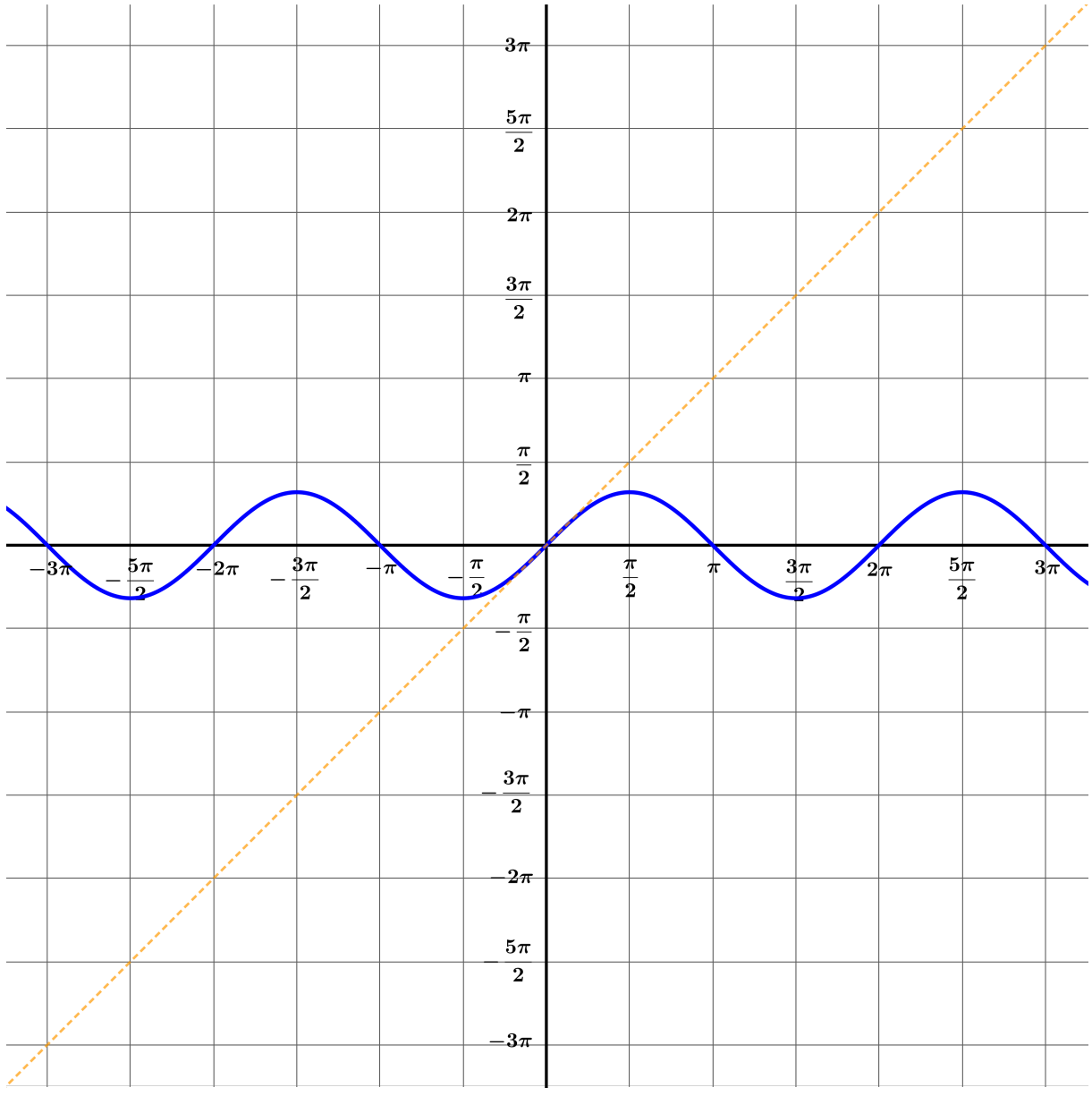
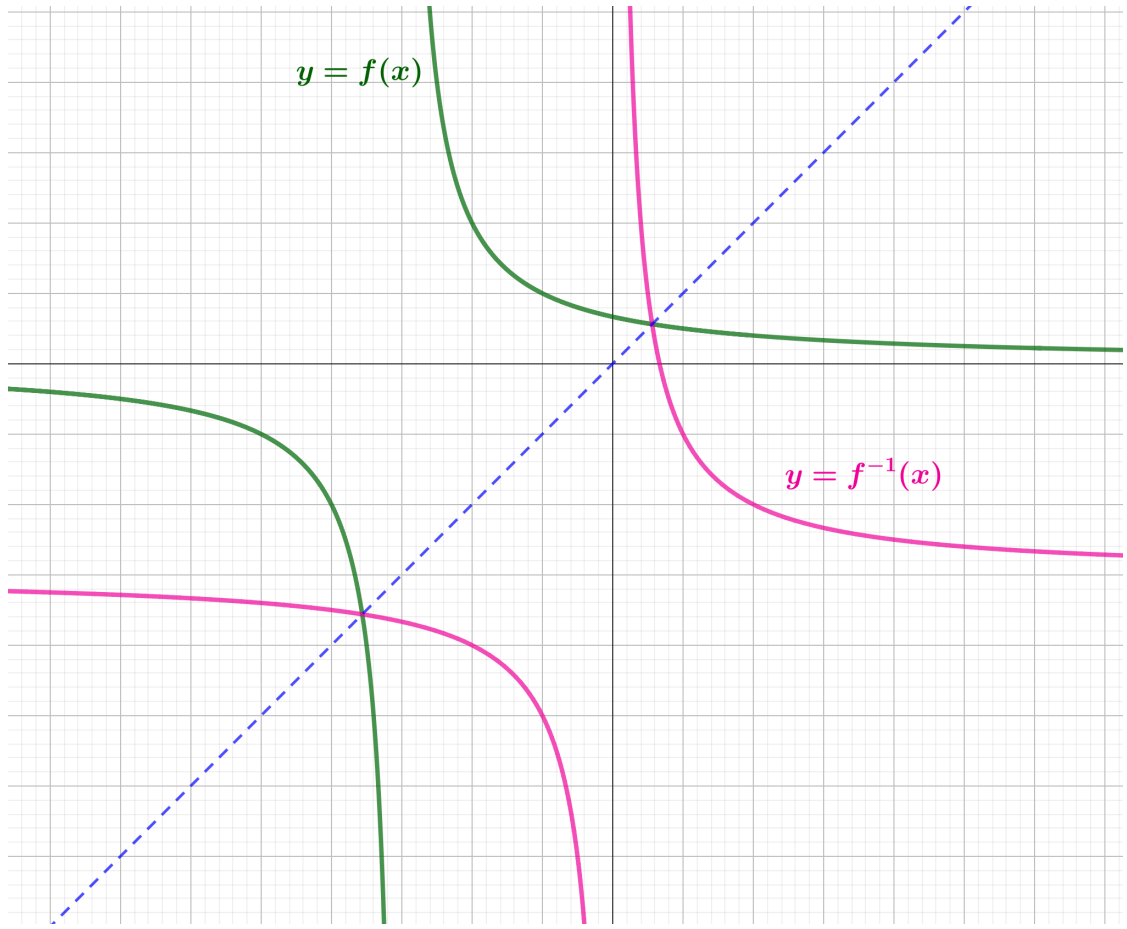
Inverse circular functions

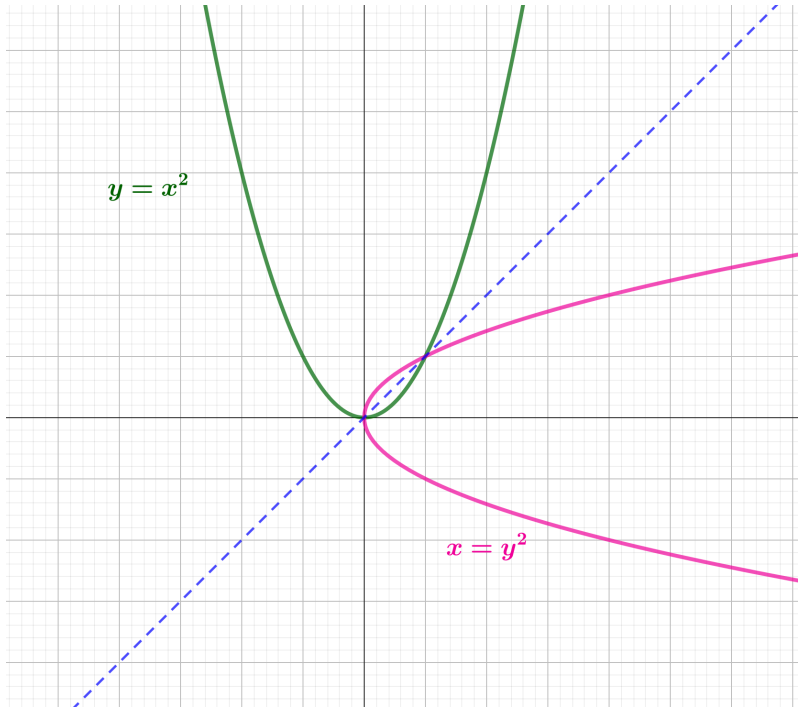
Draw the graph $x = \sin y$.

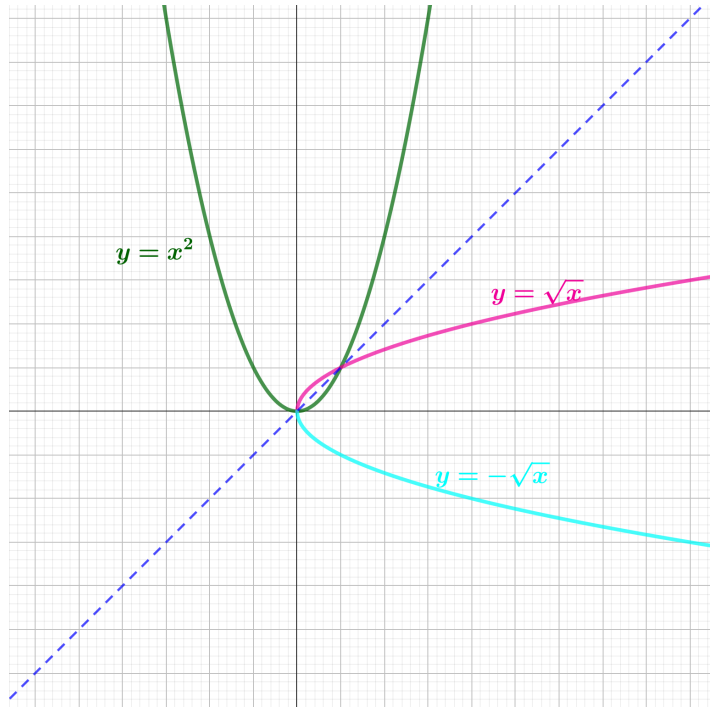


radical maths



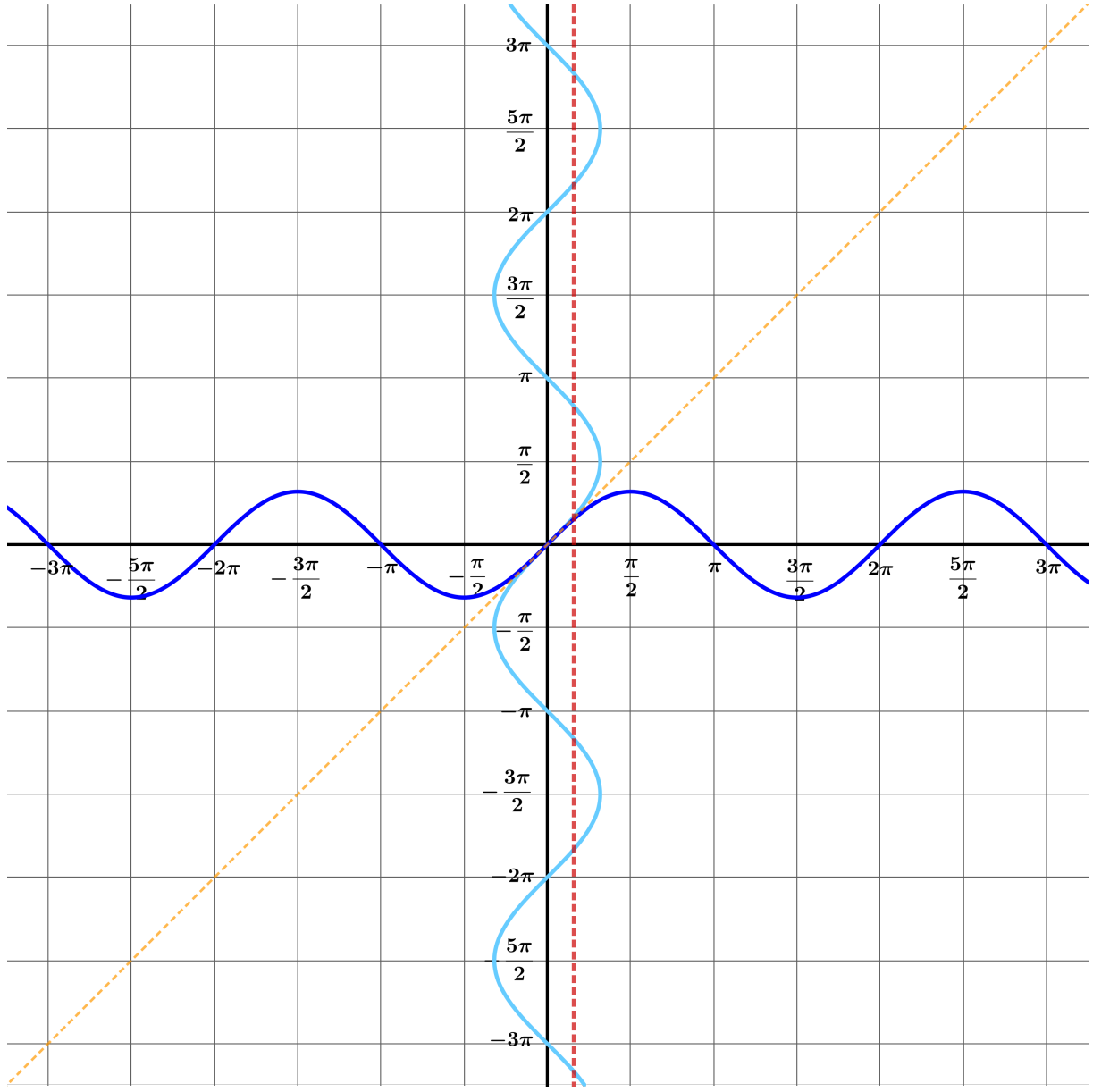
radical maths





radical maths

Solve the equation $\sin y = \frac{1}{2}$.

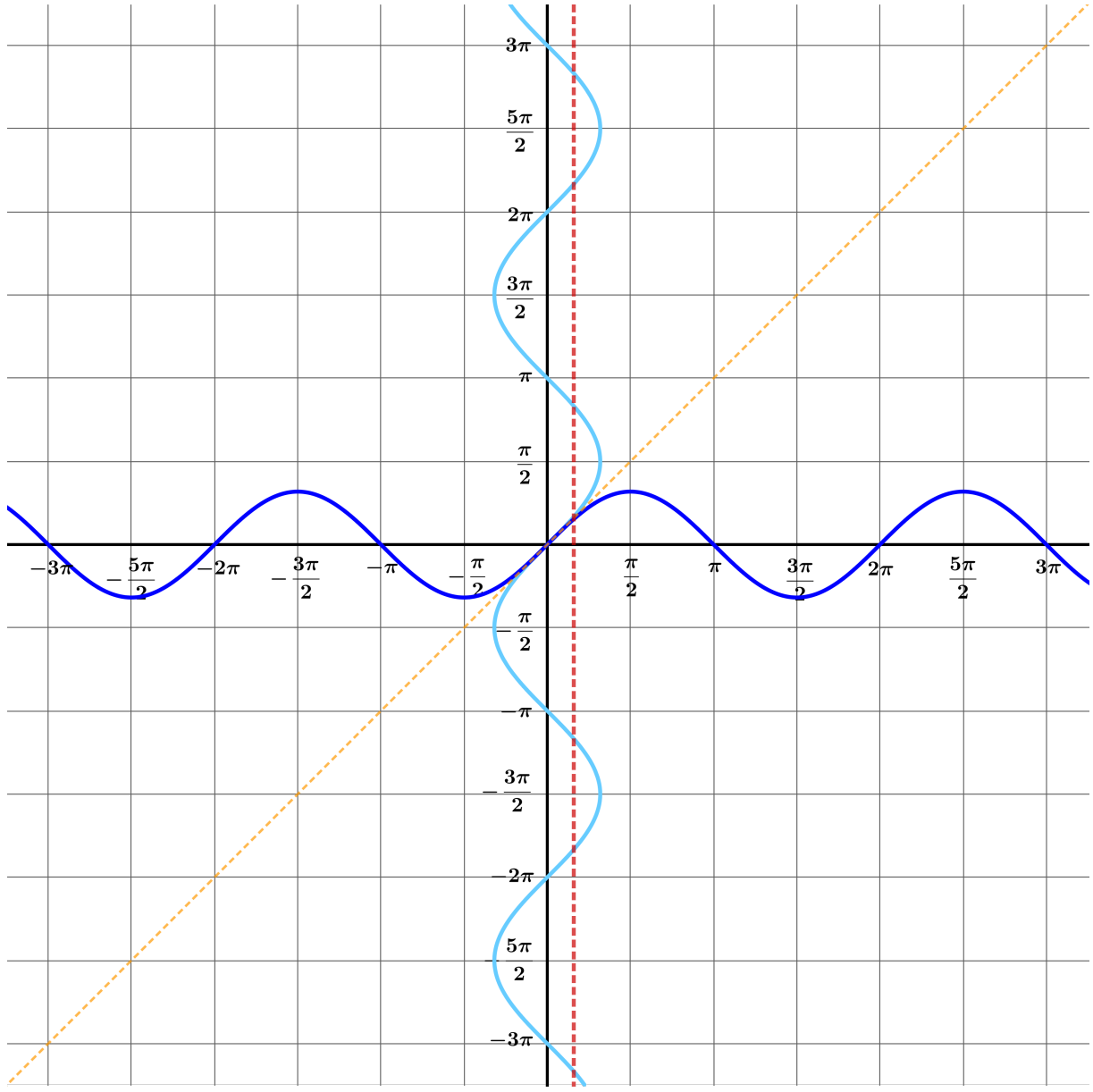


How does your answer relate to this graph?

How many values do you want for $\sin^{-1}\frac{1}{2}$?

How many times do you want the line $x = \frac{1}{2}$ to intersect with the graph $y = \sin^{-1} x$?

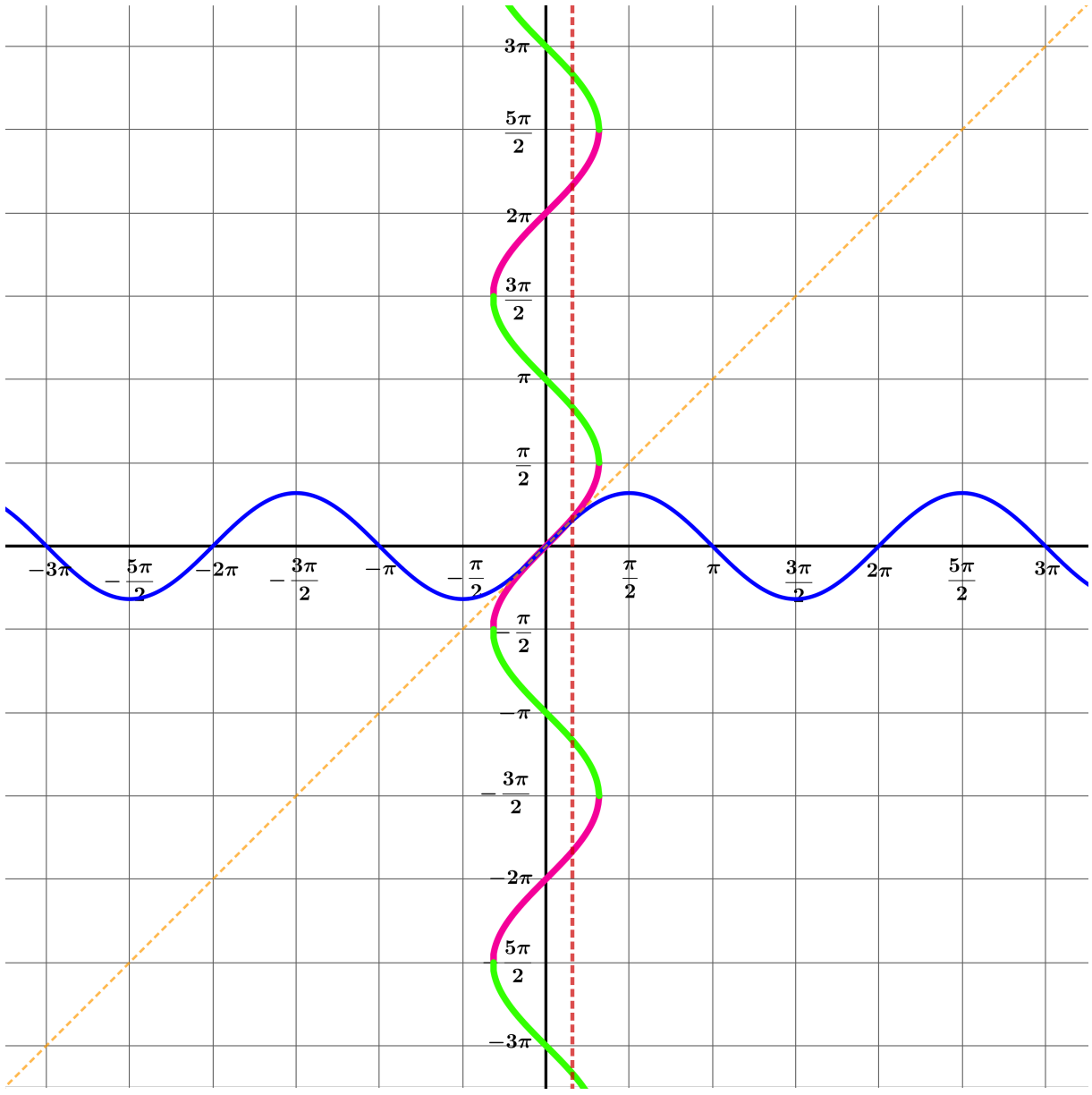
How can you adapt the graph $x = \sin y$ to ensure that any vertical line between $x = -1$ and $x = 1$ intersects it exactly once?



radical maths

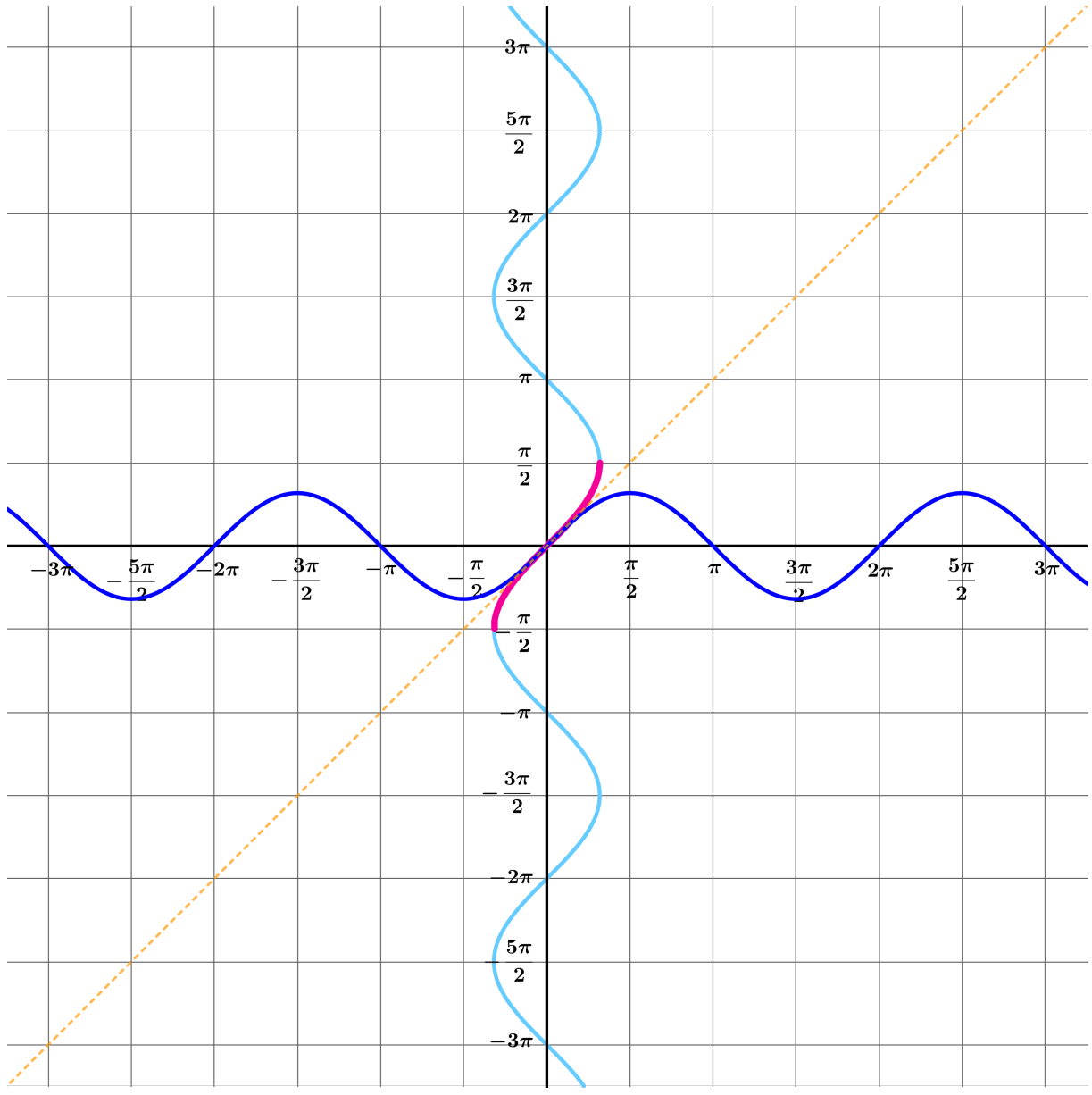
Which of these pink or green segments on the curve would correspond to a possible definition of the function $\sin^{-1} x$?

Which would be the most sensible to choose?



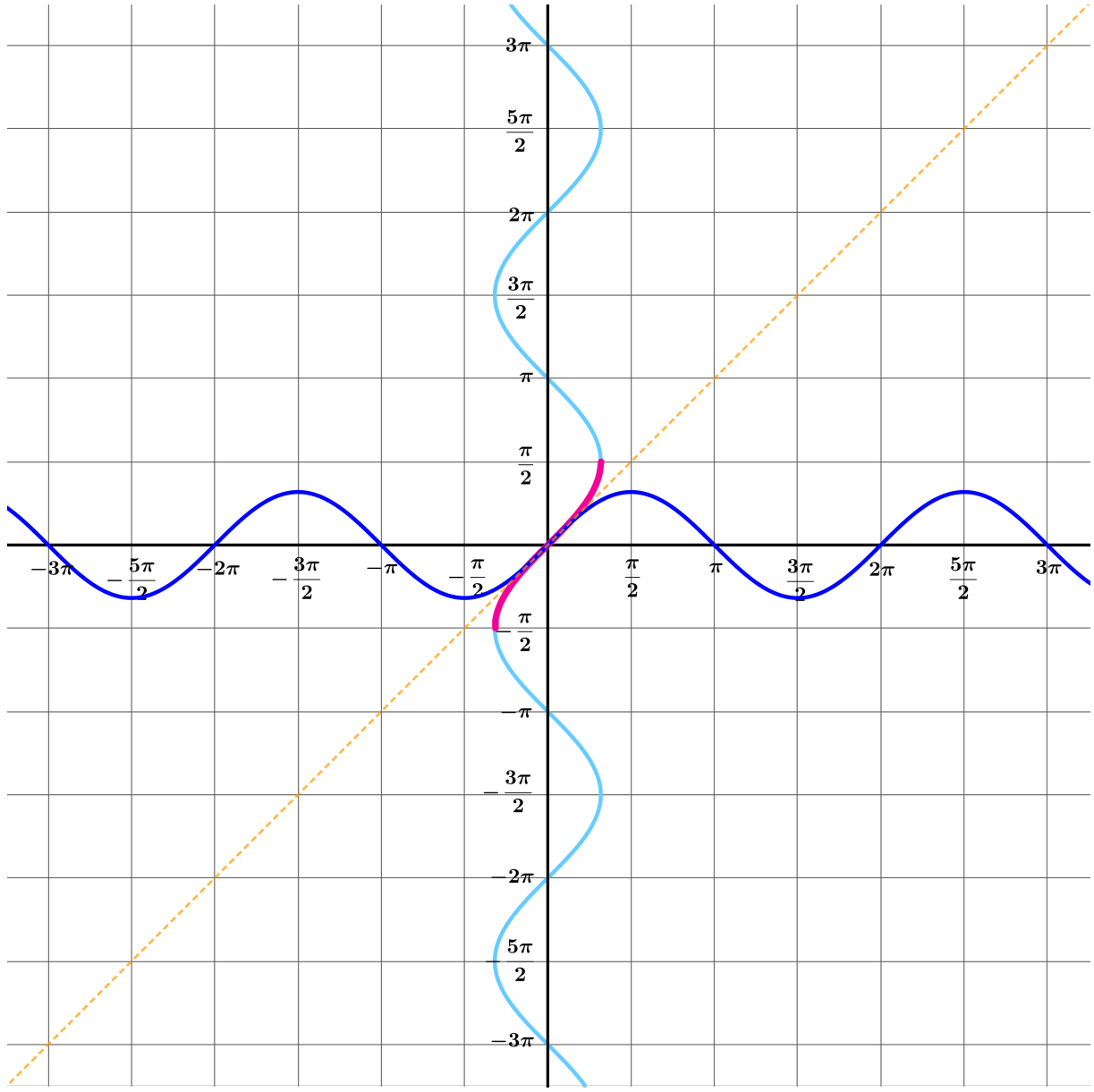
radical maths

Here it is: the graph $y = \sin^{-1} x$.



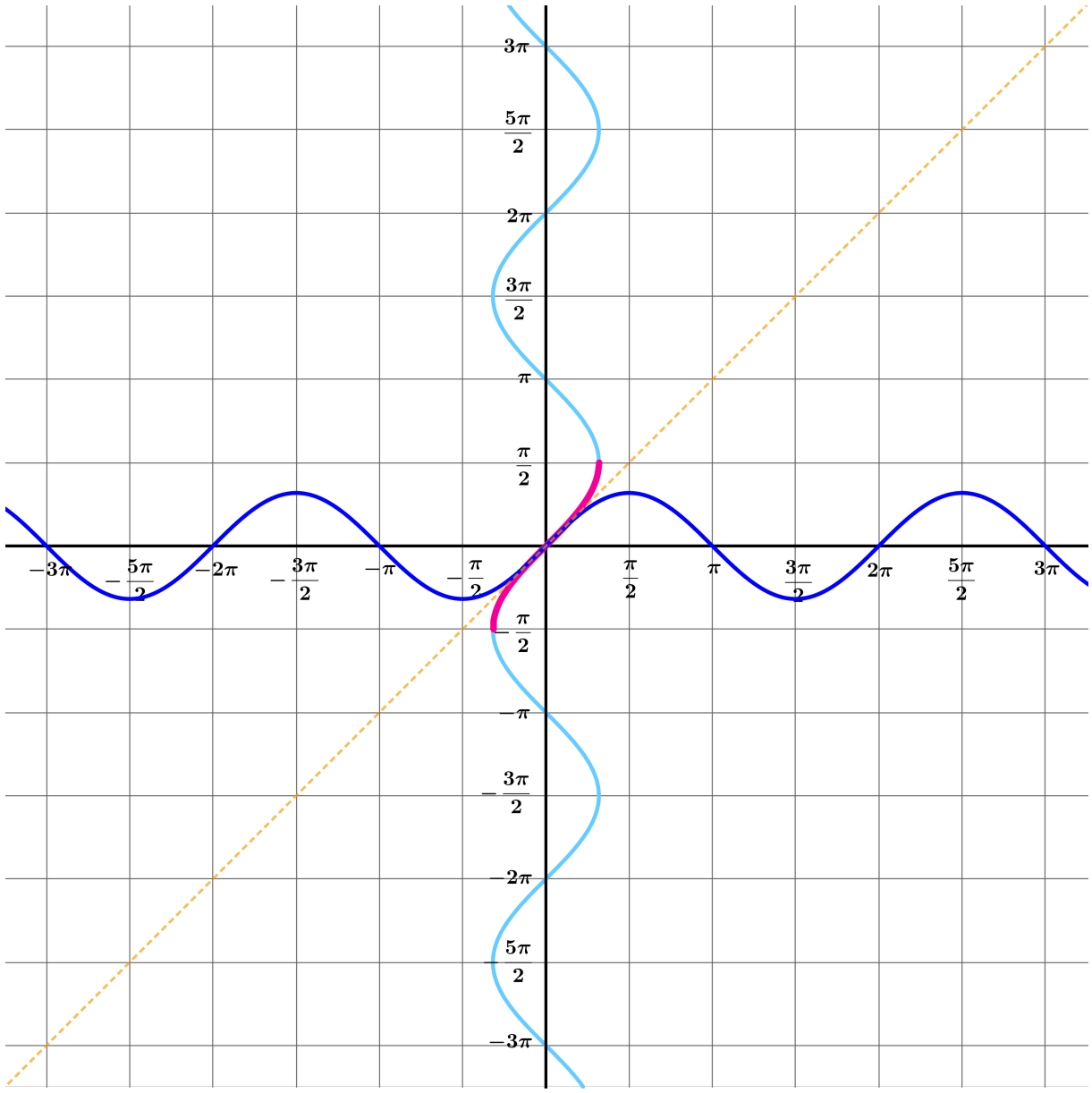
What are its domain and range?

What are $\sin\left(\sin^{-1}\frac{1}{2}\right)$ and $\sin^{-1}\left(\sin\frac{\pi}{6}\right)$?



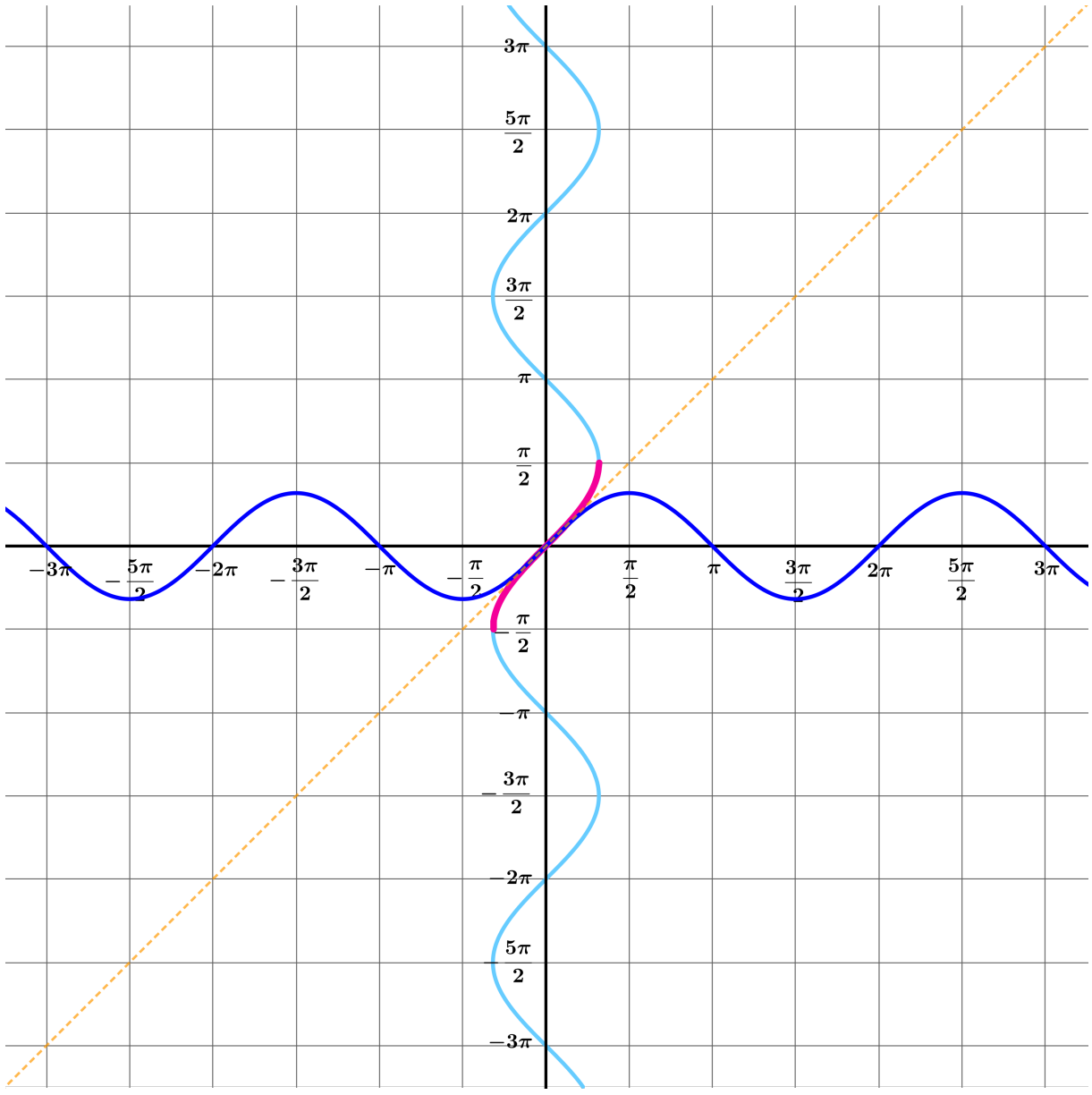
radical maths

What are $\sin\left(\sin^{-1}(-1)\right)$ and $\sin^{-1}\left(\sin\dfrac{3\pi}{2}\right)$?



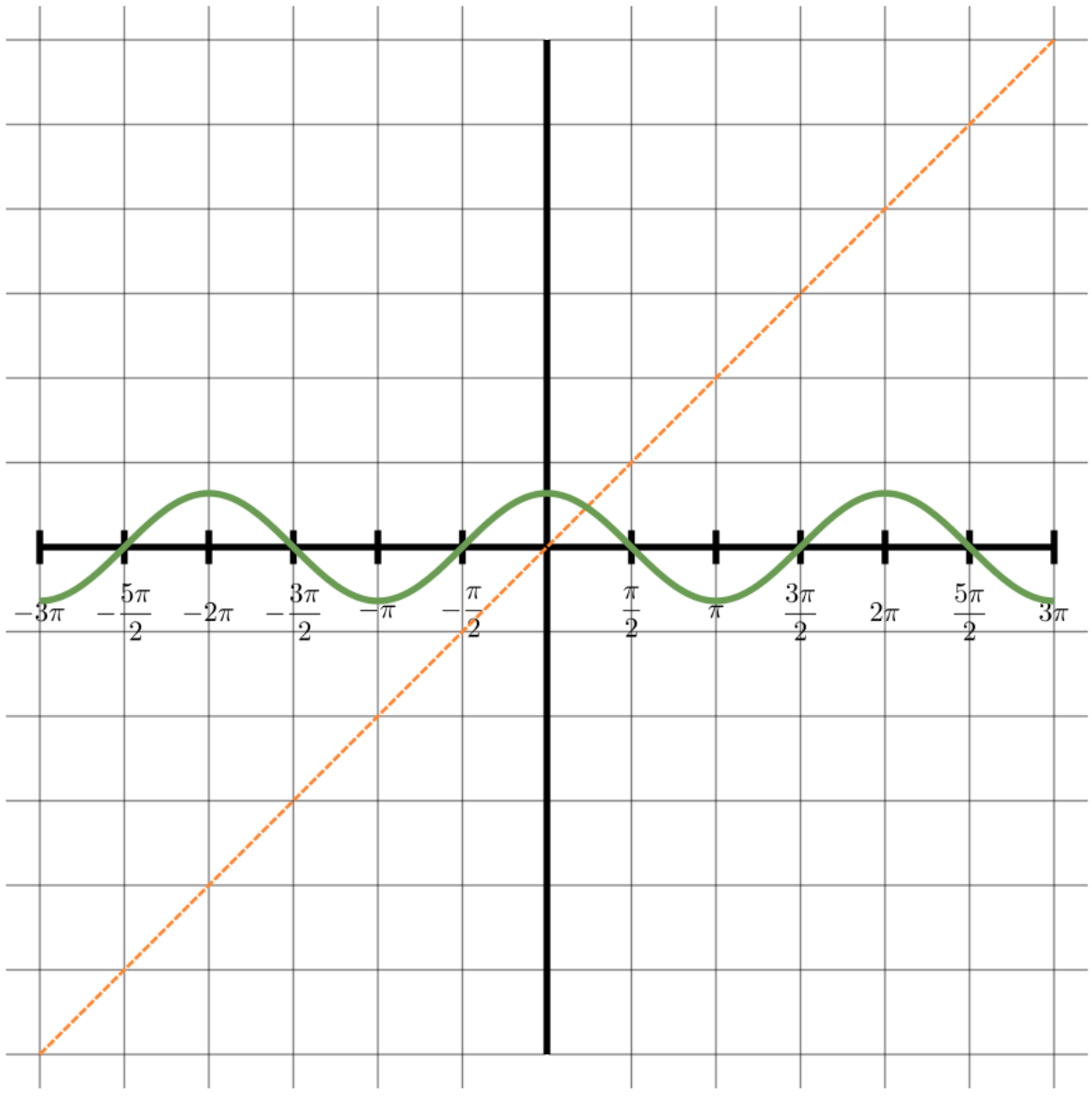
radical maths

What are $\sin\left(\sin^{-1} x\right)$ and $\sin^{-1}\left(\sin x\right)$?

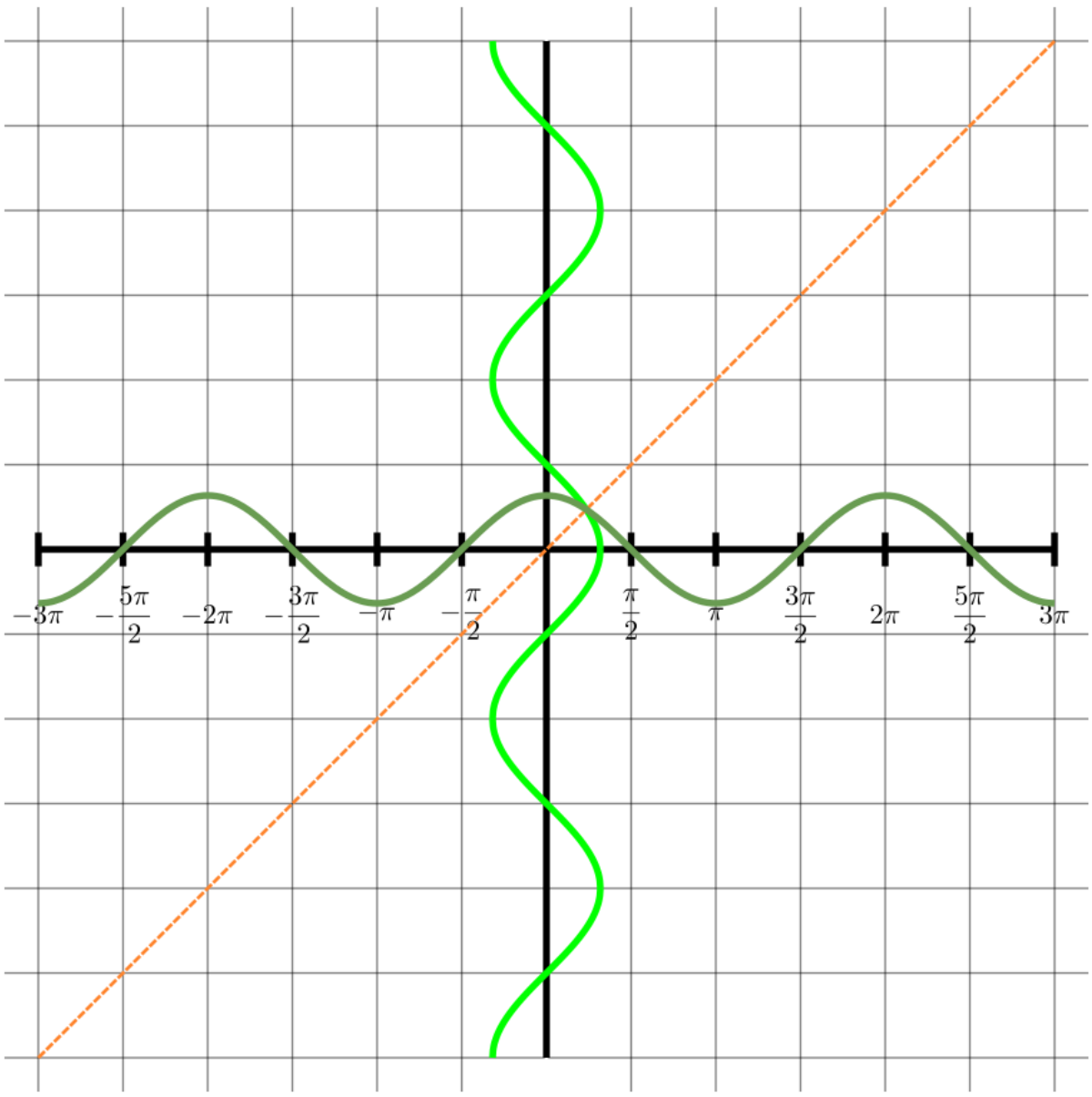


radical maths

Draw the graph $x = \cos y$.

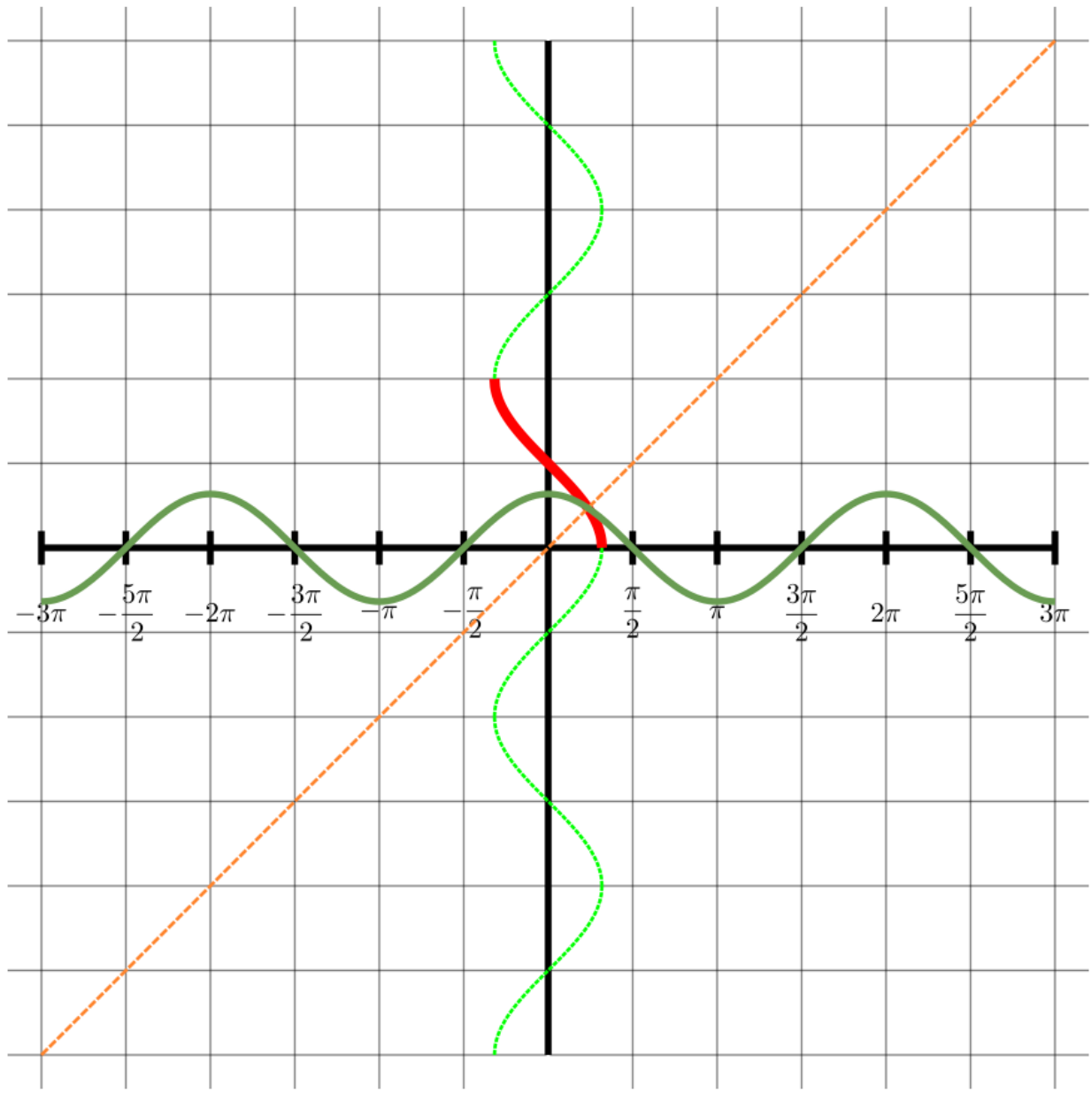


On the same axes, draw the graph $y = \cos^{-1} x$.



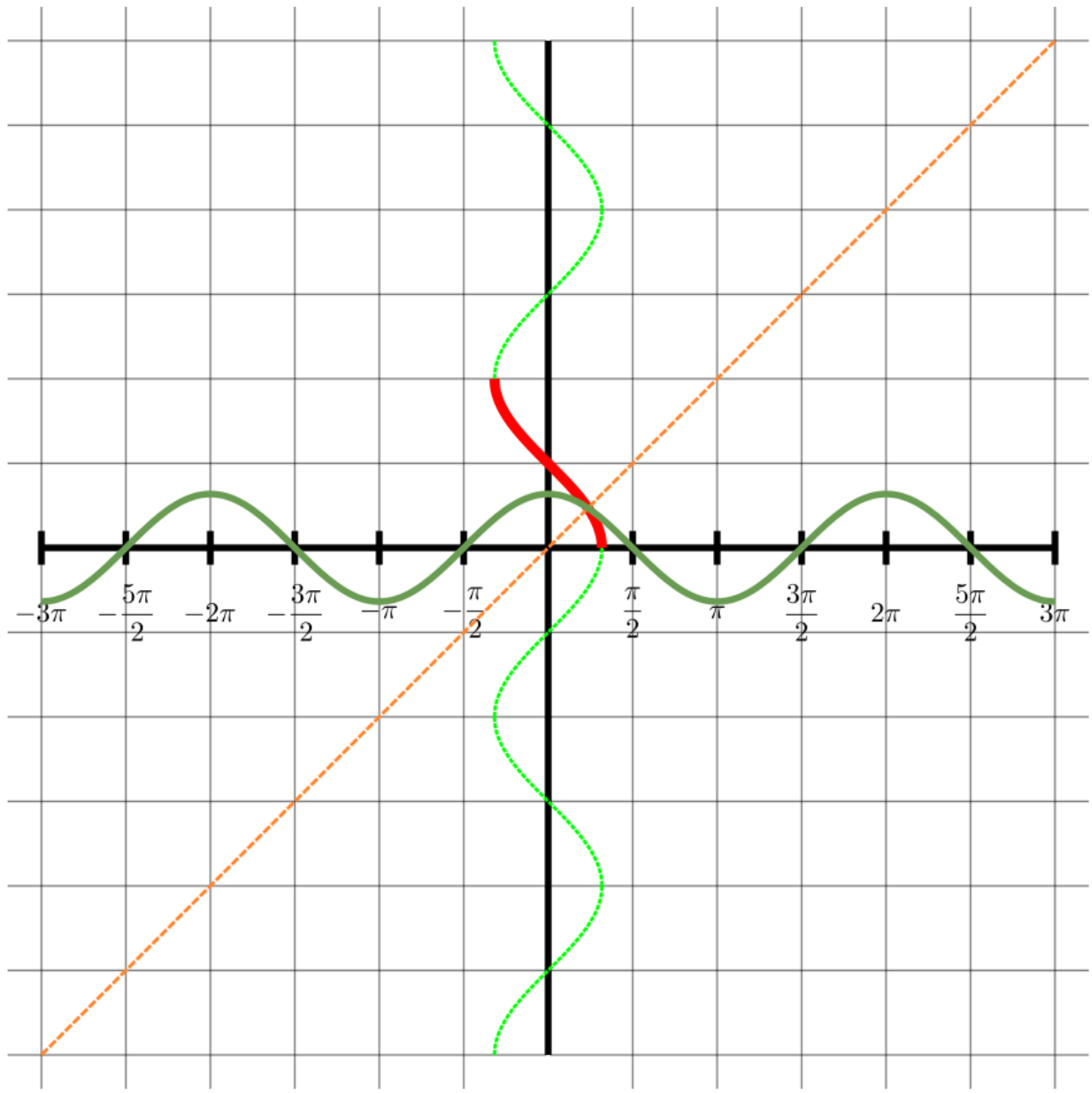
radical maths

What are the domain and range of the function $f(x) = \cos^{-1} x$



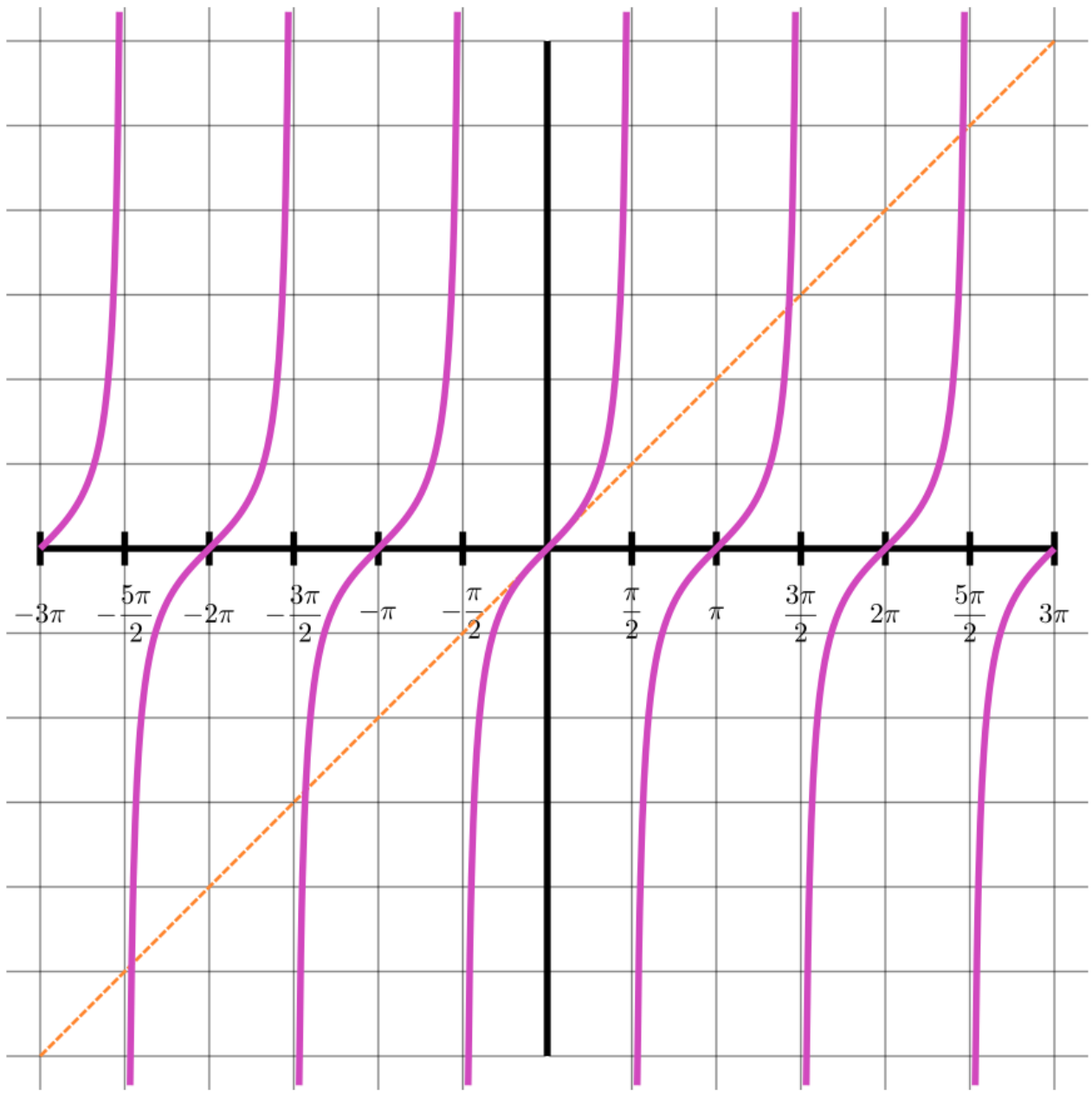
radical maths

What are $\cos\left(\cos^{-1} x\right)$ and $\cos^{-1}(\cos x)$?

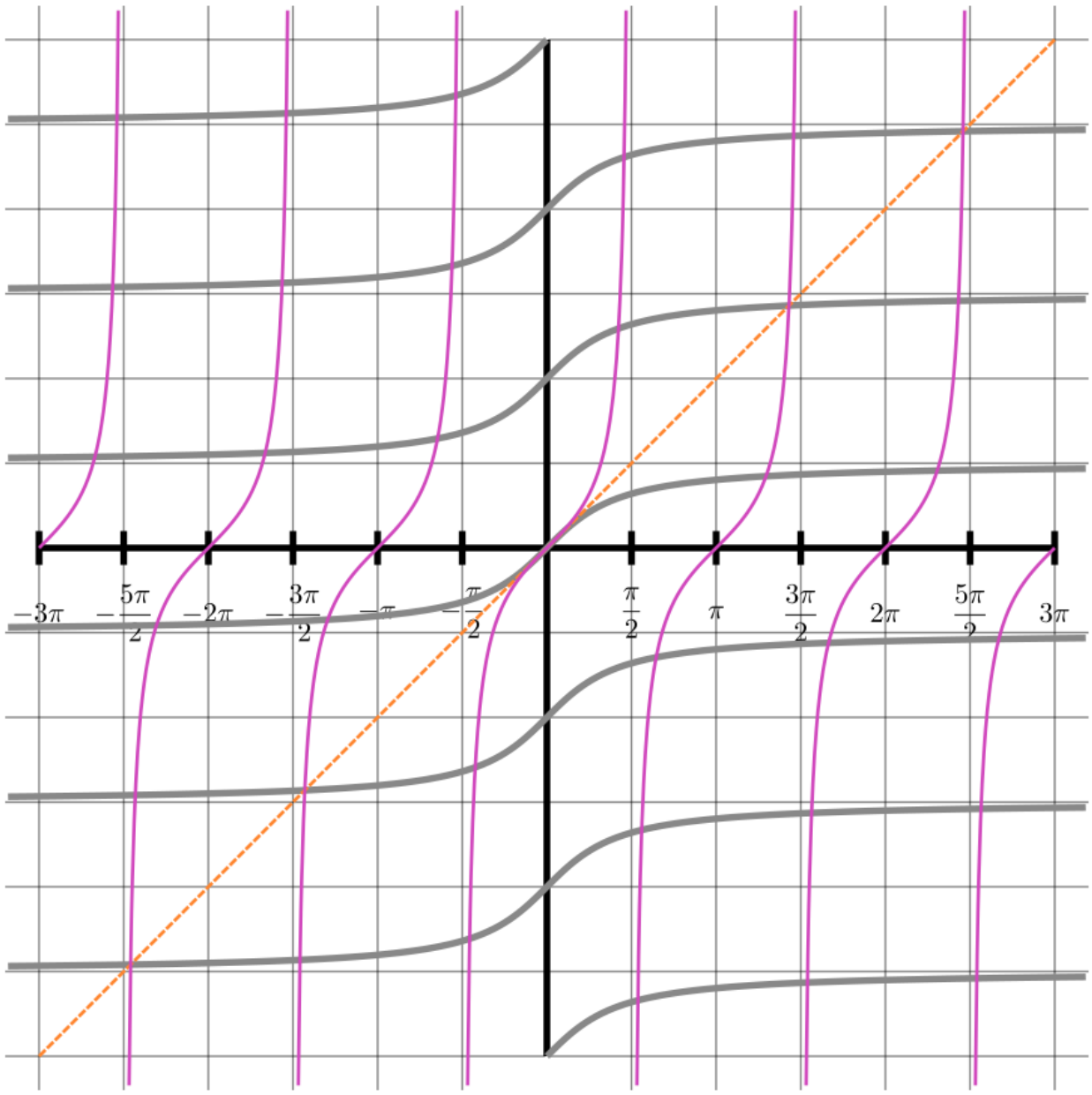


radical maths

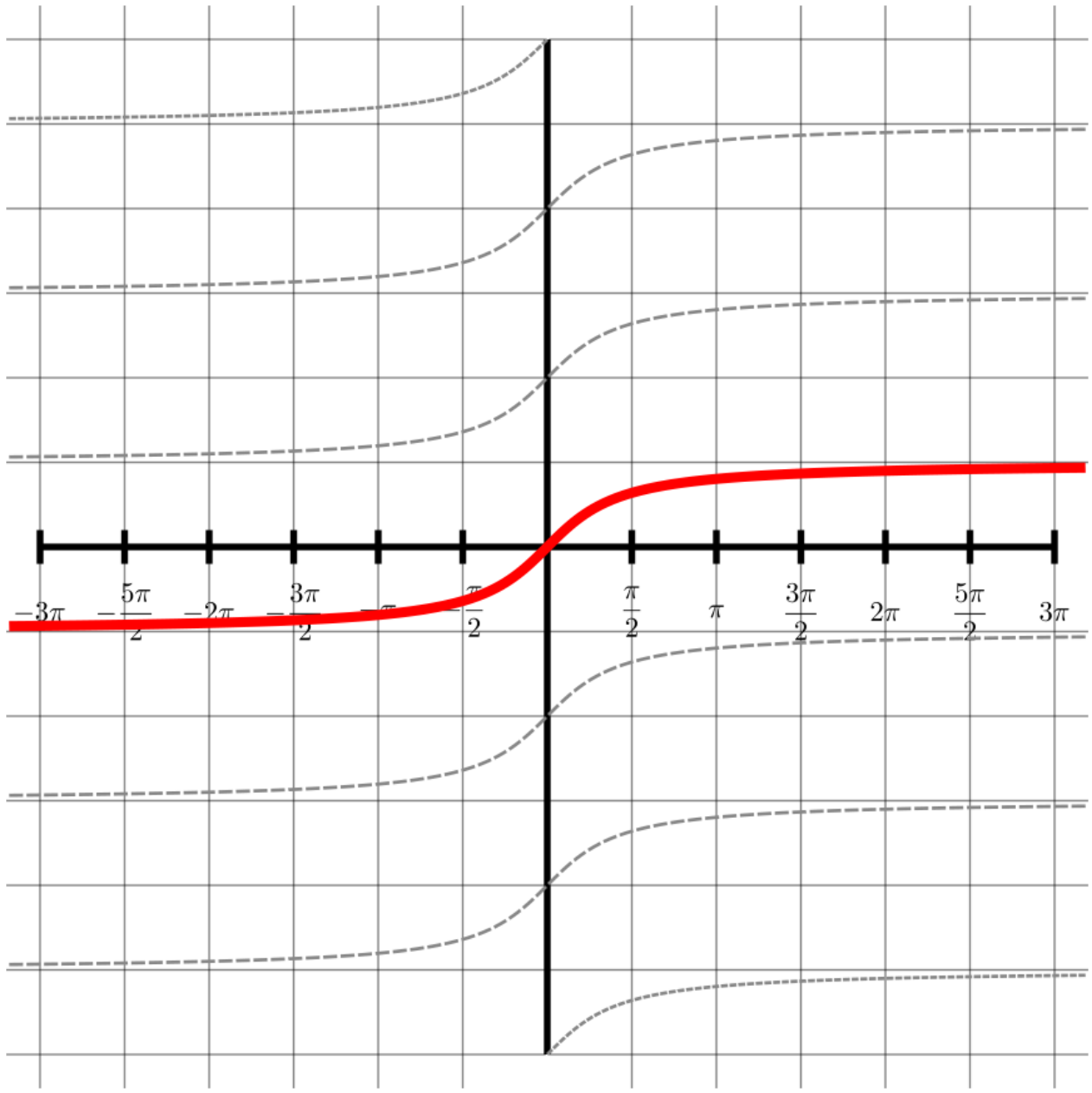
Draw the graph $x = \tan y$.



On the same axes, draw the graph $y = \tan^{-1} x$.

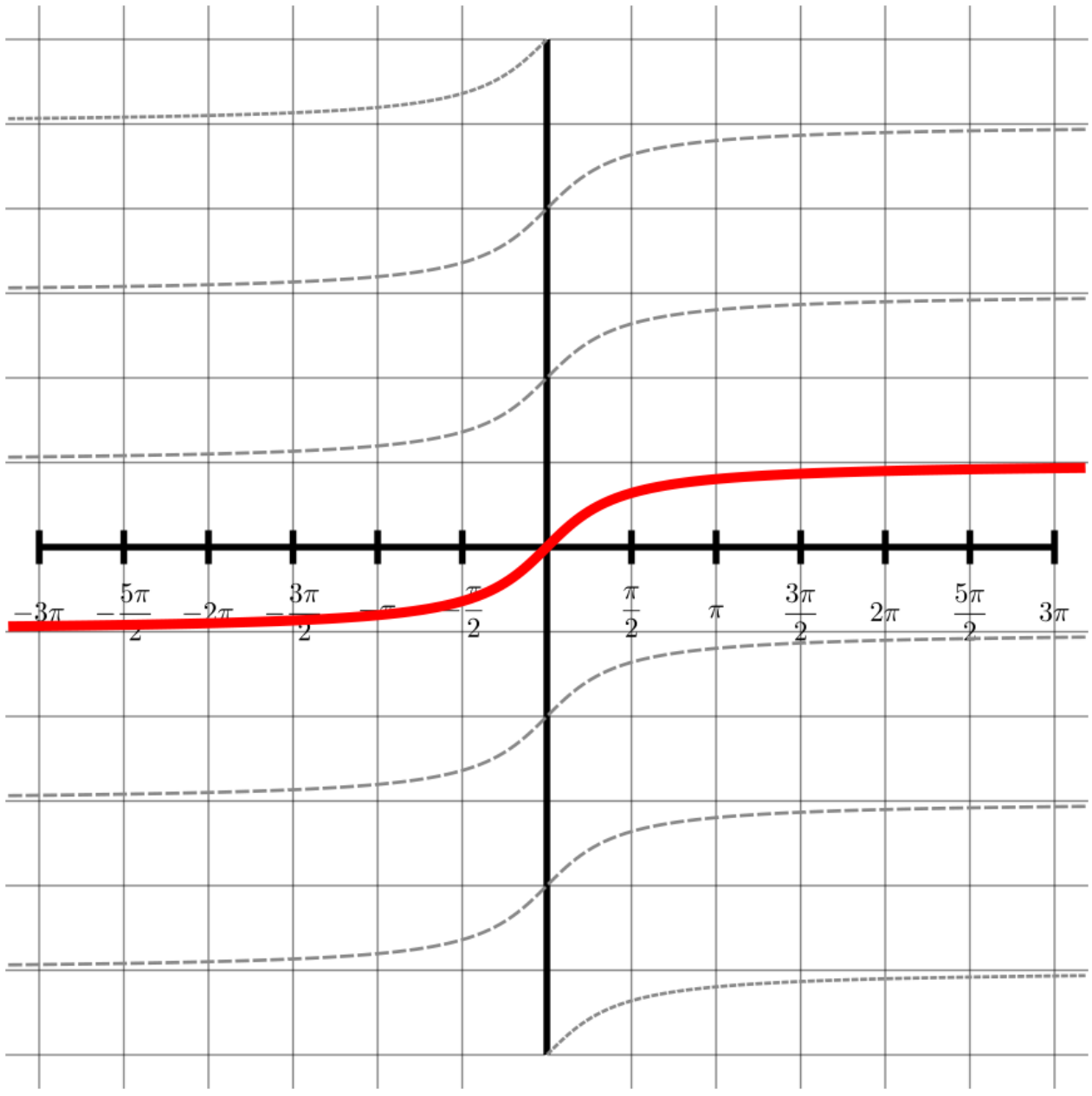


What are the domain and range of the function $f(x) = \tan^{-1} x$



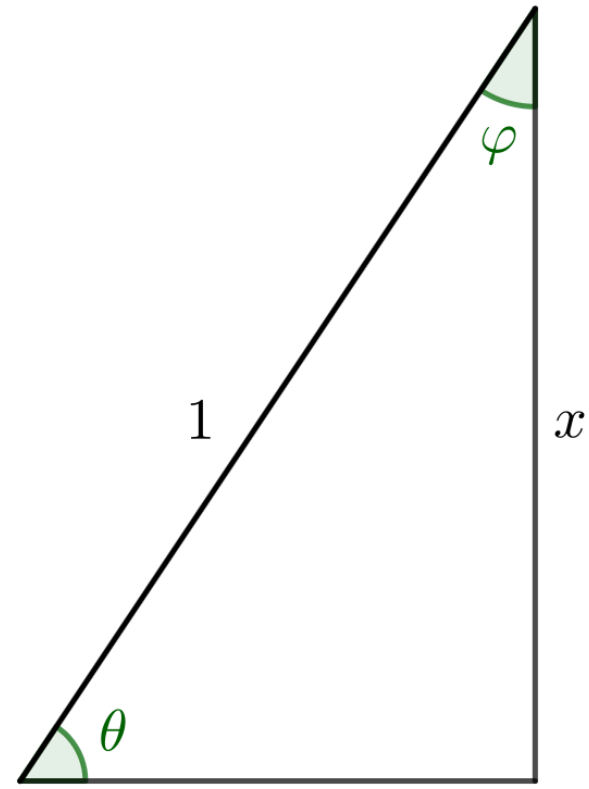
radical maths

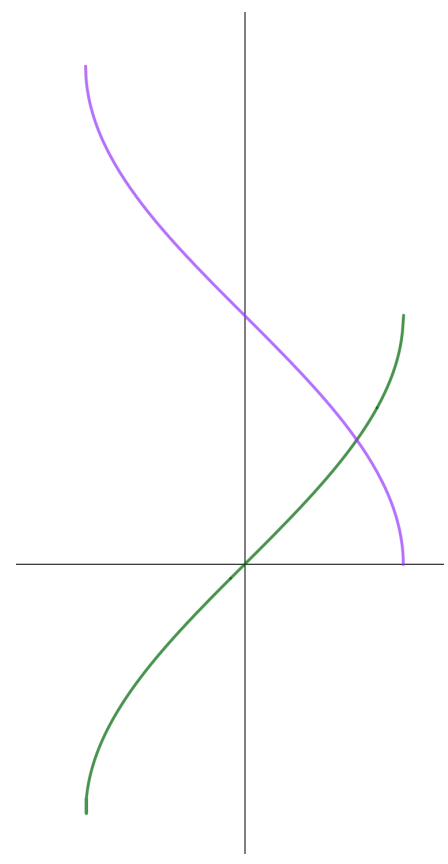
What are $\tan\left(\tan^{-1} x\right)$ and $\tan^{-1}(\tan x)$?



radical maths

What is $\sin^{-1} x + \cos^{-1} x$?

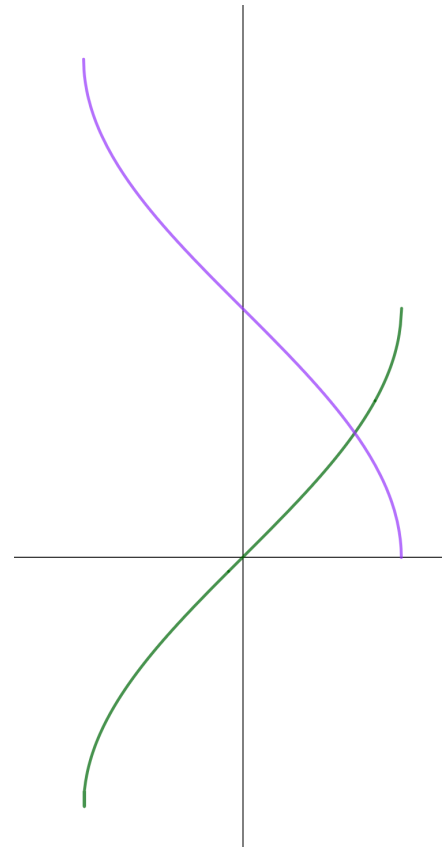




radical maths

Here are the graphs $y = \sin^{-1} x$ and $y = \cos^{-1} x$.

Where do they cross?



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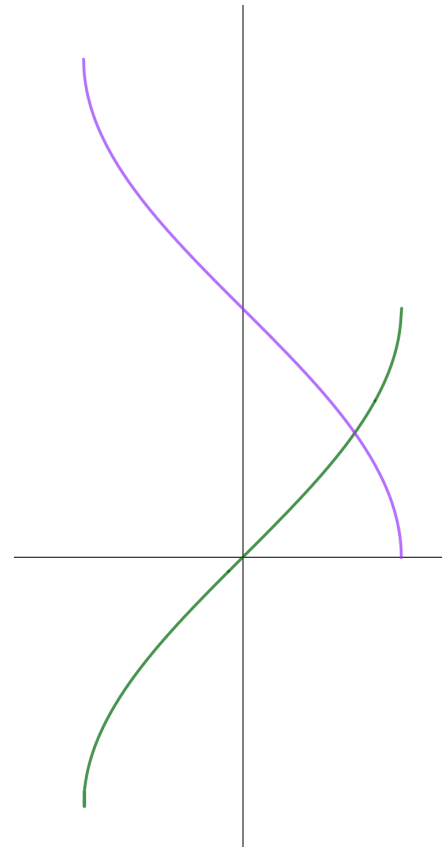
What are

$\sin^{-1} 0 + \cos^{-1} 0$

$\sin^{-1} 1 + \cos^{-1} 1$

$\sin^{-1}(-1) + \cos^{-1}(-1)$

$$\sin^{-1} \frac{\sqrt{2}}{2} + \cos^{-1} \frac{\sqrt{2}}{2} = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$



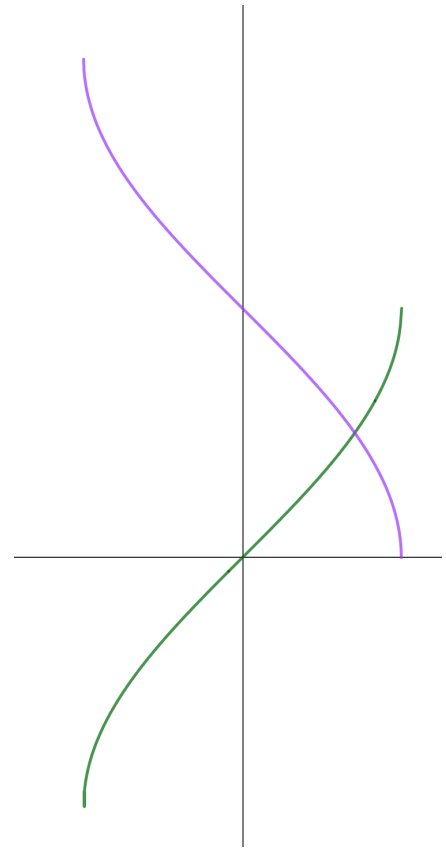
Draw any vertical line that intersects both of the curves.

What is the average of the y coordinates of the intersection points?

What is the sum of the y coordinates of the intersection points?

What is

$\sin^{-1} x + \cos^{-1} x$?



radical maths

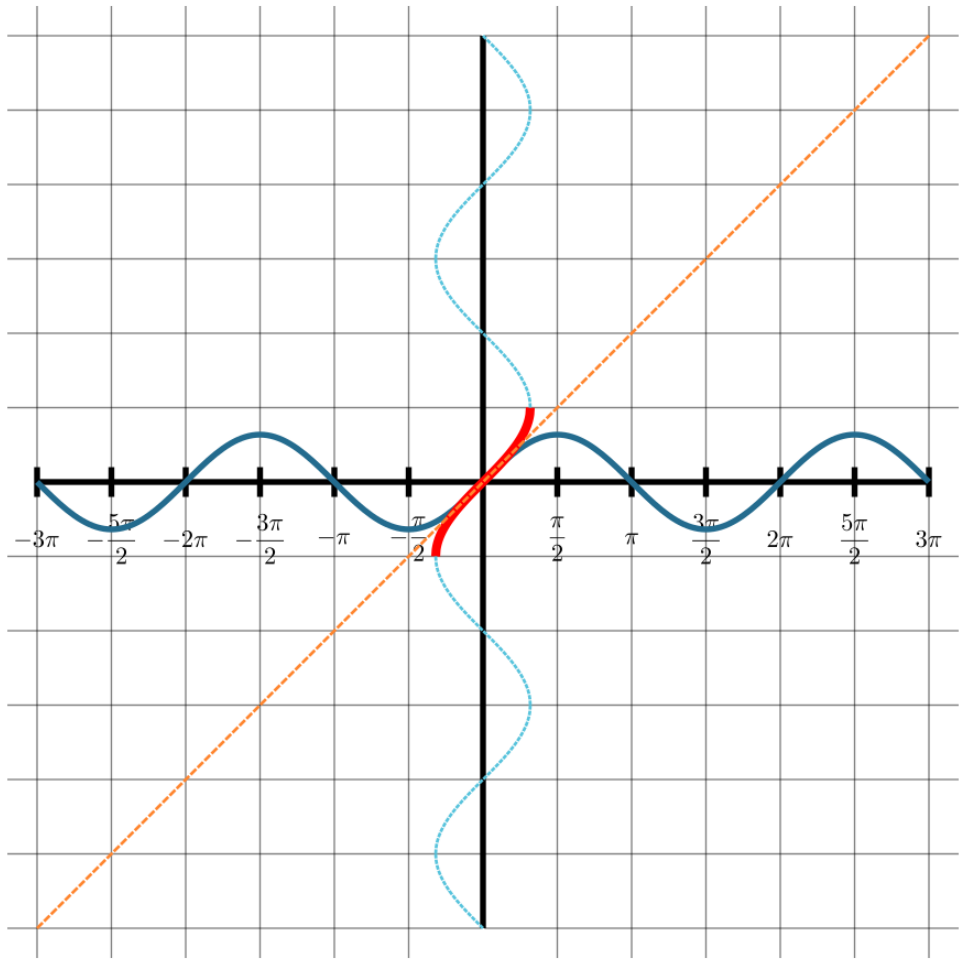
differentials of inverse circular functions

If $x = \sin y$, what is $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y ?

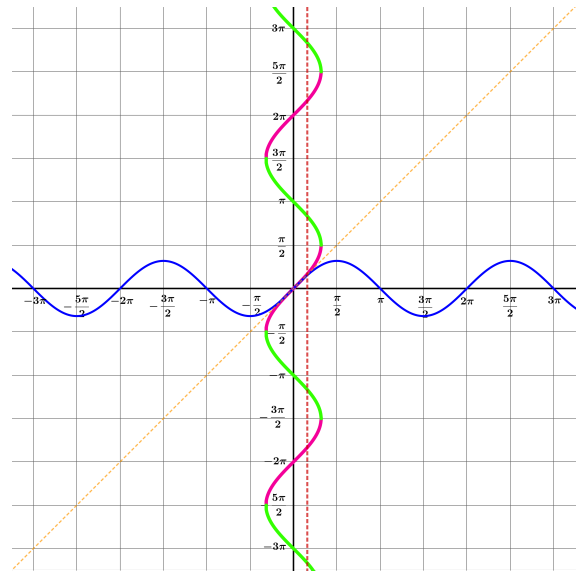
Use this to what is $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x ?

Use this to what is $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x ?

What is $\dfrac{\mathrm{d}}{\mathrm{d}x} \sin^{-1} x$?



radical maths



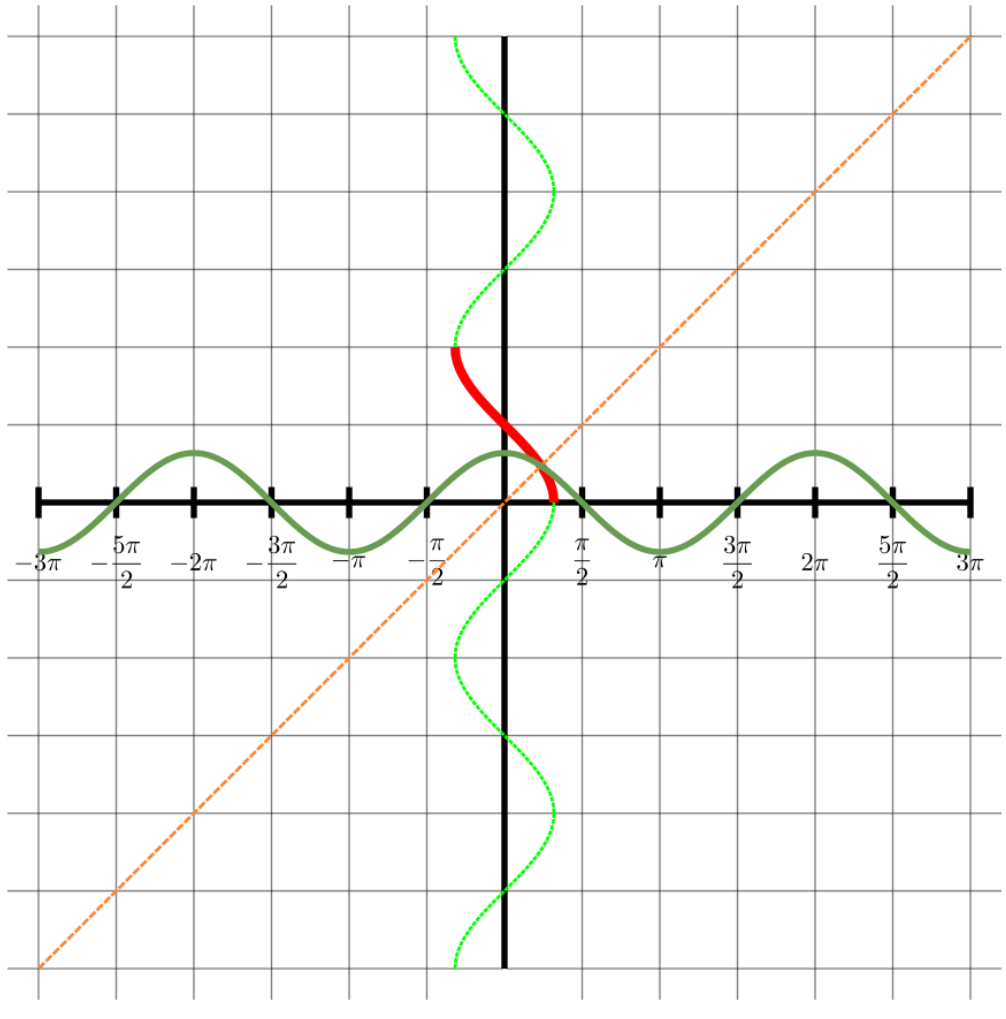
radical maths

If $x = \cos y$, what is $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y ?

Use this to what is $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x ?

Use this to what is $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x ?

What is $\dfrac{\mathrm{d}}{\mathrm{d}x} \cos^{-1} x$?



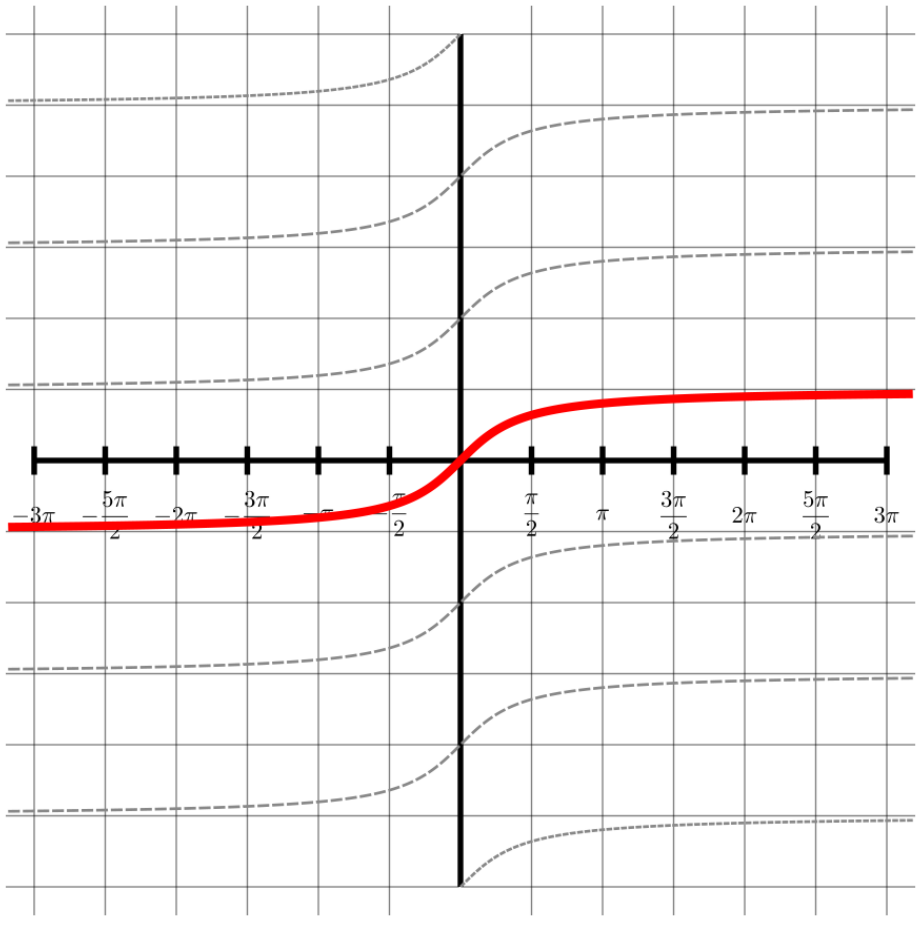
radical maths

If $x = \tan y$, what is $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y ?

Use this to what is $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x ?

Use this to what is $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x ?

What is $\dfrac{\mathrm{d}}{\mathrm{d}x} \tan^{-1} x$?



radical maths

integrals using inverse circular functions

Use the substitution $x = \sin u$ for this integral:

$$\int \frac{1}{\sqrt{1 - x^2}} \, \mathrm{d}x$$

If $y = \sin^{-1} x$, what is $\dfrac{\mathrm{d}y}{\mathrm{d}x}$?

If $y = -\sin^{-1} x$, what is $\dfrac{\mathrm{d}y}{\mathrm{d}x}$?

Use the substitution $u = \sin^{-1} x$ for this integral:

$$\int \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x$$

Use the substitution $x = \cos u$ for this integral:

$$\int \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x$$

If $y = \cos^{-1} x$, what is $\dfrac{\mathrm{d}y}{\mathrm{d}x}$?

If $y = -\cos^{-1} x$, what is $\dfrac{\mathrm{d}y}{\mathrm{d}x}$?

Use the substitution $u = \cos^{-1} x$ for this integral:

$$\int \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x$$

Show that $\sin^{-1} x + c$ and $-\cos^{-1} x + c$ are equivalent solutions for the integral

$$\int \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x$$

Use the substitution $x = \tan u$ for this integral:

$$\int \frac{1}{1+x^2}\,\mathrm{d}x$$

Use the substitution $u = \tan^{-1} x$ for this integral:

$$\int \frac{1}{1+x^2}\,\mathrm{d}x$$