2.1 # 2, 5, 7, 10, 15, 20, 22, 27, 28

2.) $A=\begin{bmatrix}2 & 0 & -1\\ 4 & -5 & 2\end{bmatrix}$, $B=\begin{bmatrix}7 & -5 & 1\\ 1 & -4 & -3\end{bmatrix}$, $C=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}$, $D=\begin{bmatrix}3 & 5\\ -1 & 4\end{bmatrix}$, $E=\begin{bmatrix}-5\\ 3\end{bmatrix}$

Compute $A+3B$, $2C-3E$, DB, EC

$$A+3B=\begin{bmatrix}2 & 0 & -1\\ 4 & -5 & 2\end{bmatrix}+\begin{bmatrix}21 & -15 & 3\\ 3 & -12 & -9\end{bmatrix}=\begin{bmatrix}23 & -15 & 2\\ 7 & -17 & -7\end{bmatrix}$$

$2C-3E$ not defined

$$DB=\begin{bmatrix}3 & 5\\ -1 & 4\end{bmatrix}\begin{bmatrix}7 & -5 & 1\\ 1 & -4 & -3\end{bmatrix}=\begin{bmatrix}21+5 & -15-20 & 3-15\\ -7+4 & 5-16 & -1-12\end{bmatrix}=\begin{bmatrix}26 & -35 & -12\\ -3 & -11 & -13\end{bmatrix}$$

EC not defined

5.) Compute AB where $A=\begin{bmatrix}-1 & 3\\ 2 & 4\\ 5 & -3\end{bmatrix}$, $B=\begin{bmatrix}4 & -2\\ -2 & 3\end{bmatrix}$

a.) by definition. $AB=[A\vec{b}_1 \;\; A\vec{b}_2]=\left[\begin{bmatrix}-1 & 3\\ 2 & 4\\ 5 & -3\end{bmatrix}\begin{bmatrix}4\\ -2\end{bmatrix} \quad \begin{bmatrix}-1 & 3\\ 2 & 4\\ 5 & -3\end{bmatrix}\begin{bmatrix}-2\\ 3\end{bmatrix}\right]=\begin{bmatrix}-10 & 11\\ 0 & 8\\ 26 & -19\end{bmatrix}$

b.) by the row-column rule. $\begin{bmatrix}-1 & 3\\ 2 & 4\\ 5 & -3\end{bmatrix}\begin{bmatrix}4 & -2\\ -2 & 3\end{bmatrix}=\begin{bmatrix}-4-6 & 2+9\\ 8-8 & -4+12\\ 20+6 & -10-9\end{bmatrix}=\begin{bmatrix}-10 & 11\\ 0 & 8\\ 26 & -19\end{bmatrix}$

7.) If a matrix A is 5×3 and the product AB is 5×7, what is the size of B? 3×7

10.) Let $A=\begin{bmatrix}3 & -6\\ -1 & 2\end{bmatrix}$, $B=\begin{bmatrix}-1 & 1\\ 3 & 4\end{bmatrix}$, and $C=\begin{bmatrix}-3 & -5\\ 2 & 1\end{bmatrix}$. Verify that $AB=AC$ and yet $B\neq C$.

Clearly $B\neq C$.

$$AB=\begin{bmatrix}3 & -6\\ -1 & 2\end{bmatrix}\begin{bmatrix}-1 & 1\\ 3 & 4\end{bmatrix}=\begin{bmatrix}-3-18 & 3-24\\ 1+6 & -1+8\end{bmatrix}=\begin{bmatrix}-21 & -21\\ 7 & 7\end{bmatrix}$$

$AB=AC$

$$AC=\begin{bmatrix}3 & -6\\ -1 & 2\end{bmatrix}\begin{bmatrix}-3 & -5\\ 2 & 1\end{bmatrix}=\begin{bmatrix}-9-12 & -15-6\\ 3+4 & 5+2\end{bmatrix}=\begin{bmatrix}-21 & -21\\ 7 & 7\end{bmatrix}$$

2.1

15.) T/F A, B, C are arbitrary and the indicated sums & products are defined.

a.) If A and B are 2×2 matrices with columns $\vec{a}_1, \vec{a}_2$ and $\vec{b}_1, \vec{b}_2$ respectively, then $AB = [\vec{a}_1\vec{b}_1 \;\; \vec{a}_2\vec{b}_2]$.

b.) Each column of AB is a linear combination of the columns of B using weights from the corresponding columns of A.

c.) $AB + AC = A(B+C)$

d.) $A^T + B^T = (A+B)^T$

e.) The transpose of a product of matrices equals the product of their transposes in the same order.

a.) False b.) False c.) True d.) True e.) False

20.) Suppose the first two columns, $\vec{b}_1$ and $\vec{b}_2$, of B are equal. What can be said about the columns of AB? Why?

The first two columns of AB are equal $[Ab_1 \; Ab_2 \; \ldots]$

22.) Show that if the columns of B are linearly dependent, then so are the columns of AB.

If the columns of B are linearly dependent, then $B\vec{x} = \vec{0}$ has a non-trivial solution. For this solution $\vec{x}$,

$$(AB)\vec{x} = A(B\vec{x}) = A(\vec{0}) = \vec{0}$$ So $\vec{x}$ is a nontrivial solution to $(AB)\vec{x} = \vec{0}$

therefore the columns of B are linearly dependent.

2.1 continued

27.) Let $\vec{u} = \begin{bmatrix} -3 \\ 2 \\ -5 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Compute $\vec{u}^T\vec{v}$, $\vec{v}^T\vec{u}$, $\vec{u}\vec{v}^T$ and $\vec{v}\vec{u}^T$.

$\vec{u}^T\vec{v} = \begin{bmatrix} -3 & 2 & -5 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = -3a + 2b - 5c$ $\quad$ $\vec{v}^T\vec{u} = \begin{bmatrix} a & b & c \end{bmatrix}\begin{bmatrix} -3 \\ 2 \\ -5 \end{bmatrix} = -3a + 2b - 5c$

$\vec{u}\vec{v}^T = \begin{bmatrix} -3 \\ 2 \\ -5 \end{bmatrix}\begin{bmatrix} a & b & c \end{bmatrix} = \begin{bmatrix} -3a & -3b & -3c \\ 2a & 2b & 2c \\ -5a & -5b & -5c \end{bmatrix}$ $\quad$ $\vec{v}\vec{u}^T = \begin{bmatrix} a \\ b \\ c \end{bmatrix}\begin{bmatrix} -3 & 2 & -5 \end{bmatrix} = \begin{bmatrix} -3a & 2a & -5a \\ -3b & 2b & -5b \\ -3c & 2c & -5c \end{bmatrix}$

28.) If $\vec{u}$ and $\vec{v}$ are in $\mathbb{R}^n$, how are $\vec{u}^T\vec{v}$ and $\vec{v}^T\vec{u}$ related? How are $\vec{u}\vec{v}^T$ and $\vec{v}\vec{u}^T$ related?

$\vec{u}^T\vec{v}$ and $\vec{v}^T\vec{u}$ are scalars, so they equal their transpose.

$\vec{u}^T\vec{v} = (\vec{u}^T\vec{v})^T = \vec{v}^T\vec{u}$ So they equal each other. $\vec{u}\vec{v}^T$ and $\vec{v}\vec{u}^T$ are both $n \times n$ matrices and are the transpose of each other.

$(\vec{u}\vec{v}^T)^T = \vec{v}\vec{u}^T$