Stat 346 Homework #4

1. Assume that the annual losses on each individual policy are either 0 with probability 0.2 or follow an exponential distribution with a mean of 1000 with probability 0.8. There are 2500 independent, identically distributed policyholders in a book of business.
   (a) What is the expected total loss from the book of business? [2,000,000]
   (b) What is the variance of that total loss? [2,400,000,000]
   (c) Using the normal approximation, what is the probability that the total loss is greater than 2,100,000 (the true probability is approximately 0.022)? [.021]
   (d) Comment on how close you expect the normal approximation and the true probability to be in this example

2. Given that $X_i \sim N(\mu_i, \sigma_i^2)$ for $i = 1, 2, \ldots, n$ are independent normal random variables, prove that $S = \sum_{i=1}^{n} X_i$ is also normally distributed. Use the moment generating function (MGF) of the normal distribution, which is $M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$, in your proof.

3. Consider a portfolio of 1000 independent policies. Each policy has a 10% chance of incurring a loss, which, when it occurs, follows a gamma distribution with $\alpha = 3$ and $\theta = 500$. Using a normal approximation, what is $\text{TVaR}_{.95}$ for aggregate losses? [184,361.35]

4. Let $S = \sum_{i=1}^{N} X_i$ be the aggregate loss, where N is the number of claims following a Poisson distribution with parameter $\lambda = 2$, and X_i are i.i.d. exponential random variables with mean 1000.
   (a) Compute the mean and variance of S.
   (b) Would this be a situation where you would use a normal approximation?

5. The cumulative loss distribution for a risk X_i is

$$F_i(x) = 1 - \frac{10^9}{(x + 10^3)^3} \qquad x > 0$$

   for all i. Assume $S = \sum_{i=1}^{N} X_i$ where N follows a negative binomial distribution with distribution with parameter $r = 4$ and $\beta = 2.5$. Calculate the mean and variance of S.

6. Consider a risk model where N, the number of claims, and X_i, the amount of each claim, are both discrete random variables. The probability distribution of N and X are given in the following tables:

n	$Pr(N = n)$
0	0.5
1	0.3
2	0.2

x	$Pr(X = x)$
100	0.5
200	0.3
300	0.2

   Let $S = \sum_{i=1}^{N} X_i$ represent the total claim amount. Calculate $Pr(S < 500)$.