3.1 # 4, 8, 11, 13, 20, 21, 31, 32, 37, 39

4.) Compute the determinant using a cofactor expansion across the first row. Also compute it by a cofactor expansion down the second column.

1st row $\begin{vmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{vmatrix} = (-1)^{1+1}(1)\begin{vmatrix} 1 & 1 \\ 4 & 2 \end{vmatrix} + (-1)^{1+2}(3)\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + (-1)^{1+3}(5)\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} = -2-3+25 = 20$

2nd column $(-1)^{1+2}(3)\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + (-1)^{2+2}(1)\begin{vmatrix} 1 & 5 \\ 3 & 2 \end{vmatrix} + (-1)^{3+2}(4)\begin{vmatrix} 1 & 5 \\ 2 & 1 \end{vmatrix} = -3+(-13)-(-36) = 20$

8.) Compute the determinant using a cofactor expansion across the first row.

$$\begin{vmatrix} 8 & 1 & 6 \\ 4 & 0 & 3 \\ 3 & -2 & 5 \end{vmatrix} = (-1)^{1+1}(8)\begin{vmatrix} 0 & 3 \\ -2 & 5 \end{vmatrix} + (-1)^{1+2}(1)\begin{vmatrix} 4 & 3 \\ 3 & 5 \end{vmatrix} + (-1)^{1+3}(6)\begin{vmatrix} 4 & 0 \\ 3 & -2 \end{vmatrix} = 48-11-48 = -11$$

11.) Compute the determinant by cofactor expansion. At each step choose a row or column that involves the least computation.

$$\begin{vmatrix} 3 & 5 & -8 & 4 \\ 0 & -2 & 3 & -7 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 2 \end{vmatrix} = (-1)^{1+1}(3)\begin{vmatrix} -2 & 3 & -7 \\ 0 & 1 & 5 \\ 0 & 0 & 2 \end{vmatrix} + (-1)^{2+1}(0) + (-1)^{3+1}(0) + (-1)^{4+1}(0)$$

$$= 3\left((-1)^{1+1}(-2)\begin{vmatrix} 1 & 5 \\ 0 & 2 \end{vmatrix} + (-1)^{2+1}(0) + (-1)^{3+1}(0)\right) = 3(-2)\begin{vmatrix} 1 & 5 \\ 0 & 2 \end{vmatrix} = -12$$

* could also have started w/ row 4.

3.1

13.) $\begin{vmatrix} 4 & 0 & -7 & 3 & -5 \\ 0 & 0 & 2 & 0 & 0 \\ 7 & 3 & -6 & 4 & -8 \\ 5 & 0 & 5 & 2 & -3 \\ 0 & 0 & 9 & -1 & 2 \end{vmatrix} = 0 + 0 + (-1)^{2+3}(2)\begin{vmatrix} 4 & 0 & 3 & -5 \\ 7 & 3 & 4 & -8 \\ 5 & 0 & 2 & -3 \\ 0 & 0 & -1 & 2 \end{vmatrix} + 0 + 0$

$= -2\left(0 + (-1)^{2+2}(3)\begin{vmatrix} 4 & 3 & -5 \\ 5 & 2 & -3 \\ 0 & -1 & 2 \end{vmatrix} + 0 + 0\right) = (-2)(3)\left((-1)^{1+1}(4)\begin{vmatrix} 2 & -3 \\ -1 & 2 \end{vmatrix} + (-1)^{2+1}(5)\begin{vmatrix} 3 & -5 \\ -1 & 2 \end{vmatrix} + 0\right)$

$= -6(4-5) = 6$ $\qquad$ * or start w/ column 2

20.) State the row operation and describe how it affects the determinant.

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}, \begin{bmatrix} a & b \\ kc & kd \end{bmatrix}$ $\qquad$ The row operation is kR_2.

$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$ $\qquad$ $\det\begin{bmatrix} a & b \\ kc & kd \end{bmatrix} = kad - kbc = k(ad-bc)$

Multiplying R_2 by k multiplies the determinant by k.

21.) $\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}, \begin{bmatrix} 3 & 4 \\ 5+3k & 6+4k \end{bmatrix}$ $\qquad$ The row operation is $kR_1 + R_2$

$\det\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} = 3(6) - 4(5)$ $\qquad$ $\det\begin{bmatrix} 3 & 4 \\ 5+3k & 6+4k \end{bmatrix} = 3(6+4k) - 4(5+3k)$

$= 3(6) + \cancel{3(4k)} - 4(5) - \cancel{4(3k)}$

The row operation $kR_1 + R_2$ has no effect on the determinant.

3.1 continued

31.) What is the determinant of an elementary row replacement matrix?

An elementary row replacement matrix has 1's in the diagonal, one other nonzero entry and the rest zeros. So it is a triangular matrix. Therefore the determinant is the product of the entries on the main diagonal which is 1.

32.) What is the determinant of an elementary scaling matrix with K on the diagonal?

An elementary scaling matrix has 1's on the main diagonal except one position and zeros everywhere else. It is a triangular matrix so its determinant is K.

37.) Let $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$. Write 5A. Is $\det 5A = 5 \det A$?

$5A = \begin{bmatrix} 15 & 5 \\ 20 & 10 \end{bmatrix}$ $\det 5A = 50$ $\det A = 2$ No $\det 5A \neq 5 \det A$

39.) True/False (A is nxn matrix)

a.) An nxn ~~de~~determinant is defined by determinants of $(n-1)\times(n-1)$ submatrices.

b.) The (i,j)-cofactor of a matrix A is the matrix A_{ij} obtained by deleting from A its ith row and jth column.

a.) True b.) False