Appendix F. L'Hôpital's Rule

Goals

- review indeterminate forms
- L'Hôpital's Rule
- summation notation

Example F.1. Find $\lim_{x\to 1} \dfrac{x^2-2x+1}{x^2-1}$ and $\lim_{x\to\infty} \dfrac{x^2-2x+1}{x^2-1}$.

Example F.2. Find $\lim_{x\to 1} \dfrac{\ln(x)}{1-x^2}$

Theorem F.3. If f and g are differentiable near $x = c$ (or ∞), and $\lim_{x\to c} f(x) = \lim_{x\to c} g(x) = 0$ or $\lim_{x\to c} f(x) = \lim_{x\to c} f(x) = \infty$, then

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)} \qquad \left(\text{similarly } \lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}\right)$$

Question F.4. Why does L'Hôpital's Rule work?

Example F.5. Find $\lim_{x\to 2} \dfrac{e^{x^2} - e^4}{x - 2}$

Example F.6. Find $\lim_{x\to \infty} \dfrac{3x - 2}{e^{x^2}}$

Example F.7. Find $\lim_{x \to 0} \dfrac{\sin(x)}{x+1}$

Example F.8. Find $\lim_{x \to 0} \dfrac{x^{100}}{x^{100} - x^{99}}$

Example F.9. Find $\lim_{x\to\infty} xe^{-x}$

Example F.10. Find $\lim_{x\to 0^+} x\ln x$

Example F.11. Find $\lim_{x\to\infty} x^{1/x}$

Question F.12. What are the indeterminate forms we've looked at and how do we find their limits?

Appendix F. Extra examples

Example F.13. (a) Find $\lim_{x\to 0} \dfrac{e^{3x}-1-3x}{e^{x^2}-\cos x}$

(b) Find $\lim_{x\to 0} \ln x \tan x$

(c) Find $\lim_{x\to\infty} \big(1+3/x\big)^x$