RJS McDonald

X4=5A X4=5A MATH 118 Lecture Notes

2.5 Inverse matrices



2.5. Key Ideas

- The inverse matrix gives $AA^{-1} = I$ and $A^{-1}A = I$.
- ullet A is invertible if and only if it has n pivots
- If $A\mathbf{x} = 0$ for a nonzero vector \mathbf{x} , then A has no inverse
- The inverse of AB is $B^{-1}A^{-1}$, and $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$.
- Reducing $[A \ I]$ to reduced row echelon form gives $[I \ A^{-1}]$.

Definition 2.5.1. An $n \times n$ matrix A is **invertible** if there is an $n \times n$ matrix C such that CA = I and AC = I, where $I = I_n$ is the identity matrix.

In this case, C is called the **inverse** of A. A matrix that is *not* invertible is called a **singular** matrix, and an invertible matrix is called a **non-singular** matrix.

Remark 2.5.2. Suppose B and C were both inverses of A. Then

$$B = BI = B(AC) = (BA)C = IC = C.$$

It turns out, that if A has an inverse, it's unique. We call this unique inverse A^{-1} .

Example 2.5.3. Let
$$A = \begin{bmatrix} 2 & 5 \\ -3 & -7 \end{bmatrix}$$
 and $C = \begin{bmatrix} -7 & -5 \\ 3 & 2 \end{bmatrix}$. Show that $C = A^{-1}$

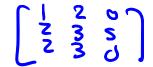
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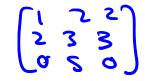
Theorem 2.5.9. An $n \times n$ matrix A is invertible if and only if A is row equivalent to \mathcal{I}_n . In this case, any sequence of elementary row operations that reduces A to I_n also transforms I_n into A^{-1} .

Procedure 2.5.10. To find A^{-1} , row reduce the augmented matrix $[A \ I]$. If A is row equivalent to I, then $[A \ I]$ is row equivalent to $[I \ A^{-1}]$. Otherwise, A does not have an inverse.

Example 2.5.11. Find the inverse of $A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$.

0 -2 | 1 0 0 |





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Example 2.5.12. Poké Balls are on sale everywhere! At the new pokéstore in Pallet Town, they're selling Poké Ball bundles of 1 Poké Ball, 2 Great Balls and 2 Ultra Balls, in Veridian, the bundles consist of 2 Poké Ball, 3 Great Balls and 3 Ultra Balls, and in Pewter City, bundles come with only 5 Great Balls. By the time he gets to Mount Moon, Ash has purchased a total of 5 Poké Ball, 15 Great Balls and 10 Ultra Balls. How many bundles did he buy in each town?

(a) Find a matrix equation Ax by whose solutions, represents bundles in each town.
(b) Find b by using the inverse of A. bundles in each town

Example 3.1.1. Let $\mathcal{M}_{m\times n}$ be the set of all $m\times n$ matrices with entries that are real numbers. Let's focus on $\mathcal{M}_{3\times 2}$, the set of all matrices with three rows and two columns. Let A, B, and C be matrices in $\mathcal{M}_{3\times 2}$, and c and d be real numbers.

(a) True or false: A + B is another matrix in $\mathcal{M}_{3\times 2}$.

>> commutative

(b) We know A+B=B+A because the entries are real numbers. For example, in the (1,1)-entry

$$A+B = \left[\begin{array}{ccc} a_{11} & * \\ * & * \\ * & * \end{array} \right] + \left[\begin{array}{ccc} b_{11} & * \\ * & * \\ * & * \end{array} \right] = \left[\begin{array}{ccc} a_{11}+b_{11} & * \\ * & * \\ * & * \end{array} \right] = \left[\begin{array}{ccc} b_{11}+a_{11} & * \\ * & * \\ * & * \end{array} \right] = \left[\begin{array}{ccc} b_{11} & * \\ * & * \\ * & * \end{array} \right] + \left[\begin{array}{ccc} a_{11} & * \\ * & * \\ * & * \end{array} \right] = B+A.$$

Use the same logic to explain how we know (A + B) + C = A + (B + C).

IS A+B+C = A+ (B+C)

- = (a11 + (b11 + C1) * *
 - (c) How could it be possible for A + B = A?

(d) Let Z be the 3×2 zero matrix. What can you say about the entries of A and B if A + B = Z?

$$A+B=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + then \quad A_{12} = -b_{12}$$

$$\vdots$$

(e) By the same logic of part (b), explain how we know c(A+B)=cA+cB and c(dA)=(cd)A.

(f) Is it possible for cA = A?