1.5 # 2, 6, 11, 15, 18, 19, 22, 23, 27, 30

2.) Determine if the system has a nontrivial solution.

$$x_1 - 2x_2 + 3x_3 = 0$$
$$-2x_1 - 3x_2 - 4x_3 = 0$$
$$2x_1 - 4x_2 + 9x_3 = 0$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ -2 & -3 & -4 & 0 \\ 2 & -4 & 9 & 0 \end{array}\right] \begin{array}{c} \\ 2R_1+R_2 \\ -2R_1+R_3 \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & -7 & 2 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right]$$

There are no free variables, so the only solution is the trivial solution.

6.) Write the solution set of the given homogeneous system in parametric vector form.

$$x_1 + 2x_2 - 3x_3 = 0$$
$$2x_1 + x_2 - 3x_3 = 0$$
$$-1x_1 + x_2 \qquad = 0$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 2 & 1 & -3 & 0 \\ -1 & 1 & 0 & 0 \end{array}\right] \begin{array}{c} \\ -2R_1+R_2 \\ R_1+R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 3 & -3 & 0 \end{array}\right] \begin{array}{c} \\ R_2/-3 \\ R_2+R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{c} -2R_2+R_1 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{cases} x_1 = x_3 \\ x_2 = x_3 \\ x_3 \text{ free} \end{cases}$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

11.) Describe all solutions of $A\vec{x} = \vec{0}$ in parametric vector form, where A is row equivalent to the given matrix.

$$\begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{c} 2R_2+R_1 \\ \\ \\ \\ \end{array} \begin{bmatrix} 1 & -4 & 0 & 0 & 3 & -7 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{c} -3R_3+R_1 \\ \\ \\ \\ \end{array} \begin{bmatrix} 1 & -4 & 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 4x_2 - 5x_6 \\ x_2 \\ x_6 \\ x_4 \\ 4x_6 \\ x_6 \end{bmatrix} = x_2 \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} -5 \\ 0 \\ 1 \\ 0 \\ 4 \\ 1 \end{bmatrix}$$

15.) Describe and compare the solution sets of $x_1+5x_2-3x_3=0$ and $x_1+5x_2-3x_3=-2$

For the nonhomogeneous equation, $x_1=-5x_2+3x_3-2$ & the solution set is

$$\vec{x}=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}-5x_2+3x_3-2\\x_2\\x_3\end{bmatrix}=x_2\begin{bmatrix}-5\\1\\0\end{bmatrix}+x_3\begin{bmatrix}3\\0\\1\end{bmatrix}+\begin{bmatrix}-2\\0\\0\end{bmatrix}$$

For the homogeneous equation, $x_1=-5x_2+3x_3$ & the solution set is

$$\vec{x}=x_2\begin{bmatrix}-5\\1\\0\end{bmatrix}+x_3\begin{bmatrix}3\\0\\1\end{bmatrix}$$

Geometrically, the solution set of the homogeneous equation is the plane through the origin spanned by $\begin{bmatrix}-5\\1\\0\end{bmatrix}$ and $\begin{bmatrix}3\\0\\1\end{bmatrix}$. The solution set of the nonhomogeneous equation is the plane through $\begin{bmatrix}-2\\0\\0\end{bmatrix}$ parallel to the solution set of the homogeneous equation.

18.) Describe the solutions of the following system in parametric vector form, and provide a geometric comparison with the solution set from exercise 6.

$$\begin{aligned}x_1+2x_2-3x_3&=5\\2x_1+x_2-3x_3&=13\\-x_1+x_2\quad\quad&=-8\end{aligned}$$

$$\left[\begin{array}{ccc|c}1&2&-3&5\\2&1&-3&13\\-1&1&0&-8\end{array}\right]\begin{matrix}\\-2R_1+R_2\\R_1+R_3\end{matrix}\left[\begin{array}{ccc|c}1&2&-3&5\\0&-3&3&3\\0&3&-3&-3\end{array}\right]\begin{matrix}\\R_2/(-3)\\R_2+R_3\end{matrix}\left[\begin{array}{ccc|c}1&2&-3&5\\0&1&-1&-1\\0&0&0&0\end{array}\right]\begin{matrix}-2R_2+R_1\\\\\\\end{matrix}$$

$$\left[\begin{array}{ccc|c}1&0&-1&7\\0&1&-1&-1\\0&0&0&0\end{array}\right]\quad\vec{x}=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}x_3+7\\x_3-1\\x_3\end{bmatrix}=x_3\begin{bmatrix}1\\1\\1\end{bmatrix}+\begin{bmatrix}7\\-1\\0\end{bmatrix}$$

This solution set is the line through $\begin{bmatrix}7\\-1\\0\end{bmatrix}$ that is parallel to the solution set in exercise 6 (also a line).

1.5 continued

19.) Find the parametric equation of the line through $\vec{a} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$ Parallel to $\vec{b} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$.

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} + t\begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

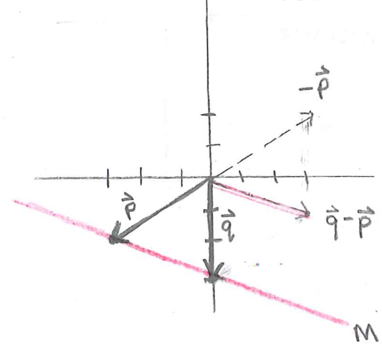
22.) Find a parametric eqn. of the line M through $\vec{p} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$ and $\vec{q} = \begin{bmatrix} 0 \\ -3 \end{bmatrix}$ (Hint: M is parallel to $\vec{q} - \vec{p}$)

oops! The book says $\vec{p} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$
Oh well, too late now!

M is parallel to $\vec{q} - \vec{p} = \begin{bmatrix} 0 \\ -3 \end{bmatrix} - \begin{bmatrix} -3 \\ -2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$$\vec{x} = \begin{bmatrix} -3 \\ -2 \end{bmatrix} + t\begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

For reference



23.) True/False

a.) A homogeneous equation is always consistent.

b.) The equation $A\vec{x} = \vec{0}$ gives an explicit description of its solution set.

c.) The homogeneous equation $A\vec{x} = \vec{0}$ has the trivial solution iff the eqn has at least one free variable.

d.) The equation $\vec{x} = \vec{p} + t\vec{v}$ describes a line through $\vec{v}$ parallel to $\vec{p}$.

e.) The solution set of $A\vec{x} = \vec{b}$ is the set of all vectors of the form $w = \vec{p} + \vec{v}_h$ where $\vec{v}_h$ is any solution of the eqn $A\vec{x} = \vec{0}$.

a.) True   b.) False   c.) False   d.) False   e.) False

1.5

27.) Suppose $A\vec{x}=\vec{b}$ has a solution. Explain why the solution is unique precisely when $A\vec{x}=\vec{0}$ has only the trivial solution.

$A\vec{x}=\vec{b}$ has a solution which is the set of all vectors of the form $\vec{w}=\vec{p}+\vec{v}_h$ where $\vec{v}_h$ is any solution of $A\vec{x}=\vec{0}$.

Therefore $A\vec{x}=\vec{b}$ has exactly one solution $\vec{w}$ when $A\vec{x}=\vec{0}$ has exactly one solution, $\vec{v}_h$.

30.) A is a 2×5 matrix with two pivot positions

a.) does the equation $A\vec{x}=\vec{0}$ have a non trivial solution?

b.) does the equation $A\vec{x}=\vec{b}$ have at least one solution for every Possible b?

a.) $\begin{bmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{bmatrix}$ Since A has 2 pivot positions, there are 3 free variables, so $A\vec{x}=\vec{0}$ has a nontrivial solution.

b.) Since A has a pivot position in every row, $A\vec{x}=\vec{b}$ is consistent regarless of $\vec{b}$.