1.1 # 1, 2, 3, 10, 12, 13, 15, 16, 21, 24, 25, 31, 32

1.) Solve using row operations. $x_1 + 5x_2 = 7$
$-2x_1 - 7x_2 = -5$

$$\left[\begin{array}{cc|c} 1 & 5 & 7 \\ -2 & -7 & -5 \end{array}\right] \xrightarrow{2R_1+R_2} \left[\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 3 & 9 \end{array}\right] \xrightarrow{R_2/3} \left[\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 1 & 3 \end{array}\right] \xrightarrow{-5R_2+R_1} \left[\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 1 & 3 \end{array}\right]$$

$\boxed{(x_1, x_2) = (-8, 3)}$

2.) $3x_1 + 6x_2 = -3$
$5x_1 + 7x_2 = 10$

$$\left[\begin{array}{cc|c} 3 & 6 & -3 \\ 5 & 7 & 10 \end{array}\right] \xrightarrow{R_1/3} \left[\begin{array}{cc|c} 1 & 2 & -1 \\ 5 & 7 & 10 \end{array}\right] \xrightarrow{-5R_1+R_2} \left[\begin{array}{cc|c} 1 & 2 & -1 \\ 0 & -3 & 15 \end{array}\right] \xrightarrow{R_2/-3} \left[\begin{array}{cc|c} 1 & 2 & -1 \\ 0 & 1 & -5 \end{array}\right] \xrightarrow{-2R_2+R_1} \left[\begin{array}{cc|c} 1 & 0 & 9 \\ 0 & 1 & -5 \end{array}\right]$$

$\boxed{(x_1, x_2) = (9, -5)}$

3.) Find the point (x_1, x_2) that lies on the line $x_1 + 2x_2 = 4$ and on the line $x_1 - x_2 = 1$.

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 1 & -1 & 1 \end{array}\right] \xrightarrow{-R_1+R_2} \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & -3 & -3 \end{array}\right] \xrightarrow{R_2/-3} \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 1 & 1 \end{array}\right] \xrightarrow{-2R_2+R_1} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right]$$

$\boxed{(x_1, x_2) = (2, 1)}$

10.) Continue row reducing

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & -2 & -7 \\ 0 & 1 & 0 & 3 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{-3R_4+R_2} \left[\begin{array}{cccc|c} 1 & 3 & 0 & -2 & -7 \\ 0 & 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{-3R_2+R_1} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -43 \\ 0 & 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{2R_4+R_1} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -47 \\ 0 & 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

12.) Solve $x_1 - 5x_2 + 4x_3 = -3$
$2x_1 - 7x_2 + 3x_3 = -2$
$-2x_1 + x_2 + 7x_3 = -1$

$$\left[\begin{array}{ccc|c} 1 & -5 & 4 & -3 \\ 2 & -7 & 3 & -2 \\ -2 & 1 & 7 & -1 \end{array}\right] \xrightarrow{R_2+R_3} \left[\begin{array}{ccc|c} 1 & -5 & 4 & -3 \\ 2 & -7 & 3 & -2 \\ 0 & -6 & 10 & -3 \end{array}\right] \xrightarrow{-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & -6 & 10 & -3 \end{array}\right] \xrightarrow{2R_2+R_3} \left[\begin{array}{ccc|c} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & 0 & 0 & 5 \end{array}\right]$$

No solution → Inconsistent

1.1

13.) $x_1 \qquad - 3x_3 = 8$
$2x_1 + 2x_2 + 9x_3 = 7$
$x_2 + 5x_3 = -2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array}\right] \xrightarrow{-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{array}\right] \xrightarrow{-2R_3+R_2} \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 0 & 5 & -5 \\ 0 & 1 & 5 & -2 \end{array}\right] \xrightarrow{R_2/5} \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 5 & -2 \end{array}\right] -5R_2+R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 3 \end{array}\right] \xrightarrow{3R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 3 \end{array}\right] \qquad (5, 3, -1)$$

15.) Determine if the system is consistent. Do not completely solve.

$x_1 - 6x_2 \qquad = 5$
$x_2 - 4x_3 + x_4 = 0$
$-x_1 + 6x_2 + x_3 + 5x_4 = 3$
$-x_2 + 5x_3 + 4x_4 = 0$

$$\left[\begin{array}{cccc|c} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ -1 & 6 & 1 & 5 & 3 \\ 0 & -1 & 5 & 4 & 0 \end{array}\right] \xrightarrow{R_1+R_3} \left[\begin{array}{cccc|c} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5 & 8 \\ 0 & -1 & 5 & 4 & 0 \end{array}\right] \xrightarrow{R_2+R_4} \left[\begin{array}{cccc|c} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 1 & 5 & 0 \end{array}\right] -R_3+R_4$$

$$\left[\begin{array}{cccc|c} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 & -8 \end{array}\right]$$

Inconsistent

16.) $2x_1 \qquad -4x_4 = -10$
$3x_2 + 3x_3 \qquad = 0$
$x_3 + 4x_4 = -1$
$-3x_1 + 2x_2 + 3x_3 + x_4 = 5$

$$\left[\begin{array}{cccc|c} 2 & 0 & 0 & -4 & -10 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ -3 & 2 & 3 & 1 & 5 \end{array}\right] \begin{array}{l} R_1/2 \\ R_2/3 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -5 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ -3 & 2 & 3 & 1 & 5 \end{array}\right] 3R_1+R_4$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -5 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 2 & 3 & -5 & -10 \end{array}\right] \xrightarrow{-2R_2+R_4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -5 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & -5 & -10 \end{array}\right] \xrightarrow{-R_3+R_4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -5 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & -9 & -11 \end{array}\right]$$

Consistent

1.1 continued

21.) Determine the value(s) of h such that the augmented matrix yields a consistent linear system.

$$\left[\begin{array}{cc|c} 1 & 4 & -2 \\ 3 & h & -6 \end{array}\right] \xrightarrow{-3R_1+R_2} \left[\begin{array}{cc|c} 1 & 4 & -2 \\ 0 & -12+h & 0 \end{array}\right]$$

All h

24.) True or False

a.) Two matrices are row equivalent if they have the same number of rows.

b.) Elementary row operations on an augmented matrix never change the solution set of the associated linear system.

c.) Two equivalent linear systems can have different solution sets.

d.) A consistent system of linear equations has one or more solutions.

a.) False b.) True c.) False d.) True

25.) Find an equation involving g, h, k that makes the following correspond to a consistent system.

$$\left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{array}\right] \xrightarrow{2R_1+R_3} \left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & -3 & 5 & 2g+k \end{array}\right] \xrightarrow{R_2+R_3} \left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & 0 & 0 & h+2g+k \end{array}\right]$$

$$\boxed{h+2g+k=0}$$

31.) Find the row operation that transforms the first matrix into the second and vice versa

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 4 & -1 & 3 & -6 \end{array}\right], \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 0 & 7 & -1 & -6 \end{array}\right]$$

$\rightarrow -4R_1+R_3$

$\leftarrow 4R_1+R_3$

32.) $\left[\begin{array}{ccc|c} 1 & 2 & -5 & 0 \\ 0 & 1 & -3 & -2 \\ 0 & 4 & -12 & 7 \end{array}\right]$, $\left[\begin{array}{ccc|c} 1 & 2 & -5 & 0 \\ 0 & 1 & -3 & -2 \\ 0 & 0 & 0 & 15 \end{array}\right]$

$\longrightarrow$ $-4R_2 + R_3$

$\longleftarrow$ $4R_2 + R_3$