

*Repetitio est mater studiorum*  
(Repetition is the mother of study).  
-Latin Wisdom

*Interest comes with success,  
and success comes with hard work.*  
-Professor Yaroslav Tagamlizki

## MATH 118, Introduction to Functions of Several Variables

Yale University

Spring 2020

### Instructor Information

**Instructor:** Bobby McDonald

**Office:** DL 429

**Location and meet time:** DL 431, Tu/Th 2:30 - 3:45p

**Email:** [robert.j.mcdonald@yale.edu](mailto:robert.j.mcdonald@yale.edu) (use Piazza, instead)

**Piazza:** <https://piazza.com/class/k53x83b5us48b>

**Office hours** (tentative until second or third week):

**Tu/Th:** 4:00 - 5:30p, **We:** 4:00 - 5:30p, (or by appointment)

**Course webpage:** <https://mathrjism.com/yale/teaching/spring2020/math118/>

**How to Contact Me:** I'd prefer all communication take place on Piazza For logistical/personal questions, you can send me a private message. For questions regarding in class material, homework, quizzes, or exam study, you can post publicly to the discussion. You can do this anonymously!

### Book

For the first portion of the course, we will use Introduction to Linear Algebra (4th edition) by Strang (recommended, not required). We will use the book Calculus, Multivariable (7th edition) by McCallum (recommended, not required) throughout the second portion of the course. *Readings will be assigned* out of Calculus and Linear Algebra Notes - Economics Applications, found at

[https://mathrjism.com/yale/teaching/spring2020/math118/shen\\_economics\\_notes.pdf](https://mathrjism.com/yale/teaching/spring2020/math118/shen_economics_notes.pdf)

### Grading (descriptions of each item follow)

Assessment	Time/Loc	Percent
Homework	due every Monday night at 11:59pm	9 + 3 (see "homework" below)
Project	Tuesday in the last week	3
Quizzes	every Tuesday at the beginning of class	15
Exams 1 & 2	Tuesdays, Feb 25 and April 8, in class	(see below)
Final	during finals week, TBA	(see below)

Your final numerical grade will be the greater of

- 30% Written Assignments and Quizzes + 30% Midterm Score + 40% Final Exam
- 30% Written Assignments and Quizzes + 40% Midterm Score + 30% Final Exam

Your final numerical grade will be converted to letter grades using the following scale:

$F \leq 65$      $50 \leq D < 65$      $65 \leq C-, C, C+ < 80$      $80 \leq B-, B, B+ < 90$      $90 \leq A-, A$

## Piazza

I've set up a course for us on Piazza. Use the Q&A section of this site for questions about material and homework assignments. This way, everyone can benefit from your question and its answer. Feel free to post anonymously if you are uncomfortable! Try to contribute answers, too! This site shines if we all work together to answer each other's questions. Please use it often!

## Homework

Homework assignments will consist of written homework assignments and assigned readings from Shen's *Calculus and Linear Algebra Notes*. Each homework assignment will be worth 10 points. Your written assignment solutions will be graded 9/12 for completion and 3/12 for accuracy using the 3-point scale in the rubric below. Late assignments can be submitted for up to a week after the due date but will receive a one point deduction on the completion grade for every day they are late.

3 points	Work is completely accurate, essentially perfect. Ideas are fully developed. Work is neat and easy to read. Complete sentences are used where appropriate.
2 points	Work is good, on the right track, but development of ideas is incomplete. Work is hard to read or disorganized.
1 point	Work is sketchy, with some correct ideas, but mostly on the wrong track. Work is messy or illegible.
0 points	Work is minimal or non-existent. No explanations are given. Answer is completely incorrect.

The *written assignments* assess your ability to synthesize information and construct arguments. Your answers will be in the form of explanations written in plain English with mathematical notations. You will be graded on the mathematical, logical and grammatical coherence of your explanations. You are encouraged to form study groups and work on homework assignments together. However, you must write your solutions independently.

**You will submit your written homework assignments as a pdf. file** by taking a picture of or scanning your written work. I recommend using the app Scannable.

## Quizzes

With a few exceptions there will be a ten minute quiz every Tuesday at the beginning of class. There are no quizzes in the first or last week of classes (check the schedule for a better picture). In general, the quizzes will be around two questions on whatever was covered the previous week.

**Conference quizzes:** For the first five minutes of class, one of the quiz questions will be posted on the board with some of the information missing (e.g. numbers). This time is to discuss the question with a partner (no writing anything down). After five minutes are up, I'll pass the quiz out and you can begin.

## Project

***The stakes for this assignment are intentionally very low.*** It's supposed to be ***fun*** and a way for you to see how relevant the material is! Past students have really enjoyed this exercise, and I hope you do, too.

I'll be grading these myself, mostly holistically, and on effort. I just want to see that you did ***something***. I'm not an English teacher! At the end of the semester, I'd like you to reflect on how what we've learned is used in either your field, or in a topic that interests you. For example, you may want to talk about how constrained optimization is used in economics, or you may be interested in reading up on how linear algebra is used in video games. I'll post a rubric later, but you can find a project description at the end of this Syllabus or on Canvas. It will be due Tuesday in the last week of classes. Again, the stakes are low.

## Exams/Final

There will be two in class exams. The first exam will take place after we finish chapter 4, the linear algebra portion of our course, roughly around week 7 (expected February 25), and the second exam will take place after we finish chapter 14 of the multivariable calculus portion of our course, roughly in week 13 (expected April 8). See the schedule at the end of the syllabus for more information.

## Calculator policy

You should feel free to use a calculator or a computer to check or investigate problems for homework, but be sure you can do the problems without. **Calculators will not be allowed on in class assessments.** We will make sure that the problems on the exams require only a moderate amount of calculation to allow you to spend most of your time demonstrating your mathematical knowledge.

## Special accommodations

Student athletes and students with disabilities should inform your instructor of your commitments as an athlete, any special needs that you have, etc. within the first three weeks of the semester. You will be expected to bring in a letter from the Athletics Department or the Center for Students with Disabilities. For conflicts with final examinations, students should, as usual, contact the Dean of Students.

## My advice for success in the course

- Work outside of class! These topics can not be learned in  $150 = 75 + 75$  minutes each week. On average, students (not just in our class, but in general) are supposed to spend an average of two hours outside of class. That would be about six hours per class. That's why they call it full time!
- Make mistakes! Particularly on the homework, you might not know what to try first. Just try something. The process of mathematics is full of mistakes. I could fill a book with all of the mistakes I've made. They're an integral part of learning!
- Work with your peers. Bouncing ideas off of each other is the best way to learn.
- **COME TO MY OFFICE HOURS!** It's free tutoring. From your instructor. You know, the one who writes the exams. Please come, or make an appointment if you can't!
- We'll be assigned a peer tutor, and when we do, I'll be sure to post relevant information about how to access them on our website. This is the best place to go to get someone else's take on the material that isn't me. They might be able to explain something differently.

## Schedule

Rather than have a set rigid schedule, I want to try to be flexible and give each topic the time it deserves. I will always give you a heads up as to what the next few topics will be, but in most cases you can safely assume we'll be doing every section from each chapter, roughly in the following order.

### **First half (roughly weeks 1-6)**

*We will start the semester with Linear algebra, from Strang's "Introduction to linear algebra", covering chapters 1-4. You do not need to buy this book, but our topic order will roughly be the same.*

**Chapter 1** talks about vectors and matrices, and how they relate to systems of linear equations.

**Chapter 2** will teach us how to simplify systems of equations, calculate matrix inverses, and factor matrices into simpler ones. This will give us the techniques to solve systems given by invertible matrices, which always have a unique solution.

**Chapter 3** studies vector spaces, concentrating on those associated with matrices. It will allow us to deal with more complicated systems, recognize when solutions may not exist, and when there might be more than one.

**Chapter 4** works with systems that do not have a solution: we will learn how to find the "best answer", one that does not fit the equations exactly, but it is as close as possible.

**Midterm 1** will cover all of the linear algebra we've learned. The expected date of this exam is Tuesday, February 25, in class. Really, we'll take it after we finish chapter 4, which is expected around week 7.

### **Second half (roughly weeks 8, 11-15)**

*Using what we have learned about matrices. We will cover chapters 12-15 in McCallum's "Calculus, Multivariable". You do not need to buy this book, but our topic order will roughly be the same.*

**Chapter 12** introduces functions of multiple variables. We will talk about equations of surfaces, draw cross-sections and topographical maps.

**Chapter 13** is a review of linear algebra: we will add only the cross product from here, which will allow us to write equations of planes in three dimensions.

**Chapter 14** is about derivatives. We will learn how to calculate  $x$  and  $y$  derivatives, as well as derivatives in other directions (for example along  $y = x$ ). The techniques are very similar to those of single variable calculus. Tangent lines to curves will be replaced by tangent planes to surfaces, and we will have three different kinds of second order derivatives instead of one; but everything in this chapter should feel familiar.

**Midterm 2:** the second exam (not cumulative) will cover the multivariable calculus we've learned up through Chapter 14. The expected date of this exam is Tuesday, April 8, in class. Again, we'll really take it after we finish chapter 14, which you should expect around week 13.

**Chapter 15** is about optimization, and finding minima and maxima of multivariable functions. Once again, the material will be quite familiar from single variable calculus, though the second derivative test will be more complicated, as will be the checking of "endpoints" (which will now be entire boundary curves of the region in question). Lastly, we will cover optimization subject to a constraint, using a technique known as Lagrange multipliers.

## **Project Description:** *Multivariable Mathematics in the Real World*

Try a google search with “How are linear algebra (or multivariable calculus) used in...” or “constrained optimization in economics” to find articles on how our topics apply in your major, area of study, or something you're interested in. Some ideas with interesting results are:

- Computer science
- Business/Economics
- Engineering
- Video games
- Cell phones
- Sports analytics
- Google’s search algorithm

Report why you chose the topic and how linear algebra applies using specific to examples. Some ideas of what you might talk about are below. These are **suggestions**, they may or may not apply to the topic you’ve chosen. Feel free to talk about anything that interests you.

### **Reason for Choosing the Topic**

- Why did you choose your particular topic?
- How does the topic relate to you?
- Was it easy to find connections to our course?

### **Connections to Linear Algebra**

- What applications do you find?
- How important are the subjects in our course to the topic you’ve chosen?
- Are there any other applications of mathematics to this topic?

### **Connections to Class**

- What things have we learned in class that are similar to what you find online?
- What parts of the applications do you feel like you understand very well?
- It might be nice to provide a few numerical examples (ask me if you need help!).

### **Other**

- Are there any specific people who use our subjects in the topic you’ve chosen?
- How does this use of mathematics affect us today?
- Talk about anything else you find relevant, interesting, or important.