

Lit Review for handling numerical ties in PCPs

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Introduction

Our data-driven modern civilization now depends heavily on the ability to precisely evaluate and exploit huge and complex datasets. A common way to visually show high-dimensional data is with parallel coordinate plots (PCPs). These plots are especially good at showing relationships and trends between multiple factors. However, seeing ties and overlapping data points can be hard when many numbers are the same or very close. This can cause important information to be lost in the research. This study fixes the problems that regular PCPs have with finding numerical ties by adding a new method that makes it easier to tell the difference between data sets that overlap. We want to create a methodology that makes it easier to tell the difference between ties by using a standard delta difference approach. Our suggested methods should make it easier to understand data, leading to more accurate research and decision-making in large, complex datasets.

The accuracy of data interpretation is highest when dealing directly with numerical values or one-dimensional visual representations (Spence 1990). That is becoming more and more impractical as the size and dimension of datasets continue to increase. To make judgments based on data, it is necessary to find methods of simplifying and consolidating datasets without sacrificing significant information. Data visualization, such as graphs or tables, is often an essential and invaluable intermediate step between raw data and an observer's decision-making process. An effective data visualization communicates the gist of a data set in a way that is easy for an observer to understand and evaluate; ideally, data visualizations slightly sacrifice accuracy (relative to a table of values) in favor of a greater understanding of the relationship between observations or variables.

Most standard data visualizations work within the Cartesian coordinate system, with variables or functions of variables mapped to the x and y axis. Additional variables can be mapped to different properties of the plotted points, bars, or lines, and analysts can also create small multiples that show subsets of the data; even with these additions, viewers quickly become overwhelmed by the amount of information when more than $p = 3$ or 4 variables are shown (including the x and y coordinates). When it is necessary to understand the relationship between more than four variables, visualizations on a Cartesian grid no longer work as well; extensions to additional dimensions are also ineffective (**reference?**). Other approaches are

necessary when it is necessary to understand $p > 4$ dimensions of data. Here, we examine parallel coordinate plots (PCPs) as a solution to $p > 4$ dimensional data visualization and assess the impact of different modifications of PCPs on their effectiveness for visualizing high N and/or high p dimensional data. We specifically evaluate the ability of each PCP version to facilitate the identification of overall trends, outliers, and clusters within $N > 4$ dimensional data across different magnitudes of N (observations) and p (variables).

Stimulus-responsive attention often arises when an observer recognizes that a particular graphical element in a stimulus signals valuable information, guiding further search to improve judgment accuracy. Spence (1990) talked about the importance of visual psychophysics in figuring out how simple parts of graphs can be used to convey information. He stressed that how well we can understand graphs relies on our cognitive and visual abilities. Cleveland and McGill's study on graphical perception in 1985 found that people have different skill levels when decoding multidimensional data (Cleveland and McGill 1984). This shows that designs like generalized parallel coordinate plots must consider cognitive and perceptual variability. These plots show many dimensions within a single visual glyph, and the small changes between them make it much harder to understand complicated visual data. Additional studies, like Heer and Bostock (2010) and Simkin and Hastie (1987), built on these ideas by showing that mistakes happen at different points in information processing and that different graphs are better for different jobs depending on the user's accuracy. It was emphasized by Carswell (1992) and Shah & Freedman (2011) that task difficulty and perceptual processes interact. These articles say differences in how people think and perceive things should be considered when making generalized parallel coordinate plots to show multidimensional data.

Parallel Coordinate Plots (PCPs)

Why?

Parallel coordinate plots (PCPs) are an excellent way to show data with many dimensions since each is shown on its plane. This means that complex relationships between different factors can be studied repeatedly. PCPs are great for drawing attention to trends, clusters, and outliers and efficiently show many variables in a small area. However, as with their Cartesian equivalents, PCPs are vulnerable to overplotting and can become difficult to read or interpret when N is large, but because they overplot, they can get messy with large datasets. It can be more challenging to understand the data than with Cartesian coordinates, and you must go through a learning process to get the majority of them as well.

Cartesian coordinates, on the other hand, are straightforward to understand because they are simple and well-known. They make it easier to tell the difference between different information points and their exact values by displaying data points in an easily understood manner. Cartesian coordinates are an excellent method to present data clearly because they can handle up to three dimensions. They also need help with growth because adding more dimensions requires a lot of subplots or complicated three-dimensional plots, which can get boring and

challenging to follow. Cartesian graphs additionally can take up a lot of room, and for data with more dimensions, they usually need more than one plot.

What?

Parallel coordinate plots (PCPs) leverage the projective space rather than the Cartesian coordinate system: each line in cartesian space is a set of points in projective space, and each point in cartesian space can be represented as a line in projection space (Inselberg 1985). The result is that a single data point is For high-dimensional data, parallel coordinate plots (PCPs) are a standard visualization tool whereby data points are shown as a line intersecting a series of vertical axes representing the different variables in the dataset, therefore indicating a dimension. Conventional PCPs can be difficult to interpret because of overplotting - lines overlap and need to be distinguishable, and when there are large datasets, even non-identical segments can be hard to identify because of the number of line segments. These improvements in clarity and usability of PCPs utilizing data pattern highlighting and visual clutter reduction help (J. Heinrich and Weiskopf 2009), for example, present several approaches to bundle comparable trajectories in PCPs, so minimizing overlap and improving pattern identification; Johannsen et al. (2012) address the advantages of dynamic axis reordering to highlight different data characteristics.

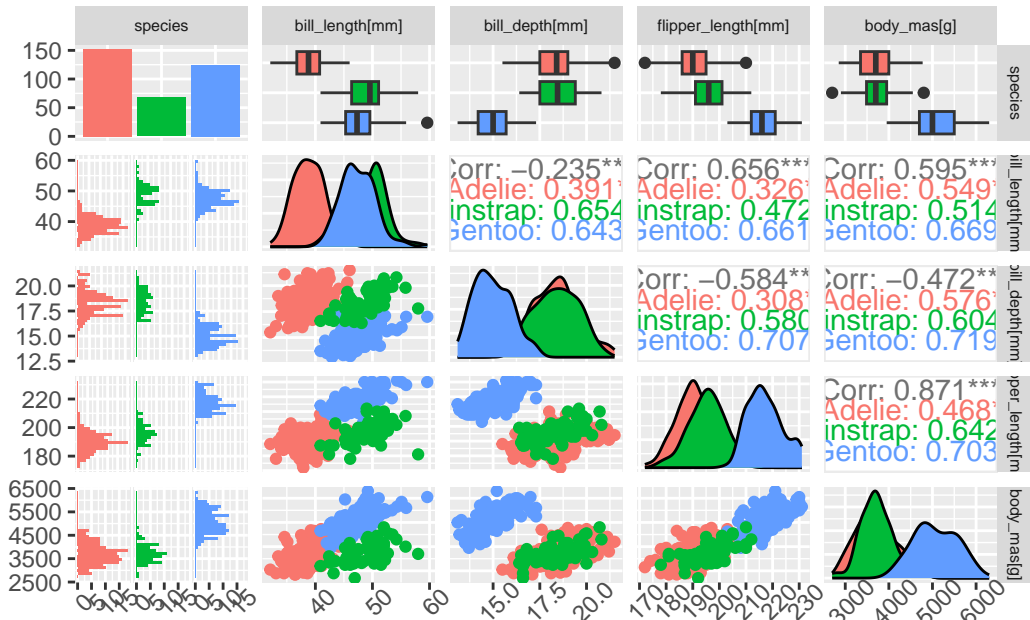


Figure 1: A generalized pairs plot

Figure 1 is a generalized pairs plot, containing histograms, density plots, scatter plots, and correlation matrices. It shows how the variables for three kinds of penguins (Adelie, Chinstrap,

and Gentoo) are distributed and how they are related. Histograms and density plots make it easy to see how each variable (bill length, bill depth, flipper length, and body mass) is spread across the different species. The density plots show clear trends for each species, regarding the length of the bills and flippers, where Gentoo penguins tend to have higher values. In the boxplots that show body mass and flipper length, outliers are clear because they show numbers very different from the rest of the data.

The scatter plots show how two variables are related to each other. There is a clear trend for overplotting in denser areas, especially for bill depth vs. bill length and flipper length vs. bill length. The correlation values show more information about these connections. They show strong positive and negative correlations, with some species having different patterns. For example, in Gentoo penguins, there is a strong positive correlation between the length of their flippers and their body mass. Other species have different association patterns, which means that the ways that traits are related can be different for each species. Overall, the matrix-style plot does an excellent job of showing the links within and between variables. However, it is hard to understand precisely what overlapping areas mean in scatter plots because individual points are hard to see.

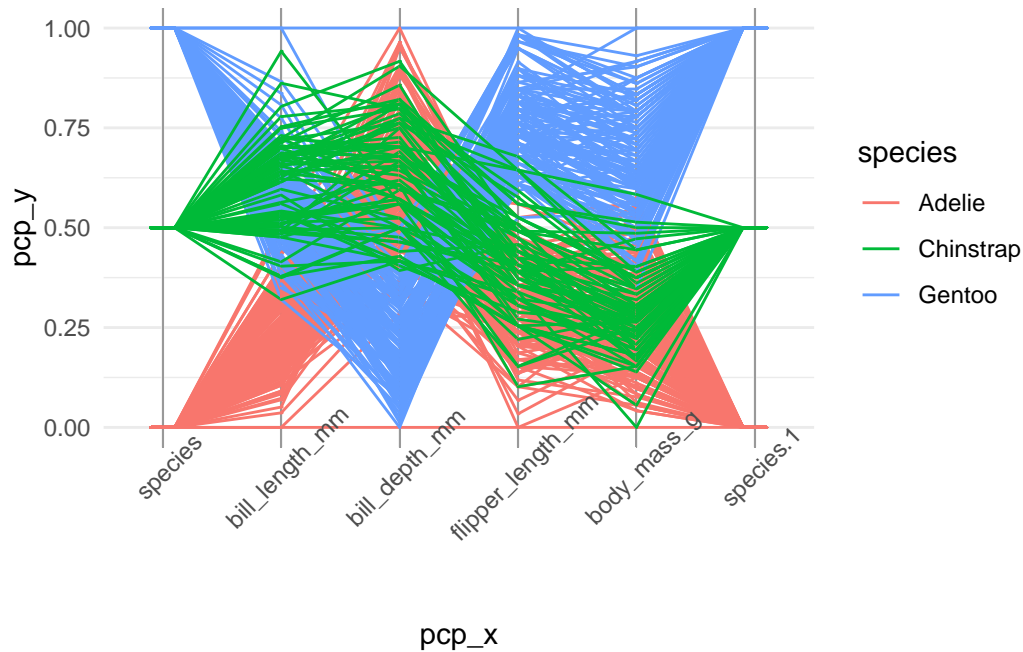


Figure 2: A parallel coordinate plot

In Figure 2, each line shows a different penguin, and the colors of the lines show the species. The Gentoo (blue) tends to have higher body mass and flipper length values, while the Adelie (red) tends to have lower values for these measurements. However, the large number of lines makes it hard to see smaller details, and it's hard to pick out individual spots or clear outliers in this visualization.

It's hard to directly find outliers because the merging lines make it hard to see individual differences. The plot doesn't show specific distribution shapes like a histogram or density plot, but how the lines are grouped at specific value ranges shows how each species generally behaves. The slope and intersection patterns of the lines suggest that the variables are related. Note that the plot demonstrates that if the length of a bird's bill goes up, other variables may go up or down similarly, based on the species. Overall, the plot works well for seeing broad patterns and trends across species, but it could be better at picking out specific data points or weak connections because it's too crowded on the screen.

Modifications to Parallel Coordinate Plots

Since Alfred Inselberg introduced parallel coordinate plots (PCPs) in 1985, considerable advances have been made to address the original method's framework and improve its capacity to depict high-dimensional data effectively. The enhancements include changes to visual representation, handling various kinds of interactive data elements, and incorporating advanced computational approaches to increase the clarity and interpretability of the visualizations. Here are some specific enhancements and modifications that have been made:

Interactive and Dynamic

The addition of interactive features has substantially increased the usefulness of PCPs. Interactive PCPs enable users to dynamically change axes, reorder them, filter data based on specified criteria, and even invert axes to study data from various angles. These characteristics enable real-time data analysis, leading to a deeper understanding of patterns and correlations. Brushing and connecting allow users to highlight specific data points across many axes, whereas filtering hides lines that do not satisfy set criteria, decreasing clutter. The interactive characteristics of PCPs make them more amenable to exploratory data analysis, particularly in big and complicated datasets (Julian Heinrich and Weiskopf 2013b).

Heinrich and Weiskopf's 2013 paper "State of the Art of Parallel Coordinates" thoroughly reviews visualization methods for parallel coordinates. It includes a taxonomy that groups the different approaches into different categories. The writers discuss various ways to model, see, and work with parallel coordinates. They also show how these methods can be used for everyday tasks in knowledge discovery, like sorting, clustering, and regression. Some of the most important advances are discussing geometric models, interpolation methods, and the point-line duality that makes up the basis of parallel coordinate plots. The study discusses a better understanding of data using density-based visualizations, axis ordering, and improvements such as brushing and bundling.

The study points out some problems with parallel coordinates, like overplotting and the need for good axis arrangement. It also suggests ways to get around these problems, such as clustering and density estimation. To make it easier to see patterns and understand data,

researchers look into different ways to map it, such as curves, shapes, and density plots. The writers also talk about how parallel coordinates can be used in real life in engineering and the life sciences. This shows how useful and flexible they are for high-dimensional data visualization tasks.

Initially, modifications in parallel coordinate plots have dynamic features, which allow users to select, highlight, and filter data dimensions or specific data points in real-time. This has facilitated more precise and focused analysis within large datasets. Interaction allows users to engage with data intuitively, selectively focusing on areas of interest. “Interactive filtering and brushing make it easier to identify patterns within large datasets, aiding discovery and hypothesis generation” (Wegman 1990).

In 1990, Wegman et al. introduced fundamental techniques for interaction with multidimensional data, laying the groundwork for modern interactive PCPs. Siirtola and Rähä developed interactive features like brushing and linking to assist in dimensional analysis, making comparing attributes within large datasets easier (Siirtola and Rähä 2006). Inselberg expanded these features by enabling dynamic filtering and axis rearrangement to tailor the display to user needs (Inselberg 2009).

Increased complexity can overwhelm novice users, requiring more computational power and potentially slowing down analysis in high-dimensional datasets. Inselberg noted, “While interactivity provides insight, it can lead to user fatigue in complex datasets due to the cognitive load required for multi-step filtering,” (Inselberg 2009).

The D3 Graph Gallery has a live parallel coordinates graph with dynamic features, making exploring and analyzing data easier. Users can move their mouse over lines to see specific data tracks. This interactivity feature makes finding and separating different data types or trends in the dataset easier. The graph also lets the user “brush” on any vertical axes, allowing users to choose and filter data points based on specific ranges or criteria. This interactivity feature changes the visualization on the fly to show the filtered group. Tooltips show up when you move their mouse over lines or axis names. They give the user more information about the data points, which helps the user understand them better. The user can change the order of the axes by dragging them horizontally. This interactivity feature lets the user change how dimensions are compared, which makes it easier to find patterns and correlations between different factors. The parallel coordinates plot is a powerful way to see and understand multidimensional data in a way that is easy to understand and use because it has these dynamic parts.

[Interactive Parallel Coordinate Plot](#)

Bundling and Curving

Bundling and curving techniques have been introduced to handle visual clutter, especially with high-dimensional data. These techniques group similar paths, making patterns more visible and reducing overplotting. Bundling and curving significantly reduce clutter, making

spotting overarching trends and common patterns easier. Holten and van Wijk noted that “bundling offers a compelling solution to mitigate the chaos often found in high-dimensional visualizations” (Holten and Van Wijk 2009).

Holten and van Wijk pioneered edge bundling for PCPs, which visually aggregates similar paths to reduce clutter. McDonnell and Mueller furthered this approach by introducing curvilinear PCPs, where curved lines help distinguish intersecting paths for clearer visual analysis (McDonnell and Mueller 2008). Johansson and Forsell evaluated the effectiveness of bundling and curving in PCPs, establishing criteria for when these modifications enhance interpretability (Johansson and Forsell 2015).

Curved lines can be altered to vary in curvature dependent on data attributes, which improves visual separation and pattern recognition, particularly in scenarios with strongly correlated dimensions. Curving in PCPs visually separates overlapping lines, making data relationships more discernible. It improves line separation by reducing intersections, making it easier to differentiate between trajectories. Curved lines are more aesthetically pleasing and reduce cognitive load, enhancing readability. They simulate a more three-dimensional space within a two-dimensional PCP, making it easier to perceive data points’ relative positioning. Research shows curving lines reduces visual clutter and improves data interpretation Qu et al. (2007). This strategy has improved PCPs’ ability to visualize complicated datasets with overlapping points (Kachhway 2013).

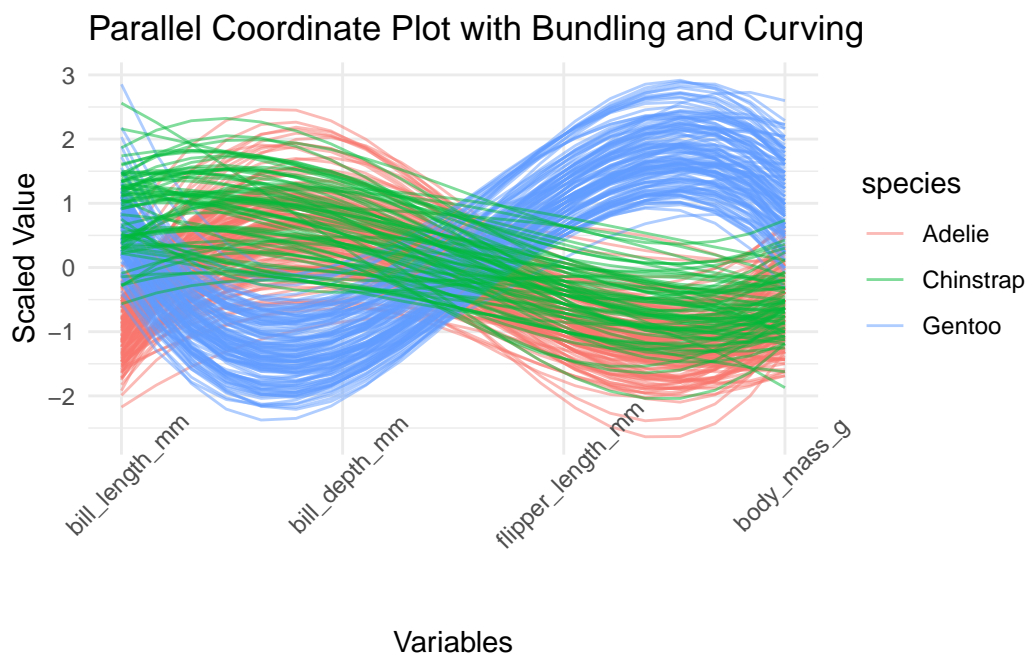


Figure 3: A Bundling and Curving Parallel Coordinate Plot

In Figure 3, the parallel coordinate plot adds bundling and curving, which makes the lines smoother, less distracting, and better able to tell the patterns between the species apart.

It's easier to see general trends within each species when the data is bundled, but it still doesn't show specific distribution shapes like a histogram might. Bundling makes it harder to see outliers in this image because the extreme values are "pulled" into the main flow, making it harder to see individual deviations. The curved lines make the relationships between the variables stand out, indicating that specific measurements may go up or down together depending on the species. Still, they make it harder to see small changes between individual points.

Moustafa et al. combine parallel coordinate plots (PCPs) with parallel coordinate density plots (PCDPs) to reduce clutter when working with big datasets. As part of Moustafa's methodology, the standard PCP is mutated into a density plot. This creates plot areas with more observations that stand out and reduce visual clutter (Rida E. Moustafa 2011).

The new approach uses density estimation methods to transform the PCP image based on polylines into a continuous, smooth depiction of data density. This change shows how to see groups and trends usually hidden in regular PCPs. To make things even better, Moustafa has interactive parts that let users experience different data dimensions in real-time. This makes PCPs even better at analysis. With its interactive nature, people who aren't experts can use this method confidently and successfully.

Moustafa's research indicates that to deal with even larger datasets more efficiently, future efforts should focus on enhancing density estimation methods. These methods can be used in biology and finance to demonstrate their usefulness and adaptability for examining real-world data (Rida E. Moustafa 2011).

Bundling groups with similar paths may also obscure individual data points or outliers, which can be crucial in some analyses. "The clarity gained in bundle simplification comes at the cost of data precision," warns McDonnell and Mueller, pointing out that outliers or unique data paths might be lost (McDonnell and Mueller 2008).

Dimension Reduction Techniques

Dimensionality reduction techniques such as Principal Component Analysis (PCA) and Independent Component Analysis (ICA) are frequently used to improve the readability of PCPs before charting. These strategies help to reduce the number of dimensions while retaining the most informative parts of the data. PCPs can be used to visualize reduced dimensions, providing a better understanding of high-dimensional interactions. Axis ordering is another important feature that has been adjusted. The relationships between variables can be highlighted by sorting the axes according to measurements such as correlation or mutual information. Automatic axis arrangement techniques have been developed to reduce line crossings, making recognizing patterns easier.

The article "Orientation-Enhanced Parallel Coordinate Plots" by Raidou et al. describes a new way to make parallel coordinate plots (PCPs) more accessible to read and understand.

The authors suggest a method that uses input about direction to solve the clutter and overlap problems with regular PCPs. This method changes the direction of the plot axes on the fly by applying both automatic orientation and user-interactive adjustments. This method reduces visual noise and makes data patterns and correlations more visible. This will make it easier for users to study and analyze large datasets, (Raidou et al. 2015).

The suggested orientation-enhanced PCPs are designed with the user’s needs at the forefront, offering a more informative visual representation. Several preprocessing steps, including principal component analysis (PCA) and multi-dimensional scaling (MDS), were employed to determine the optimal axis orientation. This ensures that data clusters are maximally separated and data lines are minimally overlapped. The interactive features allow users to adjust the orientation according to their preferences and requirements, enhancing the usability and adaptability of the plots for research needs (Raidou et al. 2015).

Dimension reduction has become critical for managing PCPs with high-dimensional data. Techniques such as principal component analysis (PCA) and clustering help to focus on the most significant data features. Reducing dimensions simplifies analysis, enabling analysts to focus on the most important features while avoiding overwhelming visual data. “Dimension reduction helps to retain essential information without drowning the user in less relevant details,” (Wegman and Luo 1997).

Wegman and Luo (1997) applied PCA within PCPs, enabling users to represent key data features while minimizing less relevant dimensions (Wegman and Luo 1997). Guo et al. (2011) utilized clustering and hierarchical dimension reduction techniques to streamline high-dimensional datasets, enhancing usability without sacrificing detail (H. Guo, Xiao, and Yuan 2012). Yang et al. (2013) demonstrated the integration of non-linear dimension reduction methods within PCPs to capture complex relationships in data more effectively (Yang et al. 2017).

Dimension reduction techniques like PCA or clustering can inadvertently remove nuances or minor patterns that might be relevant in specific cases. Guo et al. (2011) caution that, “while beneficial, these methods might lead to information loss, potentially masking relationships only visible in higher dimensions.”

In Figure 4, the parallel coordinate plot incorporates principal component analysis (PCA) components (PC1 to PC4) on the x-axis, with the scaled PCA values on the y-axis, color-coded by species (Adelie in red, Chinstrap in green, and Gentoo in blue). PCA reduces dimensionality, meaning that each PC represents a combination of the original variables, simplifying complex relationships. The lines between components show how each penguin transitions across these derived features, making it easier to observe general trends for each species in the reduced space. Due to the numerous overlapping lines, overplotting remains an issue, though species-level patterns are visible, with distinct separations in specific PCs. Outliers are challenging to detect due to the bundled nature of the lines. While distribution shapes aren’t explicitly visible, the clustered points at each PC axis indicate each species’ general concentration of values. Though individual variability remains obscured, relationships between PCs and species are

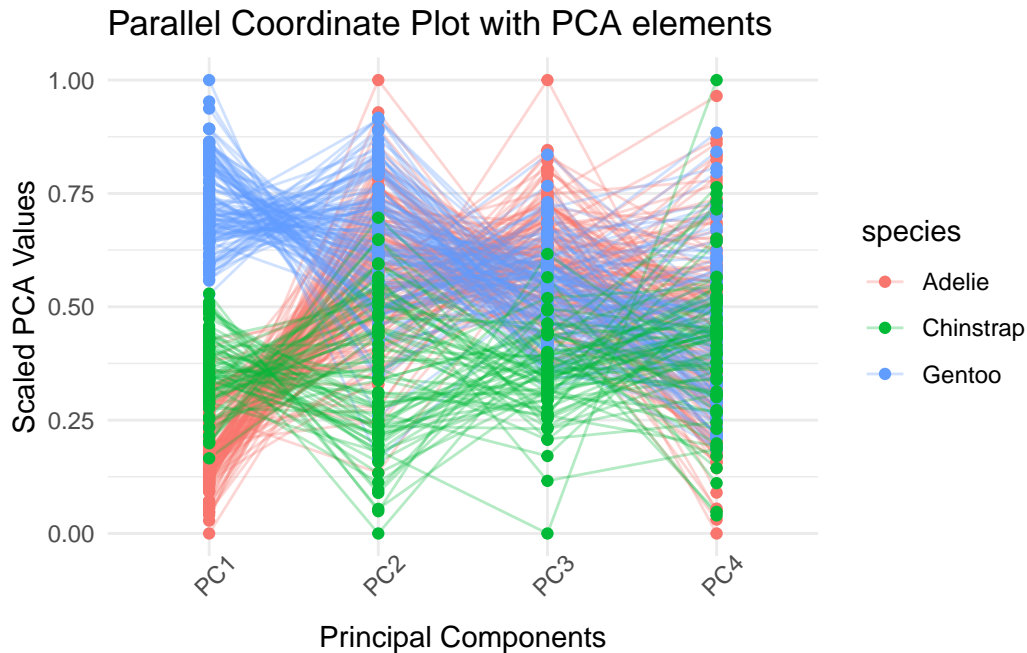


Figure 4: A PCA in Parallel Coordinate Plot

implied through color-coded clusters and trends, suggesting some separation between species along specific principal components.

Enhanced Color Encoding and Shading

Visual upgrades like color coding, opacity modifications, and changing line thickness have been implemented to solve overplotting concerns, particularly with massive datasets. Significant data trends can be highlighted using different colors or modifying line opacity in response to data density, with less relevant information deemphasized. This approach helps users identify trends and outliers that might otherwise be hidden. Line thickness can be adjusted to reflect other variables, such as data frequency or confidence intervals, adding depth to the visual depiction. Such multi-layered visualization techniques provide an additional layer of information to classic PCPs

In their 2013 work “State of the Art of Parallel Coordinates,” Heinrich and Weiskopf give a full overview. They discuss the different methods used to avoid overplotting and improve data interpretation. They discuss how to use alpha blending, which changes the transparency of lines to clear up space, and interactive line manipulation methods that let users explore different data points in real-time. These tips are important for making parallel coordinate plots easier to read, especially when working with big datasets, (Julian Heinrich and Weiskopf 2013b).

In their 2008 study, “Extensions of Parallel Coordinates for Interactive Exploration of Large Multi-Timepoint Data Sets,” Blaas et al. stress how important it is to have visual traits that can be changed, such as the colors and opacity of clusters. Their method lets users change these settings in real-time, which makes it easier to focus on specific data features and control line density (Blaas, Botha, and Post 2008). In the same way, Raidou et al., in “Orientation-Enhanced Parallel Coordinate Plots,” talk about how different levels of opacity and color can help draw attention to patterns, show line densities, and allow for interactive data study (Raidou et al. 2015). In their work, they also talk about how to deal with the problem of overplotting in complex datasets by smoothing and averaging polylines.

In their paper “Visual Clustering in Parallel Coordinates,” Zhou et al. make another important addition by suggesting a tool that lets users choose color and opacity to draw attention to clusters in the data. This method helps tell the difference between data groups and changes the shape of curves to make things clearer (Zhou et al. 2008). In their paper “Hierarchical Parallel Coordinates for Exploration of Large Datasets,” Fua et al. also talk about ways to reduce visual clutter by changing the thickness of lines and using hierarchical data representations (Fua, Ward, and Rundensteiner 1999). These methods make it possible to effectively visualize data density, which gives you a better look at the trends hidden in big datasets.

These studies show that changing how parallel coordinate plots look, such as color coding, adjusting brightness, and moving lines around, can make them much easier to read and understand. These methods are significant for dealing with thick and overlapped data, making parallel coordinate plots a more helpful tool for studying data in multiple dimensions.

By adding color gradients and shading, PCPs can encode additional data attributes, such as density or frequency, improving the ability to spot trends and correlations. Color gradients and shading effectively encode additional variables, allowing more complex data insights to be derived visually. “Color encoding adds a new layer of perception, turning a purely structural plot into a multi-dimensional analysis tool,” (Theus and Urbanek 2008).

Theus and Urbanek introduced color-coded parallel coordinates, which allow users to map additional variables using color and thereby increase interpretive power (Theus and Urbanek 2008). Bertini et al. applied density shading techniques to PCPs to make frequent patterns in large datasets stand out more prominently (Bertini, Dell’Aquila, and Santucci 2005). Novotny and Hauser developed opacity and color-blending techniques in PCPs, which help understand overlapping patterns more intuitively (Novotny and Hauser 2006).

Heavy reliance on color may lead to visual strain, especially in cases where multiple gradients overlap or in users with color vision deficiencies. Bertini et al. noted, “When too many colors are applied, the encoding becomes confusing, and can overwhelm rather than enhance the viewer’s understanding.”

In Figure 5, the parallel coordinate plot uses enhanced color coding. It might have changed the line density, which makes the differences between species (Adelie is red, Chinstrap is green, and Gentoo is blue) more obvious across the variables (bill length, bill depth, flipper length, and body mass). The plot does a good job of showing general trends for each species. For

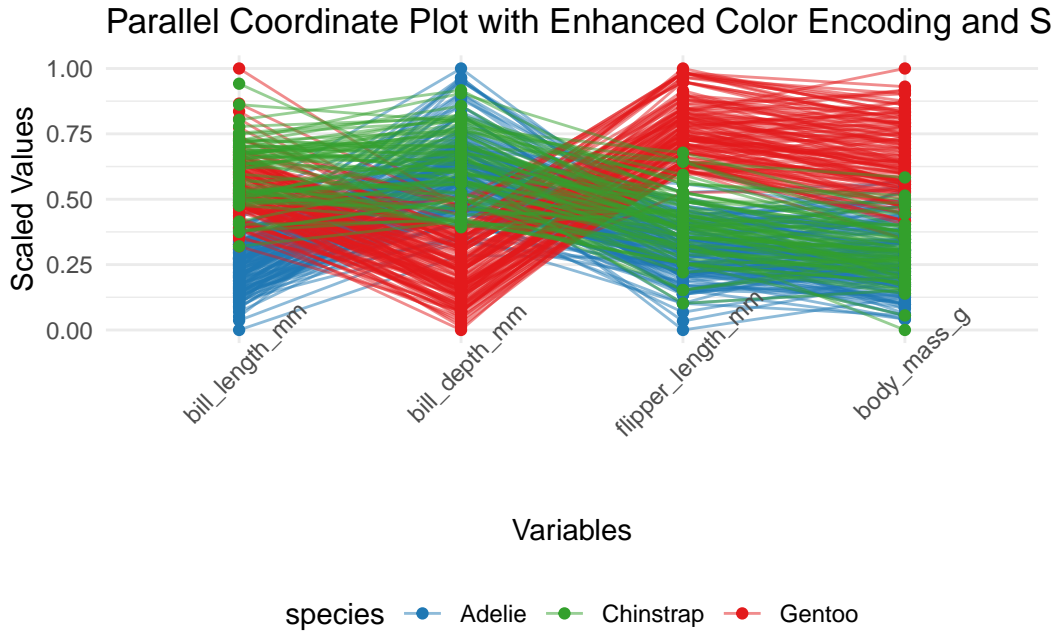


Figure 5: Enhanced Color Encoding and Shading in PCP

example, the Gentoo tends to have higher flipper length and body mass values, while the Adelie tends to have lower values. Overplotting is still a problem, especially near the middle ranges of each axis, which hides small differences and makes it hard to find outliers directly. The plot doesn't show individual distribution shapes, but how the lines are grouped on each axis suggests that each species has central tendencies. The general trends of each species' lines suggest relationships between numerical factors, but it still needs to be easier to figure out individual correlations because they overlap. This graph gives a broad picture of trends across species, but it could be better at picking out specific data points or rare observations.

Reordering and Axis Flipping

Adaptive reordering and axis flipping based on correlation measures or user-defined parameters can simplify the analysis of multidimensional relationships. Reordering and axis flipping align more relevant dimensions, minimizing intersections and making relationships more interpretable. "Reordering transforms PCPs from a cluttered tangle into a roadmap of relationships," (Inselberg and Dimsdale 1990).

Inselberg and Dimsdale proposed reordering to enhance interpretability, especially for highly interrelated variables. LeBlanc et al. introduced correlation-based reordering to position axes, reducing visual clutter by aligning more related dimensions (LeBlanc, Mellor-Crummey, and Fowler 1990). Peng et al. (2004) enabled automatic reordering algorithms that flip axes according to user-defined weights, giving users more control over PCP readability.

Changing the order of the axes in parallel coordinate plots can make them easier to read by putting variables that are strongly correlated or connected by theme next to each other. Moving the axes around reduces the amount of visual noise caused by crossings between variables that aren't strongly linked to each other. This makes patterns and clusters stand out more. The method of minimizing visual noise helps show hidden patterns in the data that would not be seen otherwise. Researchers Johansson et al. (2005) and Wegman and Luo (1997) all found that reordering is an important way to find connections in PCPs. If you use well-thought-out reordering algorithms, you can see groups, trends, and outliers more clearly and avoid the feeling of overplotting. This method works incredibly well for grouping data with many dimensions, making it easier to see.

Automatic reordering may reduce control over axis sequence, potentially misaligning dimensions relevant to a specific analytical question. LeBlanc et al. pointed out that “the gain in interpretability through automated ordering can sometimes sacrifice user intention, misaligning axes crucial to the analysis context.”

Axis flipping is a way to eliminate unnecessary line crossings and bring out trends in data by flipping the scale of one or more axes. It works well when two PCP axes next to each other have negatively correlated factors. This change allows different patterns to show up without too many crossings, giving a clearer picture of how the data is related. Axis flipping is an important tool for dealing with negative correlations because it clarifies the connection between dimensions by reducing visual interference. It also shows patterns of correlation that regular PCPs might miss, especially when working with datasets that have many factors that are unrelated to each other or are related in the opposite way. Automated axis flipping methods can change plots on the fly based on data, which lowers the chance of mistakes when users interpret PCPs. Heinrich and Dasgupta's studies both agree that axis flipping makes PCPs easier to understand by turning bad relationships into a more familiar shape Julian Heinrich and Weiskopf (2013a).

Reordering and flipping axes based on correlation or user-defined parameters simplify the interpretation of complex relationships. “Dynamic reordering optimizes the alignment of correlated variables, making it easier to identify associations and trends across dimensions” (Yuan et al. 2009). This adaptation increases flexibility, allowing the plot to respond to specific analytical needs and user preferences.

Reordering and axis shifting work well together to fix the issue of overplotting in PCPs. Rearranging linked variables makes them easier to see while flipping the axes reduces the number of times lines cross for negatively correlated variables. Combining these two tools makes looking for patterns and exploring large datasets easier.

By using these methods, users can better understand how multiple variables are connected, which helps them make better decisions based on visual information. Research shows that these methods make parallel coordinate plots easier to understand, more accurate, and better able to find groups and trends, all of which are very important for data analysts.

Continuous reordering can create inconsistency, as users may find it challenging to track relationships when axes are frequently altered. “Constant reconfiguration of the axes can disrupt the analytical flow, making it difficult to build a stable mental model of the data structure” (Julian Heinrich and Weiskopf 2013a). Additionally, for datasets with minimal correlation, this method may offer limited value, as reordering may not result in clearer insights.

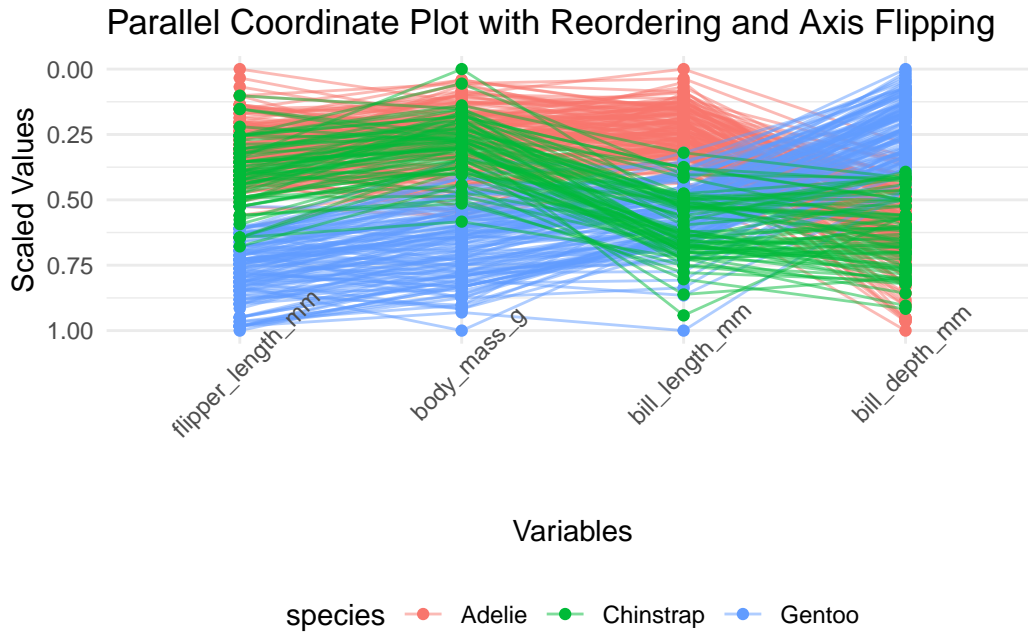


Figure 6: Reordering and Axis Flipping in PCP

In Figure 6, the parallel coordinate map changes the variables by rearranging them and flipping the axes to show patterns and connections between the penguin species more clearly. By flipping some axes and putting related factors closer together (like body mass and flipper length), the plot makes species-based trends stand out more: The Gentoo (blue) tends to have a shallower bill but a bigger body and longer flippers, while the Adelie (red) tends to have the opposite pattern. Overplotting is still a problem, especially when values are close together in the middle ranges, making it hard to tell the difference between lines or peaks. You can't see the distributions' exact forms, but the lines' density and spread along each axis give you a good idea of the range of values for each species. The new order makes relationships between variables stand out more because the changes between the axes make species-based correlations in measurement trends more apparent. However, individual relationships are still hard to see because lines cross.

Cluster-Based and Hierarchical PCPs

Clustering techniques, such as spectral clustering, have been used in PCPs to group related data points and expose the underlying structure of high-dimensional datasets. Clustering data and viewing the clusters in parallel coordinate plots makes detecting links between data points and cluster features easier. This technique is beneficial for discovering patterns not immediately evident in raw data and providing insights into the natural grouping of data points (Zhao and Kaufman 2012). Several data reduction procedures, such as hierarchical clustering and principal component analysis, are used before plot generation to improve interpretability. These strategies help to limit the number of dimensions visualized by focusing on the most important components that explain the majority of the variance in the data.

Zhao and Kaufman’s (2012) work “Structure Revealing Techniques Based on Parallel Coordinates Plot” talks about how traditional parallel coordinate plots (PCPs) can’t always show important patterns in complex, high-dimensional data because of problems like overplotting. To deal with these problems, the writers suggest new ways to sort and cluster data specifically made for PCPs. Using spectrum theory, they develop algorithms that group similar polylines and sort the axes to show hidden trends and correlations. This makes the structure of the data easier to see. A correlation-based sorting method is also added to arrange the axes to make relationships between variables stand out. This makes it easier to see trends across dimensions (Zhao and Kaufman 2012).

The study also talks about view-range metrics, which use aggregation limits to help visualize data more clearly, even when the datasets are noisy. Results from experiments show that these improvements make it much easier for PCPs to find useful patterns, trends, and correlations, which makes data analysis more efficient. The results show that the suggested approaches improve PCPs’ ability to analyze big, complicated datasets by showing important data structures that would be hard to see otherwise.

For multidimensional data, clustering and hierarchical PCPs allow data grouping and organized representation to enable easier cluster comparison. Clustering and hierarchical visualization allow for data grouping, clarifying large datasets and enhancing comparison among groups. “Hierarchical clustering in PCPs offers a clearer, more organized data narrative by showing both the big picture and finer details,” argues Fua et al. (Fua, Ward, and Rundensteiner 1999).

Fua et al. introduced hierarchical PCPs, providing a zoomable interface to represent data clusters, allowing for detailed subset examination (Fua, Ward, and Rundensteiner 1999). Geng et al. applied clustering in PCPs to group similar data points, highlighting clusters and making general patterns more accessible (Geng, Deng, and Ali 2005). Poco et al. extended these techniques by incorporating hierarchical clustering for user-defined levels of granularity (Poco et al. 2011).

Although clusters simplify the data landscape, they can obscure individual data points. Geng et al. observed that “while useful for general patterns, clustering may bury unique or outlier

data, potentially hiding significant findings in homogenous groups,” (Geng, Deng, and Ali 2005).

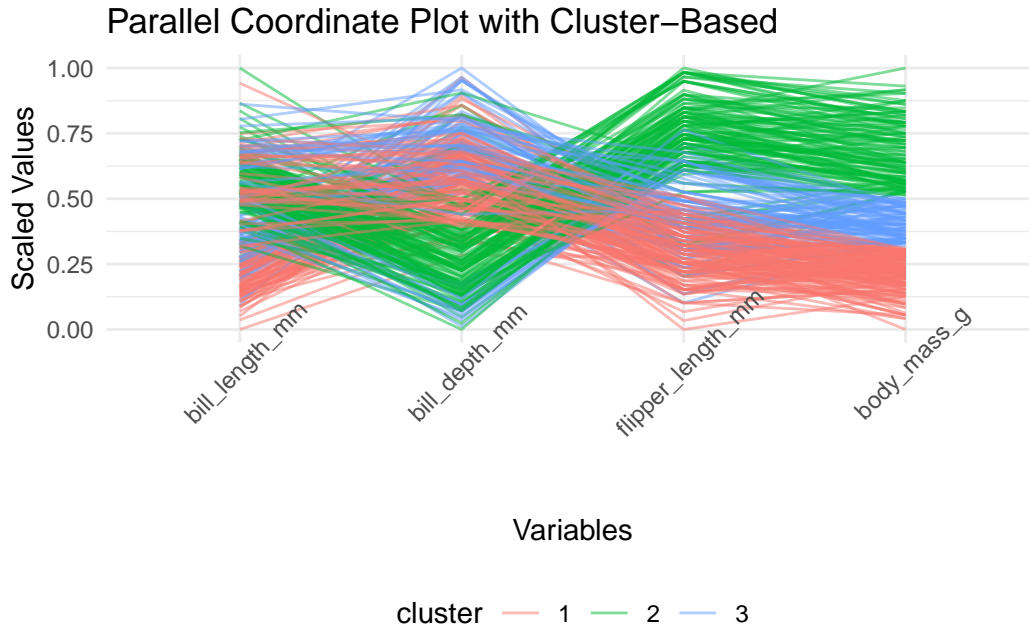


Figure 7: Cluster-Based PCP

Optimized Layout Algorithms

Layout algorithms for PCPs have been optimized to handle multi-dimensional data while minimizing line crossings, which enhances clarity and allows for smoother navigation. Optimized layouts minimize line crossings, making PCPs more straightforward, particularly with extensive, high-dimensional data. “Effective layout design in PCPs can be the difference between comprehensibility and chaos,” Ankerst et al. claim, emphasizing that thoughtful placement enhances legibility (Ankerst, Berchtold, and Keim 1998).

Ankerst et al. proposed a layout algorithm that places axes to minimize line crossings, increasing the legibility of high-dimensional PCPs. Johansson et al. optimized the axis positioning algorithm to maximize the space between overlapping lines, improving data distinction (Johansson et al. 2005). Hao et al. (2007) implemented a randomized layout approach that provides an efficient layout for PCPs, beneficial for large, high-dimensional data.

Layout optimizations sometimes follow heuristic approaches that may not always align with user-defined analysis needs, possibly overlooking preferred layouts. Johansson et al. state that “algorithmically optimized layouts prioritize minimal intersections over interpretability, occasionally misaligning with the user’s focus.” (Johansson et al. 2005).

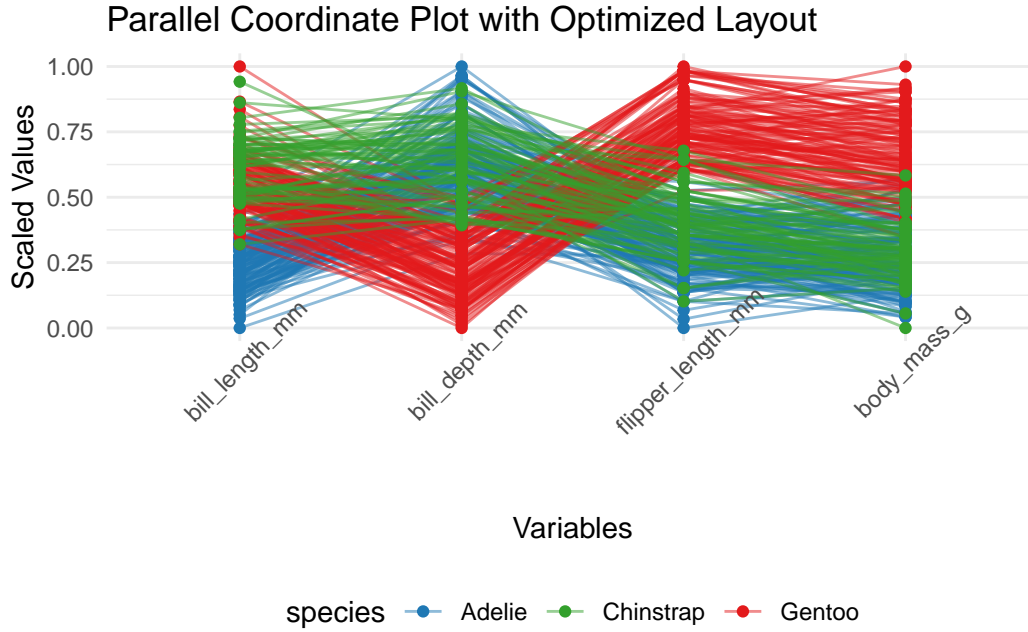


Figure 8: Algorithm Optimized Layout in PCP

Categorical and Hybrid PCPs

Traditional PCPs were primarily designed to collect continuous data. However, real-world datasets often contain a mix of continuous, ordinal, and categorical data. To solve this, changes have been made to display mixed-type data within the same PCP simultaneously. Categorical data, for example, can be visually differentiated using unique colors, symbols, or segmented lines, although continuous variables are still represented by standard lines linking the axes. Categorical Parallel Coordinate Plots (CPCPs) were created to handle the challenge of representing categorical data in PCPs, which are not suitable for continuous axis representation. Specific strategies, like adjustments to the ends of links, have been suggested to enhance how category and numerical values are connected visually. This strategy enhances the interpretability of mixed data by modifying the plotting criteria for axes representing category variables.

CPCPs use discrete axis segments or unique markers for each category, transforming categorical variables into distinguishable visual elements. Siirtola and R  ih   introduced axis segmentation, using markers and spacings to represent categorical values distinctly, improving categorical data visualization in PCPs, (Siirtola and R  ih   2006). Inselberg enhanced this representation by introducing specific encodings, which allowed categories to be differentiated visually, preserving the PCP's interpretative power (Inselberg 2009). Johansson and Forsell explored alternative segmentation methods to minimize visual clutter, enabling effective handling of multiple categorical values on the same axis, (Johansson and Forsell 2015).

The Generalized Parallel Coordinate Plot (GPCP) is a modified version of the original PCP that introduces nonlinear changes to the axes for more complex data representation. These can show more complicated relationships in the data that might not be visible in a regular PCP. Nonlinear scales such as logarithmic and exponential transformations are possible. In their work in 1997, Wegman and Luo provided a comprehensive discussion of the GPCP idea. This adjustment allows the visualization to display various data connections and handle skewed data distributions (Wegman and Luo 1997).

CPCPs adjust by organizing categorical axes more clearly or using symbols and colors to represent categories for easier interpretation. Wegman et al. (1990) suggested using symbols and color codes to represent categories, improving how categorical data is understood in parallel coordinates. Holten and van Wijk (2009) introduced color gradients for categories to facilitate distinguishing between multiple categorical values, especially in the analysis of intricate datasets. LeBlanc et al. (1990) highlighted the significance of arranging axes in CPCPs, positioning categories with strong relationships closer together to assist users in making meaningful comparisons.

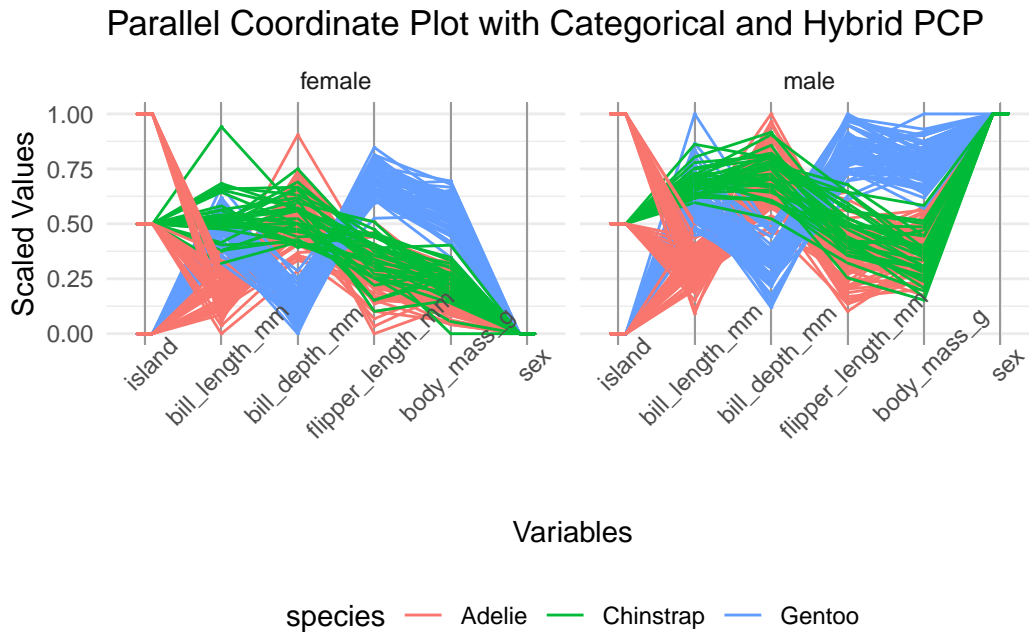


Figure 9: Categorical and Hybrid in PCP

Handling Numerical Ties

Getting rid of ties in statistical data analysis has been hard for a long time for experts who want to make visualizations more clear. Previously, statistical methods focused on rank-based approaches and binary data handling. They often needed help with tied observations, which

threw off distributions or made them more complex. Earlier methods for dealing with ties depended on changing ranking systems or adding rules to break ties, which could make results less accurate or easier to understand. As graphics data analysis grew, pioneers like Tukey, Chambers, and Cleveland created new ways to deal with ties visually. There are now several ways to deal with ties, such as slowly increasing the space between values. This makes the visualization more accurate and easier to understand without changing the structure of the data itself.

Random Jittering of Data Points

Random jittering of data points is a standard method that adds a small random value to each tied observation. This is especially helpful in scatter plots and dot plots to keep them from being too dense. This approach, discussed in “Graphical Methods for Data Analysis” by Chambers et al., is easy to use and keeps the overall distribution, making things clearer without changing the data (Chambers 1983). One problem with random jittering is that it can hide exact numbers, making it hard to understand if the jittering distance is too big. Experiments show that jittering makes visualizations much clearer when there are small overlaps, especially when used rarely.

In random jittering, we add a small random value (usually drawn from a uniform or normal distribution) to each tied observation to differentiate them.

For a set of tied values x_1, x_2, \dots, x_n , we define:

$$x'_i = x_i + \epsilon_i$$

where:

- x'_i is the jittered value of x_i ,
- $\epsilon_i \sim \mathcal{N}(0, \sigma^2)$,
- σ is the standard deviation for the normal jitter.

Rank Jittering (Rank Adjustment)

Rank jittering is another useful method for non-parametric analyses. It changes the ranks of tied values instead of the real data values. This makes it easier to see the ranks in non-parametric tests like the Wilcoxon signed-rank test. Conover’s “Practical Nonparametric Statistics” shows that rank jittering keeps the ordinal form of data while not changing the distribution much (Conover 1999). The best thing about rank jittering is that it keeps rank-based readings while making ties less noticeable. However, the method can be hard to run on

computers when dealing with big datasets, and making too many rank changes can accidentally change the test significance levels. For these reasons, it is important to be careful when using rank jittering.

In rank jittering, we adjust the ranks of tied observations by adding a small random or systematic jitter to each tied rank. This is usually applied to data that will undergo rank-based testing.

for tied ranks R_1, R_2, \dots, R_n :

$$R'_i = R_i + \delta_i$$

where:

- R'_i is the jittered rank of R_i ,
- $\delta_i \sim \mathcal{N}(0, \sigma_{rank}^2)$
- σ_{rank} are small values ensuring that the adjustment is minor.

Deterministic Jittering (Fixed Perturbation)

When you need to be clear, deterministic jitter gives you a structured way to do it by adding a small fixed value (epsilon) to each tied observation instead of depending on chance. This method explained in more detail in Tukey's "Exploratory Data Analysis", is useful when random noise is not wanted because it ensures that each observation is moved in a way that can be predicted and repeated (David and Tukey 1977). Deterministic jitter is easier to understand than random jitter, but it needs to be done carefully so that it doesn't introduce fake patterns, especially when scaling is involved. Deterministic jittering is a good choice for random noise that doesn't affect the integrity of the distribution when used carefully.

In deterministic jittering, a small, fixed value ϵ is added to each tied value to systematically spread them apart without introducing randomness.

For tied values x_1, x_2, \dots, x_n , we use:

$$x'_i = x_i + i\epsilon$$

where:

- x'_i is the adjusted value of x_i ,
- i is the index of the tied observation (e.g., $i = 1, 2, \dots, n$),
- ϵ is a small fixed distance chosen based on the data scale.

Mean or Median Splitting

One more popular method is mean or median splitting, which gives values tied to their mean or median values. This method works especially well for symmetric distributions and is perfect for images that need to show things centrally, like box plots. Wilcox’s “Modern Statistics for the Social and Behavioral Sciences” shows that splitting the mean or median gives a fair view, especially for boxplot data (Wilcox 2017). This method works best with datasets with few ties because it can hide the range and variability within tied numbers if used too much.

In mean or median splitting, we replace each tied value with the mean or median of the tied group. This technique centers tied observations on their central tendency.

For set of tied values x_1, x_2, \dots, x_n let:

$$x_{mean} = \frac{1}{n} \sum_{i=1}^n x_i \text{ or } x_{median} = median(x_1, x_2, \dots, x_n)$$

Then:

$$x'_i = x_{mean} \text{ or } x'_i = x_{median}$$

This replaces each tied value x_i with the computed mean or median, clustering them at a central point.

Kernel Density Estimation (KDE) with Bandwidth Adjustment

Kernel density estimation (KDE) with bandwidth adjustment is a powerful method for density visualizations. It spreads out overlapping observations by giving each tied value a slightly different weight. This method works especially well in density plots because it evens data distributions without changing the original numbers. Silverman’s work in “Density Estimation for Statistics and Data Analysis” shows how KDE with changed bandwidths correctly shows multimodal distributions, reducing errors caused by tied values. KDE works, but you must be careful when choosing the bandwidth because making the wrong choice can lead to misleading visual representations, especially when files are very concentrated (Silverman 2018).

For KDE with bandwidth adjustment, each data point is used to estimate a probability density function (PDF) by smoothing over a specified bandwidth h , which can be adjusted to account for ties.

The KDE for a set of observations x_1, x_2, \dots, x_n is:

$$\hat{f}(x) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x - x_i}{h}\right)$$

where:

- $\hat{f}(x)$ is the estimated density at x ,
- $K(\cdot)$ is a kernel function
- h is the bandwidth, chosen to spread tied values slightly by selecting a larger h for clusters of tied observations.

The choice of bandwidth h controls the amount of smoothing, which helps visually differentiate tied values.

Each way to show data with ties has unique benefits depending on the type of display and the type of data. Random and predictable jittering, for example, works well in scatterplots because they stop the plot from getting too big and show hidden patterns. KDE and rank-based methods, on the other hand, work well in density plots and ordinal visualizations. Researchers can improve visual clarity by using only some of these methods, ensuring the representation is correct and easy to understand. However, when parallel coordinate plots are used, where multiple directions are plotted simultaneously, the old ways of dealing with ties only sometimes work. In this case, adding the distance between values tied across multiple axes could show patterns and connections that are hard to see because the lines overlap. By creating a new method for parallel coordinate plots that adds space between ties in a planned way, we could see data trends and links across dimensions more clearly while keeping the dataset’s integrity. This new idea would make it easier to understand complex, multidimensional data.

Handling Ties in PCPs

The idea of adding distance to visual ties has changed over time as methods for visualizing data have improved to find a balance between detail and clarity in large datasets. In the early days of visualization, especially in network analysis and multidimensional scaling, closeness and alignment were the main ways to show patterns and ties between things. However, as data grew, visual clutter became a big problem. This was especially true in parallel coordinates plots (PCPs), where lines and intersections that overlapped made it hard to see the underlying data structure. To fix this problem, researchers started looking into ways to give representational parts of these graphs more “visual distance.” They did this using methods like line bundling and changing the spacing between lines to avoid overplotting. This change is emphasized by Chang et al., who say that “perceptual variations in lines not only differentiate data but also enhance recognition of broader patterns,” (Chang, Dwyer, and Marriott 2018). This shows how separating visual elements can help with cognitive grouping without breaking up the continuity of the data. In PCPs, early studies showed that visual distance could make intersections clearer and improve the overall shape of individual paths. As Guo et al. say, “Thresholding reduces noise, allowing significant intersections to remain prominent, thereby clarifying the underlying data structure without distorting continuity,” (D. Guo et al. 2005). This theory has shaped later developments, showing visual distance as a spatial tool and a key cognitive help in data interpretation needed to get around today’s more complex visualizations.

Axis Reordering

As mentioned in the section on modifications, axis reordering changes the order of dimensions to reduce the number of times lines cross between variables in PCPs, a type of data display. This method makes clusters easier to see by lining up visually linked variables, cutting down on clutter, and making patterns stand out more without overlapping. One benefit is that it works well for datasets where certain factors are strongly related, making clusters and trends easier to read without changing the data. Changing the variable order for clarity rather than data continuity could make it hard to tell if the data is in order. Blumenschein et al. say that “reordering based on correlation minimizes unnecessary line intersections, allowing natural clusters to emerge.” This shows how important changing the arrangement can be for seeing clearly. Studies in this area have shown that changing the order of dimensions makes it much easier to find patterns in datasets where dimensions are linked. However, it only works well on datasets with dimensions tied together (Blumenschein et al. 2020).

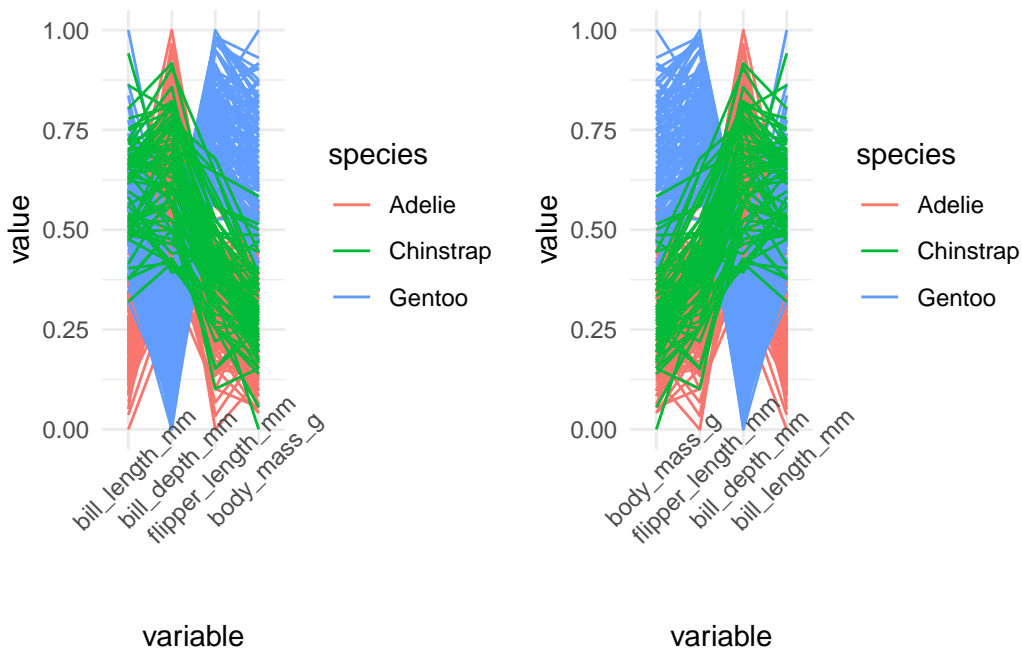


Figure 10: PCP Without Axis Reordering

Line Bundling

By putting similar lines into “bundles,” line bundling cuts down on unnecessary crossings and clutter. This method works well in dense PCPs, where many lines cross and make it hard to see individual patterns. Heinrich et al. say that “bundling lines based on value similarity reduces cognitive strain and enhances visual coherence in complex datasets.” This shows how bundling

can make complex data easier to understand without losing larger patterns. Experiments have shown that bundling makes large datasets easier to read, which makes it easier to find general trends even when the data is heavily packed (Julian Heinrich et al. 2011). It makes things easier to understand by collecting similar data lines; this works especially well for numerical ties where values are close but not exactly the same. Bundling, on the other hand, can hide individual lines within a group, making it hard to tell one data point from another.

Quantization and Aggregation

Quantization, also known as Quantized Generalized Parallel Coordinate Plots or QGPCP, groups close-together values into bins. This makes it easier to see patterns in tightly packed data. This method cuts down on the number of lines that cross each other, which makes it great for big datasets. Moustafa states, “By quantizing close data points, we can minimize overplotting and improve discernment of overall trends.” This shows that removing some details can make data easier to read. Quantization can keep important patterns while lowering noise, as tests on real-world data show. However, it may hide small but important differences in the data (Rida EA Moustafa 2009). It makes the image easier to understand by removing small details. This is helpful for high-dimensional datasets with a lot of line overlap. However, detail can be lost during aggregation, making this method less useful for situations where exact data representation is needed.

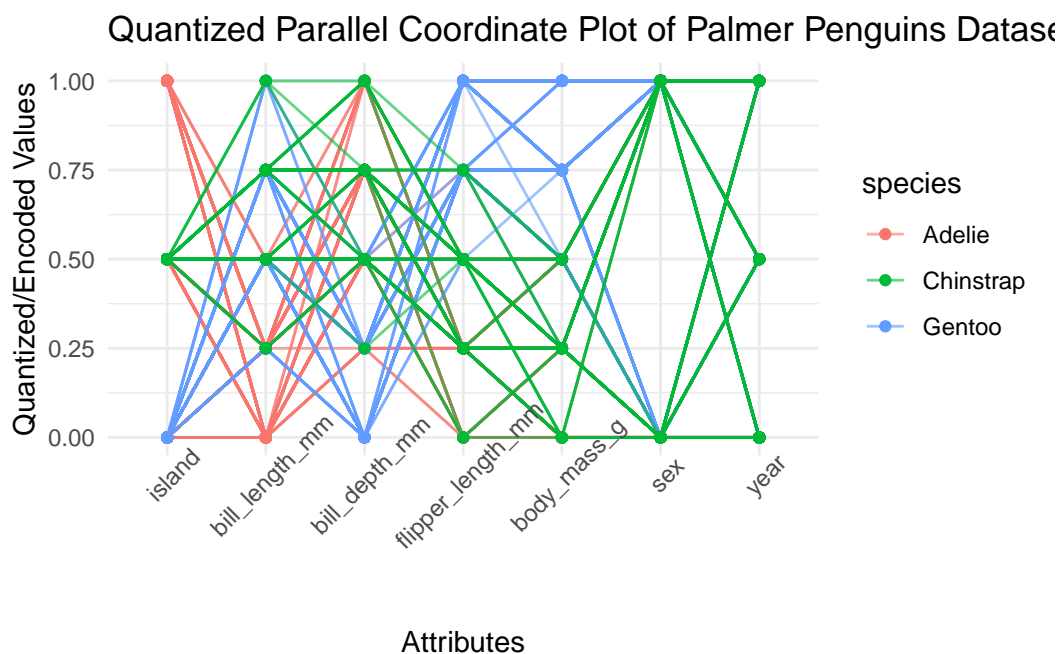


Figure 11: Quantized Generalized Parallel Coordinate Plots

In Figure 11, the overlapping of the Quantized Parallel Coordinate Plot makes it harder to

find specific individuals. Quantization makes the visualization even easier by putting numbers into groups. However, this needs to improve some of the accuracy of the original data. The plot can help you see big patterns, like how some species may follow similar paths across many traits, but it doesn't show exactly how they are related or where they live. Using jittering or interactive plots to cut down on overplotting could make things clearer and make figuring out how factors relate easier.

Color-Coding and Transparency

By changing the color and transparency of lines, you can tell them apart based on density, categories, or values. This makes it easier to tell the difference between PCP lines that meet. Firat et al. say, "Line brushing and transparency adjustments improve the gestalt of the data clusters." This means that changes in how we see things help us group data cognitively without changing the structure. Firat et al. say that user tests have shown that PCPs with color-coded and clear lines are easier to understand and recognize patterns, especially for differentiating categorical data (Firat, Swallow, and Laramée 2023). This tool can be used to handle both numerical and categorical ties. The color and transparency options make it easy to visually separate clusters without changing where the lines are placed. However, this method might only work for plots with little information because even changes in color can't fully fix heavy overplotting.

Edge Bundling for Categorical Data

Edge bundling, which comes from network visualization, groups paths for category values in PCPs. This method works best with categorical data because it lets you visually break up ties without changing other factors. Palmas et al. state that "bundling lines with similar orientations prevents overplotting, allowing clearer identification of dominant trends within complex data." This means bundling can help find trends even in spaces with many dimensions. Bundling made finding categorical clusters in complex PCPs easier and decreased users' work (Palmas et al. 2014). Edge bundling is an excellent way to clear up clutter and make categorical groups easier to see. It lets users see categorical differences without messing up numerical relationships. On the other hand, each bundle may hide small details, making them less useful for datasets that need a lot of precision.

Stacked Histograms and Density Plots

This change adds histograms or density plots on top of PCPs to show how category data is distributed in terms of frequency. This lets users see how distributions change without adding extra line clutter. According to Bok et al., adding histogram overlays gives categorical context, making variable distributions easier to see without affecting line continuity. This means that frequency-based cues can help you understand categorical distributions without adding more

visual clutter. Clear frequency-based differentiation in categorical data without too much line overlap improves the visual understanding of distributions when used with PCPs. Real-world category dataset tests showed that histogram overlays made it easier to differentiate between data sets and see what they meant (Bok, Kim, and Seo 2020). Adding histograms makes things look more complicated, which could be too much for some people if used with other PCP changes

Perceptual Cues (Line Thickness, Texture, etc.)

By changing the lines' width, texture, or other visual properties, you can tell them apart from overlapping lines. This helps people see patterns and tell values apart without separating them physically. According to Chang et al., "Perceptual variations in lines not only differentiate data but also enhance recognition of broader patterns." These cues help with cognitive grouping and pattern differentiation, even in complicated datasets. They work well in areas with a lot of visual information because they let users see trends without changing the structure or order of the data. They are helpful for both numerical and categorical ties. Studies with users showed that changes in how people saw things made finding patterns and trends in large, complex datasets easier. This made it easier for PCPs to look around (Chang, Dwyer, and Marriott 2018). In areas with many people, it could be useful because overlapped lines can make it harder to see.

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