## Local extrema

- 1. Rederive the equation of the plane tangent to the surface z = f(x, y) at  $(x_0, y_0)$  by completing the following steps.
  - (i) Recognize the surface as the level surface of a function of three variables F(x, y, z).

(ii) Find a vector perpendicular to the surface, and therefore perpendicular to the tangent plane, at  $(x_0, y_0)$ .

(iii) Find the z-coordinate of the point on the surface corresponding to  $x = x_0$  and  $y = y_0$ . Then write down the equation of the tangent plane.

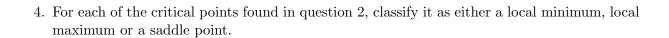
2. Find all of the critical points of the function  $f(x,y) = x^2 + 2y^2 + x^2y + 4$ .

3. To better understand the Second Derivative Test, let's look at three simple cases. For each of the following functions, find and classify all of the critical points. Then make a rough sketch of the graph of the function.

(I) 
$$f(x,y) = x^2 + y^2$$

(II) 
$$f(x,y) = 1 - x^2 - y^2$$

(III) 
$$f(x,y) = y^2 - x^2$$



5. Find and classify all of the critical points of f(x,y) = (x-2y)(4-xy).