1.3 # 3, 6, 7, 9, 12, 14, 15, 21, 22, 23, 25

3.) Display $\vec{u}, \vec{v}, -\vec{v}, -2\vec{v}, \vec{u}+\vec{v}, \vec{u}-\vec{v}$ and $\vec{u}-2\vec{v}$ on an xy-graph

$\vec{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \vec{v} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

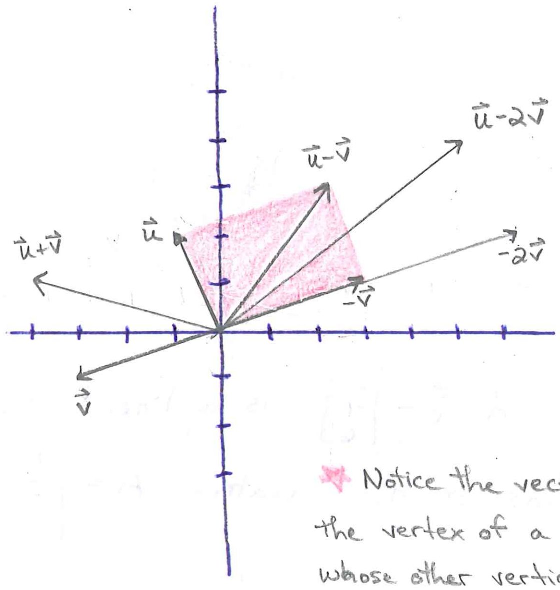
$-\vec{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$-2\vec{v} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$

$\vec{u}+\vec{v} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$

$\vec{u}-\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\vec{u}-2\vec{v} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$



* Notice the vector $\vec{u}-\vec{v}$ gives the vertex of a parallelogram whose other vertices are given by $\vec{0}$, $\vec{u}$ and $-\vec{v}$

6.) Write a system of equations that is equivalent to the vector equation

$x_1 \begin{bmatrix} 3 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} 7 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\begin{cases} 3x_1 + 7x_2 - 2x_3 = 0 \\ -2x_1 + 3x_2 + x_3 = 0 \end{cases}$$

7.) Use the figure to write each vector $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ as a linear combination of $\vec{u}$ and $\vec{v}$. Is every vector in $\mathbb{R}^2$ a linear combination of $\vec{u}$ and $\vec{v}$?

(figure on pg. 32 Lay)

$\vec{a} = \vec{u} - 2\vec{v}$, $\vec{b} = 2\vec{u} - 2\vec{v}$, $\vec{c} = 2\vec{u} - 3.5\vec{v}$, $d = 3\vec{u} - 4\vec{v}$

Yes, every vector can be written as a linear combination of $\vec{u}$ and $\vec{v}$

9.) Write a vector equation equivalent to the system

$$\begin{aligned} x_2 + 5x_3 &= 0 \\ 4x_1 + 6x_2 - x_3 &= 0 \\ -x_1 + 3x_2 - 8x_3 &= 0 \end{aligned}$$

$x_1 \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -1 \\ -8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

1.3

12.) Determine if $\vec{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$ is a linear combination of $\vec{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{a}_2 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}$, $\vec{a}_3 = \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}$.

If yes then $x_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix} + x_3 \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$ for some x_1, x_2, x_3.

The question is equivalent to solving the system corresponding to the augmented matrix

$$\left[\begin{array}{ccc|c} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 1 & -2 & 5 & 9 \end{array}\right] \xrightarrow{-R_1+R_3} \left[\begin{array}{ccc|c} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 0 & 0 & 11 & -2 \end{array}\right]$$

This has a solution, so yes $\vec{b}$ is a linear combination of $\vec{a}_1, \vec{a}_2, \vec{a}_3$.

14.) Determine if $\vec{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$ is a linear combination of the vectors formed by the colomns of the matrix $A = \begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & -6 \\ 0 & 2 & 8 \end{bmatrix}$.

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{array}\right] \xrightarrow{2R_1+R_2} \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 8 & 6 \end{array}\right] \xrightarrow{-2R_2+R_3} \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Yes, this has a solution (in fact many)

So $\vec{b}$ is a linear combination of the vectors in A.

15.) $\vec{a}_1 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$, $\vec{a}_2 = \begin{bmatrix} -5 \\ -8 \\ 2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 3 \\ -5 \\ h \end{bmatrix}$. For what value(s) of h is $\vec{b}$ in the plane spanned by $\vec{a}_1$ and $\vec{a}_2$?

Equivalent to asking for which values of h is $\vec{b}$ a linear combination of $\vec{a}_1$ and $\vec{a}_2$, or also equivalently, for which values of h does the associated system have a solution / is consistent.

$$\left[\begin{array}{cc|c} 1 & -5 & 3 \\ 3 & -8 & -5 \\ -1 & 2 & h \end{array}\right] \xrightarrow[R_1+R_3]{-3R_1+R_2} \left[\begin{array}{cc|c} 1 & -5 & 3 \\ 0 & 7 & -14 \\ 0 & -3 & 3+h \end{array}\right] \xrightarrow{R_2/7} \left[\begin{array}{cc|c} 1 & -5 & 3 \\ 0 & 1 & -2 \\ 0 & -3 & 3+h \end{array}\right] \xrightarrow{3R_2+R_3} \left[\begin{array}{cc|c} 1 & -5 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & -3+h \end{array}\right]$$

This is consistent when $-3+h=0$ ie $\boxed{h=3}$

21.) Show that $\begin{bmatrix} h \\ k \end{bmatrix}$ is in Span $\left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$ for all h, k.

$$\left[\begin{array}{cc|c} 2 & 2 & h \\ -1 & 1 & k \end{array}\right] \rightarrow \left[\begin{array}{cc|c} -1 & 1 & k \\ 2 & 2 & h \end{array}\right] \xrightarrow[2R_1+R_2]{R_1/(-1)} \left[\begin{array}{cc|c} 1 & -1 & -k \\ 0 & 4 & 2k+h \end{array}\right]$$

This is consistent regardless of the values of h and k.

Therefore $\begin{bmatrix} h \\ k \end{bmatrix}$ can be written as a linear combination of $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ which is the definition of being in Span $\left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$.

1.3 continued

22.) Construct a 3×3 matrix A, with nonzero entries, and a vector $\vec{b}$ in $\mathbb{R}^3$ such that $\vec{b}$ is not in the set spanned by the columns of A.

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & -1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

This augmented matrix corresponds to an inconsistent system.

We apply some row operations to make a more interesting matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & -1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{array}\right] \begin{array}{c} 2R_2+R_1 \\ 3R_1+R_2 \\ -R_2+R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 10 & -1 \\ 3 & 1 & 15 & -3 \\ 0 & -1 & -3 & 2 \end{array}\right]$$

$$A = \begin{bmatrix} 1 & 2 & 10 \\ 3 & 1 & 15 \\ 0 & -1 & -3 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$$

23.) True/False

a.) Another notation for the vector $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$ is $[-4\ 3]$.

b.) The points in the plane corresponding to $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} -5 \\ 2 \end{bmatrix}$ lie on a line through the origin.

c.) An example of a linear combination of vectors $\vec{v}_1$ and $\vec{v}_2$ is the vector $\frac{1}{2}\vec{v}_1$.

d.) The solution set of the linear system whose augmented matrix is $[\vec{a}_1\ \vec{a}_2\ \vec{a}_3\ \vec{b}]$ is the same as the solution set of the equation $x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3 = \vec{b}$.

e.) The set Span$\{\vec{u}, \vec{v}\}$ is always visualized as a plane through the origin.

a.) False
$(-4, 3)$
is the same

b.) False

c.) True

d.) True

e.) False

1.3

25.) $A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 3 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}$. Denote the columns of A by $\vec{a}_1, \vec{a}_2, \vec{a}_3$

and let $W = \text{Span}\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$

a.) Is $\vec{b}$ in $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$? How many vectors are in $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$

No, three (just $\vec{a}_1, \vec{a}_2, \vec{a}_3$ themselves)

b.) Is $\vec{b}$ in W? How many vectors are in W?

$$\left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ -2 & 6 & 3 & -4 \end{array}\right] \underset{2R_1+R_3}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 6 & -5 & 4 \end{array}\right] \underset{-2R_2+R_3}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 0 & -1 & 2 \end{array}\right]$$

Yes, infinitely many.

c.) Show that $\vec{a}_1$ is in W.

$\vec{a}_1 = 1\vec{a}_1 + 0\vec{a}_2 + 0\vec{a}_3$ which is a linear combination of $\vec{a}_1, \vec{a}_2, \vec{a}_3$

therefore $\vec{a}_1$ is in $\text{Span}\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$.