MATH 118
Written Assignment #1

For the written homework assignment, you are expected to provide full solutions with complete justifications. You will be graded on the mathematical, logical and grammatical coherence of your solutions. Your solutions must have your name written on the top of the first page.

Your written assignment should be submitted as a pdf electronically through Canvas. Please email me if you are experiencing any technical difficulties.

1. Let $\mathbf{u} = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -2 \\ 4 \\ 3 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} -1 \\ -1 \\ 5 \end{bmatrix}$. Calculate each of the following.

   (a) $3\mathbf{u} - \mathbf{v} + 2\mathbf{w}$.

   (b) $\|\mathbf{w}\|$

   (c) $\mathbf{v} \cdot \mathbf{w}$

   (d) $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})$

2. Compute each of the following matrix products, or explain why they don't make sense.

   (a) $\begin{bmatrix} 1 & 4 \\ -3 & 6 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}$

   (b) $\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \begin{bmatrix} -5 \\ 1 \end{bmatrix}$

   (c) $\begin{bmatrix} -2 & 0 \\ -1 & 4 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

   (d) $\begin{bmatrix} 7 & 4 & -2 \\ 1 & 6 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

3. Let $\mathbf{u} = \begin{bmatrix} 5 \\ 2 \\ 7 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 0 \\ 1 \\ 6 \end{bmatrix}$.

(a) Use vector addition and scalar multiplication to verify that $3\mathbf{u} - 5\mathbf{v} = \mathbf{w}$.

(b) Use (a) (and no row operations!) to find x and y that satisfy the equation

$$\begin{bmatrix} 5 & 3 \\ 2 & 1 \\ 7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 6 \end{bmatrix}.$$

4. Consider the following system of linear equations.

$$\begin{cases} 2x - 3y + 5z = 0 \\ 2y + z = 1 \\ x - 3z = 2 \end{cases}$$

(a) Write down a matrix equation for the system of equations.

(b) Write down the associated augmented matrix.

(c) Solve the system of equations by performing row operations on the augmented matrix in (b).

5. A coffee company owns three coffee roasting plants, each of which is powered by electricity and employs several people. The table below provides an estimate of each plant's energy consumption, the number of people it employs, and the amount of coffee it roasts each day.

Factory	Electricity (kWh)	# Employees	Amount of coffee (lbs)
Factory A	405	57	510
Factory B	175	21	290
Factory C	320	43	445

In the region where the plants are located, the unit price of electricity is $0.14/kWh. In addition, the company pays each of its employees $144/day, and pays $1.05/lb of green (unroasted) coffee.

(a) Calculate the product
$$\begin{bmatrix} 405 & 57 & 510 \\ 175 & 21 & 290 \\ 320 & 43 & 445 \end{bmatrix} \begin{bmatrix} 0.14 \\ 144 \\ 1.05 \end{bmatrix}.$$

(b) What does your result from (a) represent? (What does each component in the resulting vector from (a) represent?)

6. Breakfast cereals often list their nutritional information on their containers. The number of calories and the amounts of protein and carbohydrates for two cereals is given in the following table.

Nutrient	Cereal A	Cereal B
Calories	110	150
Protein (g)	2	4
Carbohydrates (g)	20	18

Suppose you want to eat a mixture of the two cereals that contains exactly 350 calories, 8g of protein, and 48g of carbohydrates.

(a) Set up a matrix equation for this problem. Include a statement of what the variables in your equation represent.

(b) Does such a mixture of the two cereals exist? Justify your answer.