radical maths

for independence
for confidence
for creativity
for insight

Circular functions 6

$\sin(A + B)$ etc

teacher version

Circular functions

Defining the circular functions	sin, cos, tan and the unit circle
Solving circular function equations	like $\sin\theta = 0.4$
Graphing the circular functions	graphs $y = \cos x$ and the like
Relationships between circular functions	$\sin(90° - x) = \cos x$ and the like
More circular functions	$\sec x = \dfrac{1}{\cos x}$ and so on

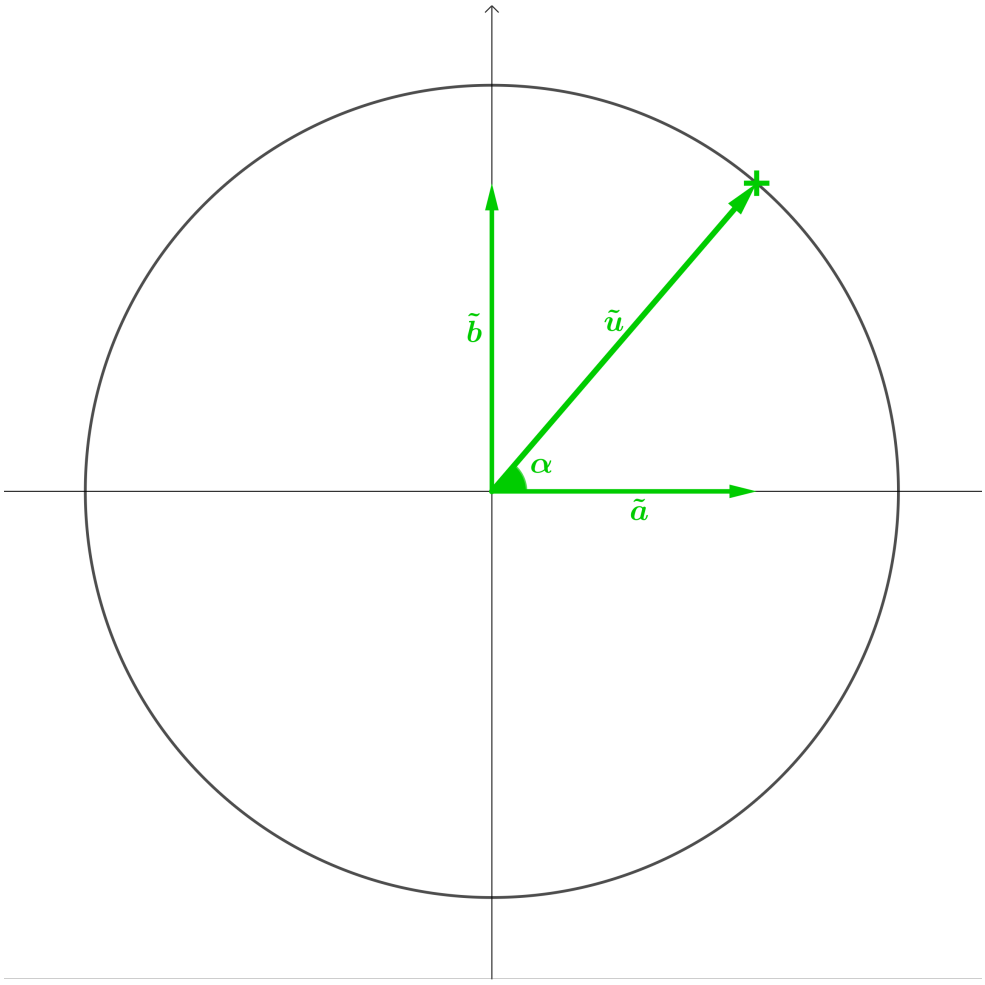
Circular functions of sums — **formulas like $\sin(A + B) = \sin A \cos B + \cos A \sin B$**

Transforming and adding circular functions	$\sin x + \cos x = \sqrt{2}\sin\left(x + 45°\right)$ and so on
Differentiating circular functions	radians, and tangents to graphs
Integrating circular functions	areas
Inverses of circular functions	$\arcsin x$, $\cos^{-1} x$, $\cot^{-1} x$ and the like, including graphs, differentials, integrals, and integration by substitution

Here is a circle radius 1.

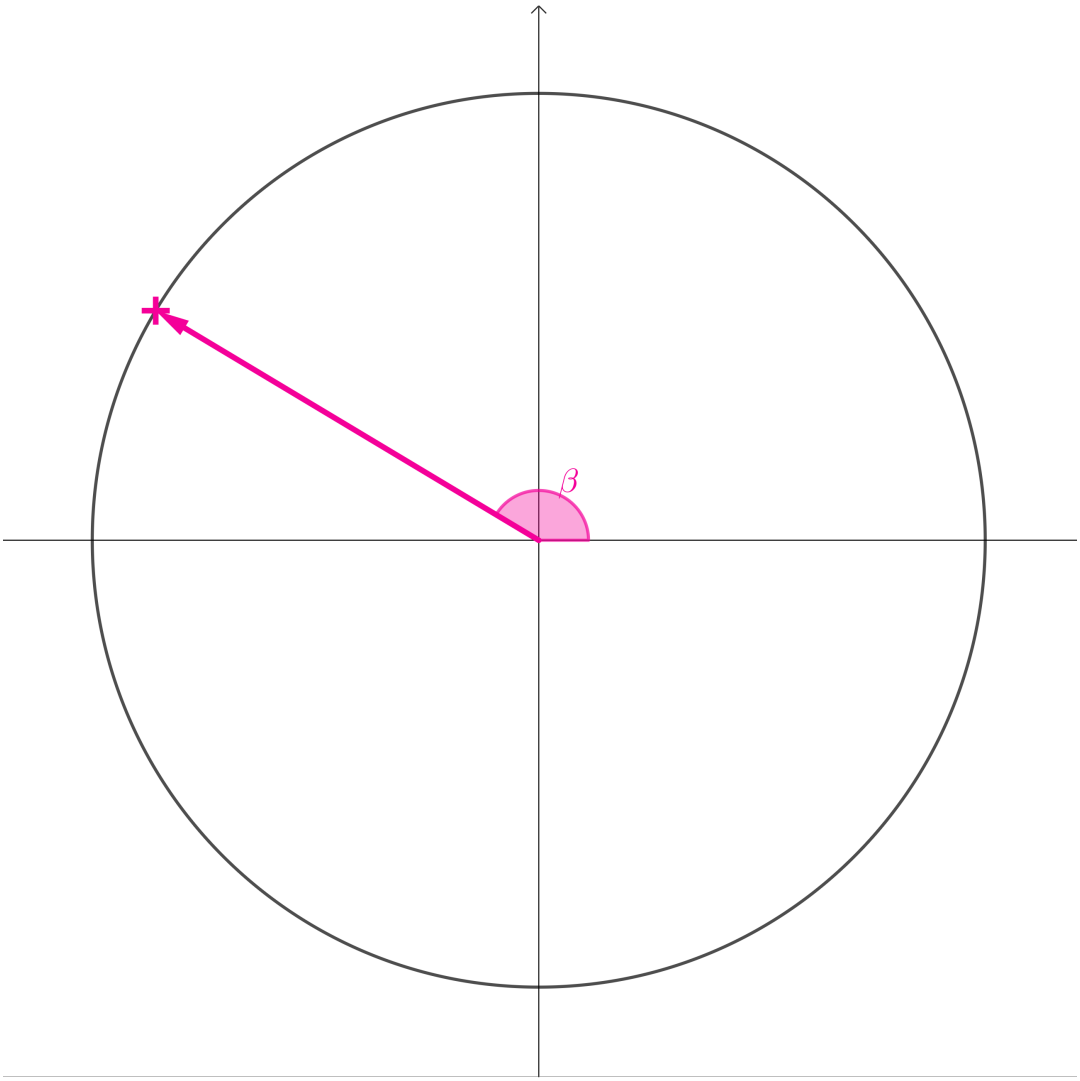
What are the coordinates of the green cross?

Write each of the vectors as column vectors in terms of α.



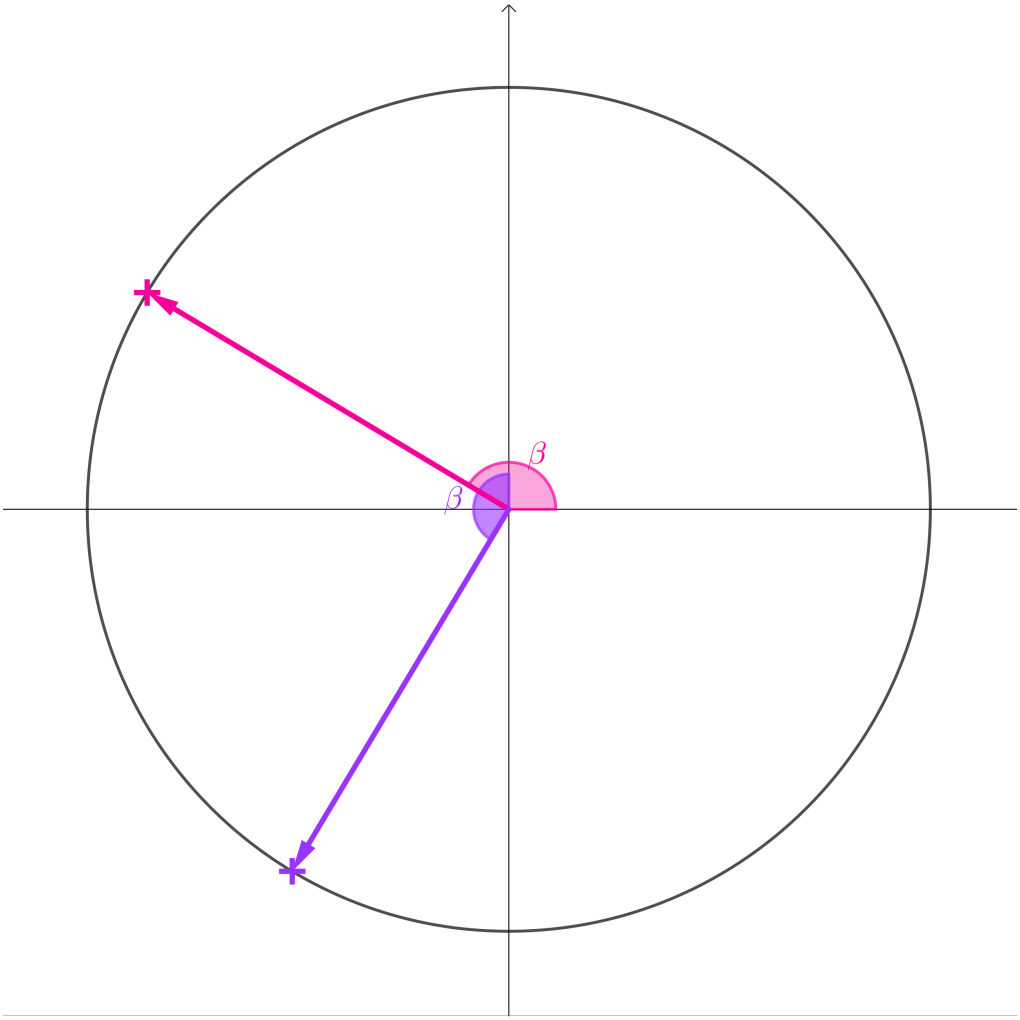
What are the coordinates of the red cross?

Write the red vector as column vector in terms of β.



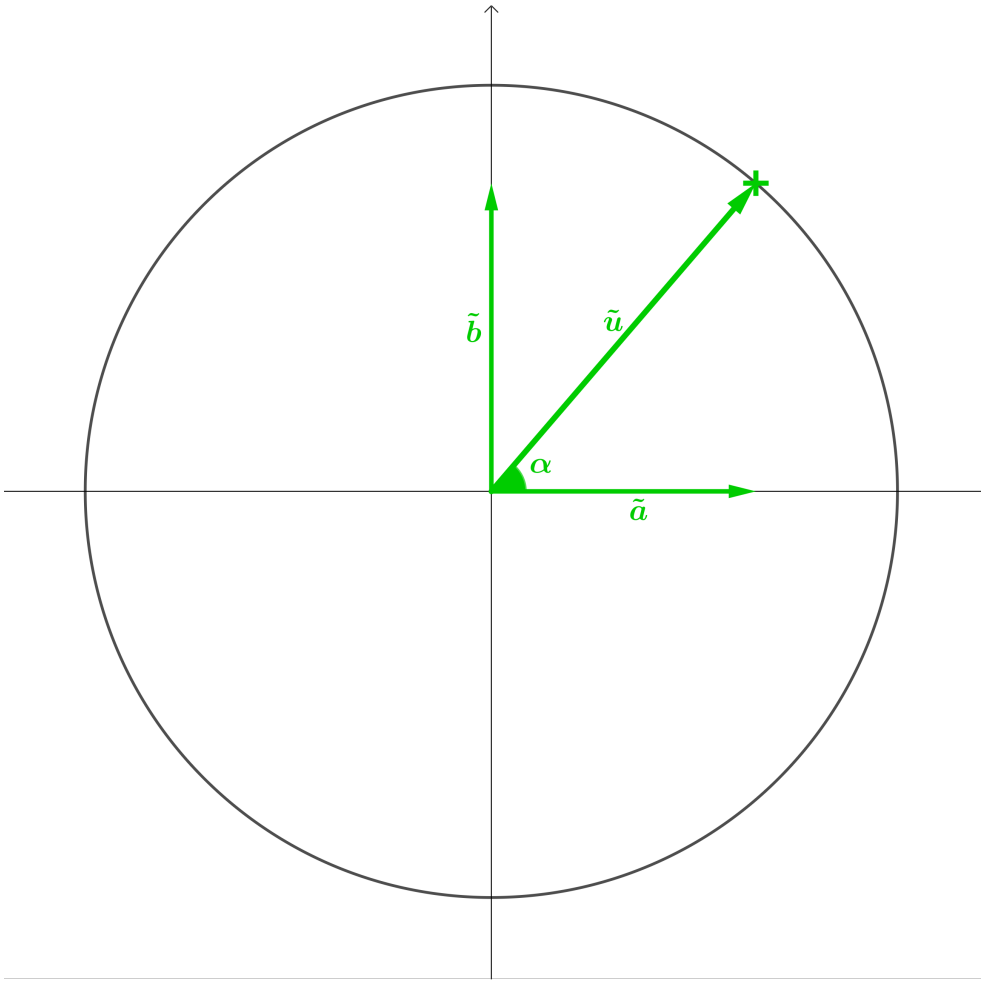
What are the coordinates of the darker cross?

Write the purple vector as column vector in terms of β.



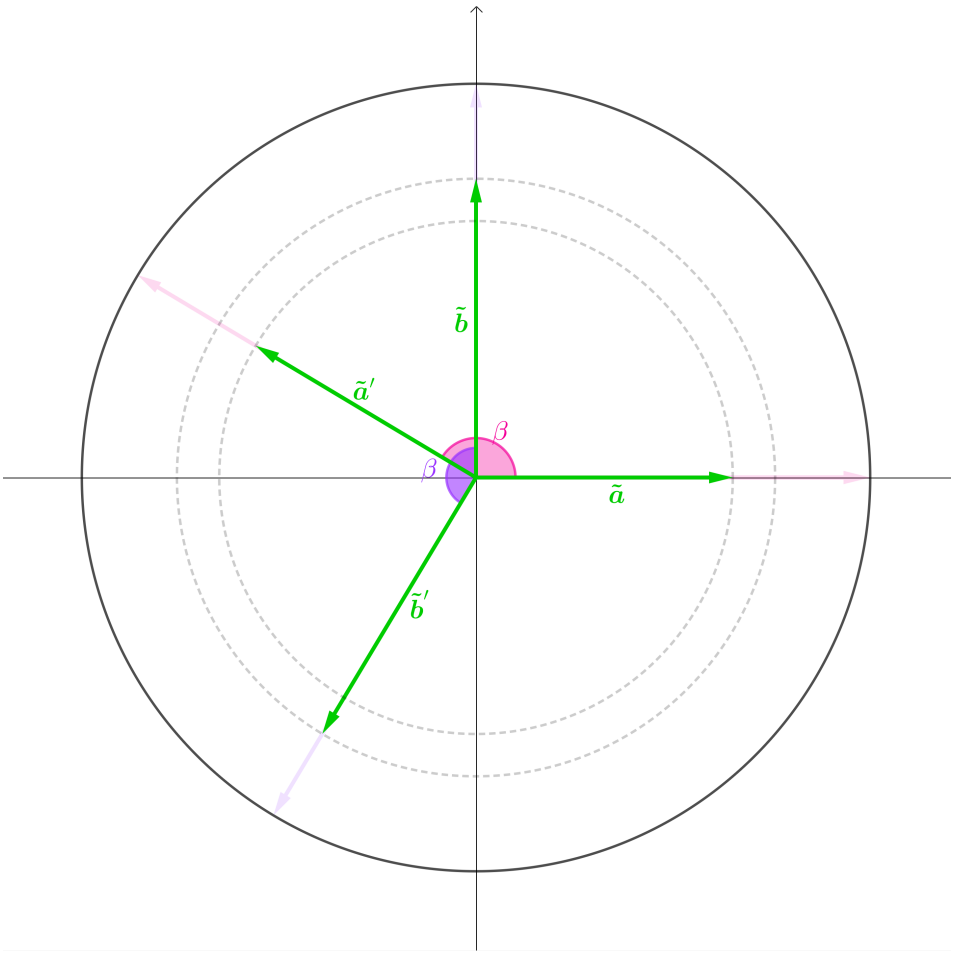
Quick reminder:

Write each of the vectors as column vectors in terms of α.

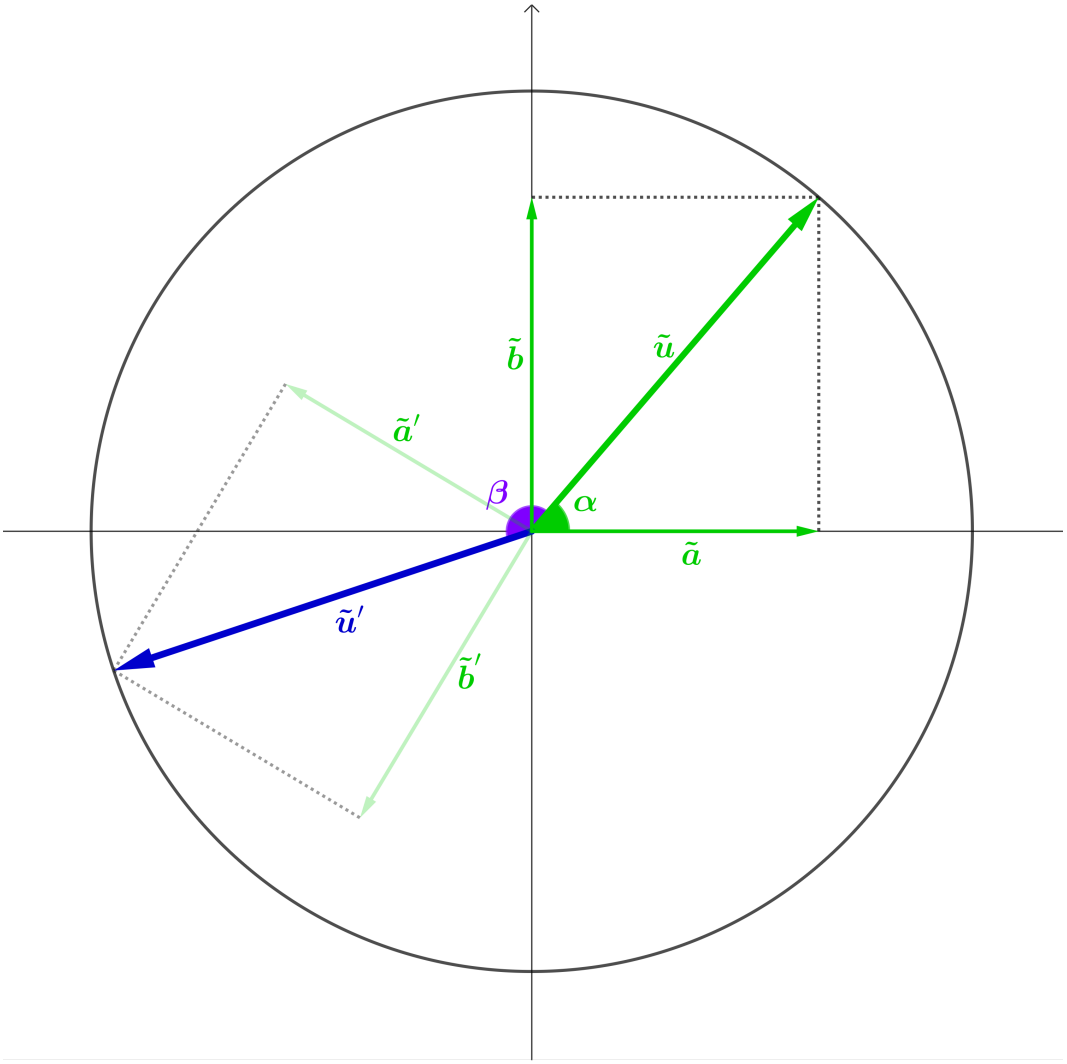


What are the magnitudes and directions of $\tilde{\boldsymbol{a}}'$ and $\tilde{\boldsymbol{b}}'$?

Write the vectors $\tilde{\boldsymbol{a}}'$ and $\tilde{\boldsymbol{b}}'$ as column vectors in terms of α and β.

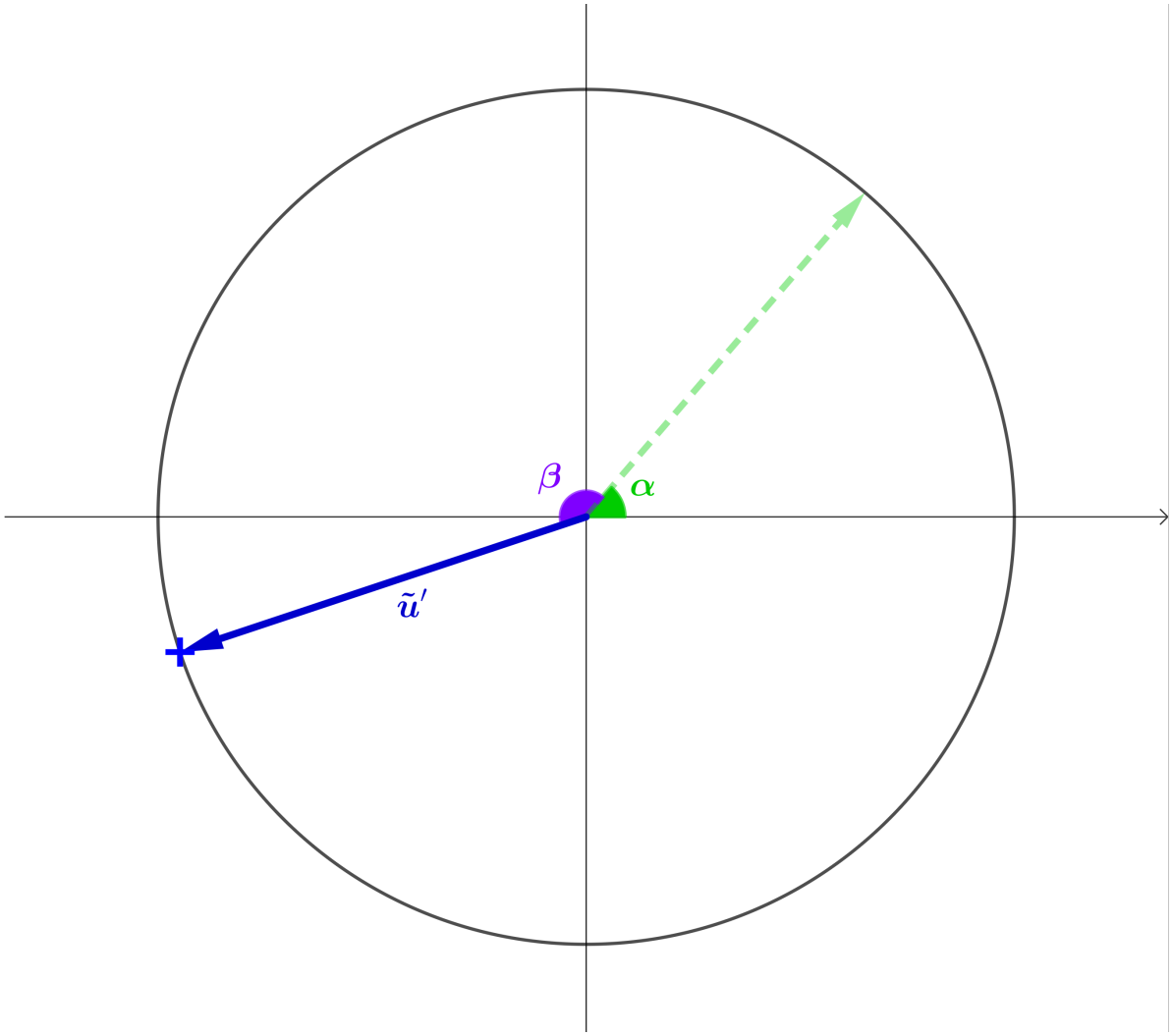


What is $\tilde{\boldsymbol{u}}'$ in terms of $\tilde{\boldsymbol{a}}'$ and $\tilde{\boldsymbol{b}}'$?

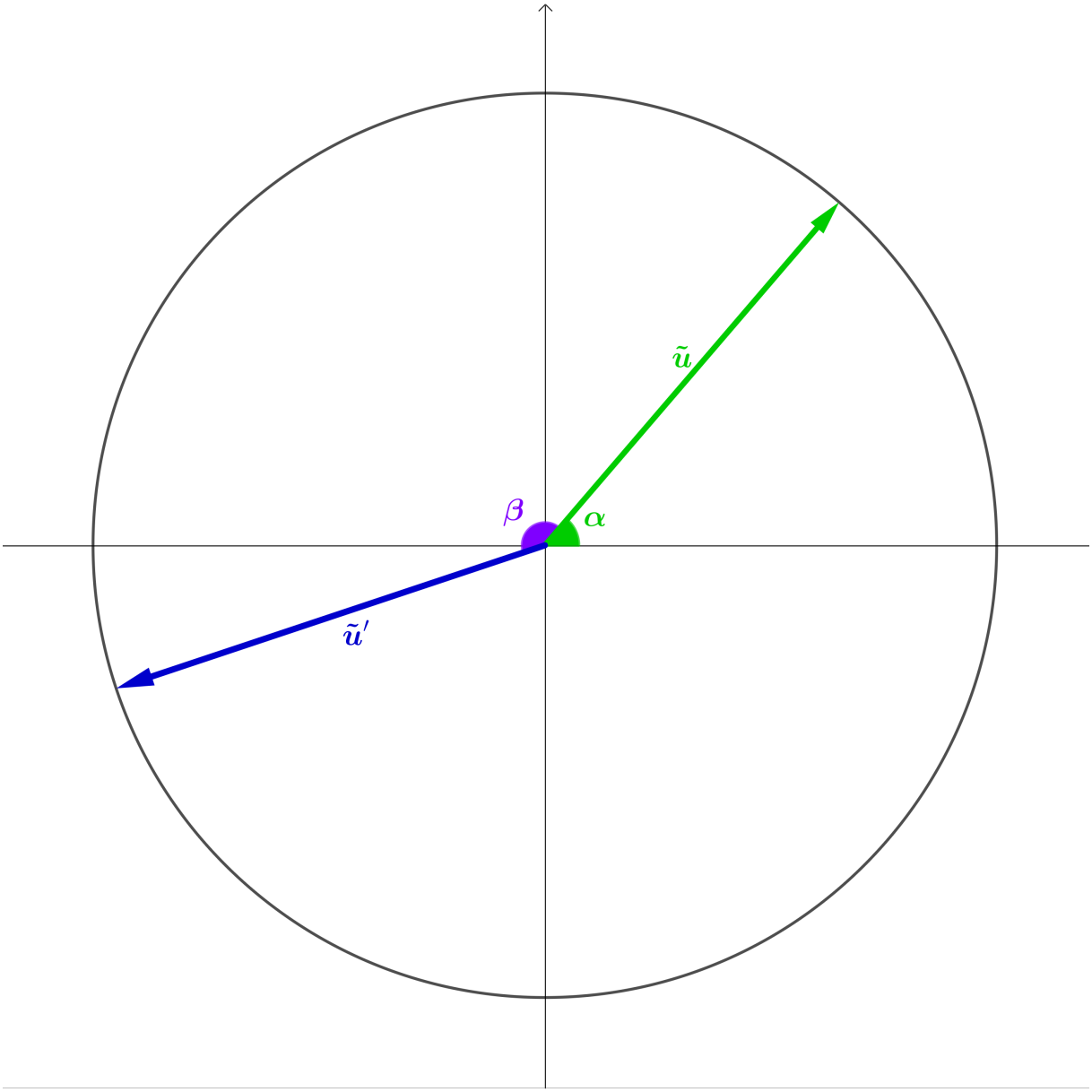


Use this to write $\tilde{\boldsymbol{u}}'$ in terms of α and β.

What are the coordinates of the blue cross?



Use the two expressions for $\tilde{\boldsymbol{u}}'$ together to find new expressions for $\cos(\alpha + \beta)$ and $\sin(\alpha + \beta)$.



Use these results to find $\tan(\alpha + \beta)$ in terms of $\tan\alpha$ and $\tan\beta$.

Use these results to find sin, cos, and tan of $\alpha - \beta$.

Use these results to find $\sin 2\alpha$, $\cos 2\alpha$, and $\tan 2\alpha$.

Use $\cos^2\alpha + \sin^2\alpha = 1$ to find two different formulas for $\cos 2\alpha$

Find $\sin 75^\circ$, $\cos 75^\circ$, and $\tan 75^\circ$.

Find $\sin 15^\circ$, $\cos 15^\circ$, and $\tan 15^\circ$.

Use the formula $\cos 2\theta = 2\cos^2\theta - 1$ to find $\cos 15°$.

Compare the two expressions you now have for $\cos 15°$.

Find $\int \sin^2 x \, \mathrm{d}x$

If $\sin\theta = \frac{2}{5}$ (θ is acute) and $\cos\varphi = -\frac{3}{4}$ (φ is obtuse)

find, **without using your calculator**:

$\cos\theta$

$\tan 2\theta$

$\tan\theta$

$\cos\frac{\theta}{2}$

$\cos 2\theta$

$\sin\frac{\theta}{2}$

$\sin 2\theta$

$\tan \varphi$

$\tan \frac{\theta}{2}$

$\cos 2\varphi$

$\sin 2\varphi$

$\sin \varphi$

$\tan 2\varphi$

$\tan \frac{\varphi}{2}$

$\cos \frac{\varphi}{2}$

$\sin \frac{\varphi}{2}$

$\cos(\theta + \varphi)$

$\cos(\theta - \varphi)$

$\sin(\theta + \varphi)$

$\sin(\theta - \varphi)$

$\tan(\theta + \varphi)$

$\tan(\theta - \varphi)$