Lecture 4 Functions of several variables

Stewart 14.1, McCallum 12.3, 12.5

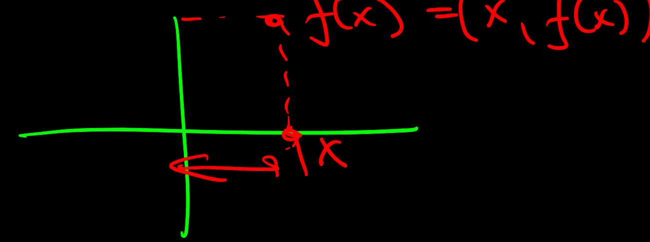
Lecture 4. Key Ideas So far, the functions that we've studied in calculus have been real-valued, taking values in $\mathbb{R}$ and outputting values in $\mathbb{R}$. In this chapter, we will study functions whose outputs are vectors, primarily in three dimensions.

- understand what a function of two variables is
- identify the domain fo a function of two variables
- find the level curves of a function of two variables
- identify graphs of paraboloids, cones, spheres, planes and cylinders

Lecture 4.1 Functions of more than one variable

Definition 4.1. A function of two variables is one whose input is two numbers and output is a single number. Similar for functions of three or more variables.

before $\mathbb{R} \to \mathbb{R}$

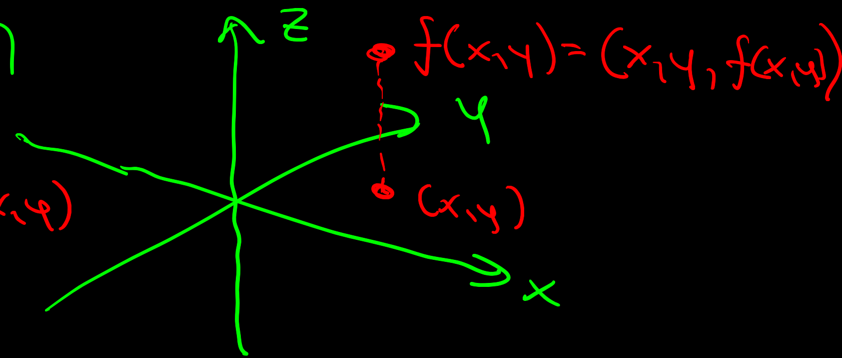


now $\mathbb{R}^2 \to \mathbb{R}$

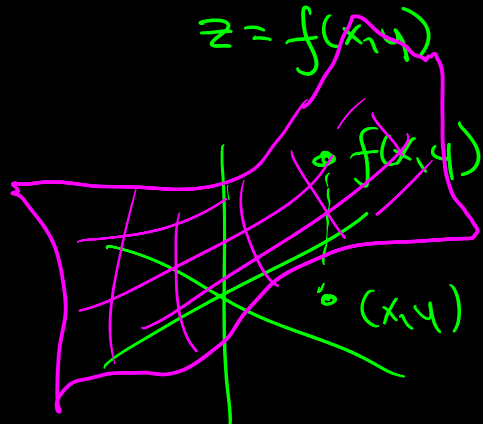
$(x,y) \longmapsto f(x,y)$

lives in the plane

$z = f(x,y)$



Example 4.2. Describe $f(x, y) = 2 - x + 2y$ geometrically.



$$z = 2 - x + 2y$$

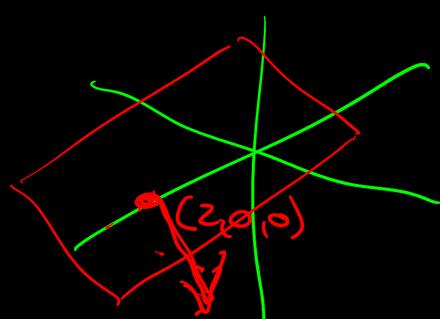
$$-(x-2) - 2y + z = 0$$

(eqn of a plane)

plane thru $(2,0,0)$

w/ normal $\langle 1, -2, 1 \rangle$

find three points



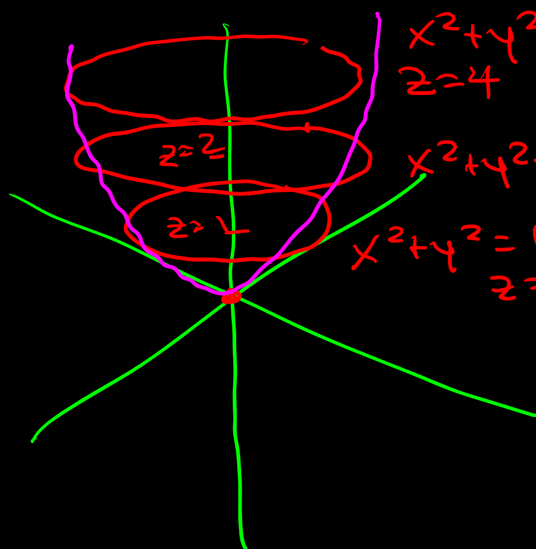
Example 4.3. What does the graphs of $f(x, y) = x^2 + y^2$ look like?

$x^2+y^2=z$

if z fixed

circle radius $\sqrt{z}$

centered at $(0,0,z)$



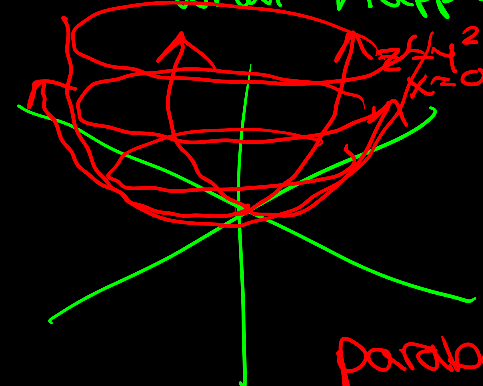
$x^2+y^2=4$
$z=4$

$x^2+y^2=2$
$z=2$

$x^2+y^2=1$
$z=1$

what happens $x=0$

$z=y^2$
$x=0$



when $y=0$

$z=x^2$
$y=0$

paraboloid

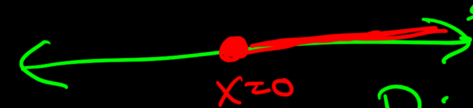
Lecture 4.2 Domain

Definition 4.4. The domain of $f(x, y)$ is the set of all (x, y) for which f is defined.

from calc 1

domain of $f(x)$ is all allowed inputs

eg $f(x) = \sqrt{x}$

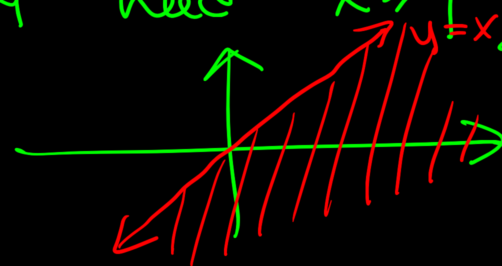


inputs from ℝ line

$D: x \geq 0$

now:

e.g. $f(x,y) = \sqrt{x-y}$ need $x \geq y$



inputs from plane

Example 4.5. Describe the function $\sqrt{1 - x^2 - y^2}$ geometrically. What is its domain?

$$\boxed{z = \sqrt{1-x^2-y^2}}$$

upper hemisphere

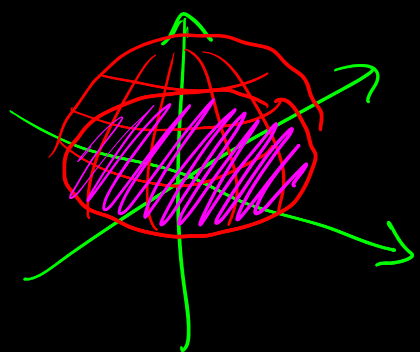
$$z^2 = 1 - x^2 - y^2$$

$$z \geq 0$$

$$(x, y, \sqrt{1-x^2-y^2})$$

$$x^2 + y^2 + z^2 = 1$$

$$z \geq 0$$



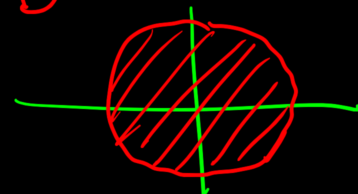
domain = all allowed inputs

if $z = \sqrt{1-x^2-y^2}$

need $1 - x^2 - y^2 \geq 0$

$\iff x^2 + y^2 \leq 1$

D



Lecture 4.3 Level Curves and Contour Maps

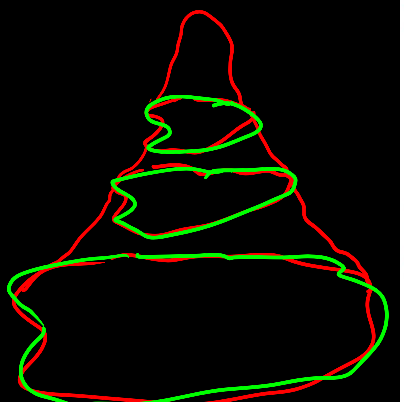
Example 4.6. Below is a map of Lonesome mountain.

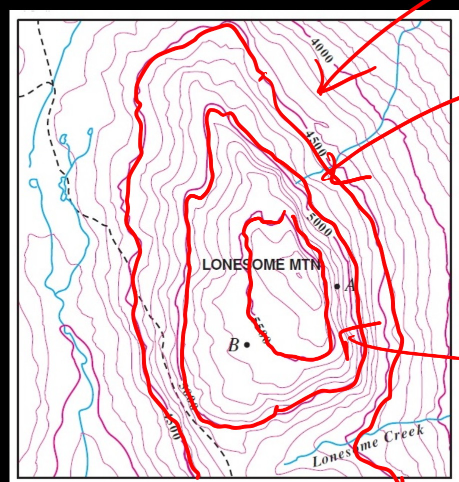
What do the lines represent?

4500ft elev

5000ft elev

$f(x,y) = e^{xy}$

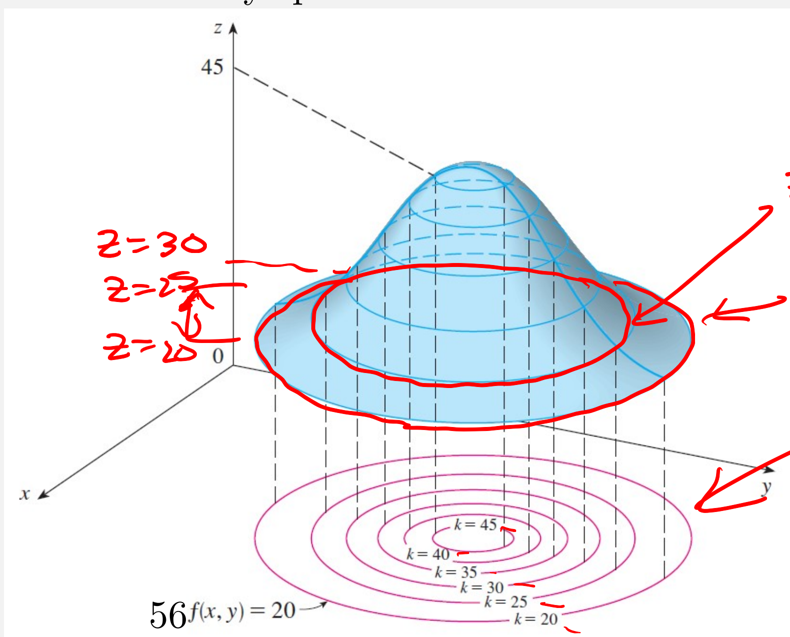




topographical map

slices of the mountain

Definition 4.7. Given a function $z = f(x, y)$, the **level curves** are the curves obtained by setting $z = c$. A **contour map** is a drawing of several evenly spaced level curves.

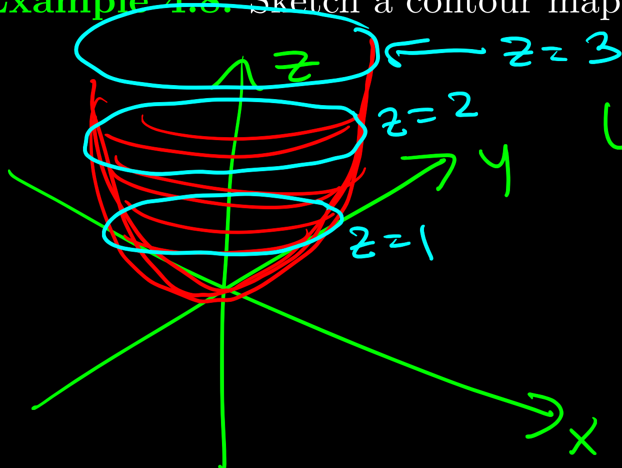


z=25

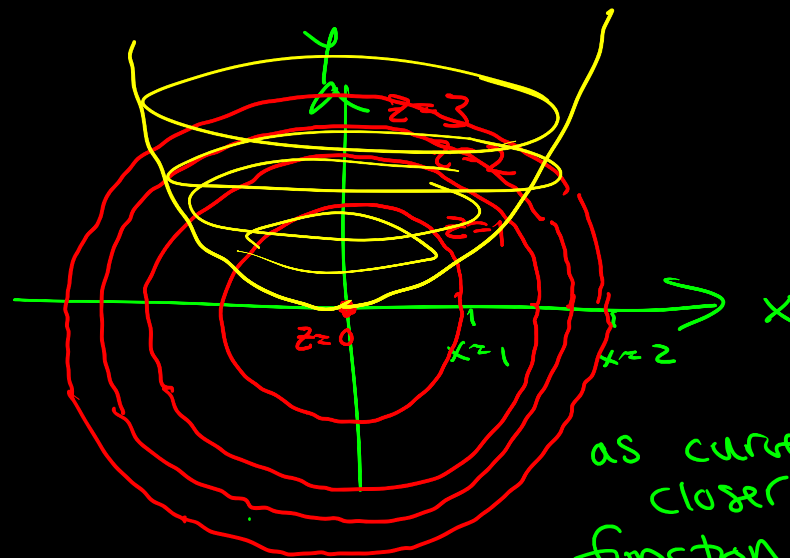
z=20

projection of level curves to plane (like topographical map)

Example 4.8. Sketch a contour map for $z = x^2 + y^2$.



level curves



$x^2+y^2=1$

$z=2 \quad x^2+y^2=2$

$x^2+y^2=3$

$x^2+y^2=4$

as curves are closer together function changes more rapidly