1.7 # 1, 2, 5, 7, 9, 15, 16, 20, 21, 32, 35

1.) Determine if the vectors are linearly independent.

$\begin{bmatrix}5\\0\\0\end{bmatrix}, \begin{bmatrix}7\\2\\-6\end{bmatrix}, \begin{bmatrix}9\\4\\-8\end{bmatrix}$

These vectors are linearly independent if $x_1\begin{bmatrix}5\\0\\0\end{bmatrix} + x_2\begin{bmatrix}7\\2\\-6\end{bmatrix} + x_3\begin{bmatrix}9\\4\\-8\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$ has only the trivial soln.

$$\left[\begin{array}{ccc|c}5 & 7 & 9 & 0\\0 & 2 & 4 & 0\\0 & -6 & -8 & 0\end{array}\right] \xrightarrow{3R_2+R_4} \left[\begin{array}{ccc|c}5 & 7 & 9 & 0\\0 & 2 & 4 & 0\\0 & 0 & 4 & 0\end{array}\right]$$

x_1, x_2, x_3 are all basic variables

Since there are no free variables $A\vec{x} = \vec{0}$ has only the trivial soln.

The vectors are linearly independent.

2.) $\begin{bmatrix}0\\2\\3\end{bmatrix}, \begin{bmatrix}0\\0\\-8\end{bmatrix}, \begin{bmatrix}-1\\3\\1\end{bmatrix}$

$$\left[\begin{array}{ccc|c}2 & 0 & 3 & 0\\3 & -8 & 1 & 0\\0 & 0 & -1 & 0\end{array}\right] \xrightarrow{-\frac{3}{2}R_1+R_2} \left[\begin{array}{ccc|c}2 & 0 & 3 & 0\\0 & -8 & -7/2 & 0\\0 & 0 & -1 & 0\end{array}\right]$$

The vectors are linearly independent.

5.) Determine if the columns in the matrix form a linearly indep. set.

$$\begin{bmatrix}0 & -3 & 9\\2 & 1 & -7\\-1 & 4 & -5\\1 & -4 & -2\end{bmatrix} \quad \left[\begin{array}{ccc|c}1 & -4 & -2 & 0\\0 & -3 & 9 & 0\\2 & 1 & -7 & 0\\-1 & 4 & -5 & 0\end{array}\right] \begin{array}{l}-2R_1+R_3\\R_1+R_4\end{array} \left[\begin{array}{ccc|c}1 & -4 & -2 & 0\\0 & -3 & 9 & 0\\0 & 9 & -3 & 0\\0 & 0 & -7 & 0\end{array}\right] \xrightarrow{3R_2+R_3} \left[\begin{array}{ccc|c}1 & -4 & -2 & 0\\0 & -3 & 9 & 0\\0 & 0 & 24 & 0\\0 & 0 & -7 & 0\end{array}\right] \begin{array}{l}\\ \\ \to x_3=0\\ \to x_3=0\end{array}$$

The columns form a linearly indep. set.

7.) $\begin{bmatrix}1 & 4 & -3 & 0\\-2 & -7 & 5 & 1\\-4 & -5 & 7 & 5\end{bmatrix}$

Since there are more columns than rows, at least one of the variables is free. Thus there is a non-trivial solution.

The columns form a linearly <u>dependent</u> set.

1.7

9.) $\vec{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 5 \\ -7 \\ h \end{bmatrix}$

a.) For what values of h is $\vec{v}_3$ in Span$\{\vec{v}_1, \vec{v}_2\}$?

b.) For what values of h is $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ linearly dependent?

a.) $\left[\begin{array}{cc|c} 1 & -3 & 5 \\ -3 & 9 & -7 \\ 2 & -6 & h \end{array}\right] \begin{array}{c} \\ 3R_1+R_2 \\ -2R_1+R_3 \end{array} \left[\begin{array}{cc|c} 1 & -3 & 5 \\ 0 & 0 & 8 \\ 0 & 0 & -10+h \end{array}\right]$ This is always inconsistent (because of R_2) So $\vec{v}_3$ is never in Span$\{\vec{v}_1, \vec{v}_2\}$. $\boxed{\text{No } h}$

b.) $\vec{v}_2 = -3\vec{v}_1$ so $\vec{v}_1$ and $\vec{v}_2$ are linearly dependent there $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent no matter what. $\boxed{\text{All } h}$

15.) Determine if the vectors are linearly independent by inspection.

$\begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 7 \end{bmatrix}$ There are more vectors than there are entries in each vector (more columns than rows).

These vectors are linearly dependent.

16.) $\begin{bmatrix} 2 \\ -4 \\ 8 \end{bmatrix}, \begin{bmatrix} -3 \\ 6 \\ -12 \end{bmatrix}$

$-3 = \frac{-3}{2}(2)$

$6 = \frac{-3}{2}(-4)$

$-12 = \frac{-3}{2}(8)$

$\begin{bmatrix} -3 \\ 6 \\ -12 \end{bmatrix} = \frac{-3}{2} \begin{bmatrix} 2 \\ -4 \\ 8 \end{bmatrix}$ Linearly dependent.

20.) $\begin{bmatrix} 1 \\ 4 \\ -7 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ Any set containing the zero vector is linearly dependent

21.) True/False

a.) The columns of a matrix A are linearly independent if the equation $A\vec{x} = \vec{0}$ has the trivial solution. a.) False

b.) If S is a linearly indep. set, then each vector is a linear combination of the other vectors in S. b.) False

c.) The columns of any 4×5 matrix are linearly dependent. c.) True

d.) If $\vec{x}$ and $\vec{y}$ are linearly independent and if $\{\vec{x}, \vec{y}, \vec{z}\}$ is linearly dependent, then $\vec{z}$ is in Span$\{\vec{x}, \vec{y}\}$. d.) True

1.7 continued

32.) Given $A = \begin{bmatrix} 4 & 3 & -5 \\ -2 & -2 & 4 \\ -2 & -3 & 7 \end{bmatrix}$, observe that the first column minus 3 times the second column equals the third column. Find a non-trivial solution of $A\vec{x} = \vec{0}$.

$$\begin{bmatrix} 4 \\ -2 \\ -2 \end{bmatrix} - 3\begin{bmatrix} 3 \\ -2 \\ -3 \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \\ 7 \end{bmatrix} \iff \begin{bmatrix} 4 \\ -2 \\ -2 \end{bmatrix} - 3\begin{bmatrix} 3 \\ -2 \\ -3 \end{bmatrix} - \begin{bmatrix} -5 \\ 4 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix}$$

35.) "If $\vec{v}_1, \dots, \vec{v}_5$ are in $\mathbb{R}^5$ and $\vec{v}_3 = 0$, then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5\}$ is linearly dependent."

True/False & Justify.

True. Any set of vectors containing the zero vector is linearly dependent. (Theorem 9)

$\vec{v}_3 = 0\vec{v}_1 + 0\vec{v}_2 + 0\vec{v}_4 + 0\vec{v}_5$ $\quad$ $\vec{v}_3$ is a linear combination of $\vec{v}_1, \vec{v}_2, \vec{v}_4, \vec{v}_5$