

for independence  
for confidence  
for creativity  
for insight

## **Circular functions 7**

**Transforming and adding circular functions**

# Circular functions

Defining the circular functions

sin, cos, tan and the unit circle

Solving circular function equations

like  $\sin \theta = 0.4$

Graphing the circular functions

graphs  $y = \cos x$  and the like

Relationships between circular functions

$\sin(90^\circ - x) = \cos x$  and the like

More circular functions

$\sec x = \frac{1}{\cos x}$  and so on

Circular functions of sums

formulas like  
 $\sin(A + B) = \sin A \cos B + \cos A \sin B$

## Transforming and adding circular functions

$$\sin x + \cos x = \sqrt{2} \sin(x + 45^\circ) \text{ and so on}$$

Differentiating circular functions

radians, and tangents to graphs

Integrating circular functions

areas

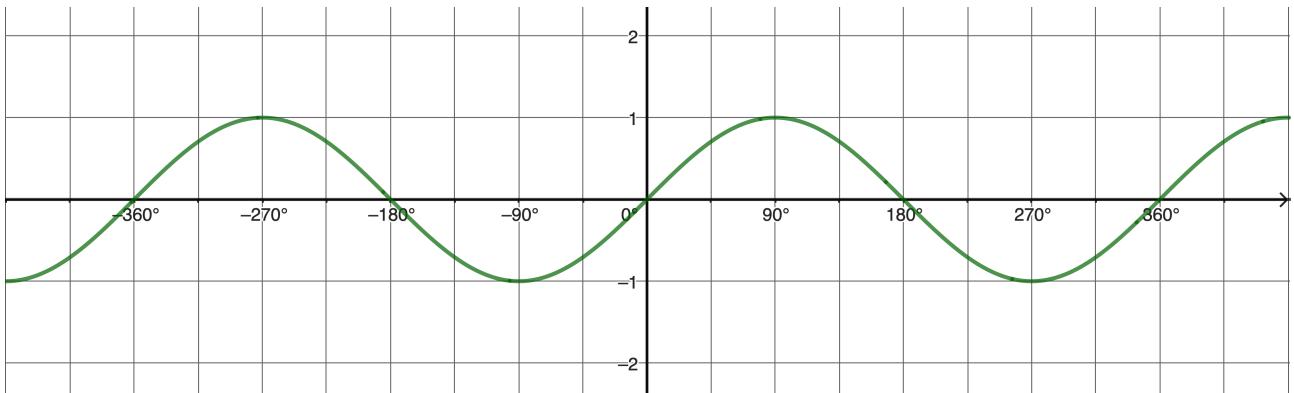
Inverses of circular functions

$\arcsin x$ ,  $\cos^{-1} x$ ,  $\cot^{-1} x$  and the like,  
including graphs, differentials, integrals,  
and integration by substitution

Here is the graph  $y = \sin x$ .

Translate the graph left by  $45^\circ$ .

What is the equation of your new graph?

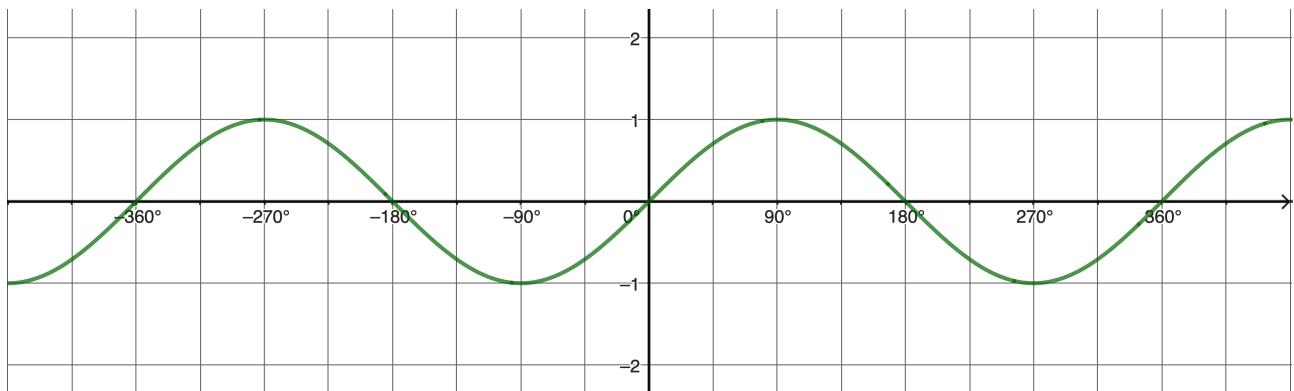


Now stretch the new graph parallel to the  $y$  axis scale factor  $\sqrt{2}$ .

What is the equation of your latest graph?

Now use a compound angle formula to expand  $\sqrt{2} \sin(x + 45^\circ)$

Draw the graph  $y = \sin x + \cos x$

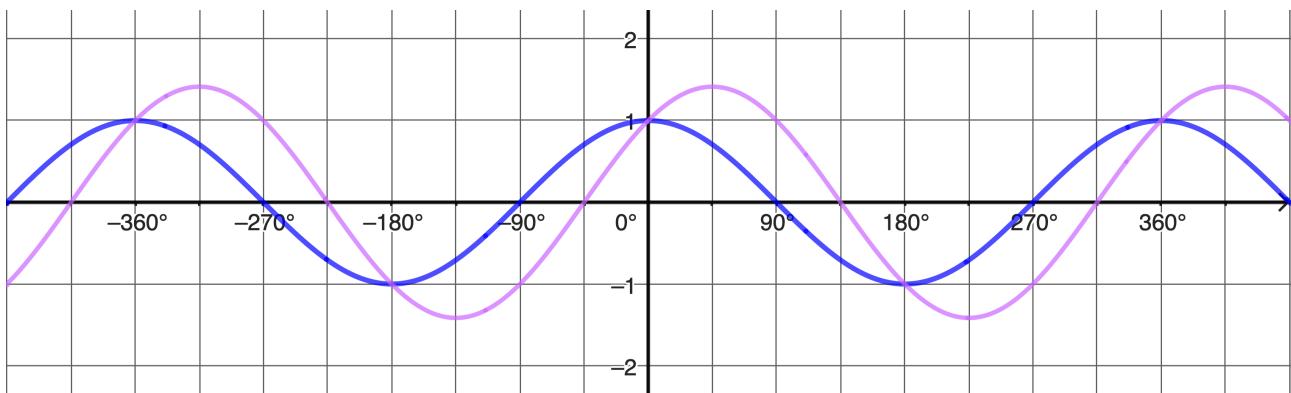


We can achieve the same graph

$$y = \sin x + \cos x$$

by applying transformations to the graph  $y = \cos x$ . Here are the two graphs together

What transformations will turn the blue ( $y = \cos x$ ) into the pink ( $y = \sin x + \cos x$ )?



Expand  $\sqrt{2} \cos(x - 45^\circ)$

What do you notice about the graphs

$$y = \sqrt{2} \sin(x + 45^\circ)$$

$$y = \sqrt{2} \cos(x - 45^\circ)$$

$$y = \sin x + \cos x$$

What transformations of  $y = \sin x$  or  $y = \cos x$  will result in the graphs

$$y = \sqrt{2} \sin(x + 405^\circ)$$

$$y = \sqrt{2} \cos(x - 405^\circ)$$

$$y = -\sqrt{2} \sin(x + 225^\circ)$$

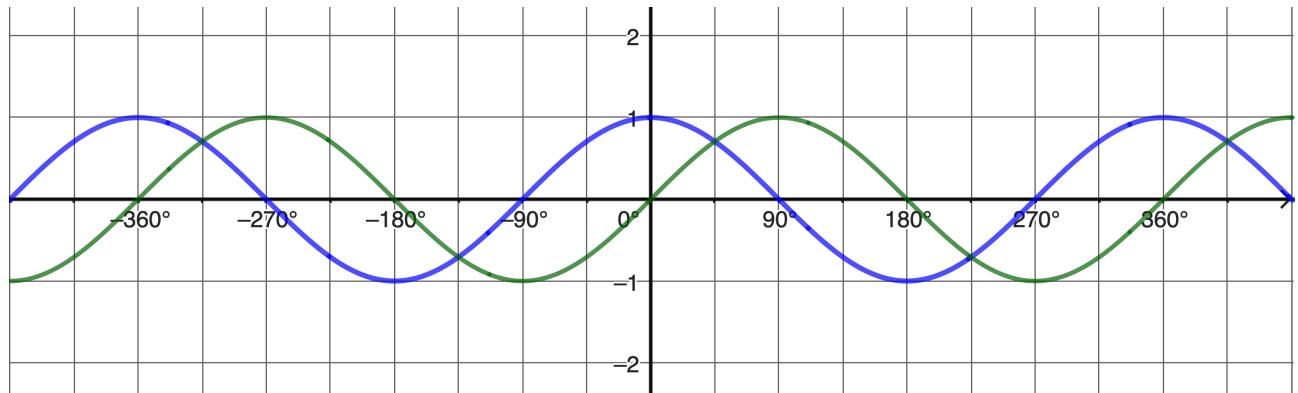
$$y = -\sqrt{2} \cos(x - 225^\circ)$$

What is the difference between these graphs and the graph  $y = \sin x + \cos x$ ?

We can also look at this from the point of view of a molecule being moved around by the two waves  $y = \sin x$  and  $y = \cos x$ .

Draw the graph obtained by “adding up” the two graphs.

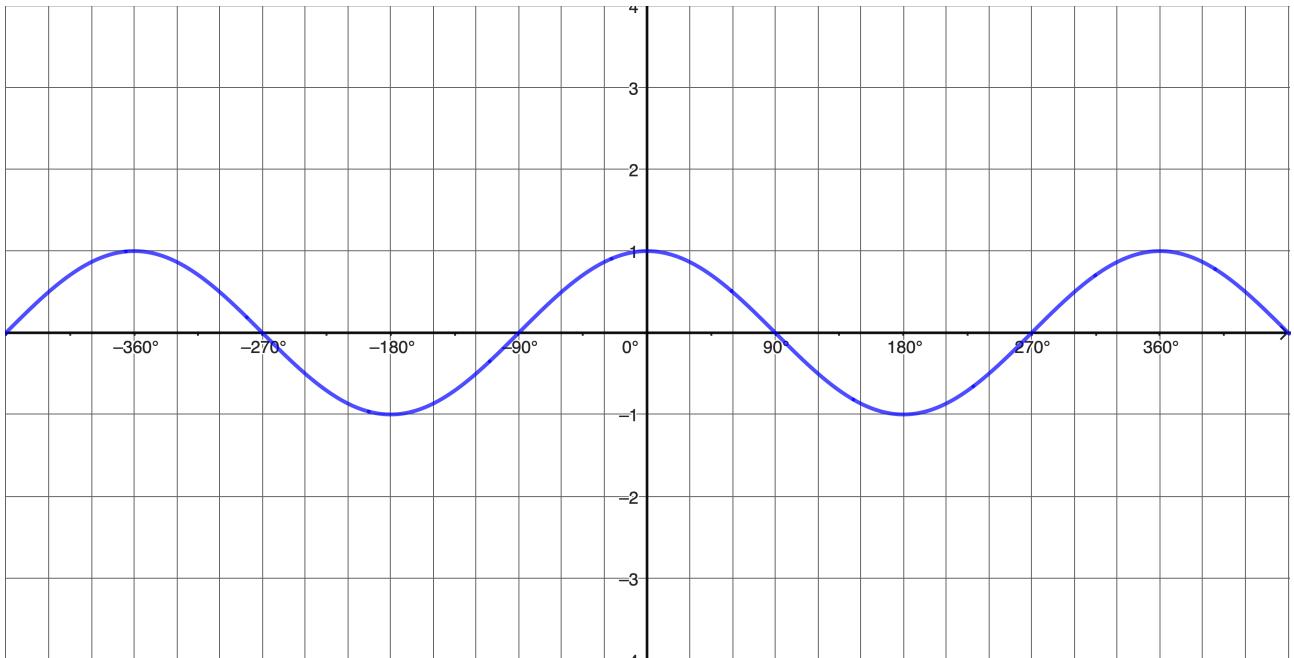
What is the equation of your new graph?



Now for a slightly trickier example.

What transformations take the graph  $y = \cos x$  to the graph

$$y = 2\sqrt{3} \cos(x + 30^\circ)?$$



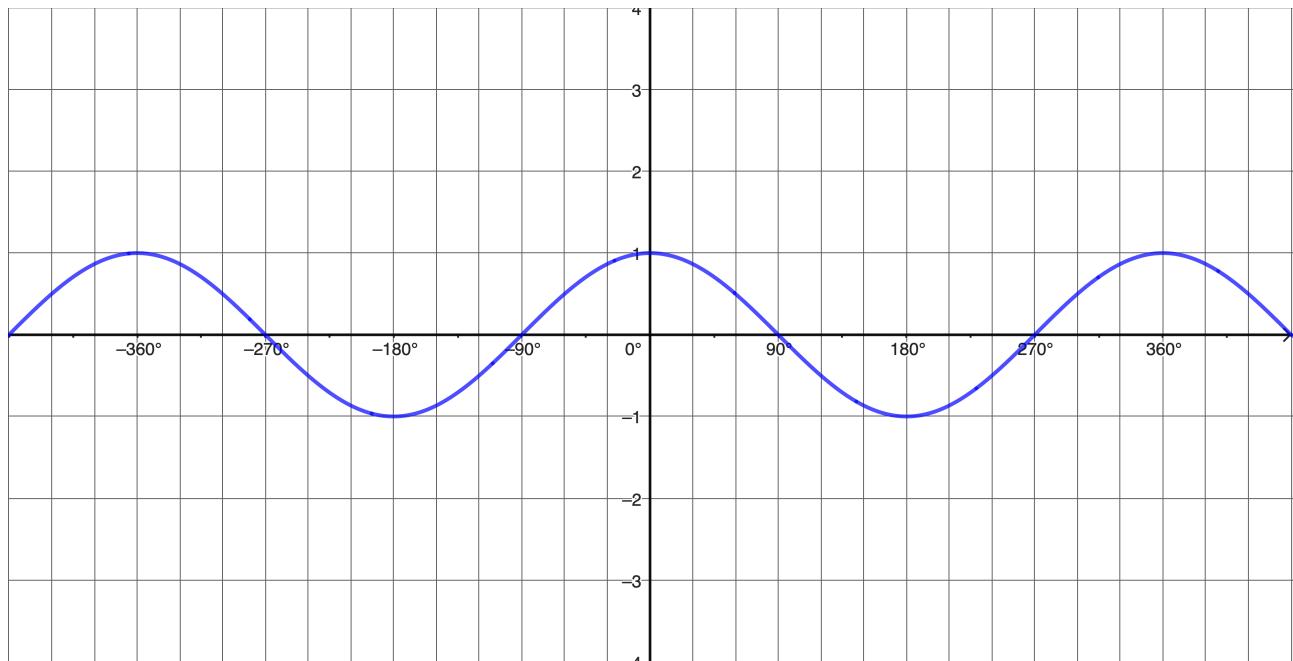
On the same axes, draw the graphs

$$y = \cos(x + 30^\circ) \text{ and } y = 2\sqrt{3} \cos(x + 30^\circ).$$

Expand the brackets in  $y = 2\sqrt{3} \cos(x + 30^\circ)$ .

Draw the graph

$$y = 3 \cos x - \sqrt{3} \sin x$$



Next, let's take a similar but very slightly different example: the wave  $y = 3 \sin x$  meets the wave  $y = \sqrt{3} \cos x$

Firstly, expand the brackets in  $y = R \sin(x + \alpha)$ .

If  $R \sin(x + \alpha) = 3 \sin x + \sqrt{3} \cos x$ , find

$$R \sin \alpha$$

$$R \cos \alpha$$

Use these results to find  $\tan \alpha$ .

Hence find the smallest positive value of  $\alpha$ .

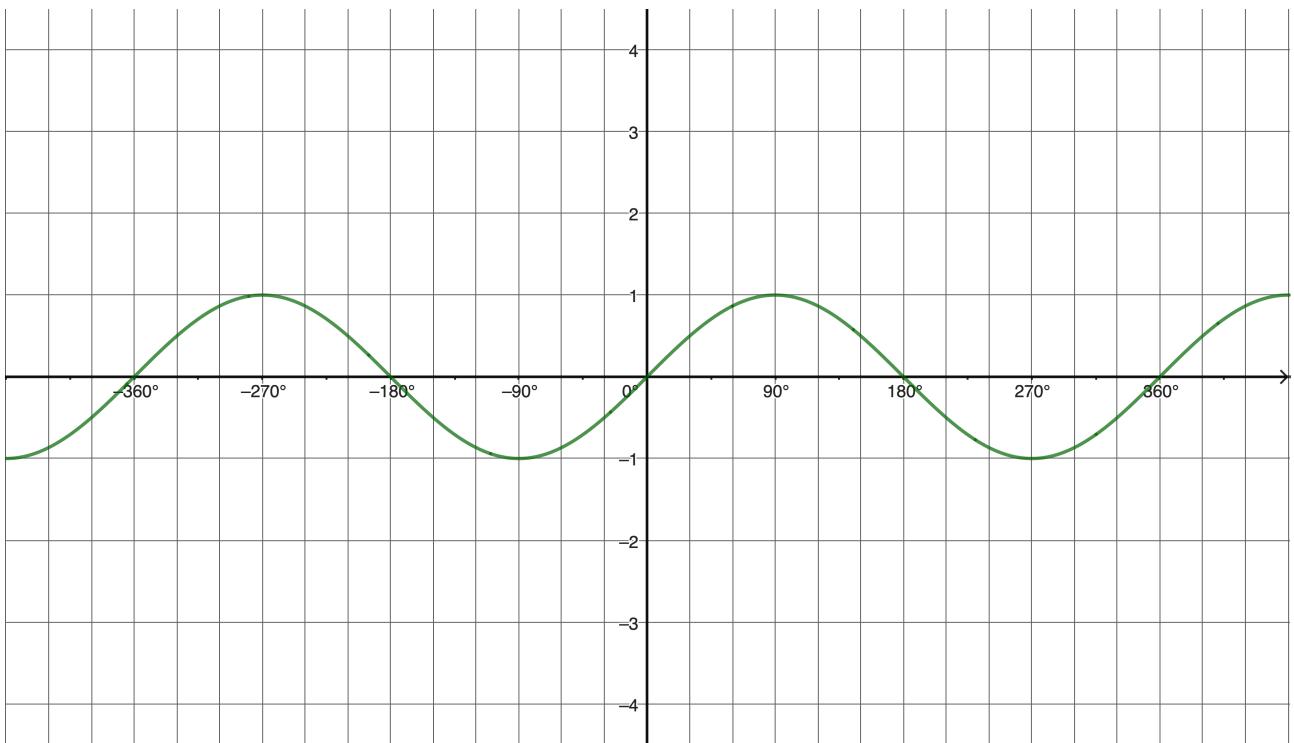
Find  $(R \sin \alpha)^2 + (R \cos \alpha)^2$ .

Hence find  $R$ .

Use these results to find the transformations that take the graph  
 $y = \sin x$  to the graph

$$y = 3 \sin x + \sqrt{3} \cos x$$

and draw this graph.



We could also do it this way.

If  $R \cos(x - \alpha) = 3 \sin x + \sqrt{3} \cos x$ , find

$$R \sin \alpha$$

$$R \cos \alpha$$

Use these results to find  $\tan \alpha$ .

Hence find the smallest positive value of  $\alpha$ .

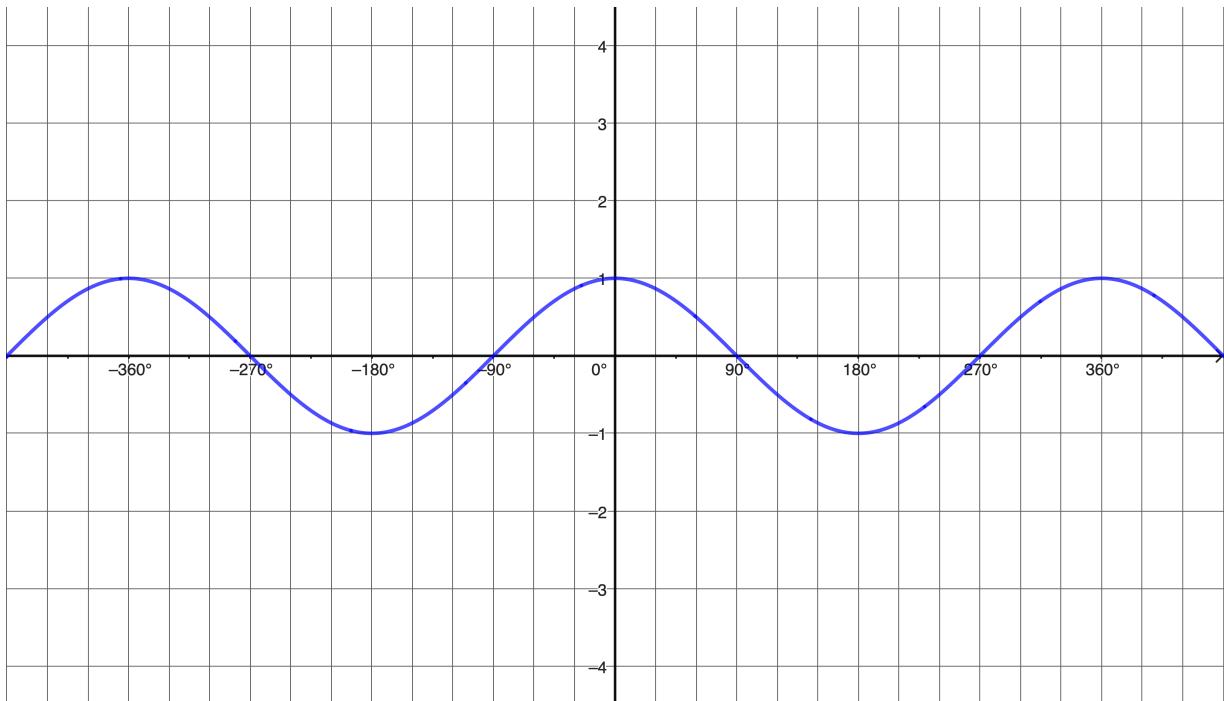
Find  $(R \sin \alpha)^2 + (R \cos \alpha)^2$ .

Hence find  $R$ .

Use these results to find the transformations that take the graph  
 $y = \cos x$  to the graph

$$y = 3 \sin x + \sqrt{3} \cos x$$

and draw this graph.



Any two waves  $y = A \sin x$  and  $y = B \cos x$  can be combined (either adding or subtracting) by these methods. Sometimes it's easier to transform the graph  $y = \sin x$  to get  $y = A \sin x \pm B \cos x$ . Other times, starting with  $y = \cos x$  is better.

Experiment with the combinations

$$y = 5 \sin x + 2 \cos x$$

$$y = 5 \sin x - 2 \cos x$$

$$y = 2 \cos x - 5 \sin x$$

using each of the following forms:

$$R \sin(x + \alpha)$$

$$R \sin(x - \alpha)$$

$$R \cos(x + \alpha)$$

$$R \cos(x - \alpha)$$

and taking  $\tan 22^\circ$  to be  $\frac{2}{5}$ .

Which  $R, \alpha$  forms work best for each of the combinations?