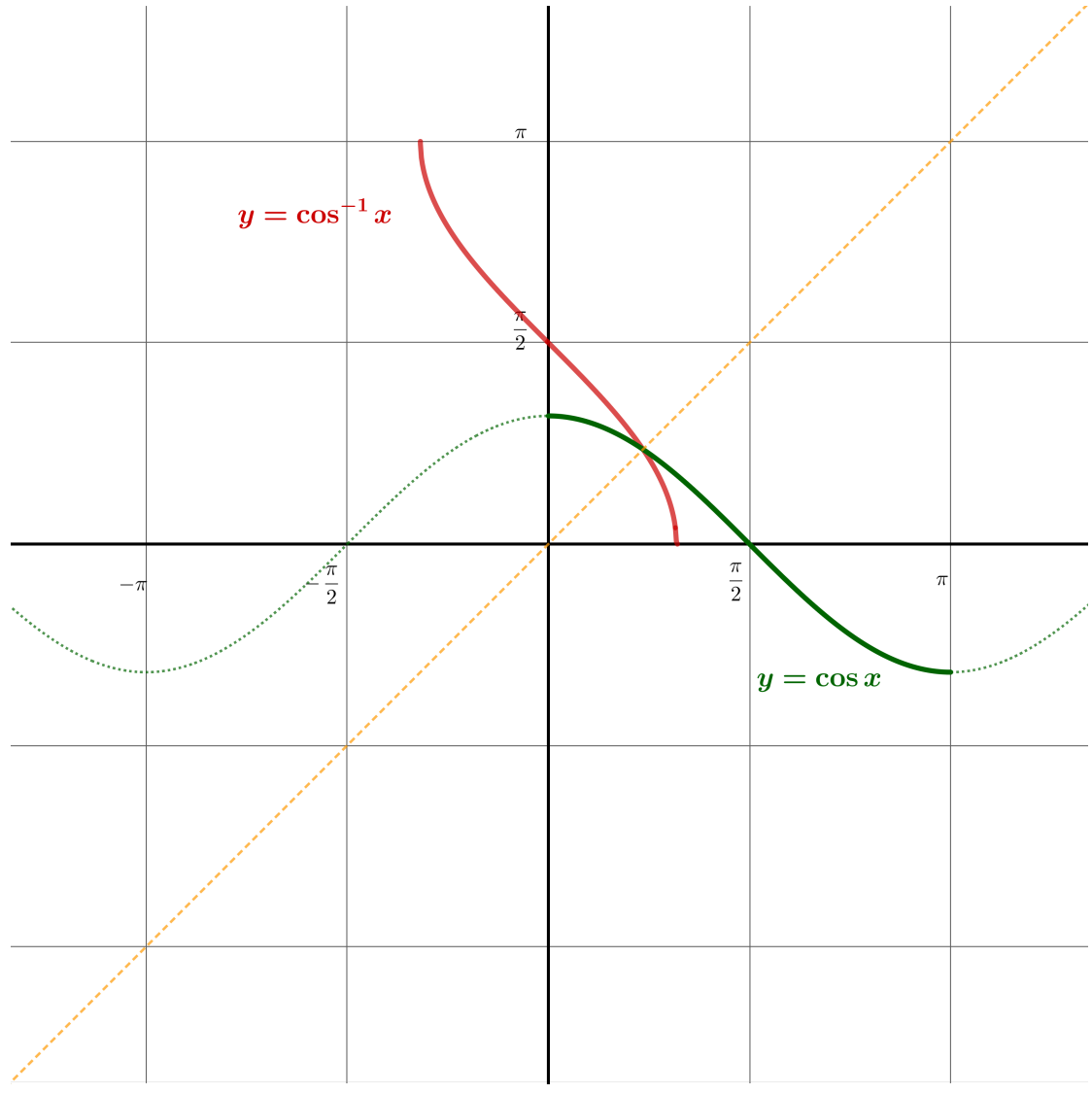
inverse circular functions: extension

What are the domain and range of $f(x) = \cos^{-1} x$?

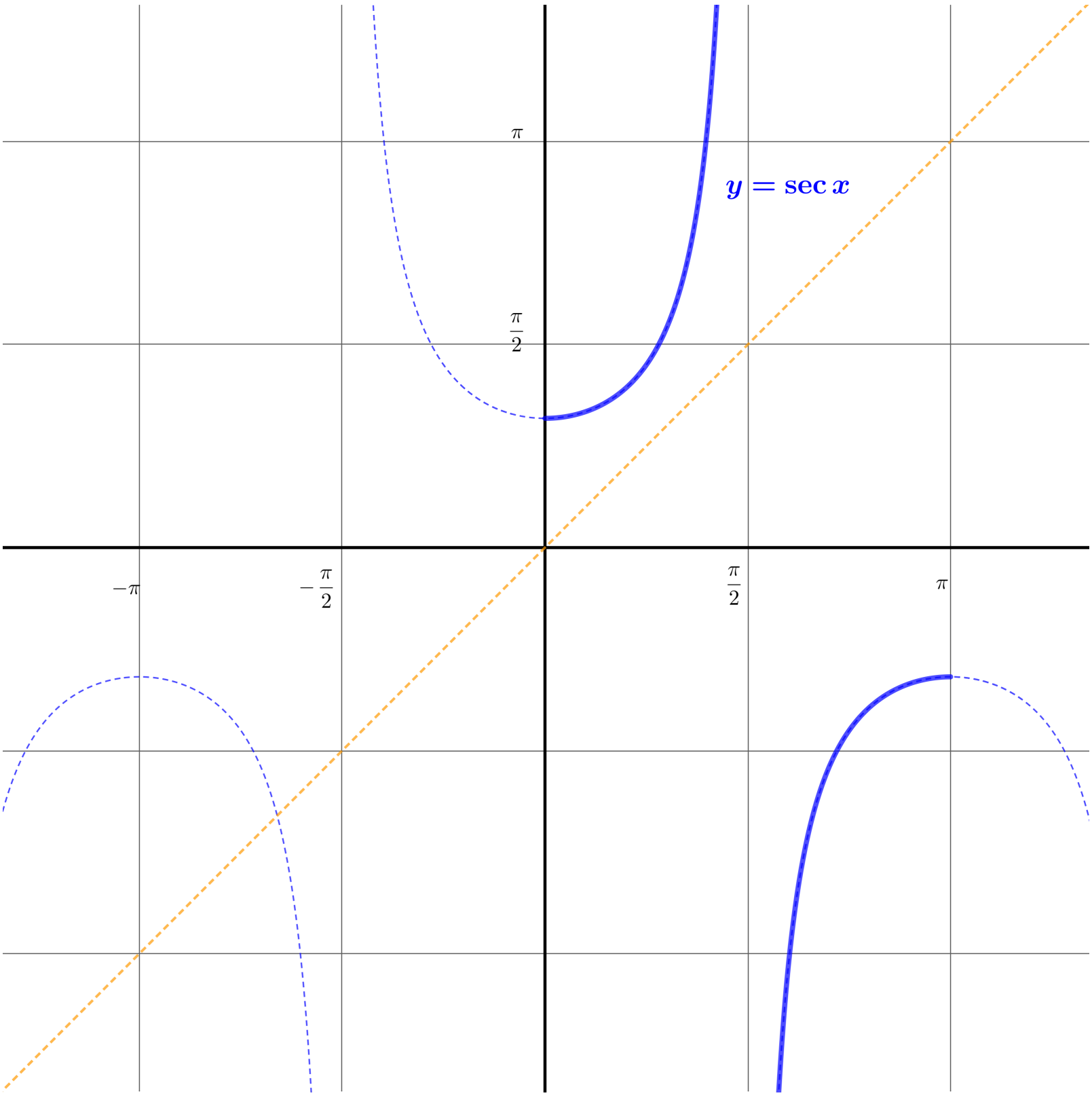


What are the domain and range of $f(x) = \sec^{-1} x$?

Here is the graph $y = \sec x$ over the domain

$$\left\{ x : 0 \leq x \leq \pi, \, x \neq \frac{\pi}{2} \right\}$$

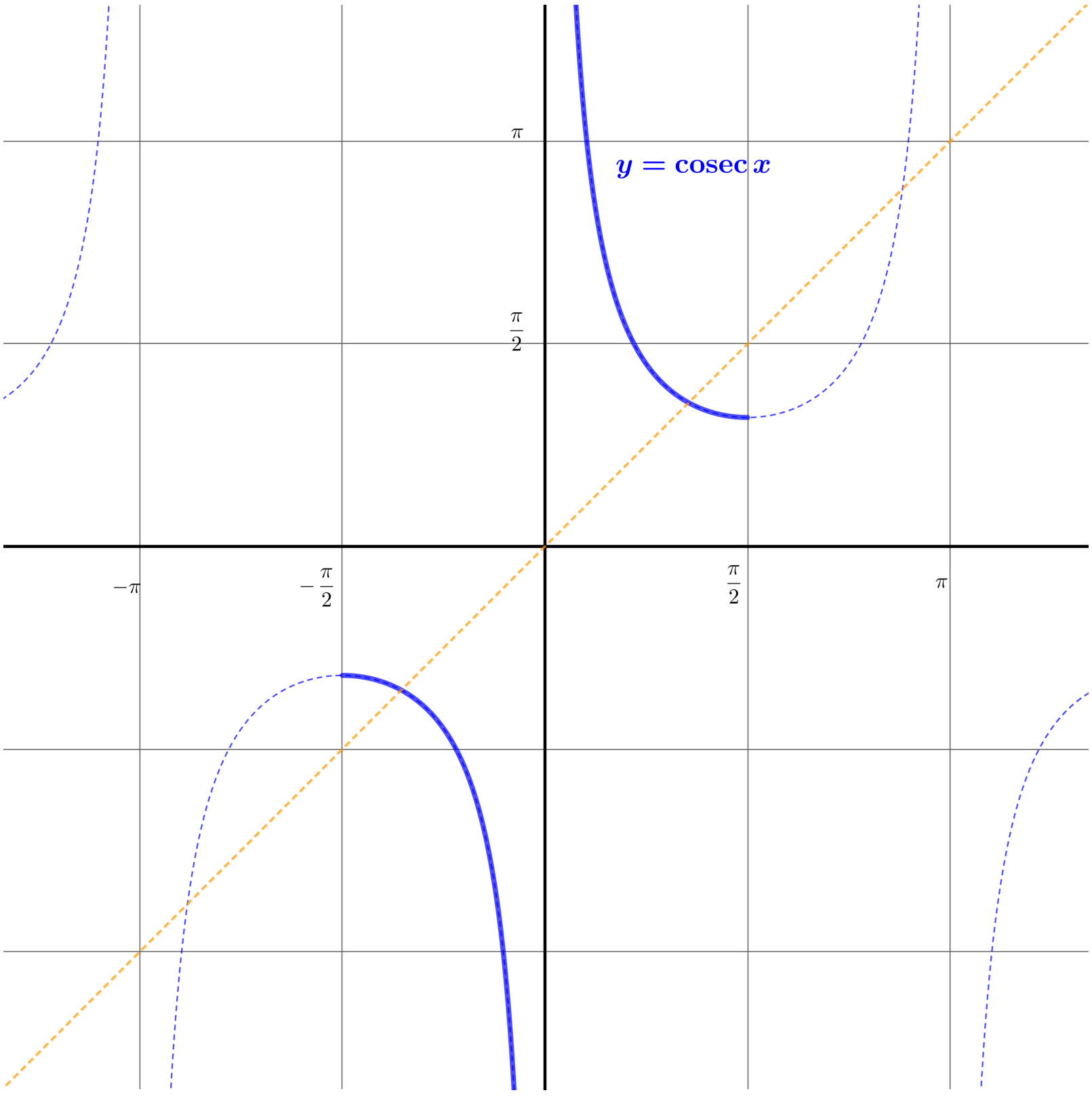
Draw the graph $y = \sec^{-1} x$.



Here is the graph $y = \operatorname{cosec} x$ over the domain

$$\left\{ x : -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}, x \neq 0 \right\}$$

Draw the graph $y = \operatorname{cosec}^{-1} x$.

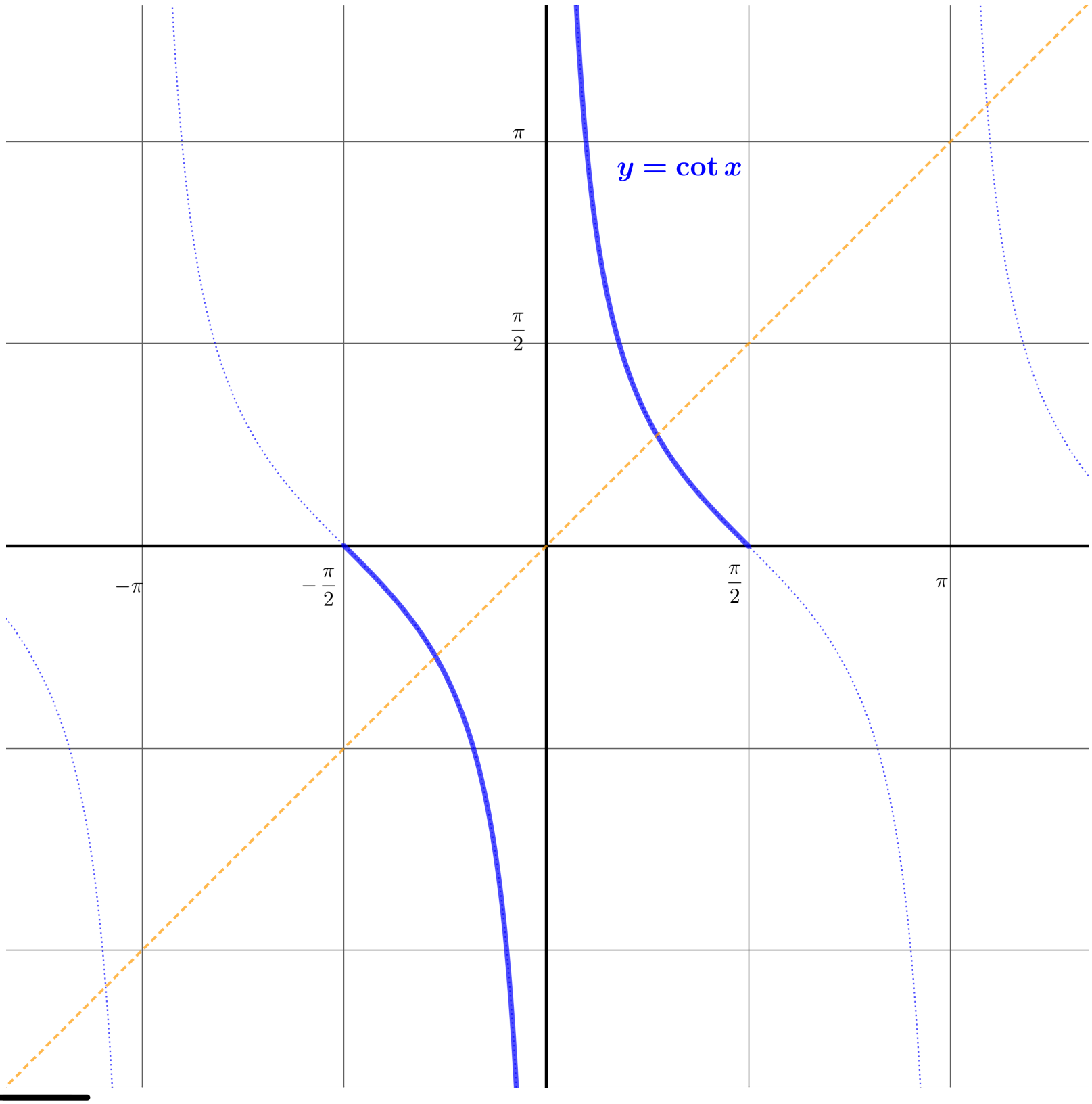


radical maths

Here is the graph $y = \cot x$ over the domain

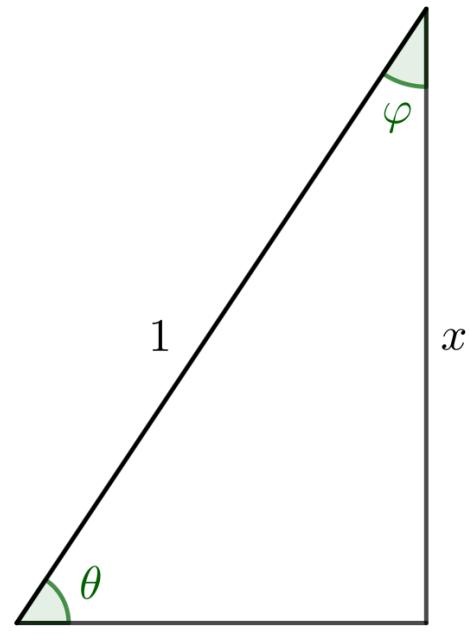
$$\left\{ x : -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}, x \neq 0 \right\}$$

Draw the graph $y = \cot^{-1} x$.



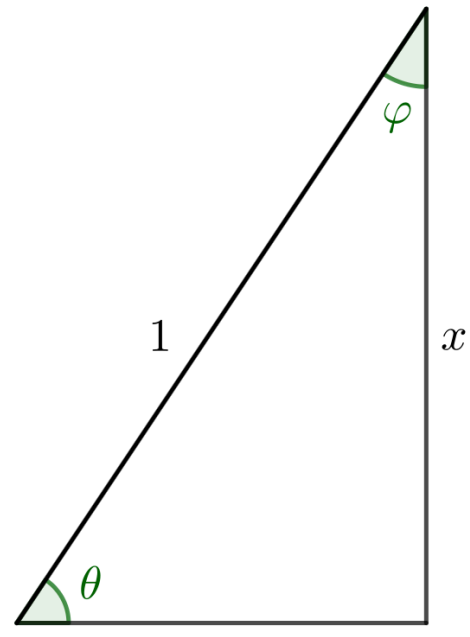
radical maths

What is $\cos\left(\sin^{-1} x\right)$?



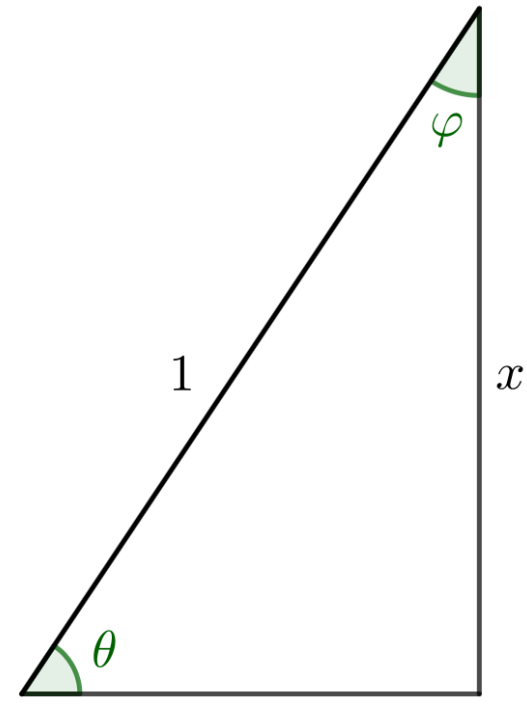
radical maths

What is $\cos^{-1}\left(\sin\theta\right)$?



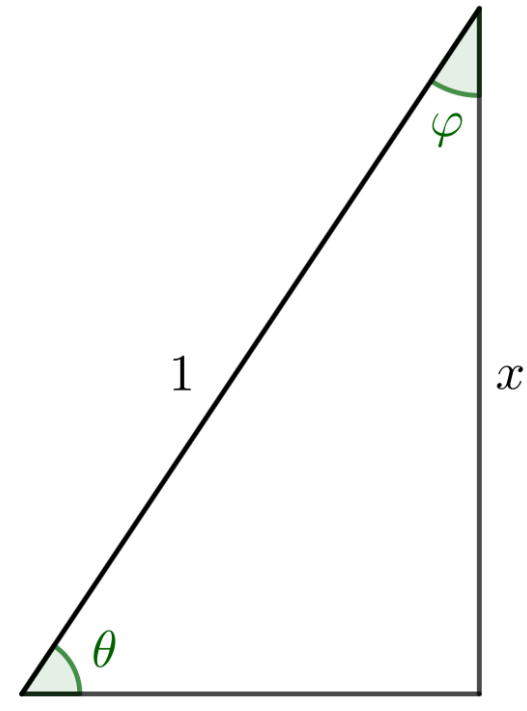
radical maths

What are $\sin\left(\cos^{-1} x\right)$ and $\sin^{-1}(\cos\varphi)$?



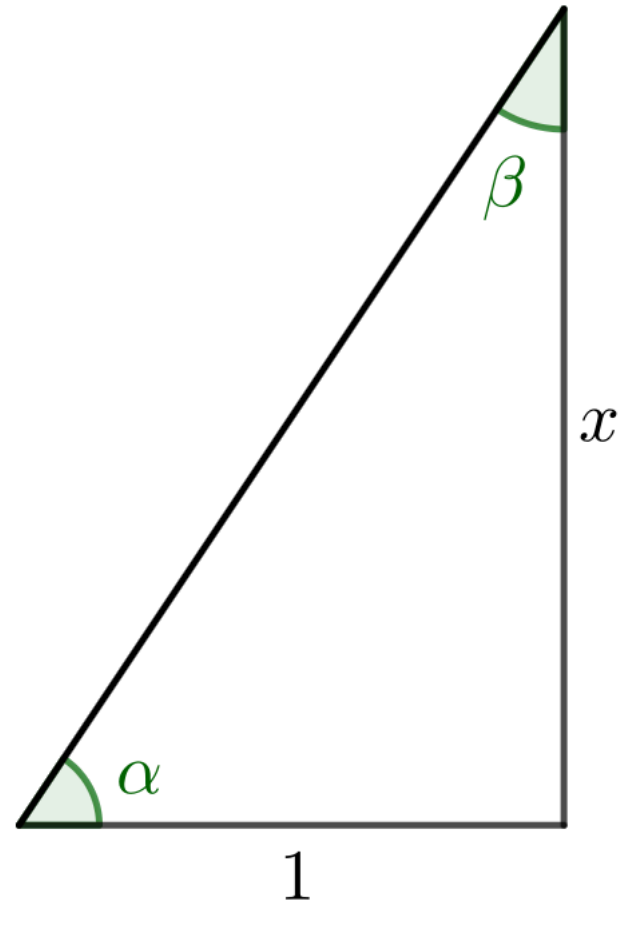
radical maths

What are $\tan\left(\sin^{-1} x\right)$ and $\tan\left(\cos^{-1} x\right)$?



radical maths

What are $\sin\left(\tan^{-1} x\right)$ and $\cos\left(\tan^{-1} x\right)$?



radical maths

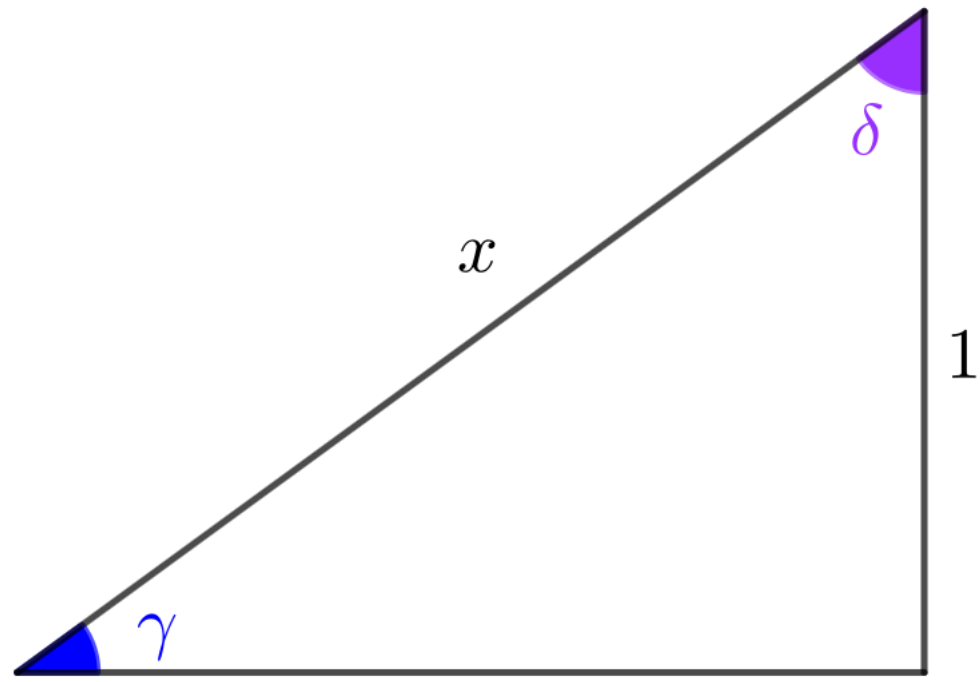
Simplify

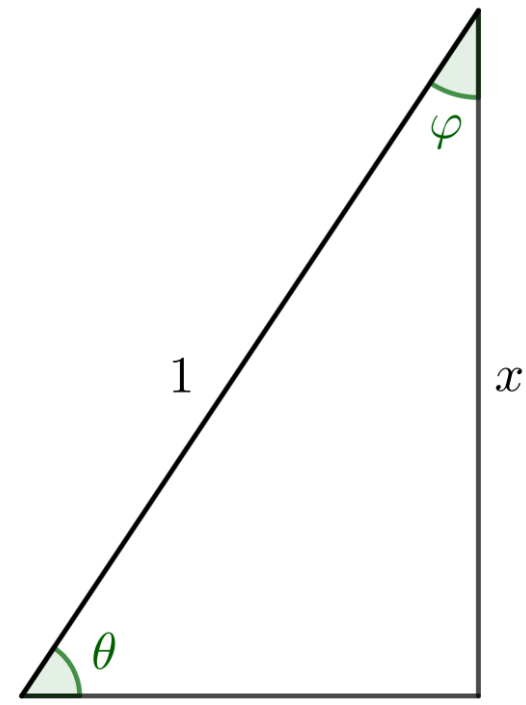
$\sin\left(\text{cosec}^{-1} x\right)$

$\text{cosec}\left(\sin^{-1} x\right)$

$\cos\left(\sec^{-1} x\right)$

$\sec\left(\cos^{-1} x\right)$



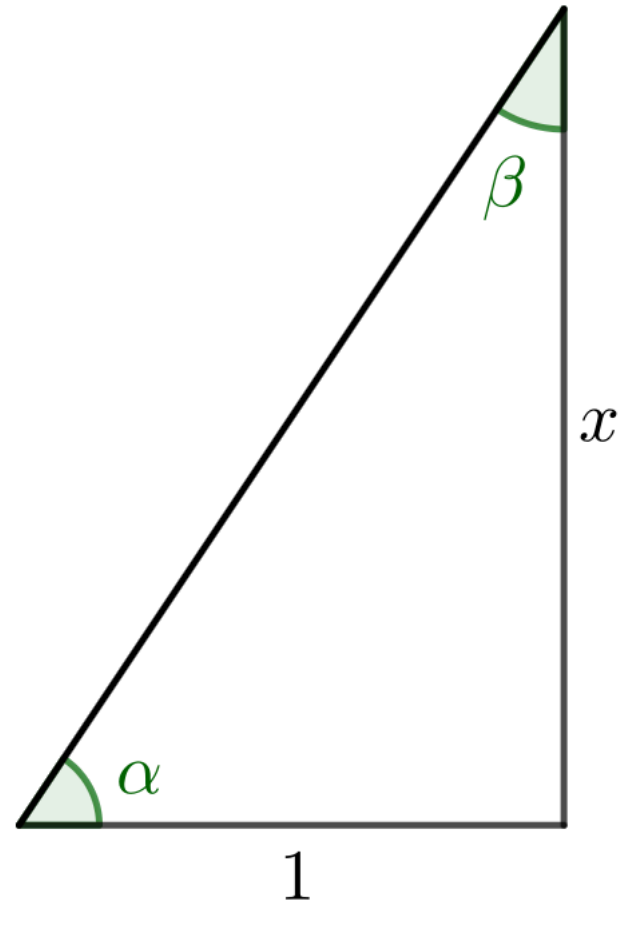


radical maths

Simplify

$\tan\left(\cot^{-1} x\right)$

$\cot\left(\tan^{-1} x\right)$



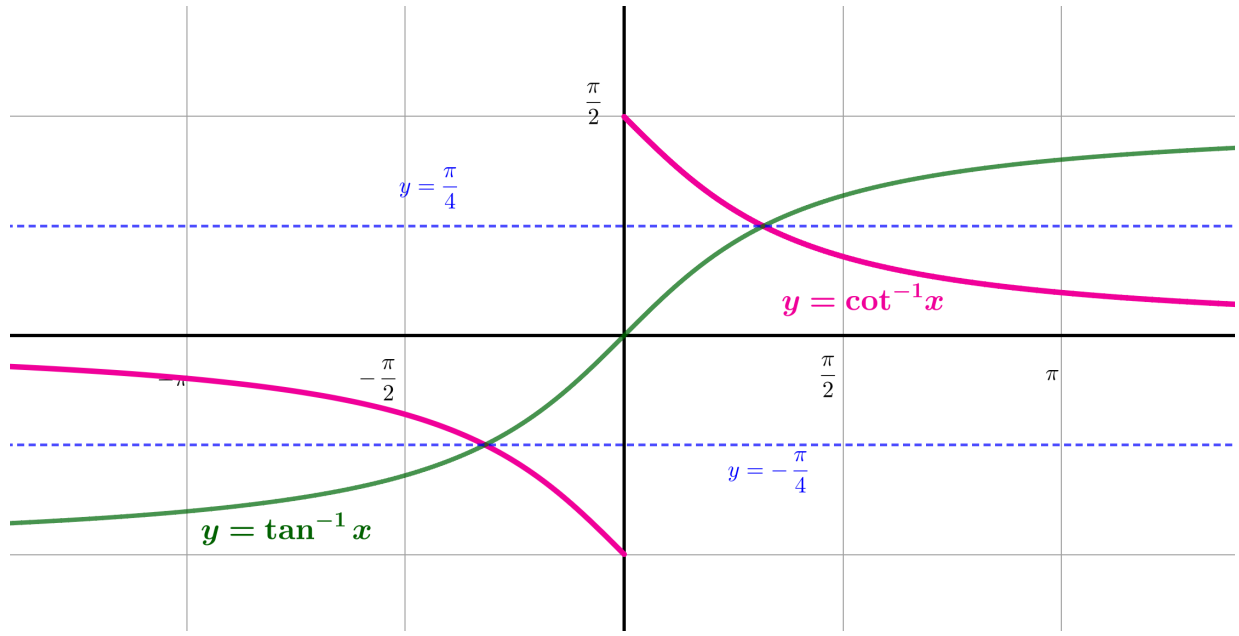
radical maths

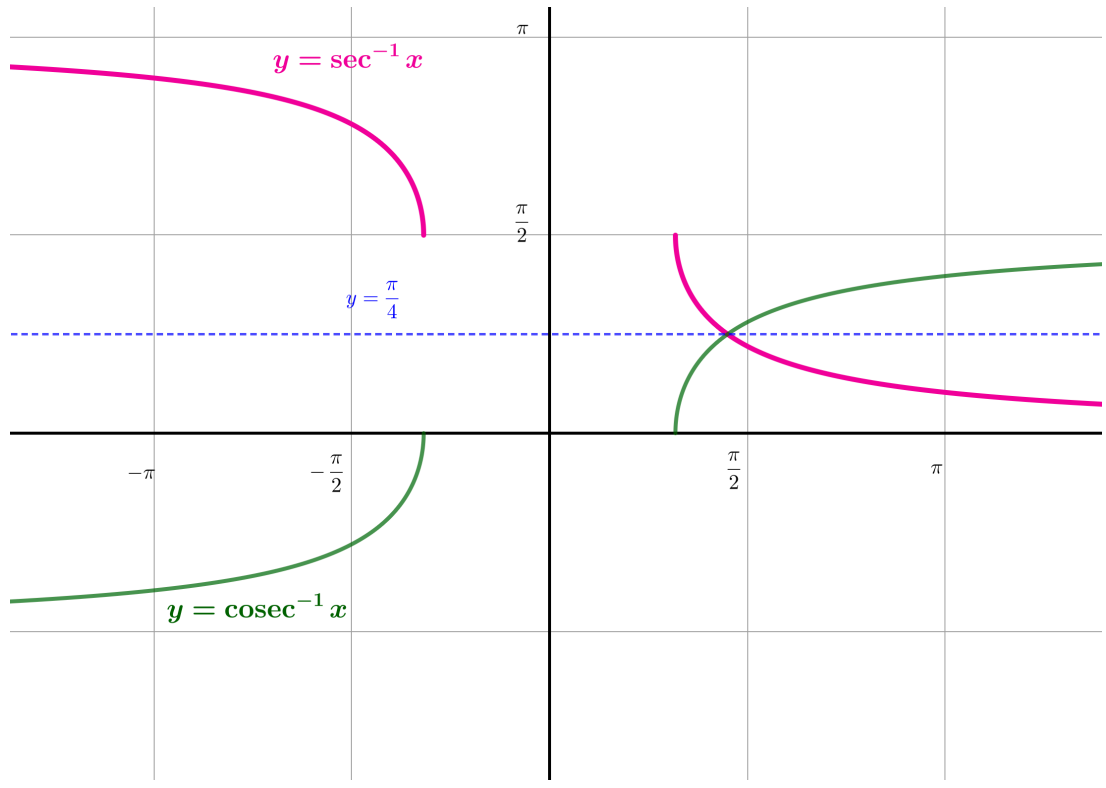
What are

$\cot^{-1} x + \tan^{-1} x$

and

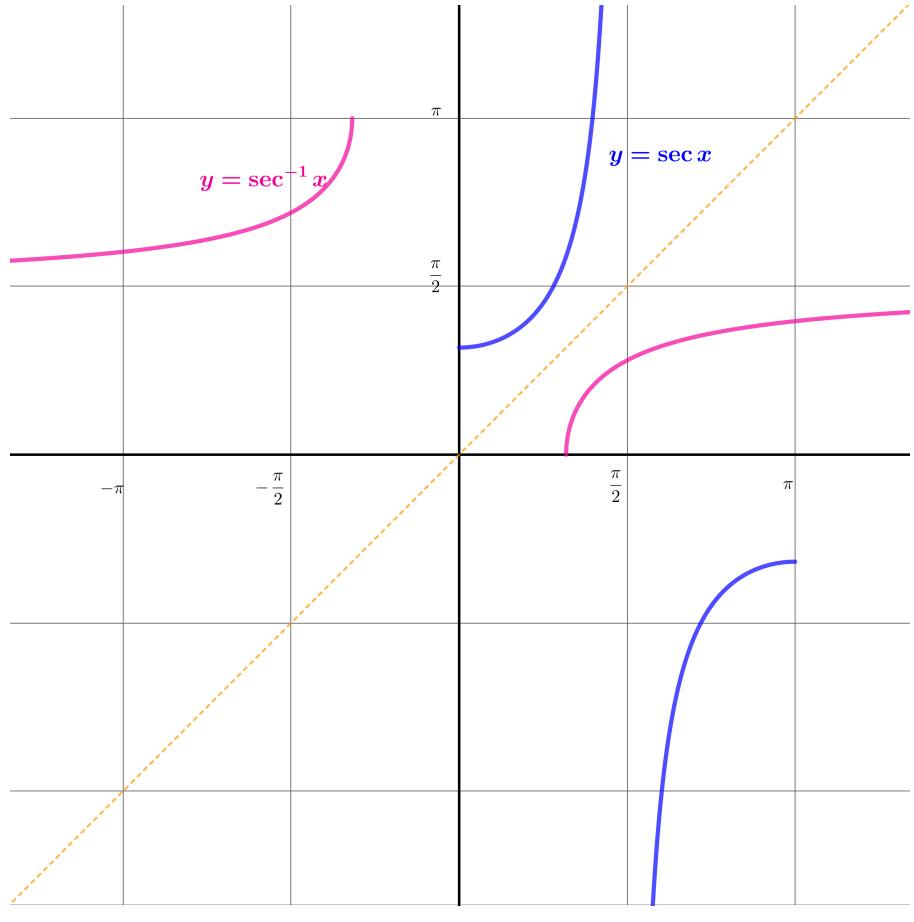
$\sec^{-1} x + \operatorname{cosec}^{-1} x$?





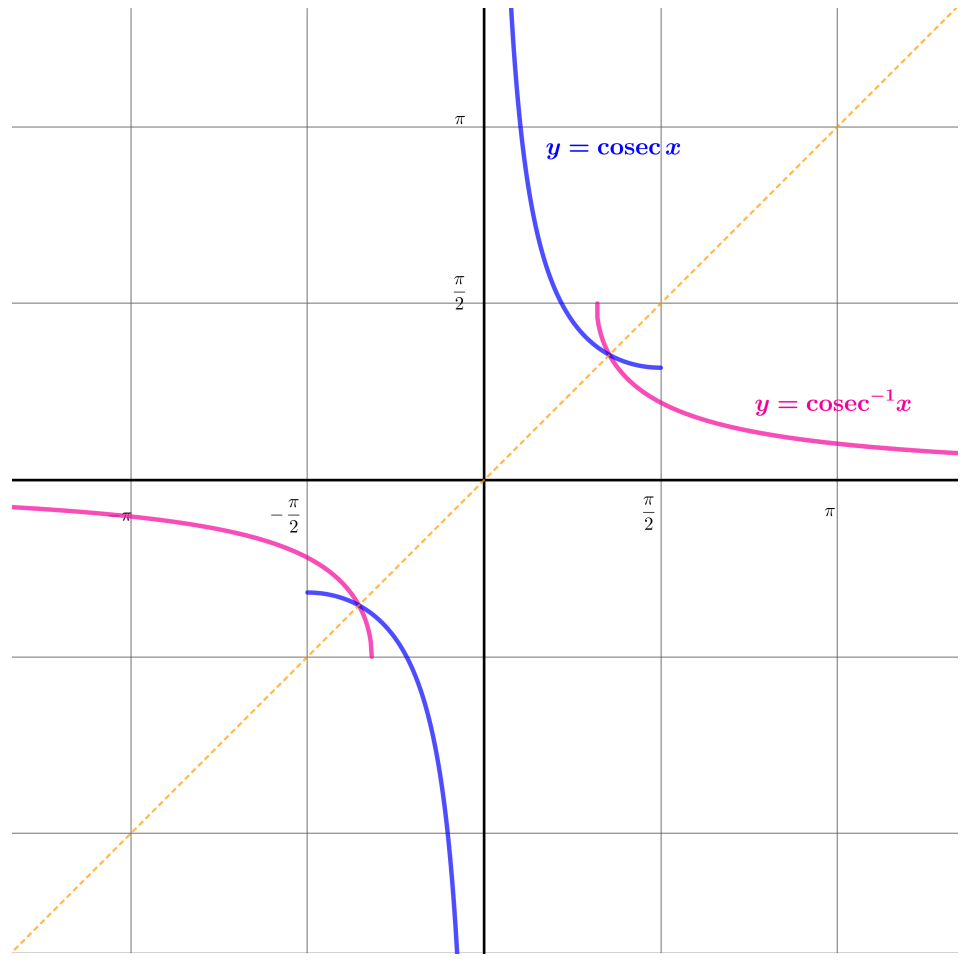
more differentials of inverse circular functions

What is $\dfrac{\mathrm{d}}{\mathrm{d}x}\sec^{-1}x$?



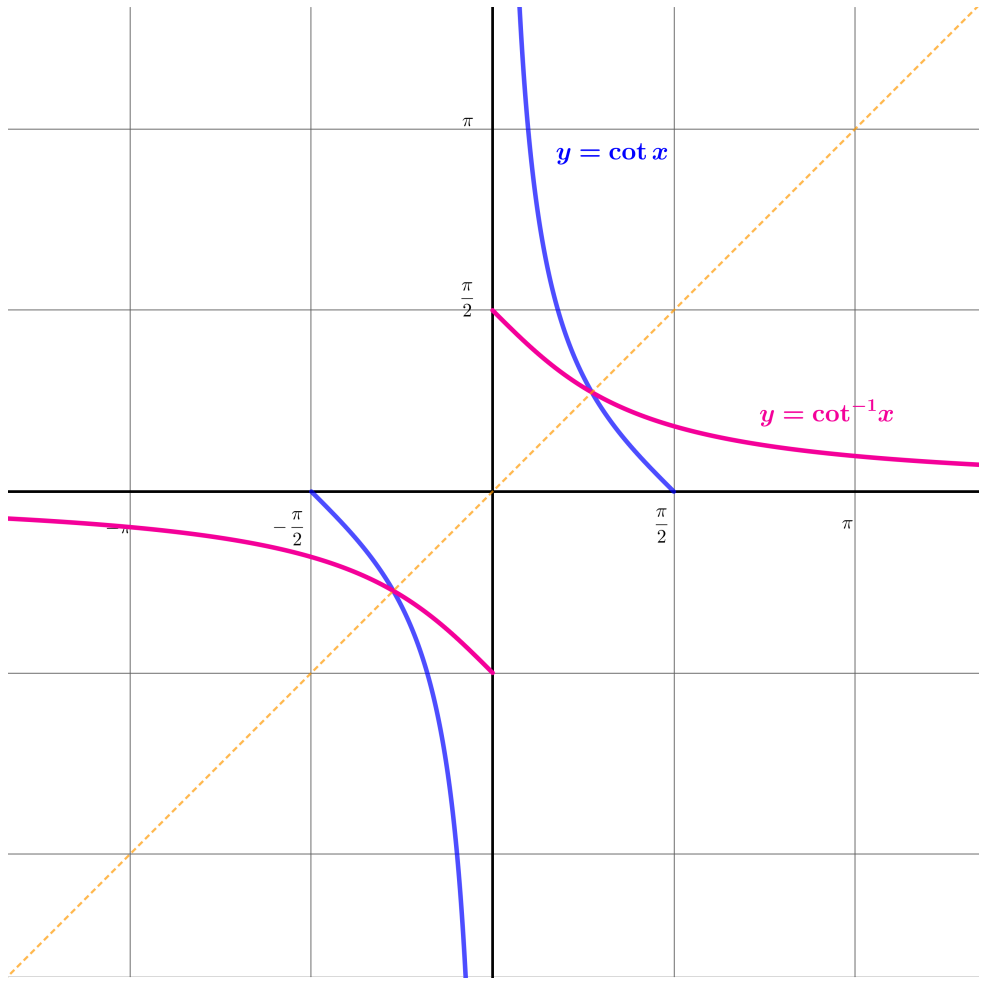
radical maths

What is $\dfrac{\mathrm{d}}{\mathrm{d}x}\mathrm{cosec}^{-1}\,x$?



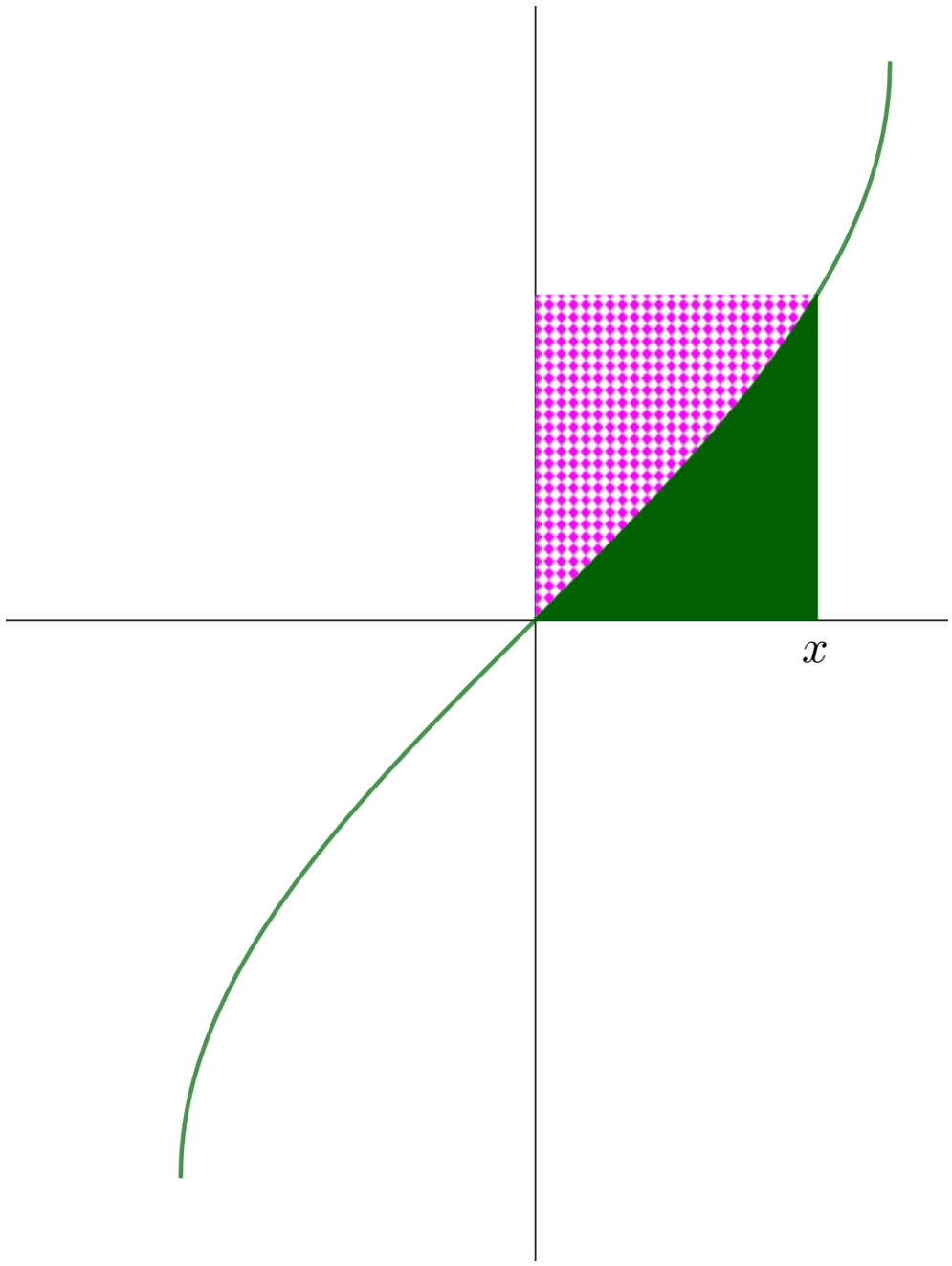
radical maths

What is $\dfrac{\mathrm{d}}{\mathrm{d}x}\cot^{-1} x$?



radical maths

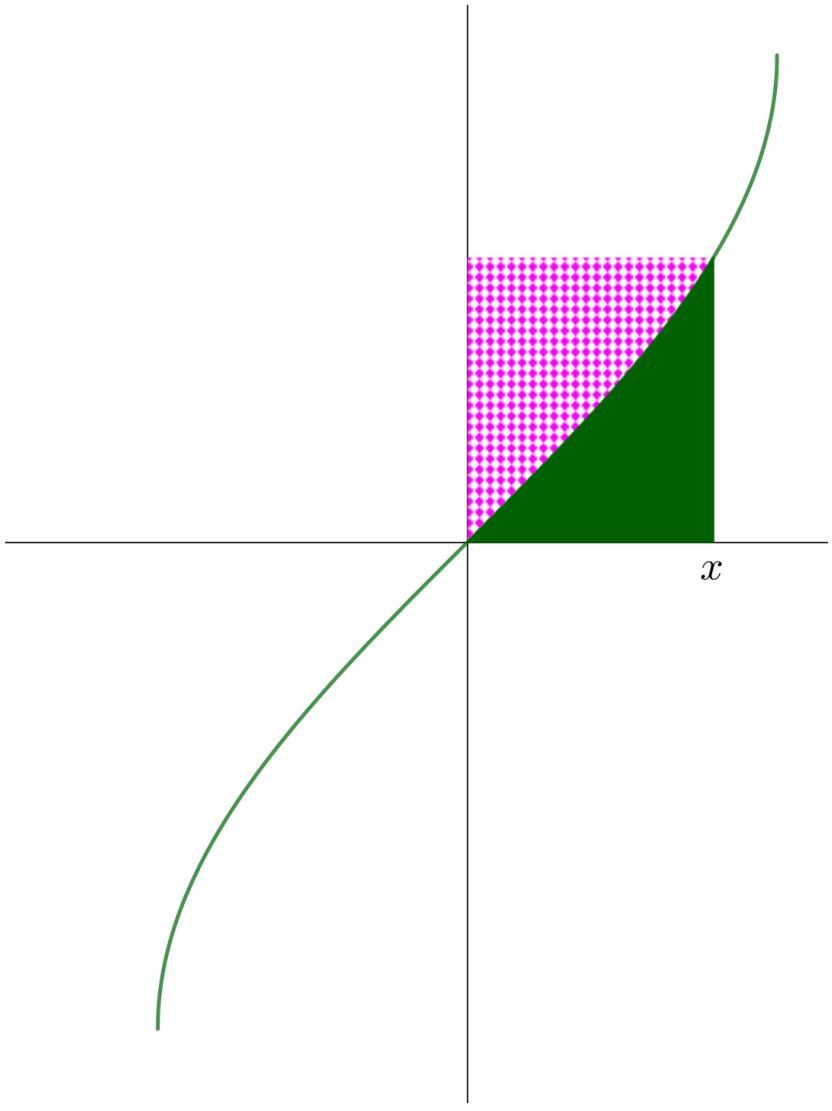
integrals of inverse circular functions



What is the area of the whole shaded rectangle?

Write the two shaded areas as integrals.

What is the area of the pink (chequered) area?



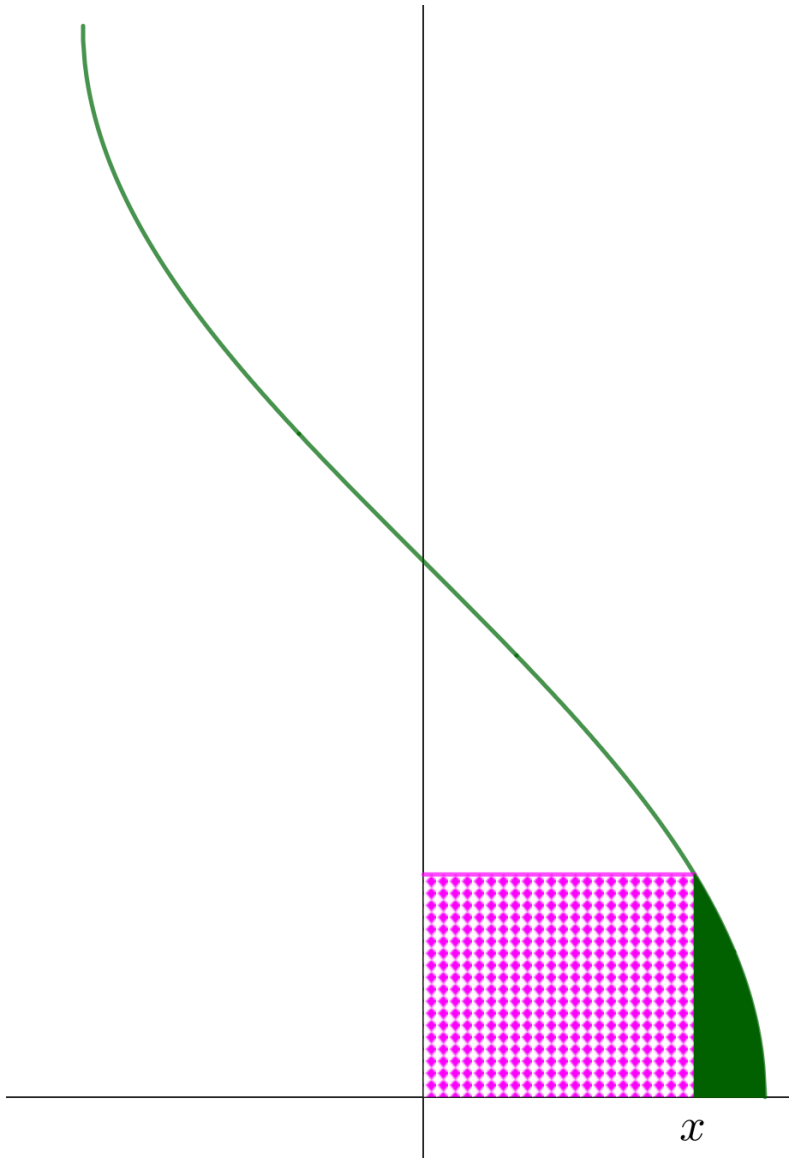
Use these results to find

$$\int_0^x \sin^{-1} x \, \mathrm{d}x$$

and

$$\int \sin^{-1} x \, \mathrm{d}x$$

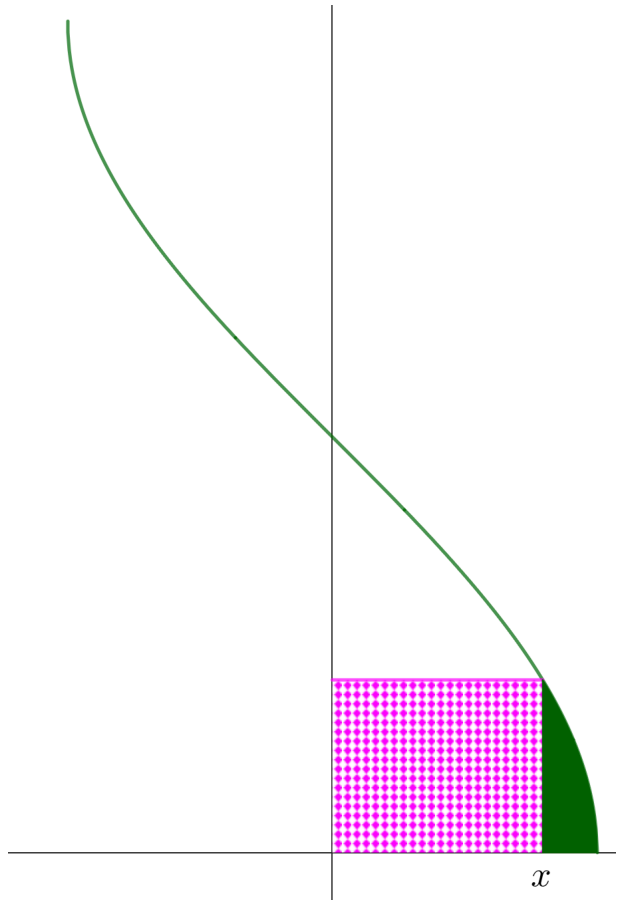
Here is the graph $y = \cos^{-1} x$.



What is the area of the pink (chequered) rectangle?

Write the entire shaded area as an integral.

Write the green (solid) shaded area as an integral.



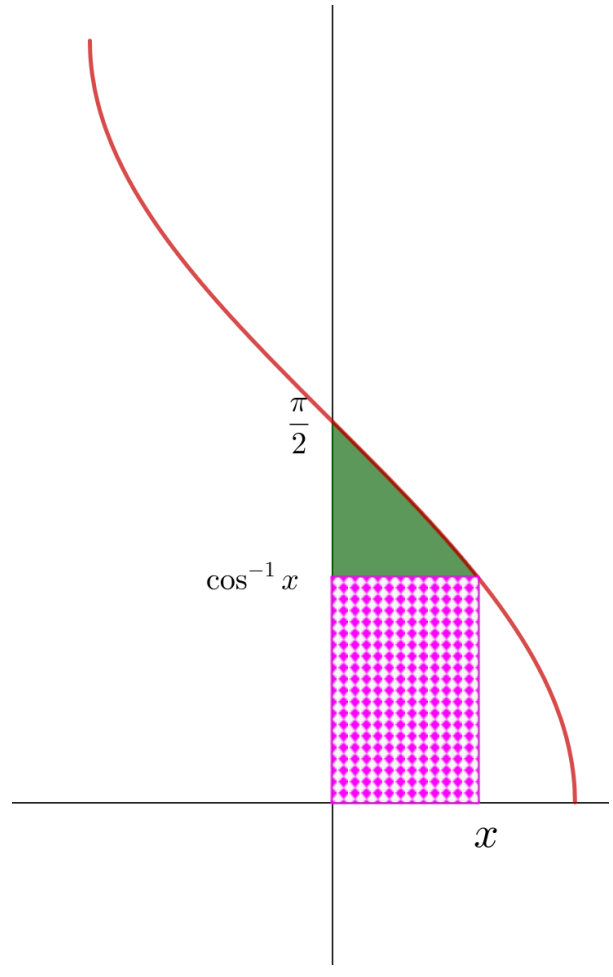
Use these results to find

$$\int_x^1 \cos^{-1} x \, \mathrm{d}x$$

and

$$\int \cos^{-1} x \, \mathrm{d}x$$

Here is the graph $y = \cos^{-1} x$.



Use these results to find

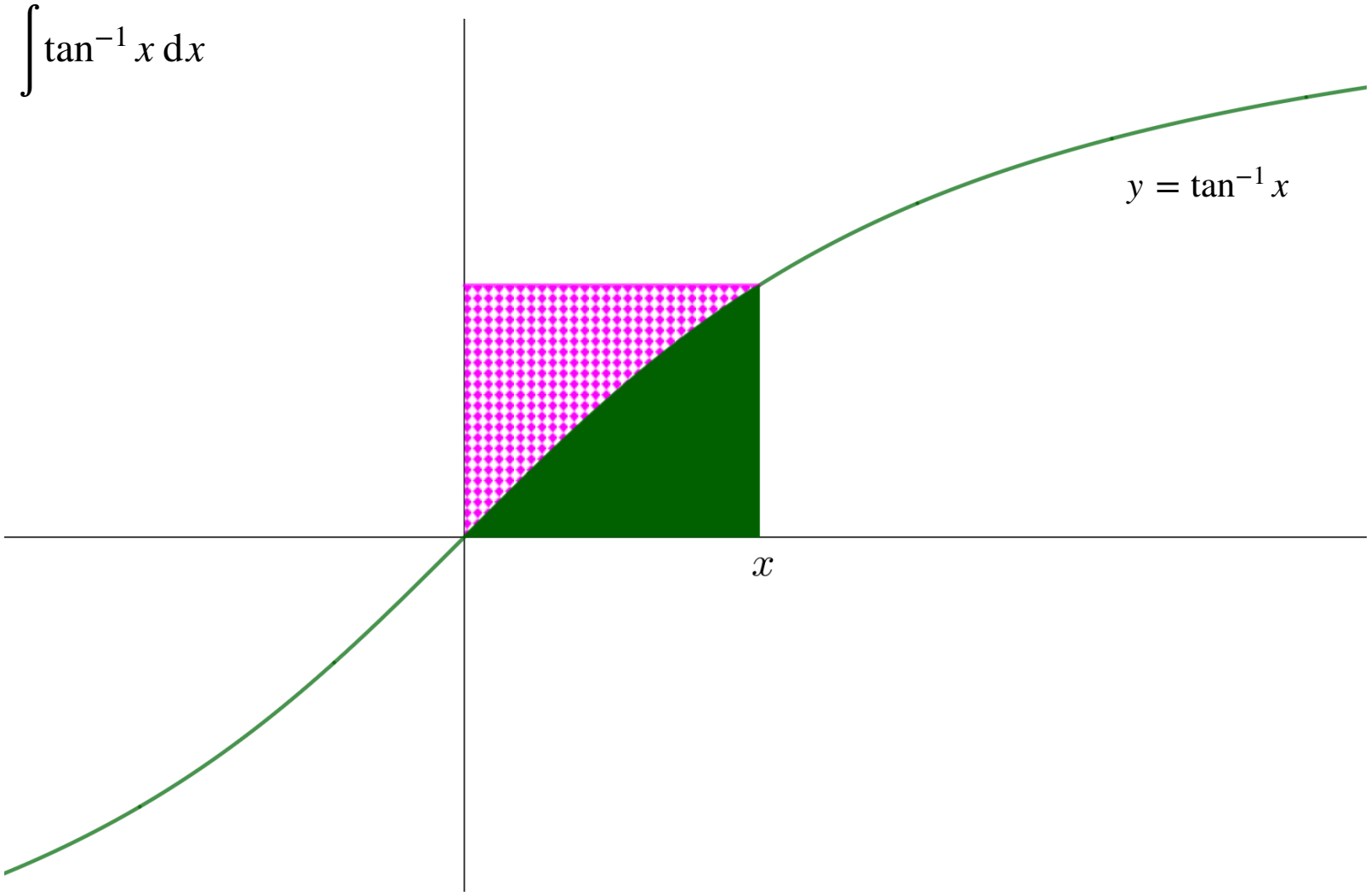
$$\int_0^x \cos^{-1} x \, \mathrm{d}x$$

and

$$\int \cos^{-1} x \, \mathrm{d}x$$

Use a similar strategy to find

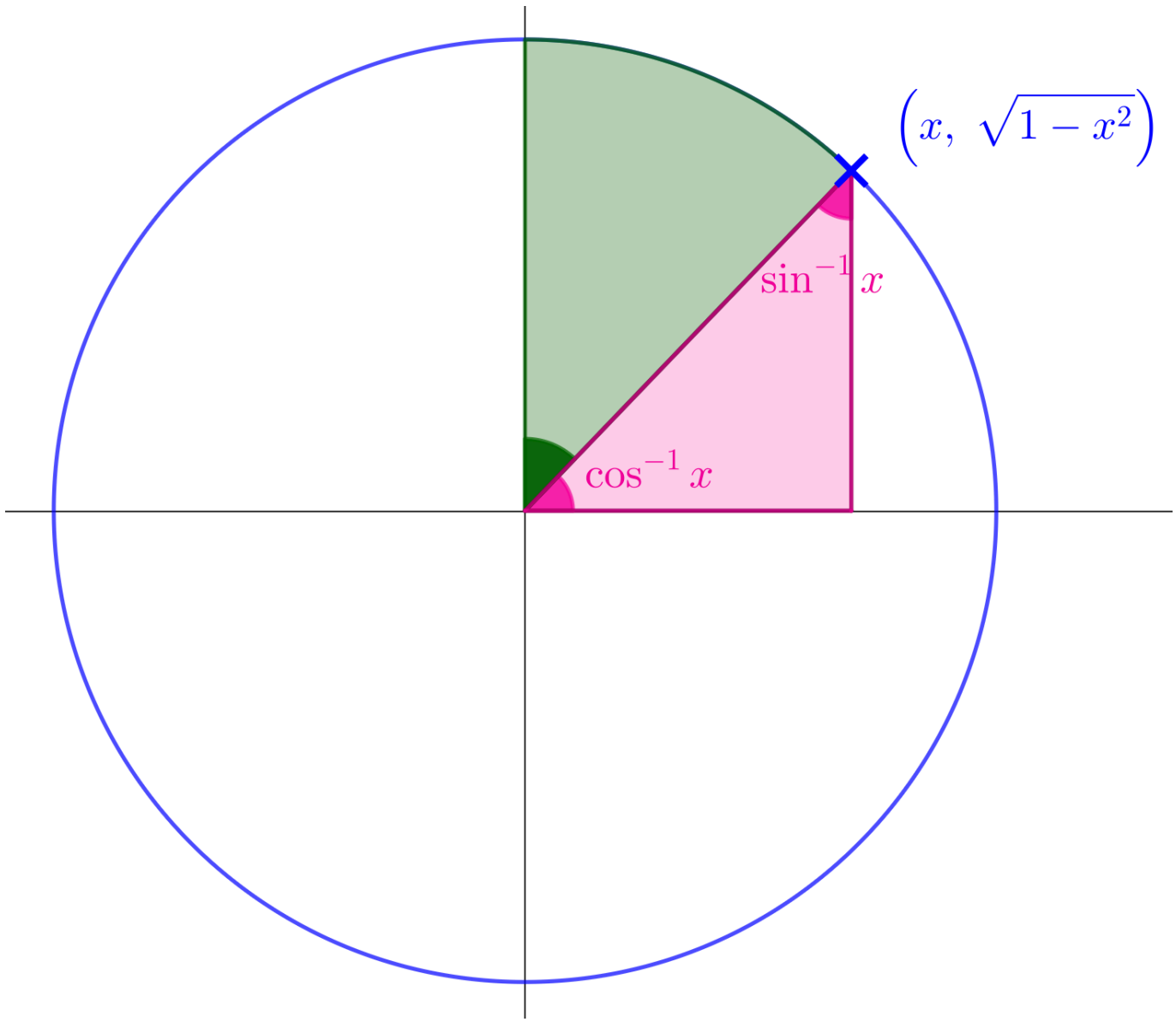
$$\int \tan^{-1} x \, \mathrm{d}x$$



Another integral using inverse circular functions

By finding the two shaded areas, find

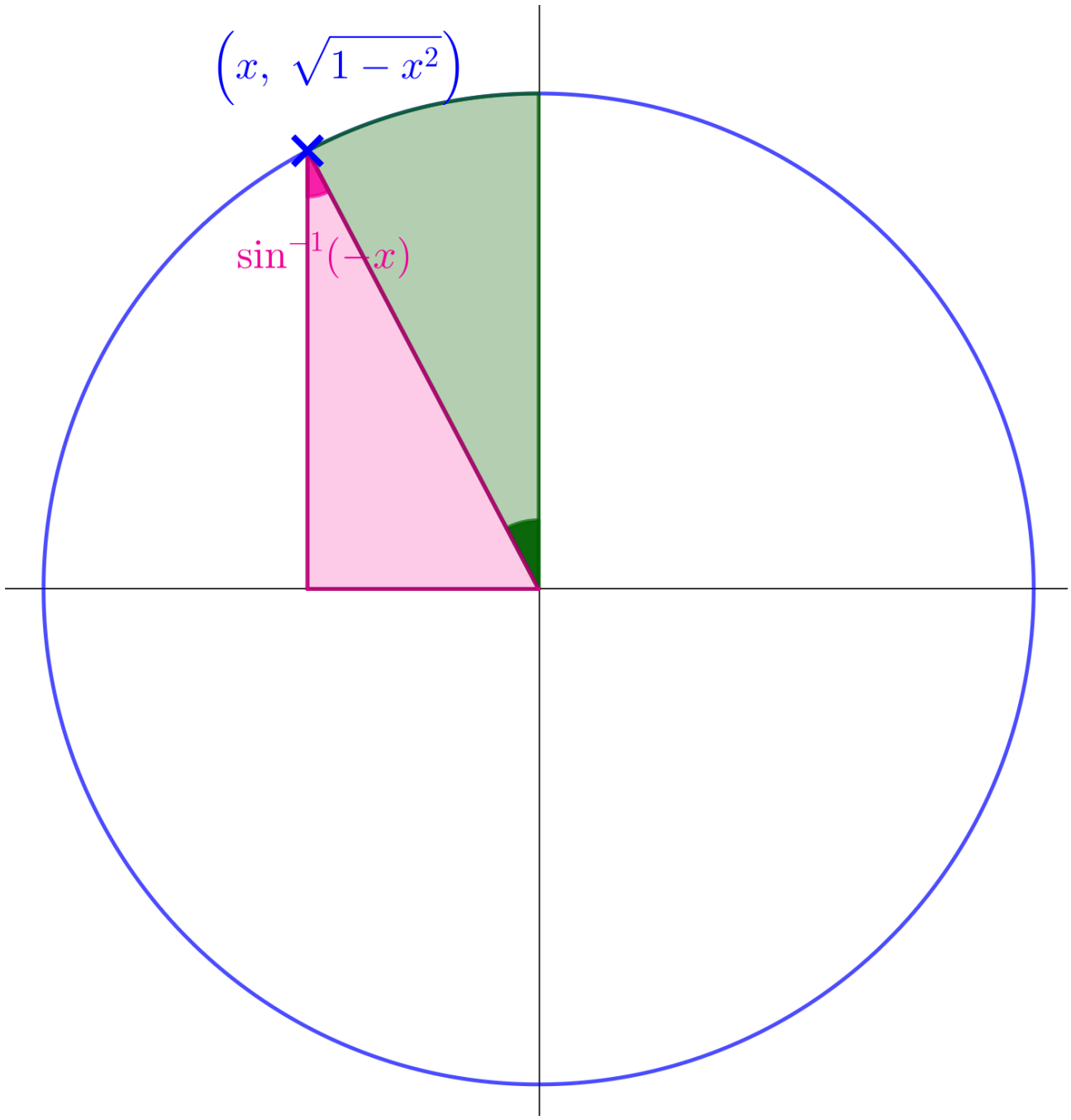
$$\int_0^x \sqrt{1-x^2}\,\mathrm{d}x \text{ and hence find } \int \sqrt{1-x^2}\,\mathrm{d}x.$$



radical maths

$x < 0$, by finding the two shaded areas, find

$\int_x^0 \sqrt{1-x^2}\,\mathrm{d}x$ and hence find $\int \sqrt{1-x^2}\,\mathrm{d}x$.



Use the substitutions $u = \sin x$ and $u = \cos x$ to find $\int \sqrt{1 - x^2}\, \mathrm{d}x$