Credibility

Chapters 17-19

Stat 346 - Short-term Actuarial Math

Why Credibility?

You purchase an auto insurance policy and it costs \$150. That price is mainly the expected cost of a policyholder with your characteristics (car make and model, age, driving record, etc.). After three years, you have no claims. You call the insurer to explain how you are better than the average driver with your characteristics, so should be charged less.

Is your record a sign of your good driving, or just random chance?

Full Credibility

If the sample mean $\bar{X}_n$ is a stable estimator of ξ, the true mean, then we should only use $\bar{X}_n$. We say the data is fully credible in this situation. More specifically,

$$Pr(-r\xi \leq \bar{X} - \xi \leq r\xi) \geq p$$

which says that the difference between the estimated mean and the true mean is proportionally small, with high probability (common choices of p and r are 0.9 and 0.05, respectively)

Full Credibility cont.

We can restate that previous formula as

$$\Pr\left(\left|\frac{\bar{X}-\xi}{\sigma/\sqrt{n}}\right| \leq \frac{r\xi\sqrt{n}}{\sigma}\right) \geq p$$

and find the minimum value

$$y_p = \inf_y \left\{\Pr\left(\left|\frac{\bar{X}-\xi}{\sigma/\sqrt{n}}\right| \leq y\right) \geq p\right\}$$

If it is continuous then

$$\Pr\left(\left|\frac{\bar{X}-\xi}{\sigma/\sqrt{n}}\right| \leq y_p\right) = p$$

Therefore, for full credibility, $y_p \leq r\xi\sqrt{n}/\sigma$

Full Credibility cont.

For full credibility,

$$y_p \leq \frac{r\xi\sqrt{n}}{\sigma} \to n \geq \left(\frac{y_p\sigma}{r\xi}\right)^2$$

In many cases, we can assume that

$$\frac{\bar{X} - \xi}{\sigma/\sqrt{n}}$$

follows a standard normal distribution. And

$$p = \Pr(|Z| \leq y_p) = 2\Phi(y_p) - 1$$

Therefore y_p is the $(1+p)/2$ percentile of the standard normal distribution. This is also called Limited Fluctuation Credibility.

Central Limit Example

Suppose 10 past years' losses are available from a policyholder

0	0	0	0	0	0	253	398	439	756

The sample mean is used to estimate $\xi = E(X_j)$. Determine the full credibility standard with $r = 0.05$ and $p = 0.9$.

$$\begin{aligned} n &\geq \left(\frac{y_p \sigma}{r\xi}\right)^2 \\ &= \left(\frac{(1.645)(267.89)}{(0.05)(184.6)}\right)^2 \\ &= 2279.51 \end{aligned}$$

Note that we used the sample mean and standard deviation to estimate ξ and σ and that 10 samples fall far short of the full credibility standard.

Poisson Example

Assume that we are interested in estimating the number of claims and assume the follow a Poisson(λ) distribution. Find the number of policies necessary for full credibility. We know that $\xi = E(N_j) = \lambda$ and $\sigma^2 = Var(N_j) = \lambda$ so the full credibility standard is

$$n \geq \left(\frac{y_p\sqrt{\lambda}}{r\lambda}\right)^2$$
$$= \frac{y_p^2}{r^2\lambda}$$

and λ will need to be estimated from the data.

Aggregate Payments Example

Assume further that each claim size has a mean of θ_Y and a variance of σ_Y^2. What is the standard for full credibility in terms of the average aggregate claim amount?

$$\xi = E(S_j) = \lambda\theta_Y$$
$$\sigma^2 = Var(S_j) = \lambda(\theta_Y^2 + \sigma_Y^2)$$

$$\begin{aligned} n &\geq \left(\frac{y_p}{r}\right)^2 \frac{\lambda(\theta_Y^2 + \sigma_Y^2)}{\lambda^2\theta_Y^2} \\ &= \frac{y_p^2}{r^2\lambda}\left[1 + \left(\frac{\sigma_Y}{\theta_Y}\right)^2\right] \end{aligned}$$

SOA Practice #2

You are given:

- The number of claims has a Poisson distribution.
- Claim sizes have a Pareto distribution with parameters $\theta = 0.5$ and $\alpha = 6$.
- The number of claims and claim sizes are independent.
- The observed pure premium should be within 2% of the expected pure premium 90% of the time.

Calculate the expected number of claims needed for full credibility.
[16,913]

Partial Credibility

Rather than either using either only the data or the manual rate, how about a weighted average? The partial credibility estimate (or credibility premium) is:

$$P_c = Z\bar{X} + (1 - Z)M$$

where Z is the credibility factor.

Partial Credibility

It can be shown that

$$
\begin{aligned}
\frac{\xi^2 r^2}{y_p^2} &= Var(P_c) \\
&= Var[Z\bar{X} + (1-Z)M] \\
&= Z^2 Var(\bar{X}) \\
&= Z^2 \frac{\sigma^2}{n} \\
\therefore Z &= \frac{\xi r \sqrt{n}}{\sigma y_p}
\end{aligned}
$$

It turns out that the credibility factor is the square root of the ratio of the actual count to the count needed for full credibility.

SOA Example #65

You are given the following information about a general liability book of business comprised of 2500 insureds:

- $X_i = \sum_{j=1}^{N_i} Y_{ij}$ is a random variable representing the annual loss of the ith insured.
- $N_1, N_2, \ldots, N_{2500}$ are independent and identically distributed random variables following a negative binomial distribution with parameters $r = 2$ and $\beta = 0.2$.
- $Y_{i1}, Y_{i2}, \ldots, Y_{iN_i}$ are independent and identically distributed random variables following a Pareto distribution with $\alpha = 3$ and $\theta = 1000$.
- The full credibility standard is to be within 5% of the expected aggregate losses 90% of the time.

Using limited fluctuation credibility theory, calculate the partial credibility of the annual loss experience for this book of business. [0.47]

Partial Credibility

The formula for calculating a partially credible estimate is:

$$\text{Estimated Value} = Z \times \mu + (1 - Z) \times M$$

where:

- Z is the credibility factor, ranging from 0 to 1, indicating the weight given to the observed data.
- μ is the mean of the observed data.
- M is the manual rate or historical rate used as a benchmark.

The credibility factor Z is determined based on the amount of data available and the level of variability in that data. As the amount of data increases and becomes more reliable, Z approaches 1, giving more weight to the observed data.

Example: Calculating Partial Credibility

Consider an insurance policy where we want to estimate the premium based on observed claim data and a manual rate. Suppose the observed mean claim amount (μ) is \$1,000, the manual rate (M) is \$1,200, and the credibility factor (Z) for this policy is 0.7 due to the limited number of claims.
Using the partial credibility formula:

$$\text{Estimated Premium} = Z \times \mu + (1 - Z) \times M$$

Substituting the given values:

$$\text{Estimated Premium} = 0.7 \times 1000 + (1 - 0.7) \times 1200 = 1060$$