Forwards and Options

DM Chapter 2

Options

An **option** is a contract that gives the owner of the option the *right* but not the *obligation* to buy or sell an asset if they wish. This can be thought of as a contract where the transaction takes place only if it is profitable for the owner of the contract.

Ownership of an option costs money upfront, called a **premium**.

Options

Option terminology:

- Premium - The price to buy an option (not the asset)
- Strike Price - The agreed upon price that the asset is bought or sold for.
- Exercise - If it is profitable for the owner of the option, they will make the transaction happen. This is called exercising the option.
- Expiration date - the date after which the the option may no longer be exercised.
- European option - This type of option can only be exercised at the expiration date
- American Option - This type of option can be exercised at any point prior to the expiration date
- Spot price - The price of the underlying asset at any given time

We will mainly deal with European options. If I don't specify, I mean European option.

Options

A **call option** is the right to buy an asset at expiration if the price is right

A **put option** is the right to sell an asset if the price is right

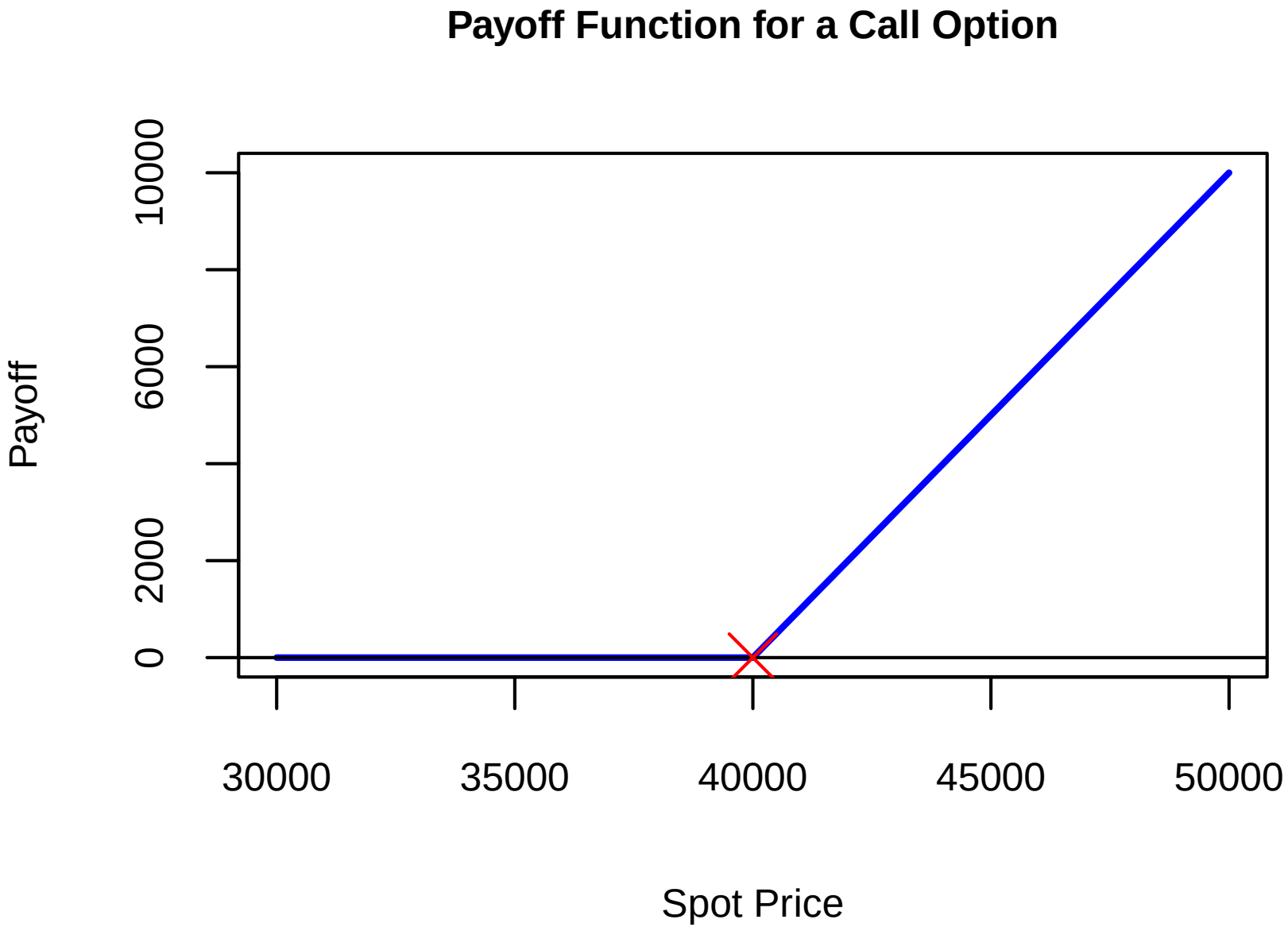
Options

Suppose the car company purchased an option to buy steel at \$400/ton at expiration.

- If the spot price of steel at expiration was more than \$400/ton, the car company would exercise the option and pay for the steel
- If the spot price of steel at expiration was less than \$400/ton, the car company would *NOT* exercise the option and make a payoff of 0

The payoff of a call option is $\max(0, \text{Spot Price} - \text{Strike Price})$

Options


Payoff Function for a Call Option
Payoff
0
2000
6000
10000
30000
35000
40000
45000
50000
Spot Price

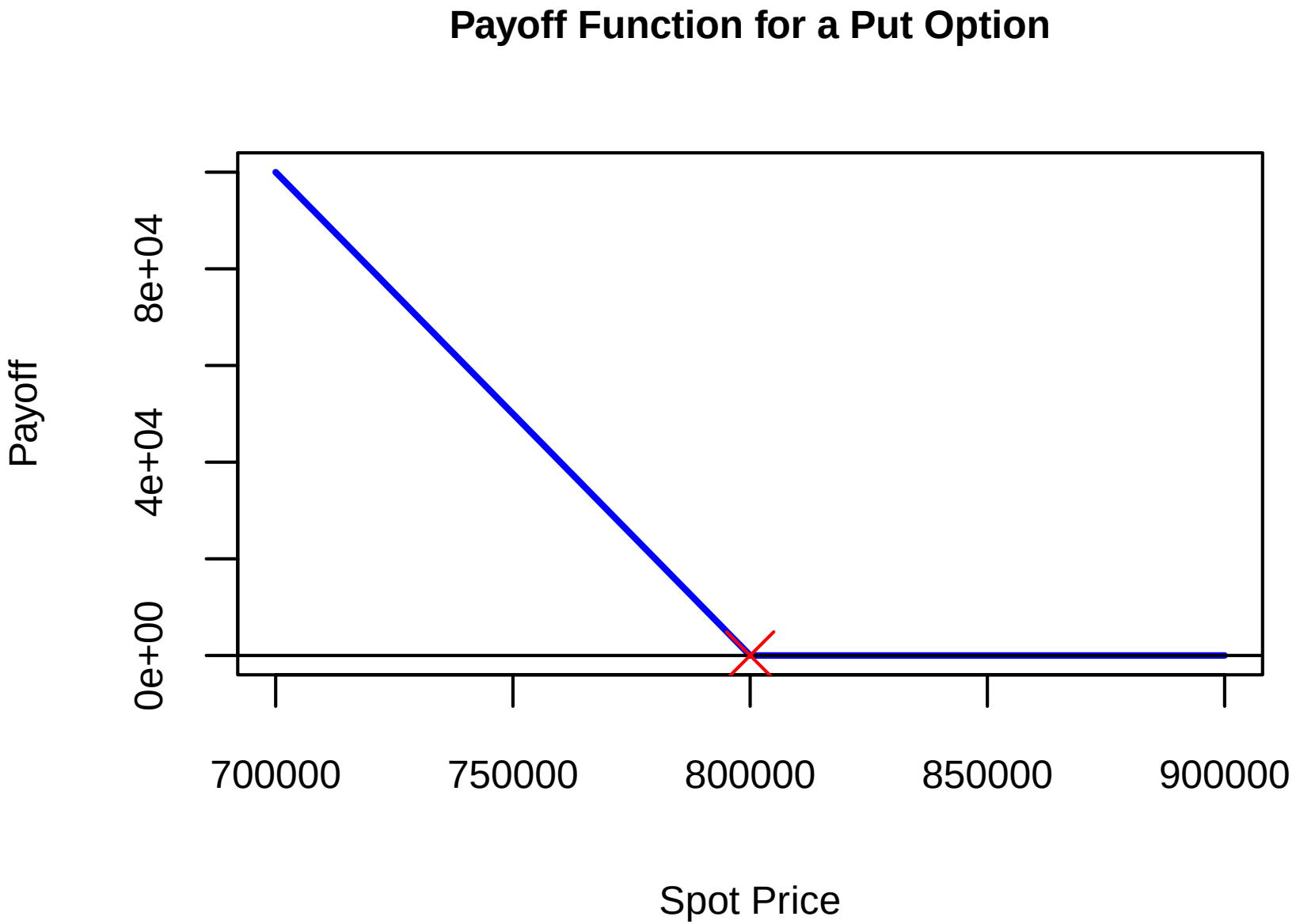
Options

Suppose that same car company purchased a put option to sell a fleet of cars in 1 year for $800,000.

- If the price of the fleet of cars at expiration is more than 800,000, the car company will *NOT* exercise the put option and have a payoff of 0.
- If the price of the fleet of cars at expiration is less than 800,000, the car company will exercise the put option sell the fleet of cars.

The payoff of a put option is $\max(0, \text{Strike Price} - \text{Spot Price})$

Options


Payoff Function for a Put Option
Payoff
0e+00
4e+04
8e+04
700000
750000
800000
850000
900000
Spot Price

Options

Options will cost a premium at time 0. While payoff doesn't consider the premium, the **profit** will consider the premium. For both call and put options

$$\text{Profit} = \text{Payoff} - \text{Future Value of Premium}$$

The logic here is that the money spent on the premium is either borrowed and accrues interest or can't be invested and the loss must be accumulated forward.

Options

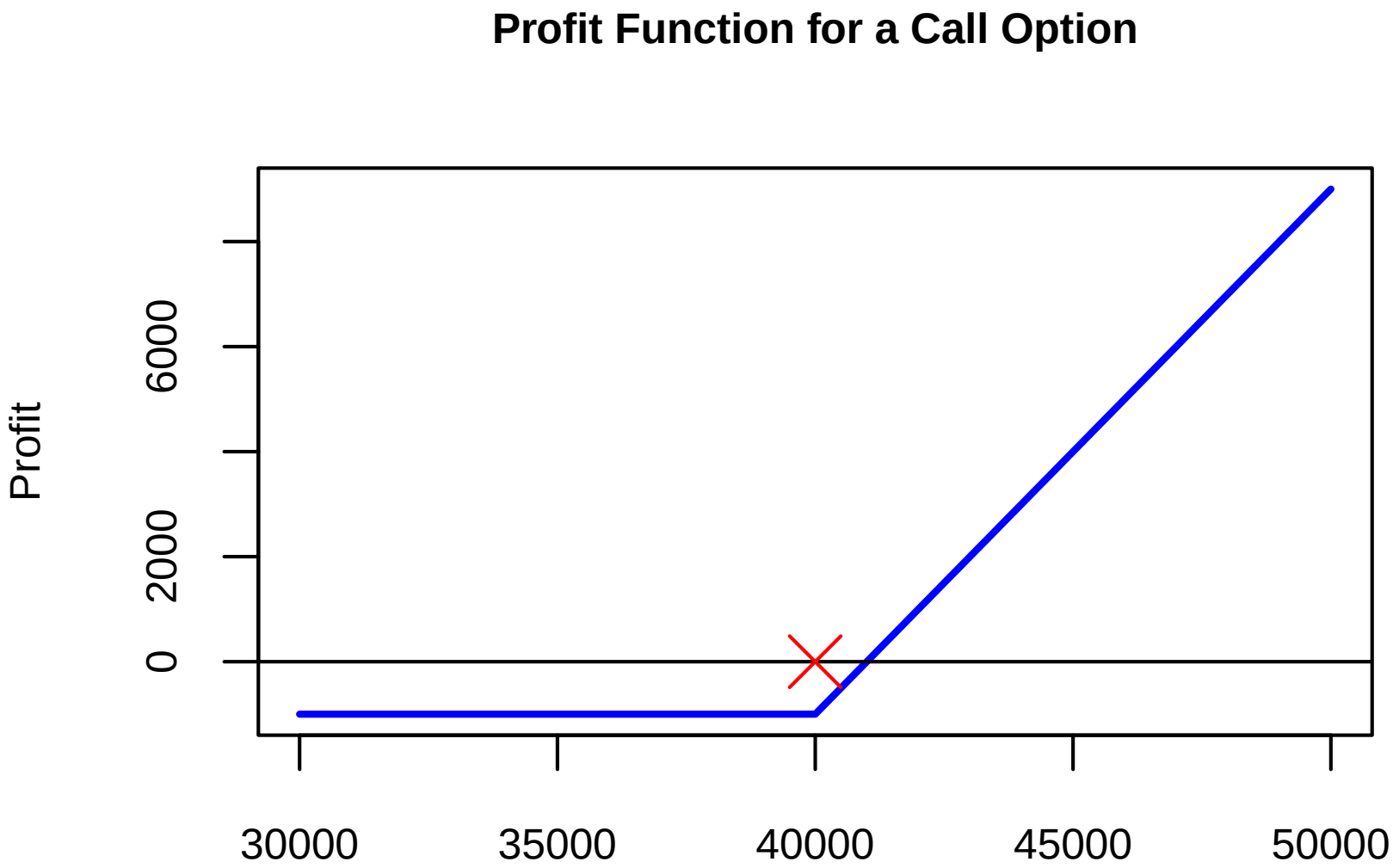
For this class, the interest rate is almost always *continuous*. So if P is the premium and the expiration time is t and the continuous interest rate is r, then the future value of the premium is

$$Pe^{rt}$$

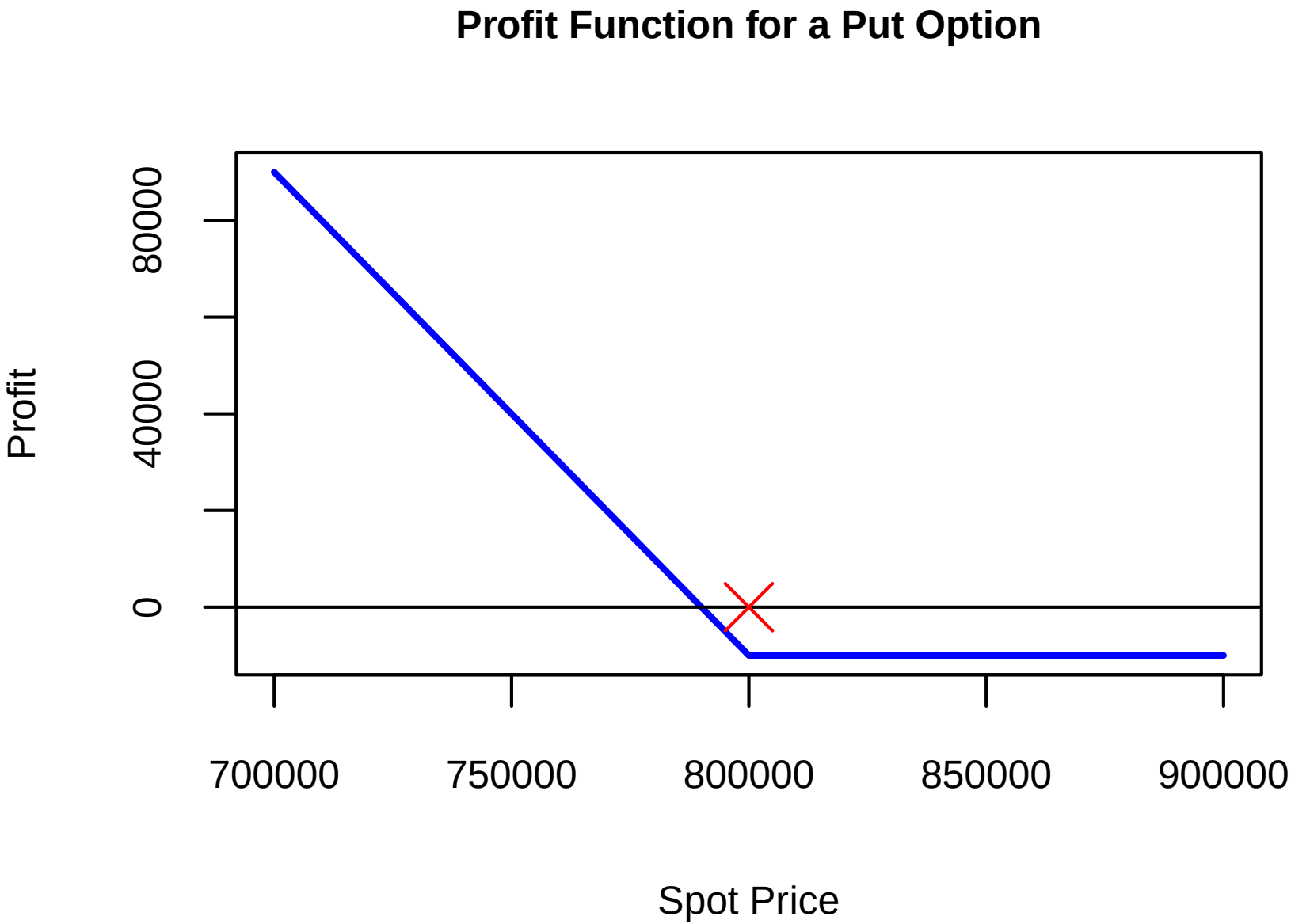
The time, t, is often measured in years. Having a premium doesn't change whether or not you exercise.

Options

Plotting profit as a function of spot price at expiration for an option must account for the premium.



Options


Profit Function for a Put Option
Profit
80000
40000
0
700000
750000
800000
850000
900000
Spot Price

Practice

A call option is purchased for \$12 to buy a stock with a strike price of 1000, an expiration date of 6 months. The risk free continuous interest rate is r = 4%. Determine the payoff and profit at expiration for the following spot prices at expiration:

- 950
- 1000
- 1010
- 1050

Practice

A put option is purchased for \$3 to sell a stock with a strike price of 40, with an expiration date of 4 months. The risk free continuous interest rate is $r = 3\%$. Determine the spot price at expiration that would make this position have 0 profit.

Moneyness

An option can be classified by its moneyness:

- In-the-money means that if the option was exercised immediately, it would make money
- Out-of-the-money means that if the option was exercised immediately, it would lose money
- At-the-money means that the option would break even if exercised immediately

The current spot price (not spot price at expiration) is compared to the strike price to determine the moneyness