

# Expected Values, Variance, and Standard Deviation

# Expected Value

For observed data, the **mean** is a measure of center. For a random variable, the equivalent measure of center is the **expected value**.

The expected value is the long-run average value if the random process were repeated many times.

$$E(X) = \sum x_i P(x_i)$$

It is a *weighted average* of all possible values, with weights given by their probabilities.

# Variance and Standard Deviation

Variance and standard deviation measure the spread of a random variable around its mean.

$$\text{Var}(X) = \sum (x_i - E(X))^2 P(x_i)$$

$$\text{SD}(X) = \sqrt{\text{Var}(X)}$$

These correspond to the sample variance and standard deviation you would expect from observed data.

## Example: Daily Sales

Suppose a small shop's daily sales revenue (in \$100s) has the following distribution:

Sales $x_i$	Probability $P(x_i)$
2	0.1
4	0.3
6	0.4
10	0.2

What is the expected daily sales revenue? What is the variance and standard deviation?

## Example: Daily Sales (continued)

**Expected value:**

$$E(X) = 2(0.1) + 4(0.3) + 6(0.4) + 10(0.2) = 6$$

So the average daily sales are \$600.

**Variance:**

$$\begin{aligned}\text{Var}(X) &= (2-6)^2(0.1) + (4-6)^2(0.3) + (6-6)^2(0.4) + (10-6)^2(0.2) \\ &= (16)(0.1) + (4)(0.3) + (0)(0.4) + (16)(0.2) = 1.6 + 1.2 + 0 + 3.2 = 6\end{aligned}$$

**Standard deviation:**

$$\text{SD}(X) = \sqrt{6} \approx 2.45$$

So daily sales typically vary by about \$245 around the mean.

# Binomial Distribution

The binomial distribution models the number of successes in  $n$  independent trials, each with success probability  $p$ .

$$P(Y = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

Moments:

$$E(Y) = np, \quad \text{Var}(Y) = np(1 - p)$$

# Geometric Distribution

The geometric distribution models the number of trials until the first success.

$$P(Z = k) = (1 - p)^{k-1}p$$

Moments:

$$E(Z) = \frac{1}{p}, \quad \text{Var}(Z) = \frac{1-p}{p^2}$$

# Scenario Practice (1)

- 1 A quality inspector checks 20 products. Each has a 5% defect rate. On average, how many defects should she expect? What is the standard deviation?
- 2 A soccer player has a 30% chance of scoring a penalty kick. On average, how many attempts until her first goal?



## Scenario Practice (2)

- ③ An online store records whether each of 15 visitors makes a purchase (probability 0.2). What is the expected number of purchases and the variability?
- ④ A die is rolled repeatedly until a six appears. What is the expected number of rolls? What is the standard deviation?