3.2 # 2, 3, 8, 10, 16, 17, 20, 26, 27, 32, 34, 40

2.) The equation illustrates a property of determinants. State the property.

$$\begin{vmatrix} 2 & -6 & 4 \\ 3 & 5 & -2 \\ 1 & 6 & 3 \end{vmatrix} = 2\begin{vmatrix} 1 & -3 & 2 \\ 3 & 5 & -2 \\ 1 & 6 & 3 \end{vmatrix}$$

If one row of A is multiplied by k to produce B then det B = k · det A.

3.) $$\begin{vmatrix} 1 & 3 & -4 \\ 2 & 0 & -3 \\ 5 & -4 & 7 \end{vmatrix} = \begin{vmatrix} 1 & 3 & -4 \\ 0 & -6 & 5 \\ 5 & -4 & 7 \end{vmatrix}$$

If a multiple of one row of A is added to another row to produce a matrix B, then det A = det B.

8.) Find the determinant by row reduction to echelon form.

$$\begin{bmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 5 & 4 & -3 \\ -3 & -7 & -5 & 2 \end{bmatrix} \begin{array}{l} \\ \\ -2R_1+R_3 \\ 3R_1+R_4 \end{array} \begin{bmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & -1 & -2 & 5 \\ 0 & 2 & 4 & -10 \end{bmatrix} \begin{array}{l} \\ \\ R_2+R_3 \\ -2R_2+R_3 \end{array} \begin{bmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Triangular matrix ⇒ multiply entries in main diagonal to get determinant. The determinant is $(1)(1)(0)(0) = 0$.

10.) $$A = \begin{bmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ -2 & -6 & 2 & 3 & 9 \\ 3 & 7 & -3 & 8 & -7 \\ 3 & 5 & 5 & 2 & 7 \end{bmatrix} \begin{array}{l} \\ \\ 2R_1+R_3 \\ -3R_1+R_4 \\ -3R_1+R_5 \end{array} \begin{bmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ 0 & 0 & 0 & 3 & 5 \\ 0 & -2 & 0 & 8 & -1 \\ 0 & -4 & 8 & 2 & 13 \end{bmatrix} \begin{array}{l} \\ \\ \\ R_2+R_4 \\ 2R_2+R_5 \end{array} \begin{bmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ 0 & 0 & 0 & 3 & 5 \\ 0 & 0 & -4 & 7 & -7 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = B$$

no change in det   no change in det

So det A = det B

Interchanging rows means det B = −det C

$$\begin{bmatrix} 1 & 3 & -1 & 0 & 2 \\ 0 & 2 & -4 & -1 & -6 \\ 0 & 0 & -4 & 7 & -7 \\ 0 & 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = C$$

Since C is triangular

$\det C = 1(2)(-4)(3)(1) = -24$

$\det A = \det B = -\det C$

$= -(-24) = \boxed{24}$

16.) Find the determinant where $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$.

$$\begin{vmatrix} a & b & c \\ 3d & 3e & 3f \\ g & h & i \end{vmatrix} = 3(7) = 21$$

3.2

17.) $\begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix} = -\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -7$

20.) $\begin{vmatrix} a+d & b+e & c+f \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$

26.) Use determinants to decide if the set of vectors is linearly independent.

$\begin{bmatrix} 3 \\ 5 \\ -6 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -6 \\ 0 \\ 7 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ -3 \end{bmatrix}$

$A = \begin{bmatrix} 0 & -2 & 2 & 3 \\ 0 & -1 & -6 & 5 \\ 0 & 3 & 0 & -6 \\ -3 & 0 & 7 & 4 \end{bmatrix} \sim \begin{bmatrix} -3 & 0 & 7 & 4 \\ 0 & -1 & -6 & 5 \\ 0 & 3 & 0 & -6 \\ 0 & -2 & 2 & 3 \end{bmatrix} \begin{matrix} \\ \\ 3R_2+R_3 \\ -2R_2+R_4 \end{matrix}$

$\det A = -\det B$

$\begin{bmatrix} -3 & 0 & 7 & 4 \\ 0 & -1 & -6 & 5 \\ 0 & 0 & -18 & 9 \\ 0 & 0 & 14 & -7 \end{bmatrix} \begin{matrix} \\ \\ R_3/(-9) \\ R_4/(7) \end{matrix} \begin{bmatrix} -3 & 0 & 7 & 4 \\ 0 & -1 & -6 & 5 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 2 & -1 \end{bmatrix} -R_3+R_4 \begin{bmatrix} -3 & 0 & 7 & 4 \\ 0 & -1 & -6 & 5 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$B \qquad C \qquad D$

$\det D = (-3)(-1)(2)(0) = 0$

$\det B = (-\frac{1}{9})(\frac{1}{7})\det C \qquad \det C = \det D$

$\det A = -\det B = (\frac{1}{9})(\frac{1}{7})\det C = (\frac{1}{9})(\frac{1}{7})\det D = 0$ Since determinant of A is zero, A is not invertible which means the vectors are linearly dependent.

27.) True/False (A, B are $n \times n$ matrices)

a.) A row replacement operation does not affect the determinant of a matrix.

b.) The determinant of A is the product of pivots in any echelon form U of A, multiplied by $(-1)^r$, where r is the number of row interchanges made during row reduction from A to U.

c.) If the columns of A are linearly dependent, then $\det A = 0$.

d.) $\det(A+B) = \det A + \det B$.

a.) True b.) True c.) True d.) False

3.2 Continued

32.) Find a formula for $\det(rA)$ when A is an $n \times n$ matrix.

rA multiplies r to each row of A and each time it multiplies the determinant of A by r. Since there are n rows,

$\det(rA) = r^n \det A$.

34.) Let A and P be square matrices, with P invertible. Show that $\det(PAP^{-1}) = \det A$.

$$\det(PAP^{-1}) = (\det P)(\det A)(\det P^{-1}) = (\det P)(\det P^{-1})(\det A)$$
$$= (\det PP^{-1})(\det A) = (\det I)(\det A) = 1 \cdot \det A = \det A$$

40.) Let A and B be 4×4 matrices, with $\det A = -1$ and $\det B = 2$. Compute:

a.) $\det AB$ b.) $\det B^5$ c.) $\det 2A$ d.) $\det A^T A$ e.) $\det B^{-1}AB$

a.) $\det AB = (\det A)(\det B) = -2$

b.) $\det B^5 = (\det B)^5 = 32$

c.) $\det 2A = 2^4 \det A = 16(-1) = -16$

d.) $\det A^T A = (\det A^T)(\det A) = (\det A)(\det A) = 1$

e.) $\det(B^{-1}AB) = \det A = -1$