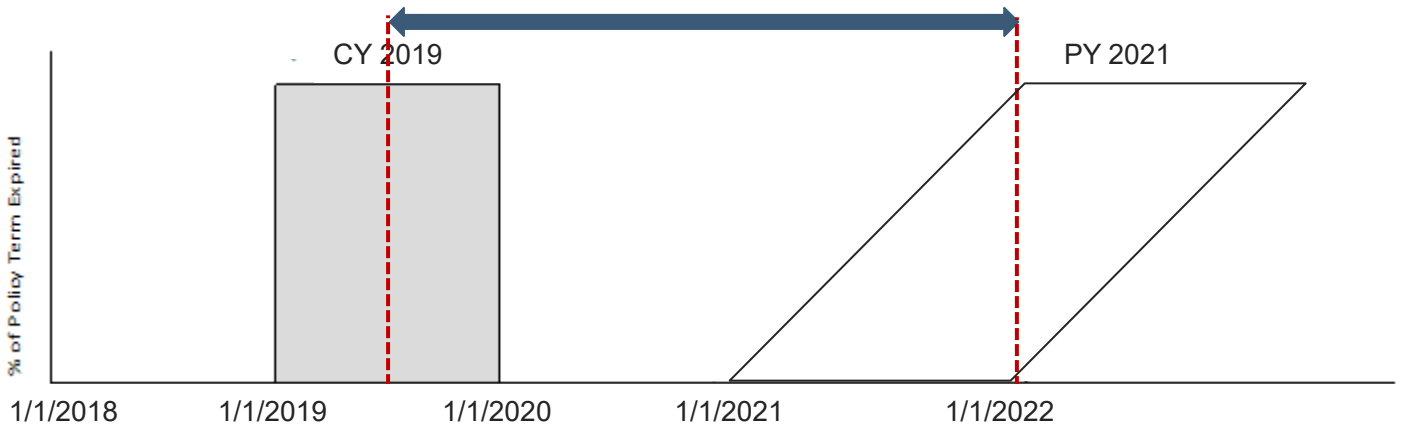
Choosing Time Points for Projection

- When projecting loss costs, it's important to choose appropriate time points.
- For data from accident years, use the average of the years.
- When projecting for a specific policy, consider that you can buy a policy at any time during the year (assumed time period for a product to be available). Person A buys a policy at the beginning of the policy year, so most of Person A's losses will occur during that same year. Person B buys the policy towards the end of the year, so most of that person's losses will actually occur the following year.
- Use a parallelogram approach
- A few specific common cases
  - For 1 year policies, project at the end of the year.
  - For 6 month policies in force for a year, project at 3/4 through the year.

Premium

One-Step Trending

- Example 1: Annual Policies, Adjusting CY2019 EP for premium trend



- Also can be determined based on the average date of premium earned in the experience period to the average date of premium earned in the projected period
  - Average earned date for CY 2019 = 6/30/2019
  - Average earned date for PY 2021 = 12/31/2021
  - Trend length is from 6/30/2019 to 12/31/2021 (**2.5 years**)
  - Apply the following trend to CY 2019 EP: *(1 + premium trend)^2.5*

Example

- Losses for 2014 were \$2,100 per exposure unit and in 2015 they were \$2,200 per exposure unit.
- It is determined that losses grow at an exponential rate with $\delta = 0.05$.
- We wish to project losses for a new 1-year policy that starts on November 1st 2016.

Question: What would the projected losses be for this policy when using 2014 data? What about using 2015 data?

- **Answer for 2014:** For 2014, the average date of accidents was midyear. We start the clock on July 1, 2014. For the new policy, the average loss will be on November 1st, 2017. Hence, $t = 3\frac{1}{3}$. The projected loss is \$2,480.86.
- **Answer for 2015:** For 2015, $t = 2\frac{1}{3}$ and projected losses will be \$2,472.24.

Example

Question: Instead of a one year policy, they want to make an 18 month policy and keep the same rate for 2 years instead of a standard one year. How would we project losses from 2015?

- **Answer:** Again the midpoint for 2015 data is July 1st, 2015. The midpoint for the policy period is 21 months into the policy (draw parallelogram). This would be August 1, 2018. Hence $t = 3\frac{1}{12}$ and the projection would be $2200e^{.05 \times 3.167} = 2577.43$ per year. Note that since this is two years for the policy, we would actually expect this loss to be doubled.

Example

- You work for a dental insurance company. You find that Losses for 2006 were \$150 per exposure unit.
- You are projecting losses use an exponential trend model.
- A new 8-month policy goes into effect February 1, 2008 for for the following year.
- You project losses for the policy to be \$182.

What δ did you use?

Trends versus rate changes

When talking about trends, you typically are talking about rising supply or medical costs, inflation, patterns in behaviors. These are things external to the insurance company. Internally, rates can be changed that will affect earned premiums. We treat these two things separately.

Premium

Parallelogram Method

- Objective: Replace the average rate level for a given historical year with the current rate level. Done through the following steps
    1. Determine the timing and amount of the rate changes during and after the experience period and group the policies into rate level groups according to the timing of each rate change.
    2. Calculate the portion of the year's earned premium corresponding to each rate level group
    3. Calculate the cumulative rate level index for each rate level group
    4. Calculate the weighted average cumulative rate level index for each year
    5. Calculate the on-level factor as the ratio of the current cumulative rate level index and the average cumulative rate level index for the appropriate year
    6. Apply the on-level factor to the earned premium for the appropriate year

Premium

Example - Standard Parallelogram Method

Step 1 ~ *Determine the timing and amount of the rate changes during and after the experience period and group the policies into rate level groups according to the timing of each rate change.*

- Effective dates and overall rate level changes for the different rate level groups:

Rate Level Group	Effective Date	Overall Average Rate Change
1	Initial	-
2	7/1/2018	5%
3	1/1/2019	10%
4	1/1/2020	-1%

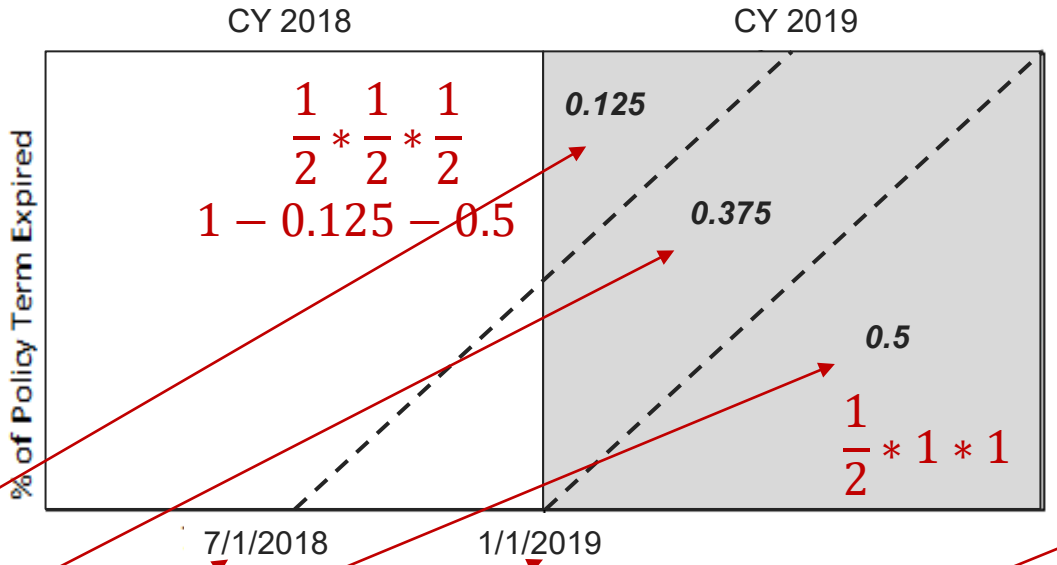
- These are annual policies with rate changes applied on or after the effective date of the policies

Premium

Example *- Standard Parallelogram Method*

Step 2 ~ *Calculate the portion of the year's earned premium corresponding to each rate level group*

- Easiest to see in a graphical format:



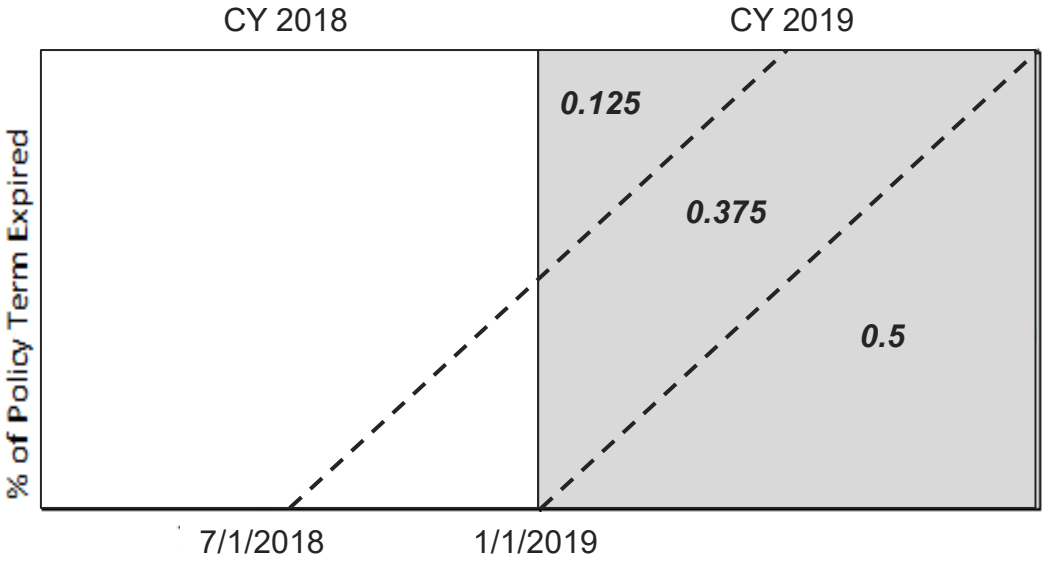
Rate Level Group	Effective Date	Overall Average Rate Change
1	Initial	-
2	7/1/2018	5%
3	1/1/2019	10%
4	1/1/2020	-1%

Annual policies

Premium

Example *- Standard Parallelogram Method*

Step 3 ~ *Calculate the cumulative rate level index for each rate level group*



Rate Level Group	Effective Date	Overall Average Rate Change	Rate Level Index	Cumulative Rate Level Index
1	Initial	-	1.00	1.0000
2	7/1/2018	5%	1.05	1.0500
3	1/1/2019	10%	1.10	1.1550
4	1/1/2020	-1%	0.99	1.1435

*= 1.0 * 1.05 * 1.1*

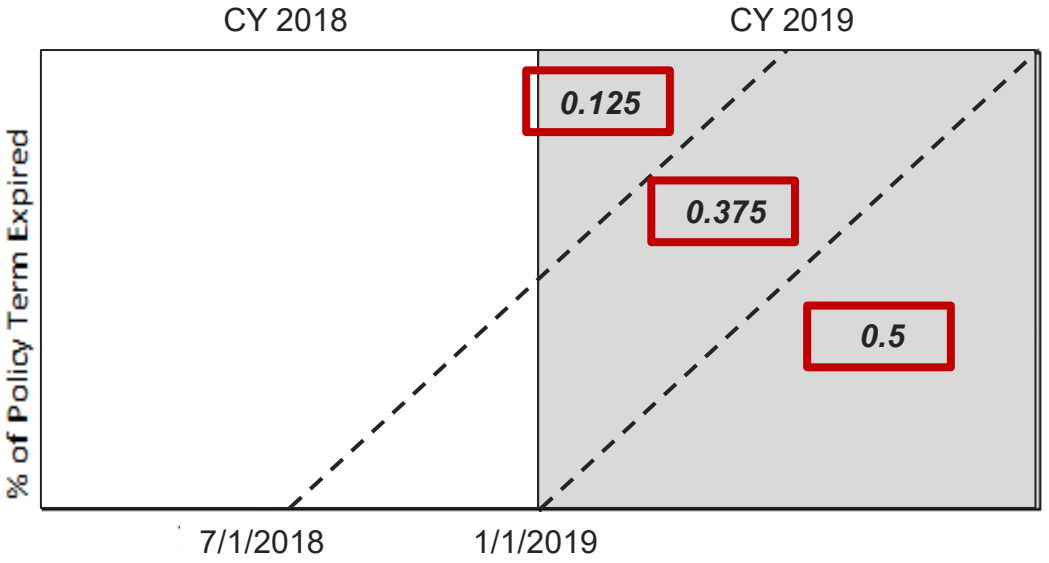
Premium

Example *- Standard Parallelogram Method*

Step 4 ~ *Calculate the weighted average cumulative rate level index for each year*

Avg rate level index for CY2019

*1.000 * 0.125*
*+ 1.050 * 0.375*
*+ 1.155 * 0.500*
= 1.0963



Rate Level Group	Effective Date	Overall Average Rate Change	Rate Level Index	Cumulative Rate Level Index
1	Initial	-	1.00	1.0000
2	7/1/2018	5%	1.05	1.0500
3	1/1/2019	10%	1.10	1.1550
4	1/1/2020	-1%	0.99	~~1.1435~~

Premium

Example - Standard Parallelogram Method

Step 5 ~ *Calculate the on-level factor as the ratio of the current cumulative rate level index and the average cumulative rate level index for the appropriate year*

Avg rate level index for CY2019

*1.000 * 0.125*
*+ 1.050 * 0.375*
*+ 1.155 * 0.500*
*= **1.0963***

- On Level Factor for Historical Period
  = Current Cumulative Rate Level Index /
  Avg Rate Level Index for Historical Period
  = 1.1435 / 1.0963
  = **1.0431**

Rate Level Group	Effective Date	Overall Average Rate Change	Rate Level Index	Cumulative Rate Level Index
1	Initial	-	1.00	1.0000
2	7/1/2018	5%	1.05	1.0500
3	1/1/2019	10%	1.10	1.1550
4	1/1/2020	-1%	0.99	1.1435

Premium

Example *- Standard Parallelogram Method*

Step 6 ~ *Apply the on-level factor to the earned premium for the appropriate year*

On Level Factor for Historical Period
= **1.0431**

CY EP @ Current Rate Level = CY2019 EP x 1.0431

Rate Level Group	Effective Date	Overall Average Rate Change	Rate Level Index	Cumulative Rate Level Index
1	Initial	-	1.00	1.0000
2	7/1/2018	5%	1.05	1.0500
3	1/1/2019	10%	1.10	1.1550
4	1/1/2020	-1%	0.99	1.1435

Practice

Mary, an actuary for Present Premium Insurance Company is trying to get the company's historical premium to the current rate level. She has collected the following information:

- *Annual policies and assume uniform writing of policies throughout the year*
- *There have been the following rate changes (effective on go-forward writings)*
- *Rate Increase of 10% on 7/1/2018*
- *Rate increase of 15% on 1/1/2019*

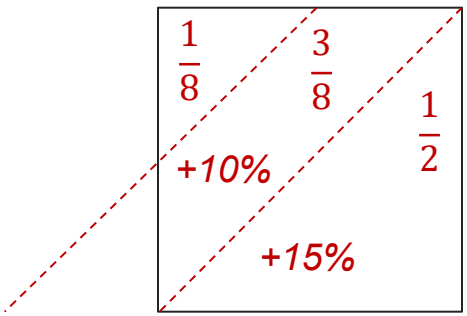
What on-level factor should be applied to the 2019 Calendar Year Earned Premium in order to bring it to the current rate level?

Practice - *Answer*

Mary, an actuary for Present Premium Insurance Company is trying to get the company's historical premium to the current rate level. She has collected the following information:

- *Annual policies and assume uniform writing of policies throughout the year*
- *There have been the following rate changes (effective on go-forward writings)*
- *Rate Increase of 10% on 7/1/2018*
- *Rate increase of 15% on 1/1/2019*

What on-level factor should be applied to the 2019 Calendar Year Earned Premium in order to bring it to the current rate level?



$$\frac{1}{8}(1.0) + \frac{3}{8}(1.0 * 1.1) + \frac{1}{2}(1.0 * 1.1 * 1.15) = 1.17$$

$$\frac{(1.0*1.1*1.15)}{1.17} = 1.08$$

Practice

Mary, an actuary for Present Premium Insurance Company is trying to get the company's historical premium to the current rate level. She has collected the following information:

- *Semi-annual policies and assume uniform writing of policies throughout the year*
- *There have been the following rate changes (effective on go-forward writings)*
- *Rate Increase of 10% on 7/1/2018*
- *Rate increase of 15% on 1/1/2019*

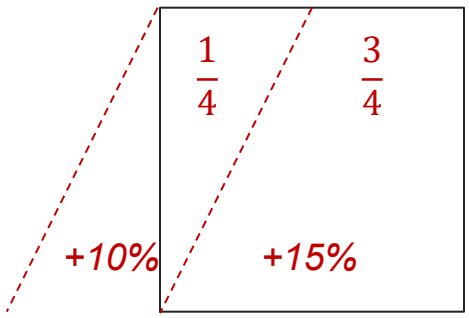
What on-level factor should be applied to the 2019 Calendar Year Earned Premium in order to bring it to the current rate level?

Practice - *Answer*

Mary, an actuary for Present Premium Insurance Company is trying to get the company's historical premium to the current rate level. She has collected the following information:

- *Semi-annual policies and assume uniform writing of policies throughout the year*
- *There have been the following rate changes (effective on go-forward writings)*
- *Rate Increase of 10% on 7/1/2018*
- *Rate increase of 15% on 1/1/2019*

What on-level factor should be applied to the 2019 Calendar Year Earned Premium in order to bring it to the current rate level?



$$\frac{1}{4}(1.1) + \frac{3}{4}(1.1 * 1.15) = 1.22 \rightarrow \frac{(1.1 * 1.15)}{1.22} = 1.04$$

or

$$\frac{1}{4}(1.0) + \frac{3}{4}(1.0 * 1.15) = 1.11 \rightarrow \frac{(1.0 * 1.15)}{1.11} = 1.04$$