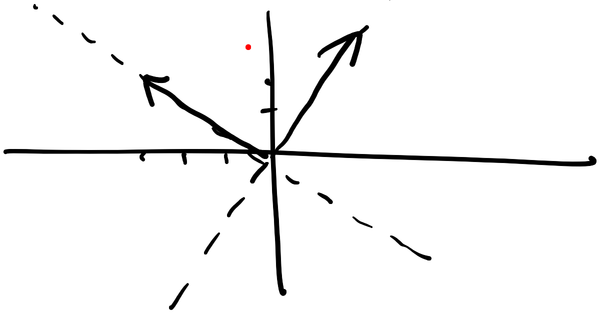
$y = \frac{b}{a} x$

Groups 1.2.12 Show that $\mathbf{u} = (2, 3)$ and $\mathbf{v} = (-3, 2)$ meet at right angles. Hint: we've already seen that (a, b) lives on the line $ay = bx$.



$\vec{v}$ lives on $y = -\frac{2}{3} x$

$\vec{u}$ lives on $y = \frac{3}{2} x$

(a, b) , $(-b, a)$

$$\langle a, b \rangle \cdot \langle -b, a \rangle = -ab + ab = 0$$

$$\langle a, b \rangle \cdot \langle -kb, ka \rangle$$

$$= kab - kab = 0$$

$$\langle 1, 2 \rangle \cdot \langle 3(-2), 6(1) \rangle$$

$$= 1 \cdot (-6) + 2 \cdot (6) = 6$$

Groups 1.2.14 Find a nonzero vector in $\mathbb{R}^3$ that is orthogonal to $\mathbf{u} = (1, 2, 3)$.

let $\langle x_1, x_2, x_3 \rangle$ be perpendicular to $\langle 1, 2, 3 \rangle$

then

$$\langle 1, 2, 3 \rangle \cdot \langle x_1, x_2, x_3 \rangle = 0$$

$$x_1 + 2x_2 + 3x_3 = 0$$

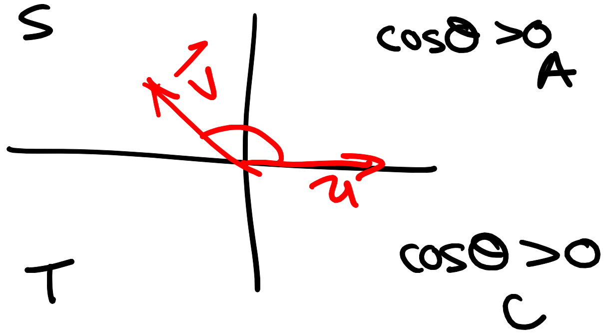
e.g. $\langle 1, 1, -1 \rangle$

$\langle 2, -1, 0 \rangle$

Question 1.2.16 What does this tell us about the *sign* of the dot product $\mathbf{u} \cdot \mathbf{v}$?

$$0 < \vec{u} \cdot \vec{v} = \|u\| * \|v\| * \cos\theta$$

$$0 < \cos\theta$$



$\cos\theta > 0$

$\Rightarrow$ acute

$\Rightarrow u \cdot v > 0$

$\Leftrightarrow \theta < 90^\circ$

acute

$u \cdot v < 0$

$\Leftrightarrow \theta > 90^\circ$

obtuse

$< 180^\circ$

Example 1.3.1 Suppose we are buying and selling candy, again. Remember, gum costs \$1.00 for a pack, chocolate is \$0.75 a bar, and hard candies are \$1.50 for a roll. Suppose

- Monday, we sell 10 packs of gum and 20 chocolate bars and buy 10 rolls of hard candy,
- Tuesday, we buy 10 packs of gum, sell 10 chocolate bars, and buy/sell no hard candies,
- Wednesday, we buy/sell no packs of gum, buy 4 chocolate bars, and buy/sell no hard candies.

What is our net profit?

on quiz...

$$\underbrace{\begin{bmatrix} 1.00 \\ 0.75 \\ 1.50 \end{bmatrix}}_{\text{prices}} \qquad \overbrace{\begin{bmatrix} 10 \\ 20 \\ -10 \end{bmatrix}}^{\text{Monday}} \qquad \overbrace{\begin{bmatrix} -10 \\ 10 \\ 0 \end{bmatrix}}^{\text{Tues}}$$

dot prod of

$$\boxed{1.00 * (10) + 0.75(20) + 1.50(-10) = 10}$$

$$1.00 * (-10) + 0.75(10) + 1.50(0) = -2.50$$

$$1.00 * 0 + 0.75(-4) + 1.50(0) = -3$$

$$1.00 * \begin{bmatrix} 10 \\ -10 \\ 0 \end{bmatrix} + 0.75 \begin{bmatrix} 20 \\ 10 \\ -4 \end{bmatrix} + 1.50 \begin{bmatrix} -10 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ -2.50 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} \text{prod A mon} \\ \text{prod A tues} \\ \text{prod A wed} \end{bmatrix}$$

Example 1.3.4 Which of the following are linear equations?

1. $4x_1 - 5x_2 + 2 = x_1$
2. $x_2 = 2(\sqrt{6} - x_1) + x_3$ $\leftarrow$ $+2x_1 + x_2 - x_3 = 2\sqrt{6}$
3. $4x_1 - 5x_2 = x_1x_2$ $\leftarrow$ not linear
4. $x_2 = 2\sqrt{x_1} - 6$ $\leftarrow$ not linear

$y = mx + b$

$xy = 1$

Example 1.3.6 Is $(5, 6.5, 3)$ in the solution set (the set of all solutions) of the system

$$\begin{aligned} 2x_1 - x_2 + 1.5x_3 &= 8 \\ x_1 \qquad - \quad 4x_3 &= -7 \end{aligned}$$

i.e. is $(5, 6.5, 3)$ in <u>both</u> lines (i.e. in intersection)

$$2(5) - 6.5 + 1.5(3) \stackrel{?}{=} 8 \checkmark$$

$$5 \qquad - 4(3) \stackrel{?}{=} -7 \checkmark$$

Example 1.3.9 What are the solution sets of the following systems?

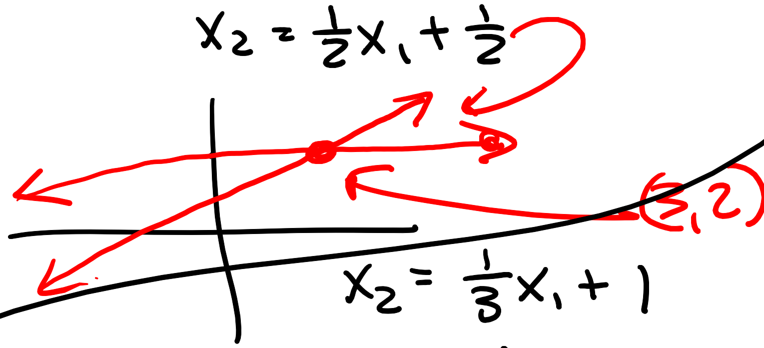
(a) $$\begin{aligned} x_1 - 2x_2 &= -1 \\ -x_1 + 3x_2 &= 3 \end{aligned}$$

(b) $$\begin{aligned} x_1 - 2x_2 &= -1 \\ -x_1 + 2x_2 &= 3 \end{aligned}$$

(c) $$\begin{aligned} x_1 - 2x_2 &= -1 \\ 2x_1 - 4x_2 &= -2 \end{aligned}$$

$$-2x_2 = -1 - x_1$$

$$x_2 = \tfrac{1}{2}x_1 + \tfrac{1}{2}$$

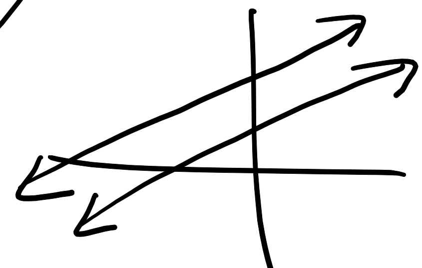


$$x_2 = \tfrac{1}{3}x_1 + 1$$

ONE SOLN

$$x_2 = \tfrac{1}{2}x_1 + \tfrac{1}{2}$$

$$x_2 = \tfrac{1}{2}x_1 + \tfrac{3}{2}$$



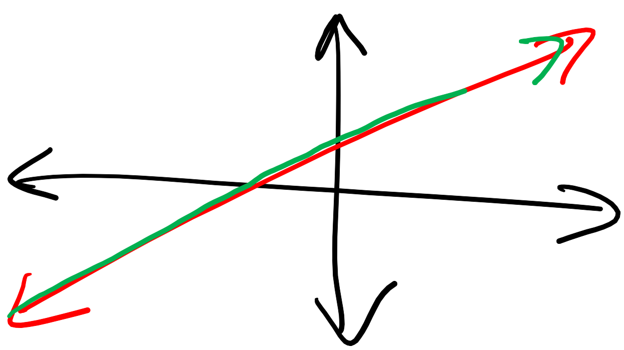
NO SOLN

$$x_2 = \tfrac{1}{2}x_1 + \tfrac{1}{2}$$

$$x_2 = \tfrac{2}{4}x_1 + \tfrac{2}{4} = \tfrac{1}{2}x_1 + \tfrac{1}{2}$$

all points

$$\left(x_1, \tfrac{1}{2}x_1 + \tfrac{1}{2}\right)$$



Example 1.3.10 What is the solution set of the following system? If we fix b_1, b_2, b_3, how many solutions will it have?

$$\begin{aligned} x_1 \qquad\qquad &= b_1 \\ -x_1 + x_2 \qquad &= b_2 \\ -x_2 + x_3 &= b_3 \end{aligned}$$

$\Longrightarrow x_1 = b_1$

$\Longrightarrow x_2 = b_2 + x_1$
$= b_1 + b_2$

$x_3 = b_3 + x_2$
$= b_1 + b_2 + b_3$

Substitution

1.3.2 Matrices

$$\begin{matrix} 1.00(10) & + 0.75(20) & + 1.50(-10) \\ 1.00(-10) & + 0.75(10) & + 1.50(0) \\ 1.00(0) & + 0.75(-4) & + 1.50(0) \end{matrix}$$

$$= 1.00\begin{bmatrix} 10 \\ -10 \\ 0 \end{bmatrix} + 0.75\begin{bmatrix} 20 \\ 10 \\ -4 \end{bmatrix} + 1.50\begin{bmatrix} -10 \\ 0 \\ 0 \end{bmatrix}$$

Example 1.3.15 Compute the product $A\mathbf{x}$ where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$A\vec{x} = \vec{a}_1 x_1 + \vec{a}_2 x_2 + \vec{a}_3 x_3$$

$$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \cdot (1) + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \cdot 2 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot 3 = \begin{bmatrix} 1+0+0 \\ -1+2+0 \\ 0-2+3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

It's a linear comb of columns of A using $\vec{x}$ as weights.

Remark: If # columns of $A \neq$ # rows of $\vec{x}$ then $A\vec{x}$ is undefined

e.g. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} * \begin{bmatrix} 1 \\ 2 \end{bmatrix} =$ undefined

---

suppose have 3 vitamins.

vitamin #1 has 10mg vitamin A
10 mg vitamin C

supplement #2 has 10 mg vit C
10 mg of iron

supplement #3 has 20 mg iron

$$\begin{array}{c|c} & \begin{array}{ccc} \#1 & \#2 & \#3 \end{array} \\ \begin{array}{c} A \\ C \\ \text{iron} \end{array} & \begin{bmatrix} 10 & 0 & 0 \\ 10 & 10 & 0 \\ 0 & 10 & 20 \end{bmatrix} \end{array} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 1 * \begin{bmatrix} 10 \\ 10 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 10 \\ 10 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix} = \begin{bmatrix} 10 \\ 30 \\ 80 \end{bmatrix} \begin{array}{c} A \\ C \\ \text{iron} \end{array}$$

how many take

what rep

Example 1.3.17 Compute the product $A\mathbf{x}$ where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_1\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + x_2\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + x_3\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 * 1 + x_2 * 0 + x_3 * 0 \\ x_1 * (-1) + x_2 * (1) + x_3 * 0 \\ x_1 * (0) + x_2(-1) + x_3 * 1 \end{bmatrix}$$

first entry is dot prod of top row and vector

ith row of $A\vec{x}$ is the dot product of ith row of A with $\vec{x}$

$$A = \begin{array}{c} \\ A \\ C \\ \text{iron} \end{array}\begin{array}{c} \#1 \quad \#2 \quad \#3 \\ \begin{bmatrix} 10 & 0 & 0 \\ 10 & 10 & 0 \\ 0 & 10 & 20 \end{bmatrix} \end{array} \qquad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

what does $A\vec{x}$ represent

$$A\vec{x} = \begin{bmatrix} 10x_1 + 0 + 0 \\ 10x_1 + 10x_2 + 0 \\ 0 + 10x_2 + 20x_3 \end{bmatrix} \begin{array}{l} A \\ C \\ \text{iron} \end{array}$$

Example 1.3.18 What if $A\mathbf{x} = \mathbf{b}$ where A and $\mathbf{b}$ are given, but $\mathbf{x}$ is unknown? How could we find $\mathbf{x}$ if we're told

$$\cancel{A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}}$$

$$\begin{array}{c} \\ A \\ C \\ \text{iron} \end{array} \begin{array}{c} \#1 \quad \#2 \quad \#3 \\ \begin{bmatrix} 10 & 0 & 0 \\ 10 & 10 & 0 \\ 0 & 10 & 20 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \\ 50 \end{bmatrix}$$

$$A\vec{x} = \begin{bmatrix} 20 \\ 30 \\ 50 \end{bmatrix}$$

want to know $\vec{x}$ (reps total of each supplement I take)

$$\begin{bmatrix} 10 & 0 & 0 \\ 10 & 10 & 0 \\ 0 & 10 & 20 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10x_1 \\ 10x_1 + 10x_2 \\ 10x_2 + 20x_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \\ 50 \end{bmatrix}$$

$$\boxed{x_1 = 2}$$

second row $10x_1 + 10x_2 = 30$

$$10(2) + 10x_2 = 30 \iff \boxed{x_2 = 1}$$

third row

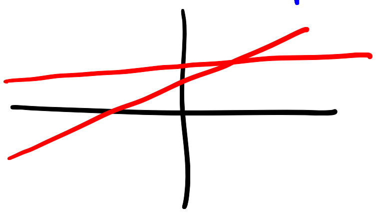
$$10x_2 + 20x_3 = 50$$
$$10(1) + 20x_3 = 50$$
$$\boxed{x_3 = 2}$$

$$\begin{aligned} 10x_1 \qquad\qquad &= 20 \\ 10x_1 + 10x_2 \qquad &= 30 \\ 10x_2 + 20x_3 &= 50 \end{aligned}$$

Example 2.1.1 How many solutions do each of the following systems have?

(a) $$\begin{aligned} x_1 - 2x_2 &= -1 \\ -x_1 + 3x_2 &= 3 \end{aligned}$$

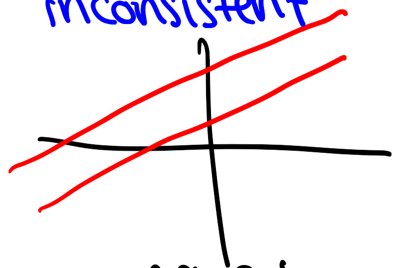
Consisi~~et~~t



one soln

(b) $$\begin{aligned} x_1 - 2x_2 &= -1 \\ -x_1 + 2x_2 &= 3 \end{aligned}$$

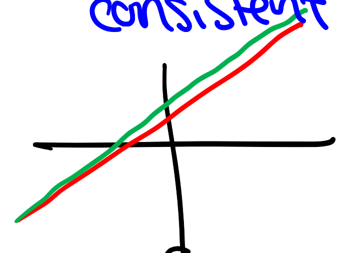
inconsistent



no soln

(c) $$\begin{aligned} x_1 - 2x_2 &= -1 \\ 2x_1 - 4x_2 &= -2 \end{aligned}$$

consistent



inf many

is it possible to get exactly two soln

$\Rightarrow$ No, not in this case

Example 2.1.5 Determine if the following system of equations is consistent.

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ 5x_1 \quad\quad - 5x_3 &= 10 \end{aligned}$$

$$\begin{aligned} & x_1 - 2x_2 + x_3 = 0 \\ +(& \quad\;\; 2x_2 - 8x_3 = 8) \end{aligned}$$

$$\boxed{x_1 \quad - 7x_3 = 8}$$

eqn 4

$5x_1 = 10 + 5x_3$

$\boxed{x_1 = 2 + x_3}$

① $(2+x_3) - 2x_2 + x_3 = 0$

$2 + 2x_3 - 2x_2 = 0$

$\Leftrightarrow \boxed{x_3 = x_2 - 1}$

② $2x_2 - 8(x_2 - 1) = 8$

$-6x_2 + 8 = 8$

$x_2 = 0$ ✓

$\Longrightarrow x_1 = 1$

$x_2 = 0$

$x_3 = -1$

Example 2.2.1 In Section 1.3 we determined whether the following systems were consistent using geometric and substitution arguments. Is there an algebraic way to do this without substitution?

(a)
$$x_1 - 2x_2 = -1$$
$$+\left(-x_1 + 3x_2 = 3\right)$$
$$0 + \boxed{x_2 = 2}$$
$$x_1 - 2(2) = -1$$
$$\Rightarrow \boxed{x_1 = 3}$$

(b)
$$x_1 - 2x_2 = -1$$
$$+\left(-x_1 + 2x_2 = 3\right)$$
$$0 + 0 = 2$$
$$\cancel{0 = 2}$$
$$5 = 6$$
no soln

~~(c)~~
$$-2\left(x_1 - 2x_2 = -1\right)$$
$$+\ 2x_1 - 4x_2 = -2$$
$$0 + 0 = 0$$

Example 2.2.3 Determine if the following system of equations is consistent without substitution

$$\begin{cases} x_1 - 2x_2 + x_3 = 0 \\ 2x_2 - 8x_3 = 8 \\ 5x_1 - 5x_3 = 10 \end{cases} \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right]$$

$\frac{1}{5}$ eqn 3

$$\sim \begin{cases} x_1 - 2x_2 + x_3 = 0 \\ 2x_2 - 8x_3 = 8 \\ x_1 - x_3 = 2 \end{cases}$$

$\frac{1}{5}R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 1 & 0 & -1 & 2 \end{array}\right]$$

eqn 3 − eqn 1

$$\begin{array}{r} x_1 \quad - x_3 = 2 \\ -(x_1 - 2x_2 + x_3 = 0) \end{array}$$

$R_3 = R_3 - R_1$

$$\sim \begin{cases} x_1 - 2x_2 + x_3 = 0 \\ 2x_2 - 8x_3 = 8 \\ 2x_2 - 2x_3 = 2 \end{cases} \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 2 & -2 & +2 \end{array}\right]$$

eqn3 = eqn 3 − eqn 2

$$\begin{array}{r} 2x_2 - 2x_3 = 2 \\ -(2x_2 - 8x_3 = 8) \\ \hline 0 \quad 6x_3 = -6 \end{array}$$

$R_3 = R_3 - R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ (0-0) & (2-2) & (-2+8) & -2-8 \end{array}\right]$$

$$\sim \begin{cases} x_1 - 2x_2 + x_3 = 0 \\ 2x_2 - 8x_3 = 8 \\ \boxed{6x_3 = -6} \end{cases}$$

$x_3 = -1$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 0 & 6 & -6 \end{array}\right]$$

what system does this rep'nt

$$\begin{array}{r} x_1 - 2x_2 + x_3 = 0 \\ 2x_2 - 8x_3 = 8 \\ 6x_3 = -6 \end{array}$$

now solve.

Example 2.2.4 Determine if the following system is consistent:

$$\begin{aligned} x_2 - 4x_3 &= 8 \\ 2x_1 - 3x_2 + 2x_3 &= 1 \\ 4x_1 - 6x_2 + 4x_3 &= 1 \end{aligned}$$

$$\sim \left[\begin{array}{ccc|c} 0 & 1 & -4 & 8 \\ 2 & -3 & 2 & 1 \\ 4 & -6 & 4 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 0 & 1 & -4 & 8 \\ 2 & -3 & 2 & 1 \\ 4-4 & -6+6 & 4-4 & 1-2 \end{array}\right]$$

$R_3 = R_3 - 2R_2$

$$\sim \left[\begin{array}{ccc|c} 0 & 1 & -4 & 8 \\ 2 & -3 & 2 & 1 \\ 0 & 0 & 0 & -1 \end{array}\right]$$

$$\sim \begin{cases} x_2 - 4x_3 = 8 \\ 2x_1 - 3x_2 + 2x_3 = 1 \\ \boxed{0x_1 + 0x_2 + 0x_3 = -1} \end{cases} \quad 0 = -1$$

$\Rightarrow$ system is inconsistent!

Example 2.3.8 Which of the following is in echelon form? Reduced echelon form? pivot

$$\begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 5/2 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 2 & 12 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

pivots pivot

* Any matrix can be row reduced
- add/sub multiples of row from another
- scale rows
- switch

to a matrix in echelon form

Example 2.3.12 Row reduce the matrix A to echelon form and locate pivot columns.

$$A = \left[\begin{array}{cccc|c} 0 & -3 & -6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 1 & 4 & 5 & -9 & -7 \end{array}\right]$$

swap R_1/R_4 swapped

$$\left[\begin{array}{cccc|c} 1 & 4 & 5 & -9 & -7 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 0 & -3 & -6 & 4 & 9 \end{array}\right] \sim$$

$R_2 = R_1 + R_2$ cancelled

$$\left[\begin{array}{cccc|c} 1 & 4 & 5 & -9 & -7 \\ 0 & 2 & 4 & -6 & -6 \\ -2 & -3 & 0 & 3 & -1 \\ 0 & -3 & -6 & 4 & 9 \end{array}\right]$$

$R_3 = 2R_1 + R_3$ cancelled

$$\left[\begin{array}{cccc|c} 1 & 4 & 5 & -9 & -7 \\ 0 & 2 & 4 & -6 & -6 \\ 0 & 5 & 10 & -15 & -15 \\ 0 & -3 & -6 & 4 & 9 \end{array}\right] \sim$$

$R_2 = \frac{1}{2}R_2$

$R_3 = \frac{1}{5}R_3$ 5

$$\left[\begin{array}{cccc|c} 1 & 4 & 5 & -9 & -7 \\ 0 & 1 & 2 & -3 & -3 \\ 0 & 1 & 2 & -3 & -3 \\ 0 & -3 & -6 & 4 & 9 \end{array}\right]$$

$R_3 - R_2$

$$\sim \begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 1 & 2 & -3 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & -6 & 4 & 9 \end{bmatrix} \sim$$

$R_4 = 3R_2 + R_4$

$$\begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 1 & 2 & -3 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & 0 \end{bmatrix}$$

swap R_3 and R_4

$$\sim \boxed{\begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 1 & 2 & -3 & -3 \\ 0 & 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}} \quad 2 \quad \checkmark$$

echelon form (infinitly many of those)

$$\begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 1 & 2 & -3 & -3 \\ 0 & 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 1 & 2 & -3 & -3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 5 & 0 & -7 \\ 0 & 1 & 2 & 0 & -3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 = 3R_3 + R_2$

$R_1 = 9R_3 + R_1$

$R_1 = R_1 - 4R_2$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 5 \\ 0 & 1 & 2 & 0 & -3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Example 2.3.16 Find the general solution of a linear system whose augmented matrix can be reduced to the matrix below.

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \begin{array}{l} x \quad -5z = 1 \\ y + z = 4 \\ 0 = 0 \end{array}$$

$x = 1 + 5z$
$y = 4 - z$

x and y depend on z

theres no restriction on z itself

$y = 4 - z$
$z = 4 - y$

$$\begin{array}{l} x + y = 2 \\ y + z = 3 \\ z = 4 \\ 0 = 0 \end{array} \qquad \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

does zero row mean that one var is all real numbers?

NOT necessarily eg.

$z = 4$
$y = 1 - 4$
$x = 5$

Example 2.3.19 Find the general solution of a system whose augmented matrix is reduced to

$$\left[\begin{array}{ccccc:c} 1 & 6 & 2 & -5 & -2 & -4 \\ 0 & 0 & 2 & -8 & -1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{array}\right]$$

to find gen soln to a system
- reduce to RREF and solve basic variables in terms of free variables