Example 3.5.7. Determine if the following sets of vectors are linearly independent.

(a) $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$ (b) $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$

dependent

Proposition 3.5.8 (Sets of two vectors). A set of two vectors $\{\mathbf{v}_1, \mathbf{v}_2\}$ is linearly independent if and only if neither of the vectors is a multiple of the other.

Question 3.5.9. What can you say about linear dependence/independence of a set of p vectors in \mathbb{R}^n if p > n?

each vectors than entires in each vector

From HW => free variable (more colon than rows)

e.g.

{ [3] [4] [3] [3]

dependent!

Converse not tre!

Theorem 3.5.10 (Too many vectors). If a set contains more vectors than there are entries in each vector, then the set is linearly dependent. That is, any set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ in \mathbb{R}^n is linearly dependent if p > n.

Definition 3.5.12. An indexed set of vectors $\mathcal{B} = \{\mathbf{b}_1, \dots, \mathbf{b}_p\}$ in V is a basis for V if

- (a) \mathcal{B} is a linearly independent set, and
- (b) \mathcal{B} spans all of V; that is,

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$$V = \operatorname{Span}(\mathcal{B}) = \operatorname{Span}\{\mathbf{b}_1, \dots, \mathbf{b}_p\}$$

The **dimension** of a vector space is the number of vectors in any basis for the space.

Example 3.5.13. Let
$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ 0 \\ -6 \end{bmatrix}$$
, $\mathbf{v}_2 = \begin{bmatrix} -4 \\ 1 \\ 7 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$. Is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ a basis for \mathbb{R}^3 ?

Check In indep by reducing $\begin{bmatrix}
U_1 & U_2 & U_3
\end{bmatrix} = \begin{bmatrix}
3 & -4 & -2 \\
0 & 1 & 1 \\
-6 & 7 & 5
\end{bmatrix}$ Proof in every column \Rightarrow linearly indep \checkmark

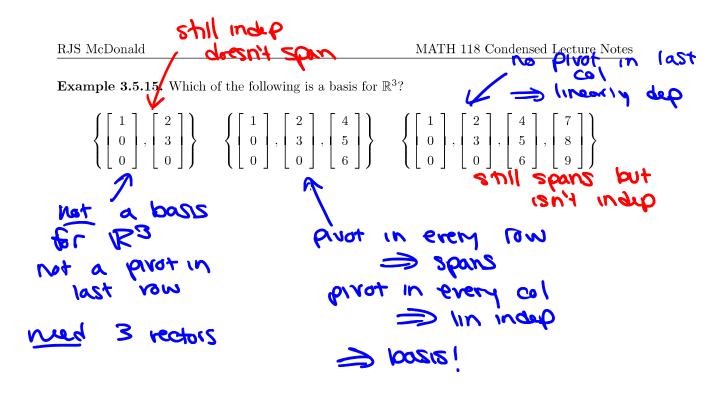
then yes Ax=to previous man solution for all to (prev HW)

columns of A span IR's if There's a proof in every row

Question 3.5.14. What is the dimension of \mathbb{R}^n ? suppose 36, ..., be sis a boss - ce. B spans IRⁿ - B is linearly indep · A= [b, bz ... bp) has n rows linearly indep => or equal
less columns than rows • b,, b,,... be spans Rn only if

A has prosts in each row ⇒ p≥n 30 dm of Pn 15

Remark: for the columns of A to be a basis for IRN, A has to be nxn invertible mitx (pivot in every 10m)



Remark 3.5.16. In one sense, a basis for V is a spanning set of V that is as small as possible. In another sense, a basis for V is a linearly independent set that is as large as possible.

Example 3.5.17. Find a basis for Col U, where $U = \begin{bmatrix} \mathbf{u}_1 \end{bmatrix}$ En, 22, 23, 24, 25} - spans be Span & W. Wz W3 W4 Wsg = Span & 21, 42, 713, 714, 745 } = Span & 11, 713, 714, 715 } also Span & 11, 213, 24, 24, 25 = Span & 21, 213, 215 } is lin indep

Example 3.5.18. Below, A is row equivalent to U from the last example. Find a basis for $\operatorname{Col} A$.

What is the dimension of Col(A)?

we did row ops on A to get U Q In indep of Cal A (there's no does not spon Cal A (there's not ast e

or operations preserve linear independence! $a_2 = 4a_1$ (bic $u_2 = 4u_1$)

a4 = 20, - a3 (b/c 24 = 24, -23)

[2][2]] spons Col A
and linearly indep!

Theorem 3.5.19. The pivot columns of a matrix A form basis for Col A. The dimension of Col(A) is the number of pivots.

Col A = Col N

Example 3.5.20. Find a basis for Nul A, where A is the same as the previous example:

What is the dimension of Nul(A)?

start with a spanning set throw out any appendent vectors!

solve (A o)

a basis

Theorem 3.5.21. The dimension of \mathbf{Y} ul(A) is the number of free variables of A. In other words, the dimension of Nul(A) is the number of columns minus the number of pivots.