1.2 # 2, 10, 13, 14, 19, 21, 24, 29, 31

2.) Which are in reduced echelon form, which in echelon form?

a.) $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ reduced echelon form

b.) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ echelon form

c.) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ not echelon form

d.) $\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ echelon form

10.) Find the general solution

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{array}\right] \xrightarrow{2R_1+R_2} \left[\begin{array}{ccc|c} 1 & -2 & -1 & 4 \\ 0 & 0 & -7 & 14 \end{array}\right] \xrightarrow{R_2/-7} \left[\begin{array}{ccc|c} 1 & -2 & -1 & 4 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

x_1, x_3 are basic

x_2 is free

$x_1 - 2x_2 - x_3 = 4$

$$\begin{cases} x_1 = 2x_2 + 2 \\ x_2 \text{ free} \\ x_3 = -2 \end{cases}$$

13.) $$\left[\begin{array}{ccccc|c} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{3R_2+R_1} \left[\begin{array}{ccccc|c} 1 & 0 & 0 & -1 & -12 & 1 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_3+R_1} \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -3 & 5 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

x_1, x_2, x_4 basic

x_3, x_5 free

$$\begin{cases} x_1 = 3x_5 + 5 \\ x_2 = 4x_5 + 1 \\ x_3 \text{ free} \\ x_4 = -9x_5 + 4 \\ x_5 \text{ free} \end{cases}$$

14.) $$\left[\begin{array}{ccccc|c} 1 & 0 & -5 & 0 & -8 & 3 \\ 0 & 1 & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{8R_3+R_1} \left[\begin{array}{ccccc|c} 1 & 0 & -5 & 0 & 0 & 3 \\ 0 & 1 & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

x_1, x_2, x_5 basic

x_3, x_4 free

$$\begin{cases} x_1 = 5x_3 + 3 \\ x_2 = -4x_3 + x_4 + 6 \\ x_3 \text{ free} \\ x_4 \text{ free} \\ x_5 = 0 \end{cases}$$

1.2

19.) Choose h, k such that the system has a.) no solution, b.) a unique soln, c.) many solutions

$x_1 + hx_2 = 2$

$4x_1 + 8x_2 = k$

$$\left[\begin{array}{cc|c} 1 & h & 2 \\ 4 & 8 & k \end{array}\right] \xrightarrow{-4R_1+R_2} \left[\begin{array}{cc|c} 1 & h & 2 \\ 0 & -4h+8 & -8+k \end{array}\right]$$

a.) no solution if $-4h+8=0$ and $-8+k \neq 0$ ie. $h=2,\ k \neq 8$

b.) one solution if $-4h+8 \neq 0$ ie. $h \neq 2$

c.) many solutions if $-4h+8=0$ and $-8+k=0$ ie. $h=2, k=8$

21.) True/False

a.) In some cases, a matrix can be row reduced to more than one matrix in reduced echelon form, using different sequences of row operations.

b.) The row reduction algorithm applies only to augmented matrices for a linear system.

c.) A basic variable in a linear system is a variable that corresponds to a pivot column in the coefficien matrix.

d.) Finding a parametric description of the solution set of a linear system is the same as solving the system.

e.) If one row in an echelon form of an augmented matrix is [0 0 0 5 0], then the associated linear system is inconsistent.

a.) False b.) False c.) True d.) True e.) False

24.) Suppose a system of linear equations has a 3x5 augmented matrix whose fifth column is not a pivot column. Is the system consistent? why/why not?

$$\left[\begin{array}{cccc|c} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{array}\right]$$

The 5th column is not a pivot column, meaning it doesn't contain a leading 1 in any of the rows (when in reduced echelon form) Thus it is impossible to get all zeros and then a nonzero number in any row [0 0 0 0 | #]. Since this is the only way the system could be inconsistent, the system is consistent.

1.2 continued

29.) A system of linear equations with fewer equations than unknowns is sometimes called an undetermined system. Can such a system have a unique solution? Explain.

Since there are more variables than equations, there are more variables than rows in the augmented matrix. This means at least one of the variables must be free. For each value the free variable takes on, there is a different solution. Therefore the system has many solutions and not one unique solution.

31.) A system of linear equations with more equations than unknowns is sometimes called an overdetermined system. Can such a system be consistent? Illustrate your answer with a specific system of three equations and two unknowns.

Yes, the matrix in reduced echelon form would have a row of all zeros.

$$\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

to make this a system of three equations, each in 2 unknowns, I'll apply some row operations

$$\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right] \begin{array}{l} 2R_2+R_1 \\ \\ 3R_1+R_3 \end{array} \left[\begin{array}{cc|c} 1 & 2 & 6 \\ 0 & 1 & 2 \\ 3 & 0 & 12 \end{array}\right] \begin{array}{l} \\ -R_1+R_2 \\ 2R_2+R_3 \end{array} \left[\begin{array}{cc|c} 1 & 2 & 6 \\ -1 & -1 & -4 \\ 3 & 2 & 16 \end{array}\right]$$

$$\begin{cases} x_1 + x_2 = 6 \\ -x_1 - x_2 = -4 \\ 3x_1 + 2x_2 = 16 \end{cases}$$

has the solution $(x_1, x_2) = (4, 2)$