Black Scholes

Binomial Tree Limit

Consider a 100-strike option with $S = 100$, $r = .06, \delta = .06, \sigma = .1$ and $t = 1$. What happens as the number of binomial periods increases?

Num of Periods	Premium
1	4.70
2	3.32
3	4.07
4	3.53
5	3.94
6	3.60

Binomial Tree Limit

Continued ...

Num of Periods	Premium
7	3.89
8	3.64
9	3.86
10	3.66
11	3.84
12	3.67
⋮	⋮
50	3.73
100	3.75
1000	3.755

Binomial Tree Limit

Eventually it settles near a specific value. As fun as it would be to do a 1000 period binomial tree, there is an easier way to find out what that limit is.

A set of equations gives us the limit. They are called the **Black-Scholes** equations.

Black-Scholes

The Black-Scholes equations:

$$\begin{aligned} d_1 &= \frac{\log(S/K) + (r + .5\sigma^2)T}{\sigma\sqrt{T}} \\ d_2 &= d_1 - \sigma\sqrt{T} \\ C &= SN(d_1) - Ke^{-rT}N(d_2) \\ P &= Ke^{-rT}N(-d_2) - SN(-d_1) \end{aligned}$$

where $N(\cdot)$ is the normal CDF function, found from a table.

Black-Scholes

Consider a 100-strike option with $S = 100$, $r = .06$, $\sigma = .1$ and $T = 1$. Assume dividend rate $\delta = 0$. What is the Black-Scholes price?

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$$\begin{aligned} d_1 &= \frac{\log(S/K) + (r + .5\sigma^2)T}{\sigma\sqrt{T}} \\ &= \frac{\log(100/100) + (.06 + .5 \times .1^2) \times 1}{.1 \times \sqrt{1}} \\ &= 0.65 \\ d_2 &= d_1 - \sigma\sqrt{T} \\ &= 0.65 - 0.1 \times \sqrt{1} \\ &= 0.55 \end{aligned}$$

Black-Scholes

Consider a 100-strike option with $S = 100$, $r = .06$, $\sigma = .1$ and $T = 1$. What is the Black-Scholes price?

$$\begin{aligned} d_1 &= 0.65 \\ d_2 &= 0.55 \end{aligned}$$

The call option price is:

$$\begin{aligned} C &= 100N(d_1) - 100e^{-0.06}N(d_2) \\ &= 100N(0.65) - 100e^{-0.06}N(0.55) \\ &= 7.459322 \end{aligned}$$

Black-Scholes Practice

The current price of a stock is \$40, the risk free rate is $r = .03$, and the volatility of the stock is $\sigma = .1$. Using Black-Scholes, what is the price of a call option that expires in 9 months to purchase the stock at a strike price of 39?

$$\begin{aligned} d_1 &= 0.595 \\ d_2 &= 0.509 \end{aligned}$$

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$$\begin{aligned} d_1 &= 0.595 \\ d_2 &= 0.509 \end{aligned}$$

The call option price is:

$$\begin{aligned} C &= 40N(d_1) - 39e^{-0.03 \cdot 0.75} N(d_2) \\ &= 2.483579 \end{aligned}$$

Black-Scholes Practice for Put Option

Using the same parameters, what is the price of a *put* option?

Black-Scholes Practice for Put Option

Using the same parameters, what is the price of a *put* option? The put option price is:

$$\begin{aligned} P &= 39e^{-0.03 \cdot 0.75} N(-d_2) - 40N(-d_1) \\ &= 0.6158774 \end{aligned}$$