5.2 # 2, 5, 9, 12, 15, 19, 20, 21

2.) Find the characteristic polynomial and the real eigenvalues of the matrix.

$\begin{bmatrix} -4 & -1 \\ 6 & 1 \end{bmatrix}$ Characteristic polynomial $= \det(A-\lambda I) = \begin{vmatrix} -4-\lambda & -1 \\ 6 & 1-\lambda \end{vmatrix} =$

$= (-4-\lambda)(1-\lambda)+6 = \boxed{\lambda^2+3\lambda+2} = (\lambda+1)(\lambda+2)$

λ is an eigenvalue of A iff $\det(A-\lambda I)=0$

$(\lambda+1)(\lambda+2)=0 \Rightarrow \boxed{\lambda=-1,\ \lambda=-2}$

5.) $\begin{bmatrix} 8 & 4 \\ 4 & 8 \end{bmatrix}$ $\det(A-\lambda I) = (8-\lambda)(8-\lambda)-16 = \boxed{\lambda^2-16\lambda+48}$

set equal to zero: $\lambda^2-16\lambda+48 = (\lambda-4)(\lambda-12) = 0$ $\boxed{\lambda = 4, 12}$

9.) Find the characteristic polynomial of $\begin{bmatrix} 4 & 0 & -1 \\ 0 & 4 & -1 \\ 1 & 0 & 2 \end{bmatrix}$.

(cofactor expansion 2nd column)

$\begin{vmatrix} 4-\lambda & 0 & -1 \\ 0 & 4-\lambda & -1 \\ 1 & 0 & 2-\lambda \end{vmatrix} = (4-\lambda)\begin{vmatrix} 4-\lambda & -1 \\ 1 & 2-\lambda \end{vmatrix} = (4-\lambda)[(4-\lambda)(2-\lambda)+1] = (4-\lambda)[\lambda^2-6\lambda+9]$

$= 4\lambda^2-24\lambda+36-\lambda^3+6\lambda^2-9\lambda = \boxed{-\lambda^3+10\lambda^2-33\lambda+36}$

12.) $\begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ $\begin{vmatrix} -1-\lambda & 0 & 2 \\ 3 & 1-\lambda & 0 \\ 0 & 1 & 2-\lambda \end{vmatrix} \overset{(1^{st}\text{ row})}{=} (-1-\lambda)\begin{vmatrix} 1-\lambda & 0 \\ 1 & 2-\lambda \end{vmatrix} + 2\begin{vmatrix} 3 & 1-\lambda \\ 0 & 1 \end{vmatrix}$

$= (-1-\lambda)(1-\lambda)(2-\lambda)+6 = \boxed{-\lambda^3+2\lambda^2+\lambda+4}$

15.) List the real eigenvalues repeated according to their multiplicities.

$\begin{bmatrix} 5 & 5 & 0 & 2 \\ 0 & 2 & -3 & 6 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 5 \end{bmatrix}$ A−λI is also triangular, so det(A−λI) is the product of its diagonal entries.

triangular

$\det(A-\lambda I) = (5-\lambda)(2-\lambda)(3-\lambda)(5-\lambda)$

$\boxed{\lambda = 5, 5, 2, 3}$

19.) Let A be an $n \times n$ matrix, and suppose A has n real eigenvalues, $\lambda_1, \lambda_2, \ldots, \lambda_n$, repeated according to multiplicities, so that $\det(A-\lambda I) = (\lambda_1 - \lambda)(\lambda_2 - \lambda)\ldots(\lambda_n - \lambda)$. Explain why $\det(A)$ is the product of the n eigenvalues of A.

This is obvious if A is triangular, but we are not assuming that here.

$\det(A) = \det(A-\lambda I)$ when $\lambda = 0$. Therefore $\det(A) = (\lambda_1 - 0)(\lambda_2 - 0)\cdots(\lambda_n - 0)$
$= \lambda_1 \lambda_2 \ldots \lambda_n$.

20.) Use a property of determinants to show that A and A^T have the same characteristic Polynomial.

we Know $\det(A) = \det(A^T)$ for all square matrices. So $\det(A-\lambda I) = \det((A-\lambda I)^T)$
$= \det(A^T - \lambda I^T) = \det(A^T - \lambda I)$.

21.) True/False. A and B are $n \times n$ matrices.

a.) The determinant of A is the product of the diagonal entries in A.

b.) An elementary row operation on A doesn't change the determinant.

c.) $(\det A)(\det B) = \det(AB)$

d.) If $\lambda + 5$ is a factor of the characteristic polynomial of A, then 5 is an eigenvalue of A.

a.) False
b.) False
c.) True

These are all review from previous sections

d.) False (-5 is an eigenvalue)