

Stat 346: Homework 1

1. Let X be continuous with pdf:

$$f(x) = 3x^2, 0 < x < 1.$$

Find:

- (a) $E(X)$
- (b) $Var(X)$
- (c) $E(X^k)$
- (d) $E(3X - 5X^2 + 1)$

$$Answers : a = \frac{3}{4} \quad b = \frac{3}{80} \quad c = \frac{3}{k+3} \quad d = \frac{1}{4}$$

2. Show that the following relationship holds:

$$E(X) = e(d)S(d) + E(X \wedge d).$$

3. Assume random variable X has the following pdf,

$$f(x) = (1 + 2x^2)e^{-2x}, x \geq 0.$$

Determine the following:

- (a) $S(x)$
- (b) $e(x)$
- (c) $\lim_{x \rightarrow \infty} e(x)$

$$Answers : a = e^{-2x}(x^2 + x + 1) \quad b = \frac{\frac{1}{2}x^2 + x + 1}{x^2 + x + 1} \quad c = \frac{1}{2}$$

4. For a random variable X you are given that

- (i) The mean is 4.
- (ii) The variance is 2.
- (iii) The raw third moment is 3.

Determine the coefficient of skewness of X . [-30.052]

5. A Pareto distribution has parameters $\alpha = 4$ and $\theta = 2$. Determine its skewness. [7.071]

6. Claim size for an insurance coverage follows a lognormal distribution with mean 1000 and median 800. Determine the probability that a claim will be greater than 1200. [0.2709]
7. Let each $Y_i, i = 1, \dots, n$ follows a $Gam(2, 4)$ distribution and are independent.
 - (a) Derive, using the pdf, $E(Y_i^4)$. [30720]
 - (b) Further assume that $Z = \sum_{i=1}^n Y_i$, find $Var(Z)$. [32n]
8. Using the distribution function

$$F_X(x) = \begin{cases} 0 & \text{for } x < 0 \\ .001x^3 & \text{for } 0 \leq x < 10 \\ 1 & \text{for } x \geq 10 \end{cases}$$

determine the difference between the mean excess loss function at $d = 1$ and the mean of the left censored and shifted random variable with $d = 1$.