3.3 # 4, 5, 6, 22, 23, 26, 29, 30

4.) Use cramer's rule to compute the solution.

$-5x_1 + 3x_2 = 9$
$3x_1 - x_2 = -5$

$A = \begin{bmatrix} -5 & 3 \\ 3 & -1 \end{bmatrix}$ $\det A = -4$ $\vec{b} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$

$A_1(\vec{b}) = \begin{bmatrix} 9 & 3 \\ -5 & -1 \end{bmatrix}$ $\det A_1(\vec{b}) = 6$ $A_2(\vec{b}) = \begin{bmatrix} -5 & 9 \\ 3 & -5 \end{bmatrix}$ $\det A_2(\vec{b}) = -2$ $\vec{x} = \begin{bmatrix} -6/4 \\ 2/4 \end{bmatrix} = \begin{bmatrix} -3/2 \\ 1/2 \end{bmatrix}$

5.) $2x_1 + x_2 = 7$
$-3x_1 + x_3 = -8$
$x_2 + 2x_3 = -3$

$A = \begin{bmatrix} 2 & 1 & 0 \\ -3 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ $\det A = 2(-1) - 1(-6) + 0 = 4$

$A_1(\vec{b}) = \begin{bmatrix} 7 & 1 & 0 \\ -8 & 0 & 1 \\ -3 & 1 & 2 \end{bmatrix}$ $\det A_1(\vec{b}) = 7(-1) - 1(-13) + 0 = 6$

$A_2(\vec{b}) = \begin{bmatrix} 2 & 7 & 0 \\ -3 & -8 & 1 \\ 0 & -3 & 2 \end{bmatrix}$ $\det A_2(\vec{b}) = 2(-13) + 3(14) + 0 = 16$

$\vec{x} = \begin{bmatrix} 6/4 \\ 16/4 \\ -14/4 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 4 \\ -7/2 \end{bmatrix}$

$A_3(\vec{b}) = \begin{bmatrix} 2 & 1 & 7 \\ -3 & 0 & -8 \\ 0 & 1 & -3 \end{bmatrix}$ $\det A_3(\vec{b}) = 2(8) + 3(-10) + 0 = -14$

6.) $2x_1 + x_2 + x_3 = 4$
$-x_1 + 2x_3 = 2$
$3x_1 + x_2 + 3x_3 = -2$

$A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 0 & 2 \\ 3 & 1 & 3 \end{bmatrix}$ $\det A = 1(-9) - 0 + 1(5) = -4$

$A_1(\vec{b}) = \begin{bmatrix} 4 & 1 & 1 \\ 2 & 0 & 2 \\ -2 & 1 & 3 \end{bmatrix}$ $\det A_1(\vec{b}) = 1(10) - 0 + 1(6) = 16$

$A_2(\vec{b}) = \begin{bmatrix} 2 & 4 & 1 \\ -1 & 2 & 2 \\ 3 & -2 & 3 \end{bmatrix}$ $\det A_2(\vec{b}) = 2(10) + 1(14) + 3(6) = 52$

$\vec{x} = \begin{bmatrix} 16/-4 \\ 52/-4 \\ 4/-4 \end{bmatrix} = \begin{bmatrix} -4 \\ -13 \\ -1 \end{bmatrix}$

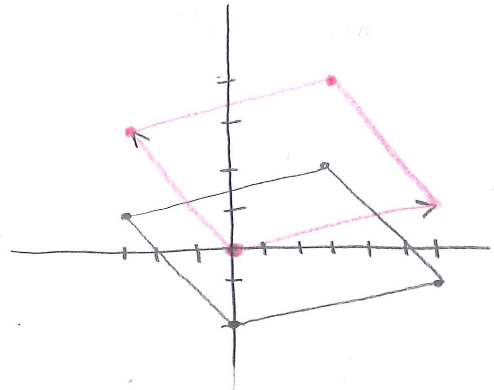
$A_3(\vec{b}) = \begin{bmatrix} 2 & 1 & 4 \\ -1 & 0 & 2 \\ 3 & 1 & -2 \end{bmatrix}$ $\det A_3(\vec{b}) = 1(-4) - 0 + 1(8) = 4$

3.3

22.) Find the area of the parallelogram whose vertices are $(0,-2),(6,-1),(-3,1),(3,2)$

$(0,0),(6,1),(-3,3),(3,4)$

$A=\begin{bmatrix}-3 & 6\\ 3 & 1\end{bmatrix}$ $\quad |\det A| = |-21| = 21$



The area of the parallelogram is 21

23.) Find the volume of the parallelepiped with one vertex at the origin and adjacent vertices at $(1,0,-2),(1,2,4)$ and $(7,1,0)$.

$A=\begin{bmatrix}1 & 1 & 7\\ 0 & 2 & 1\\ -2 & 4 & 0\end{bmatrix}$ $\quad |\det A| = |1(-4)-0+(-2)(-13)| = 22$

26.) Let $T:\mathbb{R}^m \to \mathbb{R}^n$ be a linear transformation, and let $\vec{p}$ be a vector and S a set in $\mathbb{R}^m$. Show that the image of $\vec{p}+S$ under T is the translated set $T(\vec{p})+T(S)$ in $\mathbb{R}^n$.

Let $\vec{v}$ be in S. Then $\vec{p}+\vec{v}$ is in $\vec{p}+S$ and $T(\vec{p}+\vec{v}) = T(\vec{p})+T(\vec{v})$ which is in $T(\vec{p})+T(S)$. For the other direction any vector in $T(\vec{p})+T(S)$ is of the form $T(\vec{p})+T(\vec{v})$ for some v in S. $T(\vec{p})+T(\vec{v}) = T(\vec{p}+\vec{v})$ So this vector is the image of something in $\vec{p}+S$ under T.

3.3 continued

29.) Find a formula for the area of the triangle whose vertices are $\vec{0}$, $\vec{v}_1$, and $\vec{v}_2$ in $\mathbb{R}^2$.

$\frac{1}{2}\det[\vec{v}_1, \vec{v}_2]$

30.) Let R be the triangle w/ vertices at (x_1, y_1), (x_2, y_2) and (x_3, y_3). Show that $\{\text{area of triangle}\} = \frac{1}{2}\det\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$

$(0,0), (x_2-x_1, y_2-y_1), (x_3-x_1, y_3-y_1)$

$$\text{area} = \frac{1}{2}\det\begin{bmatrix} x_2-x_1 & x_3-x_1 \\ y_2-y_1 & y_3-y_1 \end{bmatrix} = \frac{1}{2}\det\begin{bmatrix} x_2-x_1 & y_2-y_1 \\ x_3-x_1 & y_3-y_1 \end{bmatrix}$$

since $\det A = \det A^T$ when A is $n \times n$

$$= \frac{1}{2}\det\begin{bmatrix} x_1 & y_1 & 1 \\ x_2-x_1 & y_2-y_1 & 0 \\ x_3-x_1 & y_3-y_1 & 0 \end{bmatrix} = \frac{1}{2}\det\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$$

row operations R_1+R_2
R_1+R_3