Teaching Liberal Arts Mathematics Courses

Dale R. Buske
drbuske@stcloudstate.edu

Saint Cloud State University

July 31, 2013

Welcome

Outline

- Icebreaker
- Topics
- Goals
- Structure
- Audience
- Activities
- Interactive Demos
- Projects
- Developing an MLA course

Your Turn

- Name
- Institution
- The most interesting topic (to a layperson) you have ever taught in a mathematics course
- Have you taught a liberal arts math course before?

Topics found in MLA courses

- Apportionment
- Counting methods
- Descriptive statistics
- Exponential growth
- Fair division
- Fibonacci numbers and ϕ
- Fractals
- Game theory
- ***Geometry** (e.g. symmetry, tiling, perspective, non-Euclidean)
- ***Graph theory and applications**
- ID numbers (e.g. UPCs)
- Infinity
- Knot theory
- Linear growth
- Linear programming
- Logic
- ***Money** (e.g. interest, loans, annunities)
- Normal distributions
- Number theory
- Numeration systems
- Planning and scheduling
- Precision and scale
- ***Probability and expected value**
- Puzzles and games
- Regression
- Set theory (e.g. Venn diagrams)
- ***Sampling (e.g. survey methodology, clinical studies)**
- Topology
- Voting including preference ballots and weighted voting systems

Facts about MLA courses

- In 2005, MLA was the fourth-largest college mathematics cohort [CBMS report, 2007].
- In 2005, 55% of courses for MLA students at four-year colleges were taught by faculty who were neither tenured nor tenure track [CUPM, 2007].
- In 2005, 79% of MLA courses taught at two-year colleges used the lecture mode [CBMS, 2007][1]. In 2010, this number was 88%.
- Enrollments in MLA classes have increased 100% since 1995 while enrollments in all introductory level (below calculus) courses has only grown 40% over that time [CBMS, 2012].
- In 2010, about 17% of MLA students at two-year colleges were taking the course online [CBMS, 2012].

[1] This was 81% in calculus 1 and 93% in differential equations.

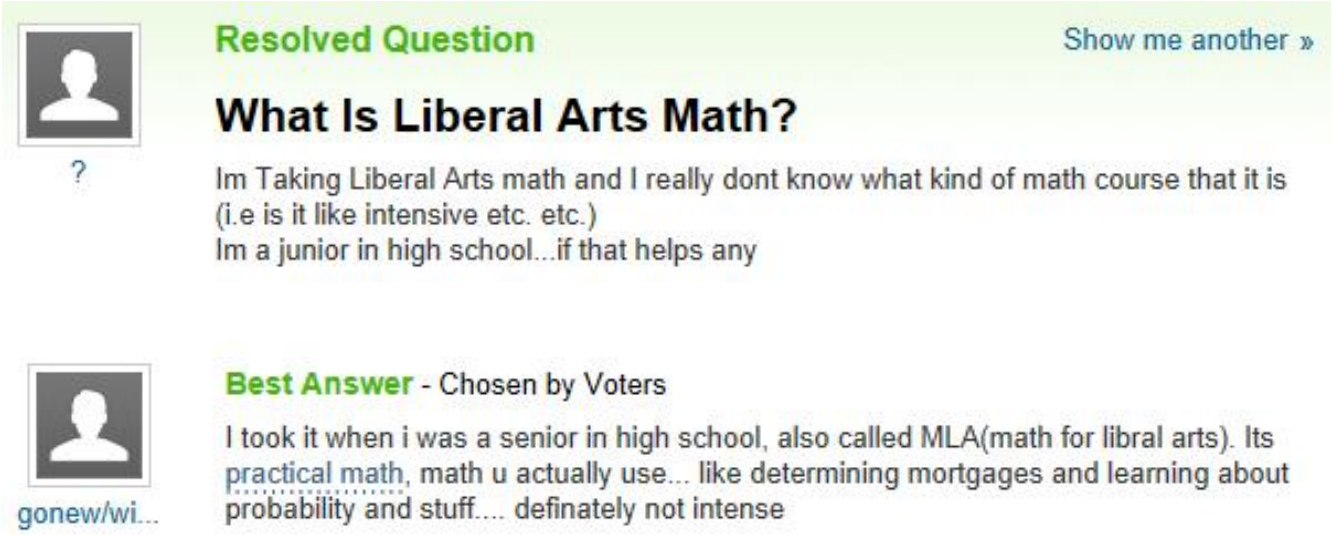
Topics found in MLA courses

Reva Kasman of Salem State College describes her experience organizing a special session on Teaching Mathematics for Liberal Arts.[2]

"For better or for worse, there is no such thing as a standard curriculum for a liberal arts math course...Course topics ranged from Renaissance paintings to airplane flight, democracy to investment banking. This was not your mothers math course...(but) MAT 120 has become one of my favorite courses to teach, and watching my students develop positive attitudes about mathematics (as well as their own capacity to do mathematics) has been extremely rewarding."

[2]Kasman, Reva, *Not Your Mothers Math Course: Mathematics for the Liberal Arts*, Salem State College Aspect newsletter, March 2010.

What is Liberal Arts Mathematics?



Resolved Question — Show me another »

What Is Liberal Arts Math?

?

Im Taking Liberal Arts math and I really dont know what kind of math course that it is (i.e is it like intensive etc. etc.)
Im a junior in high school...if that helps any

gonew/wi...

Best Answer - Chosen by Voters

I took it when i was a senior in high school, also called MLA(math for libral arts). Its practical math, math u actually use... like determining mortgages and learning about probability and stuff.... definately not intense

Handouts: 1) Data on this group. 2) A February 2000 article in the Notices of the AMS gives the opinion of Ed Burger and Mike Starbird.

Your Turn

In one-sentence, write what you believe to be the most important goal of a liberal arts math class. This can be either for a class you have taught, are about to teach, or is taught at your institution.

Goals of an MLA course - Examples

Northern Kentucky

There are three basic goals for this course[a]:

1. To reach a better understanding of several rich mathematical ideas.
2. To develop sharper skills for analyzing life issues that transcend mathematics.
3. To change the way you view the world!

Although you will be challenged, the hope behind this course is that you will gain an appreciation for mathematics, and discover the power of mathematical thinking in your everyday life.

[a] taken from *Heart of Mathematics*

Goals of an MLA course - Examples

Austin Peay State University (TN)

1. Learn how to learn both independently and collectively from peers rather than passive learning
2. Develop skills of inquiry, abstract and logical thinking, and critical analysis
3. Develop the ability to understand and use numbers and statistics
4. Develop ability in writing, reading, listening and speaking about mathematics
5. Learn, enjoy, and apply some basic concepts of elementary mathematics
6. Appreciate and feel comfortable with mathematics

Goals of an MLA course - Examples

Hawaii (Thomas Hoover)

The goal of the course is to increase your understanding of the nature of mathematics and its role in the modern world. We will look at some nice examples of mathematical reasoning and of applications of mathematics. You may learn some mechanical procedures, but a big emphasis will be on conceptual understanding. Learn the vocabulary; otherwise, you might not learn anything.

University of Hawaii - Manoa

...for students to learn, mostly through a guided direct experience with mathematics, about

- the nature of mathematics as a human activity, and
- the role of mathematics in the world, from intellectual history to modern technology

Goals of an MLA course - Examples

Robert Blitzer

1. To help students acquire knowledge of fundamental mathematics.
2. To show students how mathematics can solve authentic problems that apply to their lives.
3. To enable students to develop problem-solving skills, foster critical thinking, within a varied, interesting, and contemporary setting.

UTEP

This course is designed to introduce you to the big picture of what mathematics is, and what it means to do mathematics. In contrast (probably) to your previous experiences with mathematics, this means more than applying rote formulas or watching someone else think. You will be actively engaged in (guided) discovery, retracing for yourself the highlights of some of the major developments in mathematics.

Goals of an MLA course - Examples

University of Maryland Baltimore County

The primary goal of this course is to gain knowledge and appreciation of contemporary mathematics. More specific goals are listed below.

1. Gain an understanding of how mathematical processes shape our world.
2. Enhance precision, computational skills, and other core mathematical qualities.
3. Acquire content- and technique-based knowledge (e.g. statistics) that can be practically applied outside this course.
4. Appreciate the use of the computer as an experimental tool
5. Learn to think mathematically.
6. Learn about broader issues and open questions in mathematics.

Goals of an MLA course - Examples

Norfolk State University

The following are the major goals to be achieved by students taking this course:

- Increase quantitative reasoning/ literacy skills needed for informed citizenship, advancement, and strategic problem solving
- Develop an understanding about the fundamentals of probability and statistics as well as understand how these numbers are used to make decisions
- Gain an appreciation for the importance of financial health, investing, and real estate
- Develop an understanding of the fundamentals of measurement and geometry for the purpose of improving proficiency on standardized tests
- Build a repertoire of problem solving strategies such as the use Venn diagrams, proportional reasoning, and logic for improved reasoning ability
- Improve ability to analyze and interpret graphic representations of information for decision making and improving proficiency on standardized tests

Course Objectives

Nicholls State University (Brian Heck)

Students will be able to:

- **(Calculation)** Perform calculations with integers, fractions (rational numbers), decimals, ratios and percents.
- **(Application)** Use arithmetic, algebraic, and/or geometric methods to solve problems.
- **(Data Analysis)** Demonstrate understanding of the terms and symbols used to generated, present, an analyze data.
- **(Interpretation)** Interpret and evaluate quantitative or symbolic models such as graphs, tables, and units of measurement.
- **(Communication)** Represent and communicate quantitative or symbolic information.
- **(Pattern Recognition)** Generate and apply conclusions based on pattern recognition.

The Audience

Your Turn

As a small group, come up with three adjectives that best describe a student taking a liberal arts math class.

CUPM Curriculum Guide 2004

General education and introductory courses enroll almost twice as many students as all other mathematics courses combined. They are especially challenging to teach because they serve students with varying preparation and abilities who often come to the courses with a history of negative experiences with mathematics. Perhaps most critical is the fact that these courses affect life-long perceptions of and attitudes toward mathematics for many students – and hence many future workers and citizens.

UTEP

Prerequisites: An open mind, a healthy curiosity, and a willingness to learn new ideas.

The Audience

Michael Starbird at UT-Austin

There is a broad spectrum of students who take M302. Some are quite good at math and may even have had some calculus in high school. These, however are greatly outnumbered by the students who have weak math skills and poor backgrounds. It is not at all uncommon for the students to exhibit a fear of and dislike for math and most have very low self-confidence about their ability to succeed in a math class. In answer to this, the goal of the course should be to demonstrate that math is not about memorizing formulas, but is rather a process of thinking which is relevant to them on a daily basis.

Northern Kentucky

Expected Background: A math ACT score of 18 or better, an open and curious mind, and the willingness to put aside any preconceived prejudices or dislikes for mathematics.

The Audience

My Experiences

- Many students have been turned off to the subject from their high school experience. Developmental coursework at the university has not helped.
- Generally speaking, work ethic is good. But perhaps this is due to a heavily female audience?
- Many students view the course as their most significant hurdle to college graduation. For many of them, it is just that.
- You will see both solid raw talent and a complete lack of talent. Those lacking both talent and background skills require great advising skills.

Course Structure

Your Turn

As a small group, come up with two instructor behaviors you believe should be emphasized in teaching this course. In what ways do you wish for these behaviors to influence the actions of your students?

Course Structure

Judith Grabiner of Pitzer College encourages faculty to design their own Liberal Arts Math course. She has five principles to guide the design.[3]

1. Draw on the interests of each individual student.
2. Teach important mathematics.
3. Go slowly enough so students have a sense of mastery.
4. Encourage the students to use the mathematics they already know.
5. Let students create projects on topics they choose and then share their projects with the class.

[3] Grabiner, Judith V.. 2011. How to Teach Your Own Liberal Arts Mathematics Course. Journal of Humanistic Mathematics, 1 (1): 101-118.

Course Structure

Carmen Laterell of the University of Minnesota Duluth advocates for mathematical inquiry.[4]

"Some non-STEM majors take college algebra or another course meant to lead into higher-level mathematics, even though these students will not take higher-level mathematics. Fortunately, mathematics departments are starting to realize the foolishness of that approach.
One alternative is a survey course designed to demonstrate mathematical concepts and applications - a "math-in-our-world" approach. Another is a "mathematics appreciation" course in which students attempt mathematical thinking by looking at some of the "big ideas" in mathematics that may or may not have real-life applications. This approach presents mathematics as an art – no real justification for it is needed – and a beauty in and of itself."

[4] Laterell, Carmen. Summer/Fall 2011. Should Liberal Arts Math Courses Be Taught Through Mathematics Inquiry? Liberal Education, 60-64.

Course Structure

Laterell, ctd.

I suggest that non-STEM majors should actually do some mathematics, but not mathematics that will lead to the next course (that liberal arts majors never take anyway) or even to real-life applications. Rather, I advocate for students experiencing mathematical inquiry, without regard to any particular content. Some mathematics professors refer to the inquiry style of teaching as the "Modified Moore Method."

Course Structure

Laterell, ctd.

The instructor could set up, say, five or six investigations that the students would then have to investigate through a process of mathematical inquiry. For example, a colleague of mine defined the term "factor" for a group of non-STEM majors, and then asked them to take positive whole numbers (any numbers they wanted) and give all the factorizations of the numbers. The students were engaged by the unstructured nature of the assignment...Once students had many examples, they were asked to look for patterns and try to describe the patterns. Soon, the group had "discovered" prime numbers and what that meant. Then someone said "look, everyone has a factorization made up of only primes." Next the group started examining numbers together, and was able to perfectly describe the fundamental theorem of arithmetic.

Course Structure

Laterell, ctd.

Conducting investigations around applied problems can be equally effective. Bring three different-sized cereal boxes to class, and tell the students that you want to get the most amount of cereal for your money. Investigations would begin with a decision regarding what "most" and "for your money" really mean.

Course Structure

Discovering the Art of Mathematics - **www.artofmathematics.org**

The Vision: Mathematics for Liberal Arts (MLA) students will be actively involved in authentic mathematical experiences that are both challenging and intellectually stimulating, that provide meaningful cognitive and metacognitive gains, and that nurture healthy and informed perceptions of mathematics, mathematical ways of thinking, and the ongoing impact of mathematics not only on STEM fields but also on the liberal arts and humanities.

The Discovering the Art of Mathematics team is Julian F. Fleron, Volker Ecke, Christine von Renesse and Philip K. Hotchkiss.

Course Structure

M.B. Ulmer (emeritus) at the University of South Carolina Upstate suggests a problem-based approach to MLA[5][6].

Problem-Based Learning Approach

The Problem

Most of the students we try to reach in College Mathematics are products of pedagogical styles which depend on the learning of algorithmic processes for success. Few, if any, have been expected to think mathematically. As do many of their peers around the nation and the world, they view math and statistics as lists of rules to be memorized, selected and applied in response to problem types whose solutions are demonstrated in immediately adjacent sections of a text.

[5] http://faculty.uscupstate.edu/mulmer/PBI_Index.shtml

[6] More on problem-based learning at http://www.udel.edu/inst/

Problem-Based Learning Approach - Ulmer

A Solution?

- Uses regression modeling as a tool for describing real data which students perceive as applicable to their lives.
- Makes maximum use of available technology.
- No required textbook. A packet of activities and projects accompanies a forty-plus page booklet containing the topics to be covered.
- Intentional omission of drill problems (forces practice to occur within context of an activity involving real data).
- Many of the activities were developed with the assistance of faculty or practitioners in corresponding disciplines, ensuring that students are exposed to realistic multidisciplinary thinking.

Course Structure

Karl Dovermann of the University of Hawaii at Manoa describes the course *Math 100: Topics in Mathematics* offered within the UH system.[7]

1. Math 100 is a terminal course; it is not designed to prepare students for other courses.
2. With the start of each new topic the students have a chance to make a fresh start themselves. This feature of the course helps to keep the class together and to prevent students from falling hopelessly behind.
3. In the discussion of each individual topic we generally follow the path of encounter, exploration, formulation, struggle, solution, application and generalization.
4. We want the students to experience mathematics beyond the numbers, to explore, discover, and reason. We want discuss topics in depth and expose their beauty, and still not be overwhelming. We want to share some of the excitement of mathematics.

[7] http://math.hawaii.edu/system_wide_math/math100.pdf

Course Structure

Reva Kasman

I desire to showcase a variety of characteristics of mathematics through topics which students are unlikely to have encountered previously in their formal education. Despite the depth of our work, the technical prerequisites are minimal, and all our calculations can be done on a cheap calculator from Walgreens. The atmosphere in the class is dynamic, and active explorations are used in almost class meeting. I want my students to understand that they are "doing math – and from a mathematicians perspective, that involves asking questions, making and testing conjectures, experimenting with examples, and arguing (logically, of course!) with each other.[a]

[a] http://aspectwebsite.com/not-your-mothers-math-course/

Course Structure (Western Carolina University)

As a terminal course in mathematics, and in accordance with the Liberal Studies philosophy of trusting faculty to use their expertise and creativity in providing an excellent learning experience, broad discretion is given to each faculty member in the particular content to be covered the individual course sections, subject to the following guiding principles:

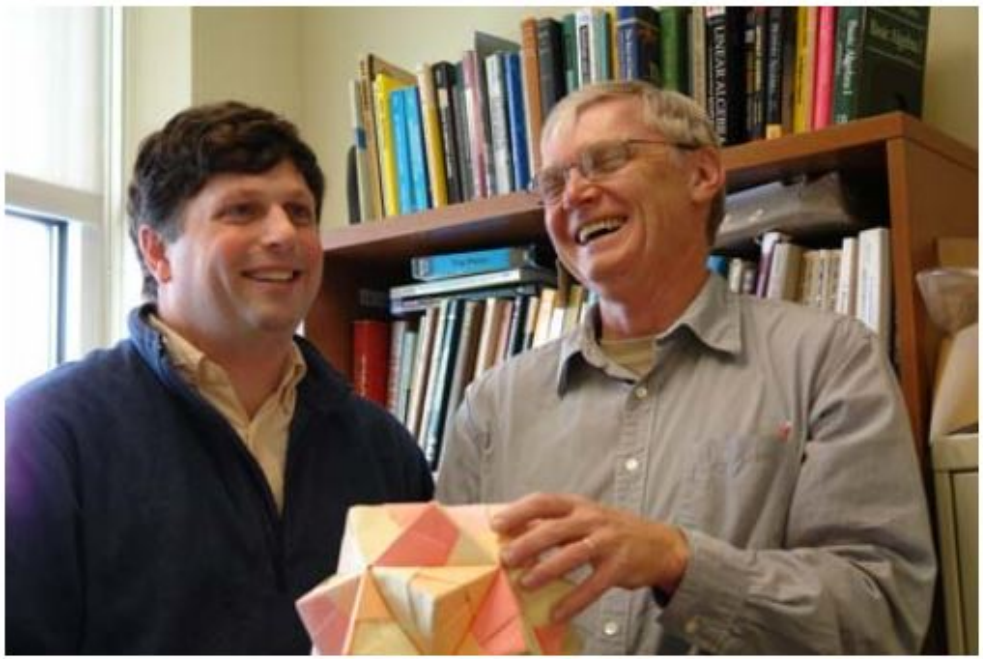
1. The course should consist of an in-depth exploration of a few (three or four) topics, as opposed to a broad but shallow coverage of numerous topics.
2. The one content area which should be common to all sections is statistical literacy.
3. The instructor is not restricted to the topics included in the text. Following the principle that faculty should be encouraged to use their creativity to provide the best possible learning experience, the instructor may (but is not required to) devote a module to a topic outside of the textbook, provided that the instructor provides suitable resource materials (handouts, homework problems, etc.) for the class.
4. One component of the course grade should be a student project.

Advise on course structure

Revamped math class for liberal arts majors

Why the last math course of their life could be the best

"Heart" is not a term often paired with "math." But three years ago, two Saint Joseph's professors adopted a radically different approach to the math course for liberal arts majors, one where equations don't fill up the board and students don't try to memorize them. What they do is learn to think in a new way - one embraced in a book called The Heart of Mathematics.



Math professors David Pinchbeck and Scott Balcomb share a laugh while trying to explain the many-sided stellated icosahedron to the author.

Dr. David Pinchbeck calls the book a gem. In his restructured Contemporary Math course, students learn about big ideas in math, like infinity and the fourth dimension, like symmetry and chaos theory. Or how any type of vote counting system becomes imperfect once there are more than two candidates.

"They learn how to think mathematically, which turns out to be a practical skill," says Pinchbeck.

Advise on course structure

Robert DeLiberato, St. Joseph's U.

- Be open to digressions.
- Decide ahead of time that you will learn something new from your students.

Margaret Morrow, SUNY Plattsburgh

- Make the course as entertaining and lighthearted as you can.
- Give writing assignments in which students reflect on the relevance of course material to their lives.

Advise on course structure

Ryan Sieve, University of Kansas

- Be interactive: Class time should not always be lecture.
- Tell stories: Sometimes, this is the best way to convey a point during lecture.

Deirdre Smith, University of Arizona

- Most students think that math is about mindlessly manipulating symbols, like they did in algebra. Emphasize that most of the math you will do will involve little algebra.
- Group work and projects.
- Use a variety of teaching styles.

Advise on course structure

Fred Richman, Carol L. Walker, Robert J. Wisner, James W. Brewer (Florida Atlantic University et. al.)

- Allow for leisurely exploration in place of drill—this is a course in mathematics appreciation.
- The spirit of mathematics can be communicated by means of simple ideas and problems without scaring or boring the students.
- It is sometimes difficult for a mathematician to realize that an idea, depending on a parameter, may be understood in general by examining it for 5, yet be totally opaque when stated in terms of n.
- Instructors should choose whatever topics please them. If the teacher doesn't like the material, the students certainly will not.

The Audience

A few means to engage this audience include

- activities
- worksheets (active learning)
- interactive demonstrations
- mathematics in context
- inquiry-based approaches
- puzzles and games
- technology

Definition

A **gnomon** to a figure F is a connected figure G which when attached (without overlap) to F produces a new figure similar to F.

Example

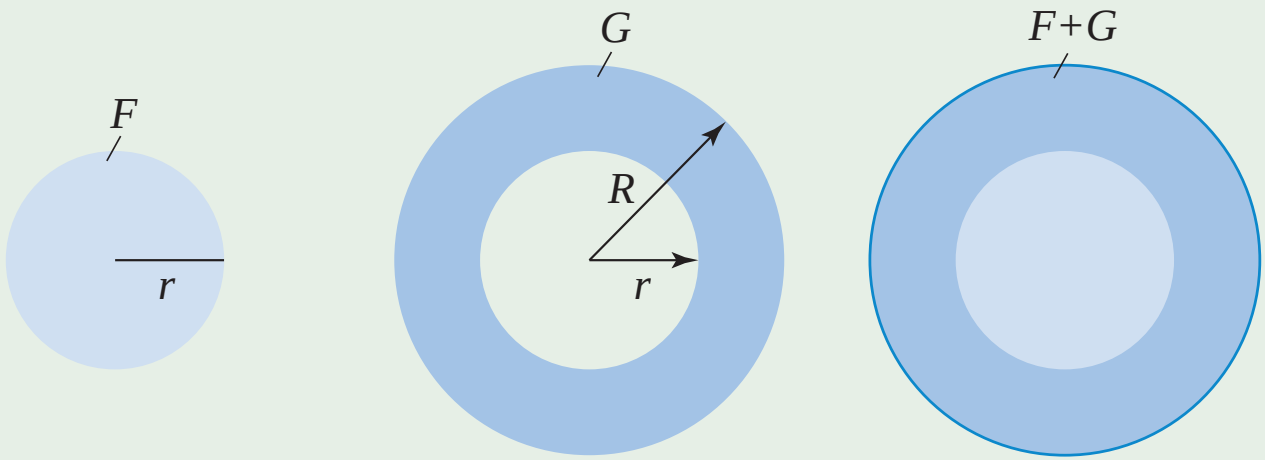


Figure: G is a gnomon to F

Activities

22 Using grid paper and scissors, cut out a 3 by 3 square. Then, cut out as many different *types* of gnomons to the square as you can.

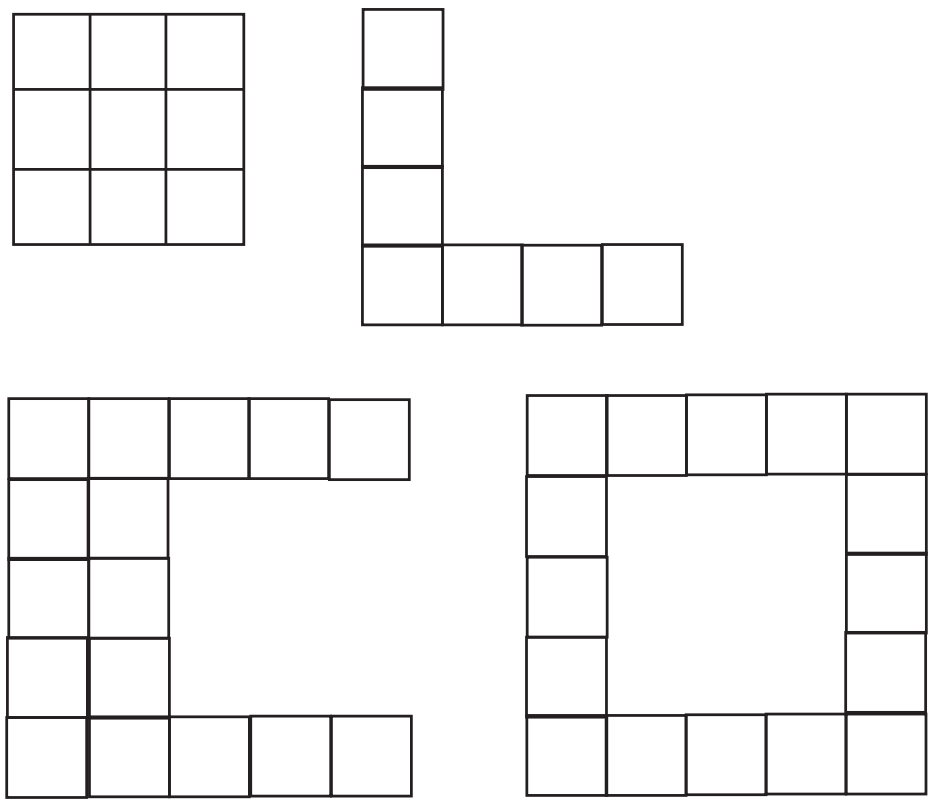


Figure: The three types of gnomons to a 3 by 3 square.

Activities

23 Repeat the above process with a 3 by 6 rectangle.

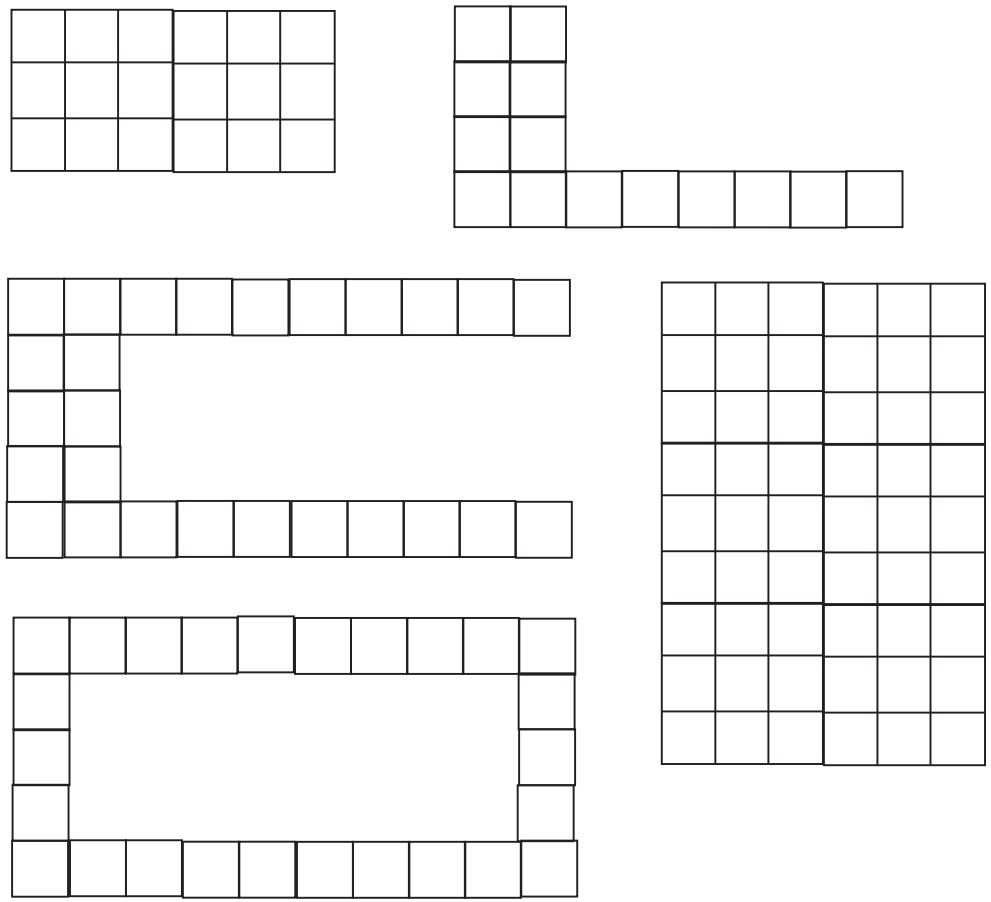


Figure: The four types of gnomons to a 3 by 6 rectangle.

Activities

24 (a) Do squares have square gnomons?
(b) Do any squares have rectangular gnomons?
(c) Does a 3 by 6 rectangle have a square gnomon?

25 3 by 6 rectangles do not have square gnomons.
(a) Why not?
(b) Is there *any* rectangle which does have a square gnomon? If so, determine the ratio of its longer side to its shorter side. If not, explain why not.

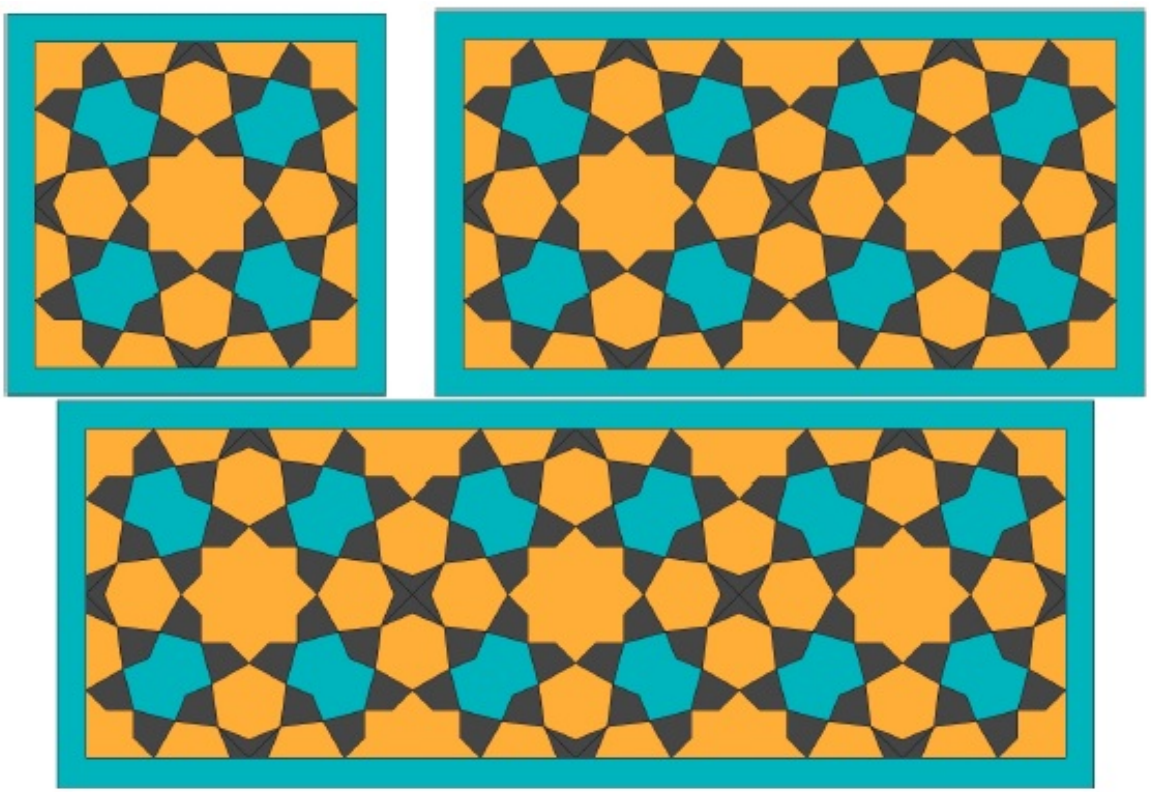
Random Sampling is Good

Rectangle Activity (handout)

Activities

Linear Growth

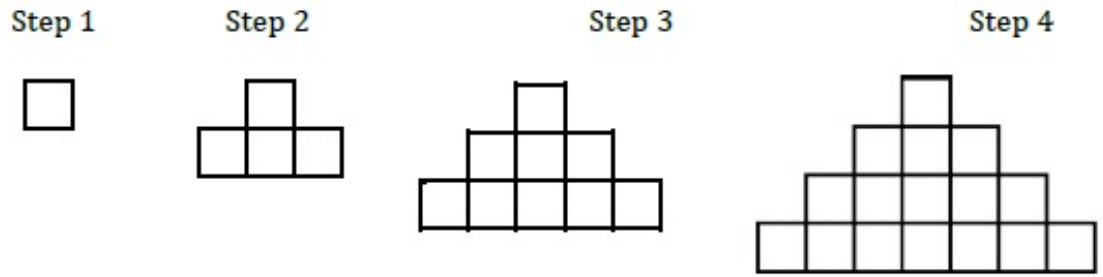
The Art of Mathematics introduces linear growth via the frieze pattern on the Darbi-i Imam shrine (count the turquoise octagons).



Activities

Linear Growth

Of course, there are an infinite number of ways to introduce linear growth. Consider the perimeters in the sequence shown (4, 10, 16, 22, ...).



Linear Growth

Other standard examples:

- A car rental costs \$b per day plus \$m per mile.
- Service charges on a checking account are \$b per month plus \$m per check.
- Joe descends from Mount X having height of b at a rate of m feet per minute.

Activities

Linear vs. Exponential Growth - Mathematics Vision Project, 2012

Our family has a small pool for relaxing in the summer that holds 1500 gallons of water. I decided to fill the pool for the summer. When I had 5 gallons of water in the pool, I decided that I didnt want to stand outside and watch the pool fill, so I wanted to figure out how long it would take so that I could leave, but come back to turn off the water at the right time. I checked the flow on the hose and found that it was filling the pool at a rate of 2 gallons every 5 minutes. How many gallons of water will be in the pool after 50 minutes? How many minutes will it take to fill the pool? Justify your answer with a mathematical model of the problem situation.

Activities

Linear vs. Exponential Growth - Mathematics Vision Project, 2012

At the end of the summer, I decide to drain the swimming pool. I noticed that it drains faster when there is more water in the pool. That was interesting to me, so I decided to measure the rate at which it drains. I found that it was draining at a rate of 3% every 5 minutes. How many gallons are left in the pool after 50 minutes? About how many minutes will it take to have less than 1000 gallons in the pool? Justify your answer with a mathematical model of the problem situation.

Activities

Linear vs. Exponential Growth - Mathematics Vision Project, 2012

My little sister, Savannah, is three years old and has a piggy bank that she wants to fill. She started with five pennies and each day when I come home from school, she is excited when I give her three pennies that are left over from my lunch money. How much money will Savannah have after 10 days? How many days will it take for her to have at least $1.50? Justify your answer with a mathematical model of the problem situation.

Activities

Linear vs. Exponential Growth - Mathematics Vision Project, 2012

I am more sophisticated than my little sister. I save my money in a bank account that pays me 3% interest on the money in the account at the end of each month. (If I take my money out before the end of the month, I don't earn any interest for the month.) I started the account with $50 that I got for my birthday. How much money will I have in the account at the end of 10 months? How many months will it take to have at least $100? Justify your answer with a mathematical model of the problem situation.

Your Turn

The goal of your next unit is to have students understand the difference between linear and exponential growth. What strategies would you use to help students meet this goal? Draft a class activity to do this.

Linear vs. Exponential Growth

22 Determine a pattern and predict what numbers come next. Which of these sequences would you say are similar to each other? What properties do they have in common?

(1) 2, 4, 6, 8, _____, _____, _____, _____, ...
(2) -1, 1, -1, 1, _____,_____, _____, _____, ...
(3) -1, -4, -7, _____,_____, _____, _____, ...
(4) 1, 4, 9, 16, _____,_____, _____, _____, ...
(5) 2, 4, 8, 16, _____,_____, _____, _____, ...
(6) 1, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, _____,_____, _____, _____, ...
(7) 1, 0, 1, 0, 1, _____,_____, _____, _____, ...
(8) 1,$\frac{1}{4}$, $\frac{1}{16}$, _____,_____, _____, _____, ...
(9) 2, -6, 18, _____,_____, _____, _____, ...
(10) $5\frac{1}{2}$, 6, $6\frac{1}{2}$, _____,_____, _____, _____, ...

Linear vs. Exponential Growth

23 Find the next term in each sequence. Then, categorize each sequence as either arithmetic, geometric, or neither.

$P_1 = 3$
$P_2 = 5$
$P_3 = 7$
$P_4 = 9$
$P_5 = 11$
$P_6 =$ _____

$A_1 = 6$
$A_2 = 3$
$A_3 = 3/2$
$A_4 = 3/4$
$A_5 = 3/8$
$A_6 =$ _____

$C_1 = 8$
$C_2 = 7$
$C_3 = 6$
$C_4 = 5$
$C_5 = 4$
$C_6 =$ _____

$H_1 = 2/9$
$H_2 = 2/3$
$H_3 = 2$
$H_4 = 6$
$H_5 = 18$
$H_6 =$ _____

$M_1 = 4$
$M_2 = 5$
$M_3 = 7$
$M_4 = 10$
$M_5 = 14$
$M_6 =$ _____

$D_1 = 3$
$D_2 = 3$
$D_3 = 6$
$D_4 = 18$
$D_5 = 72$
$D_6 =$ _____

Linear vs. Exponential Growth

24 Find the indicated term of each sequence.

$P_1 = 3$
$P_2 = 5$
$P_3 = 7$
$P_4 = 9$
$P_5 = 11$
$P_{20} =$ _____

$A_1 = 6$
$A_2 = 3$
$A_3 = 3/2$
$A_4 = 3/4$
$A_5 = 3/8$
$A_{14} =$ _____

$C_1 = 8$
$C_2 = 7$
$C_3 = 6$
$C_4 = 5$
$C_5 = 4$
$C_{30} =$ _____

$H_1 = 2/9$
$H_2 = 2/3$
$H_3 = 2$
$H_4 = 6$
$H_5 = 18$
$H_{16} =$ _____

$M_1 = 4$
$M_2 = 5$
$M_3 = 7$
$M_4 = 10$
$M_5 = 14$
$M_{15} =$ _____

$D_1 = 3$
$D_2 = 3$
$D_3 = 6$
$D_4 = 18$
$D_5 = 72$
$D_{10} =$ _____

Linear vs. Exponential Growth

25 Find both an explicit and recursive description of each sequence.

$P_1 = 3$
$P_2 = 5$
$P_3 = 7$
$P_4 = 9$
$P_5 = 11$

Recursive:
$P_N =$ _____

Explicit:
$P_N =$ _____

$A_1 = 6$
$A_2 = 3$
$A_3 = 3/2$
$A_4 = 3/4$
$A_5 = 3/8$

Recursive:
$A_N =$ _____

Explicit:
$A_N =$ _____

$C_1 = 8$
$C_2 = 7$
$C_3 = 6$
$C_4 = 5$
$C_5 = 4$

Recursive:
$C_N =$ _____

Explicit:
$C_N =$ _____

Linear vs. Exponential Growth

26 Find both an explicit and recursive description of each sequence.

$H_1 = 2/9$
$H_2 = 2/3$
$H_3 = 2$
$H_4 = 6$
$H_5 = 18$

Recursive:
$H_N =$ _____

Explicit:
$H_N =$ _____

$M_1 = 4$
$M_2 = 5$
$M_3 = 7$
$M_4 = 10$
$M_5 = 14$

Recursive:
$M_N =$ _____

Explicit:
$M_N =$ _____

$D_1 = 3$
$D_2 = 3$
$D_3 = 6$
$D_4 = 18$
$D_5 = 72$

Recursive:
$D_N =$ _____

Explicit:
$D_N =$ _____

Context: Speeding Tickets

Huskyville police cited 70 people for speeding in the year 2000. Due to growth in the town, there are two predictions as to how the number of such citings will grow over the next 25 years.

Prediction A: The number of people cited will grow at a rate of 4 people per year.

1. According to prediction A, how many people will be cited by the Huskyville police in the year 2025?
2. According to prediction A, how many people will have been cited by the Huskyville police during the time period 2000-2025?

Prediction B: The number of people cited will grow at a rate of 4% per year.

1. According to prediction B, how many people will be cited by the Huskyville police in the year 2025?
2. According to prediction B, how many people will have been cited by the Huskyville police during the time period 2000-2025?

Context: Flor-Mart

3 (a) Flo, a Flor-mart employee, earns \$40,000 during her first year of employment. Flo gets a fixed increase in salary each year of \$500. Model Flo's salary F_N during her N^{th} year working both recursively and explicitly. Project Flo's salary during her 40th year of employment.

(b) Mo, a manager, is offered a different compensation package from Flor-Mart. He also receives \$40,000 during his first year of employment. He receives an increase in salary each year of 1.25%. Model Mo's salary S_N after N years both recursively and explicitly.

(c) Compare salaries for Flo and Mo during their first year of employment.

(d) Project each worker's salary during their 40th year of employment.

Extension: Lifetime Earnings?

If Flo and Mo both retire after 40 years of work at Flor-mart, what are their lifetime earnings?

Context

Even-Numbered Fall Terms

Polling (1998 St. Cloud Times)
- Keep it local; keep it current; keep it real

Roth or Regular IRA?

You have $2000 with which to start either a regular IRA or a Roth IRA.

- The money will be invested at an 5% annual interest rate.
- In the **Roth IRA**, you pay tax now and let the money accumulate tax-free.
- In the **regular IRA**, you let the money accumulate tax-deferred.
- Assume you will be 'middle class' (i.e. in a 28% tax bracket) both now and when you retire 45 years from now.

Which is the better IRA?

Activities

Fun data to summarize

An interesting data set to analyze is how old students believe you are. Have them guess your age on a post-it note and paste it on a numberline on the board (i.e. order the data set in form of a bar graph). This (sorted) data set allows for easy summary via median, mode, five-number summary (box plot). Repeat on an older person of interest. Compare summaries.

Is It Fair?

There are many games to propose to students that may or may not be fair (or in their favor). Write several of these and hold a mini-competition where students go to different 'stations' with these games and write whether it is fair to them and they want to play and, if so, how much they are willing to bet (up to a max bet...allows them to hedge if they are unsure). Best expected outcome wins.

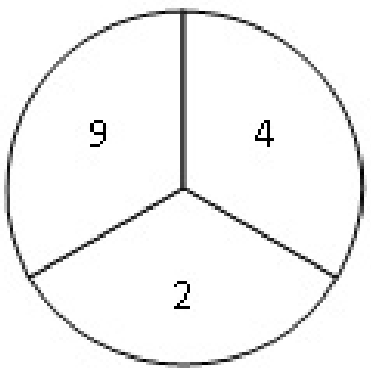
Activities

Is It Fair?

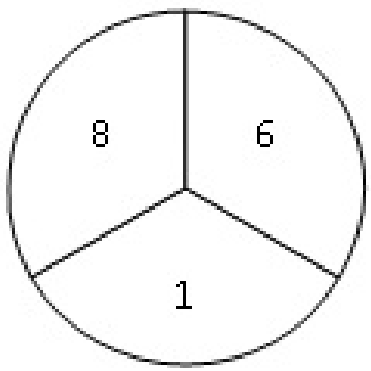
Game A. Two players each roll a standard six-sided die. Player 1 will win if the difference of the two rolls is 0, 1, or 2. Player 2 will win if the difference of the two rolls is 3, 4, or 5.

Game D. A fair coin is flipped nine times. Player 1 will win if it lands heads exactly 4 or 5 times. Otherwise player 2 will win.

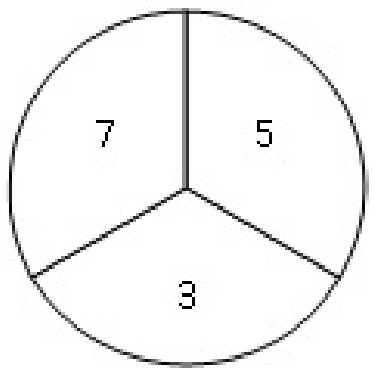
Game E. Player 1 spins wheel 1, player 2 spins wheel 2, and player 3 spins wheel 3. The player spinning the highest number wins.



Wheel 1



Wheel 2



Wheel 3

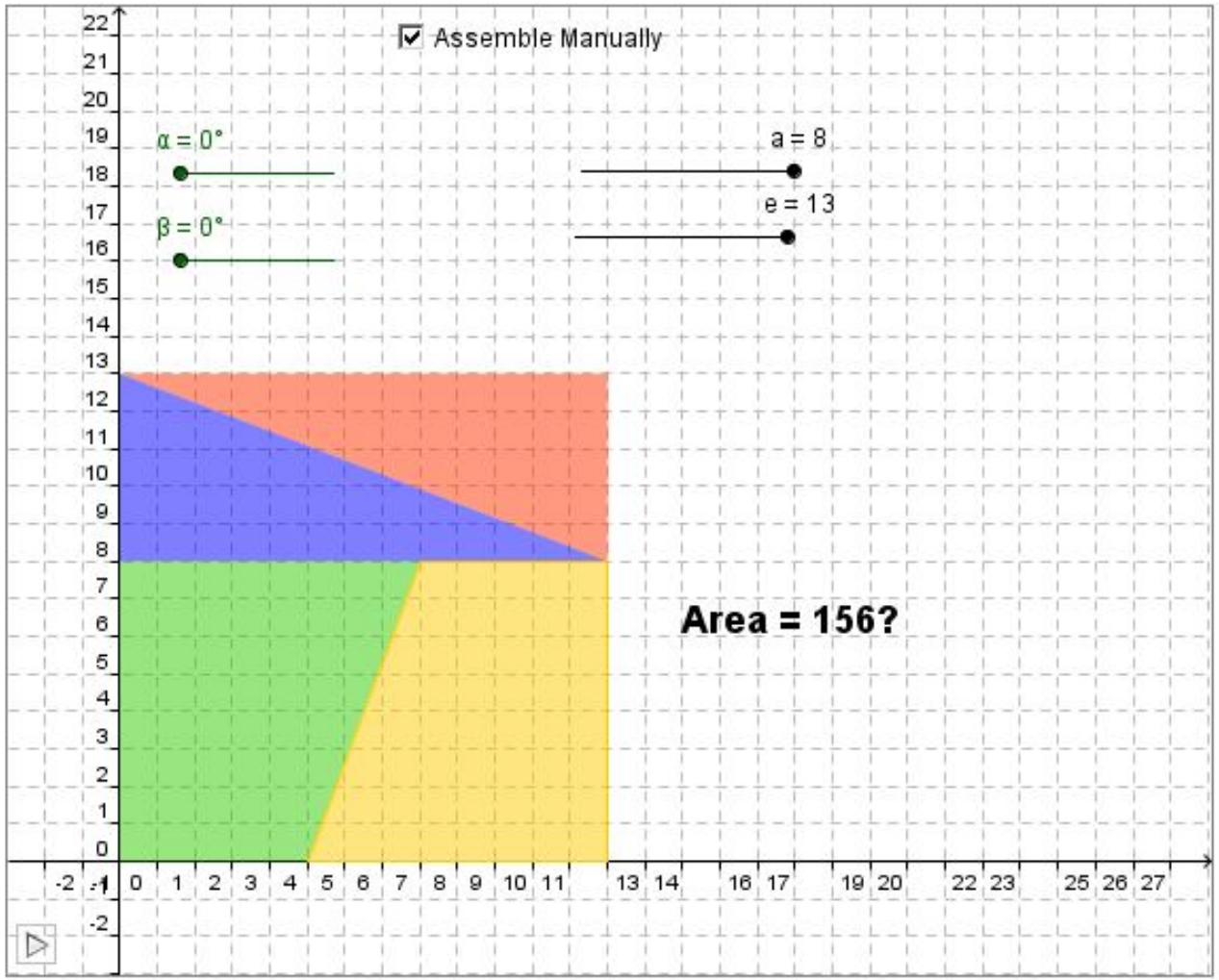
Activities

An activity/topic that didn't work well

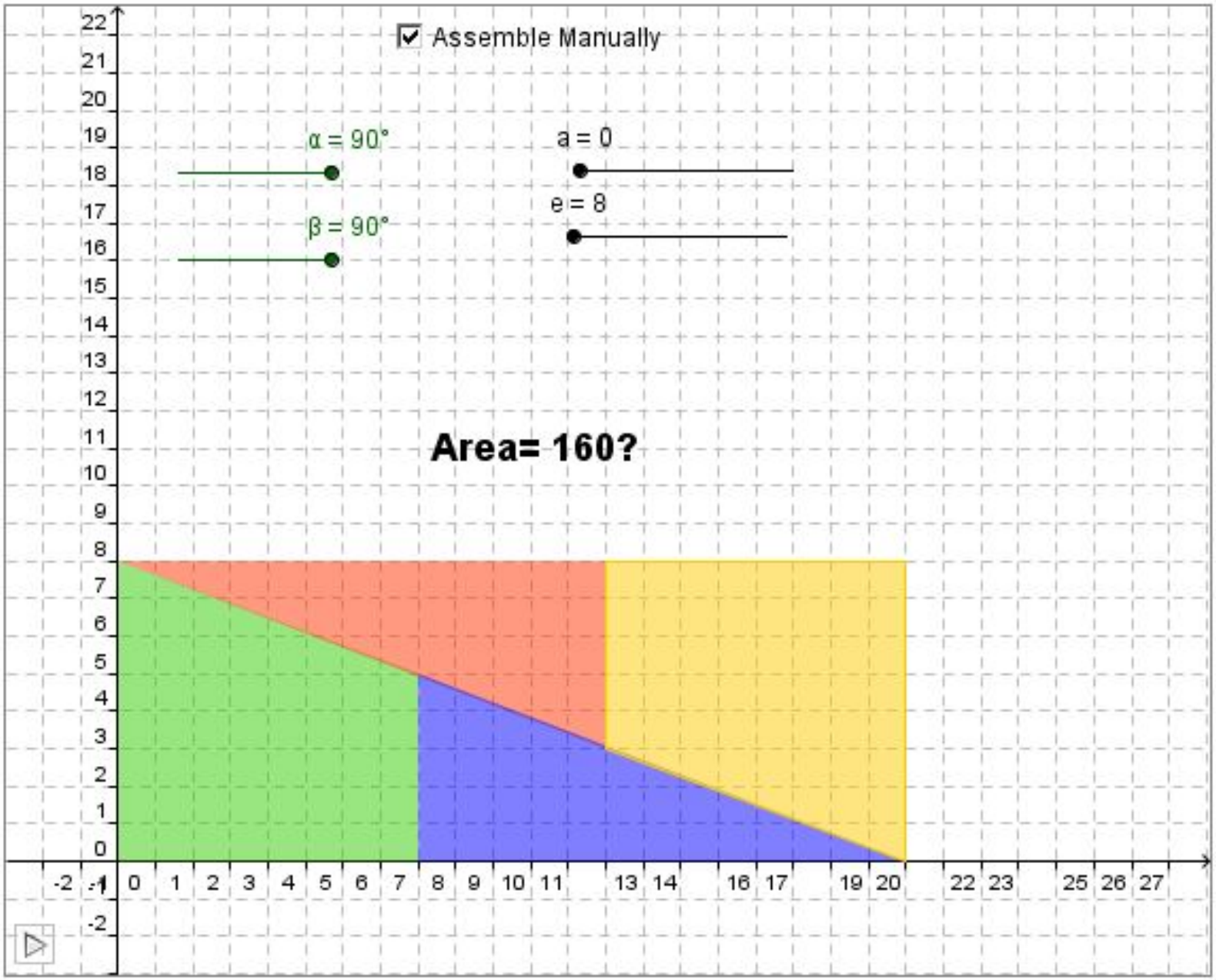
Twice I had students try to measure how noncommutative the dihedredral groups D_N are. They computed some entries and then looked for patterns in the Cayley tables (that I highlighted!) for these groups. Students were able to compute entries, but struggled to find patterns in the Cayley tables. Worse, they were somewhat unmotivated by the very question (which the rest of us find pretty interesting).

Puzzles

For a Fibonacci unit...Fibonacci's Missing Area also called The Triangle Paradox



Puzzles



Interactive Demonstrations

Capture-Recapture

Counting beans.

Fair Division

Make a large batch of play dough (2 or 3 colors). Illustrate division of the play dough using various fair-division algorithms (Divider Chooser, Lone Divider with N players, Lone Chooser with N players, and the Last Diminisher Method) with randomly called upon students in the class. Along the way, have participants explain WHY fair shares result.

Frieze Patterns

A type 1g. Have students use drawing software to construct each type of frieze or wallpaper pattern. Or have them collect examples.

Interactive Demonstrations

Traveling-Salesman Problem

Find a mileage table like that shown below for cities in your state. One by one, have students implement the nearest-neighbor algorithm to find an approximate solution to the TSP using this data. Post a large map of your state with pins identifying locations of these cities. With several pieces of string, students can model the solution. Compare with optimal. If optimal, have carefully chosen example to turn to.

Appleton											
235	Blackduck										
252	349	Chatfield									
130	121	306	Detroit Lakes								
253	161	242	202	Duluth							
145	298	105	248	232	Mankato						
147	244	106	201	156	77	Minneapolis					
110	226	143	183	182	79	37	Montrose				
180	95	264	88	140	213	159	141	Pine River			
79	172	215	103	185	145	110	88	101	Sauk Centre		
157	250	102	207	151	81	10	47	165	119	St Paul	
178	259	107	216	143	102	31	68	174	129	21	Stillwater

Interactive Demonstrations

Monty Hall

The controversial *Let's Make a Deal problem* began when Marilyn vos Savant published a puzzle in her Parade Magazine column. One of her readers posed the following question: "Suppose you're on a game show, and you're given a choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say number 1, and the host, who knows what's behind the doors, opens another door, say number. 3, which has a goat. He says to you, 'Do you want to pick door number 2?' Is it to your advantage to switch your choice of doors?" Ms. Savant, who's listed in the Guinness Book of World Records Hall of Fame for "Highest IQ" (228), answered "Yes." She received an estimated 10,000 letters in response, many arguing that she was wrong.... Was she?

Describe two interactive demonstrations you could use here.

Let's Make a Deal applets like this are easy to find.

Interactive Demonstrations

The Staged Voting Argument

Before class, select three students to stage an election argument.

Number of Voters	7	7	8	5
1st choice	A	C	D	B
2nd choice	B	B	C	A
3rd choice	C	A	B	C
4th choice	D	D	A	D

Start by illustrating to the class how A is clearly the best candidate using a 'survival of the fittest method' (i.e. plurality-with-elimination).

- Student 1 responds with "C'mon man, this is America. D has the most first-place votes and they should win. You mathematicians are nuts."
- Student 2 challenges this with something like "D!?! Most people seem to hate D. Look, in head-to-head races (i.e. pairwise comparisons), D would be beat by any other candidate. C is best."
- Student 3 then proposes a simple weighted points method (i.e. Borda count) showing how B should win.

Projects

Written projects, presentations, and posters are all common to this type of course.

Counting Funky Yahtzee Outcomes

Funky Yahtzee consists of using six (rather than five) dice each having N rather than six sides. N is the last digit of your student ID# + 7. There are 12 possible *types* of outcomes (Yahtzee, Five of a kind, Four and a Pair, Two Triples, Large Straight, etc.). The task is to determine (with explaination) the how many of each type of outcome are possible. An Excel spreadsheet (the Funky Checker) provides immediate feedback.

Funky Poker

Funky Poker is similar to Funky Yahtzee. Instead of 4 suits and 13 values in the deck, there are N values and k suits. Also, hands have six, rather than five, cards. [These parameters keep the web from being much help.]

Projects

Patterns in Famous Sequences

Guide students to use spreadsheets to investigate patterns and form conjectures regarding the Fibonacci, Lucas, and Padovan numbers. The Inverse Symbolic Calculator can help.

Buying a Home

Have students do a series of calculations related to the purchase of a home. For example,

- 28% of their monthly income based on median starting salary for their planned degree field.
- How much would need to be saved each month for last 5 years to have a 20% down payment on a specific home they have located? (+closing costs?)
- Assuming 20% down, find monthly payment (15 and 30 year options). Is the home affordable? (PITI?)
- Based on starting median income, determine maximum price of starter home possible.

Projects

Jackie Hall, Longwood University

Students are asked to prove Pythagoras' theorem to someone else and to write about the experience. One grade is for the proof itself. It is graded for correctness just as if they were answering a test question "Prove the Pythagorean Theorem." The other grade is given for the story they tell of proving the theorem.

Four-Color Problem

Choose a state with 15-24 counties (Arizona, Maine, Nevada, New Jersey, Wyoming, Maryland). Draw a graph model showing the counties as vertices. Connect counties with edges with those counties that share a common boundary. Develop a coloring scheme based on your graph and color your map.

Technology

Spreadsheets

- Apportionment is a popular topic in a liberal arts math class. Using spreadsheets is the only way to go in doing the calculations. One can use a computer lab, assign homework, or guide a student in front of the class through a demonstration.
- Linear vs. Exponential Growth (simple vs. compound interest)
- Computing the standard deviation makes for an interesting lesson in definitions and software (STDEV vs. STDEVP). It also helps students to 'visualize' the algorithm used to compute and not get lost in computation.
- Computing quartiles and percentiles demonstrate the lack of a uniform definition.

GeoGebra/Sketchpad

1. Illustrate the construction of a rotocenter.
2. Steiner Points

Technology

Applets

Computing weighted voting power indices.

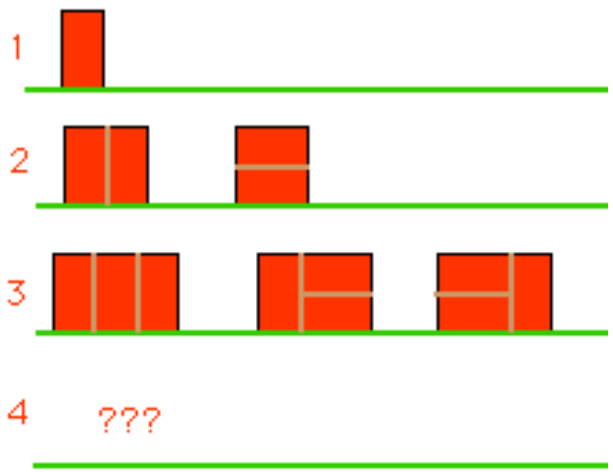
Web-based homework (mymathlab, webassign, etc.)

In general, these systems are fine, but not as valuable as they are to a more traditional algebra or calculus course.

Making Connections

Example

1 We want to build a brick wall using bricks of the usual size which are 2 units by 1 unit. If our wall is to be two units tall, how many patterns using 13 bricks are possible?

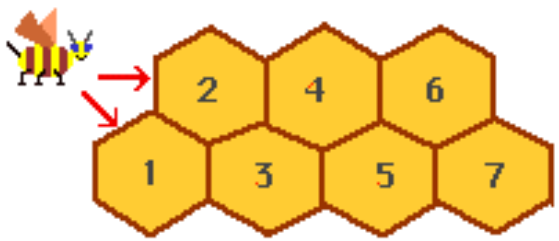


1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back

Making Connections

Example

2 A bee starts at the end of some cells in its hive. It can either start at cell 1 or 2 and moves only to the right (that is, only to a cell with a higher number in it). How many paths are there from the start to cell number 13?



1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back

Storytelling

A man is driving his car down a dirt road when all of a sudden the car starts limping along, as one of the tires is flat. He stops the car by a tree, under which a person is reading. The driver gets out of the car and takes the hubcap and the four lugnuts off the flat tire. He puts the four lugnuts in the hubcap and places the hubcap on the ground. He then goes to remove the flat tire. As he is removing the flat tire, a strong wind blows the hubcap and four lugnuts down a hill, into a deep river below. The upset driver asks the reader where the nearest gasoline station is. The reader answers that it is ten miles down the road. The driver says: What am I going to do?

The reader says: Why dont you take one of the four lugnuts off of each of your other three tires, thereby giving you three lugnuts to put on the spare, leaving three lugnuts on each of the other three tires.

The driver looks up at the reader and says: Thats brilliant! How did you think of that?

The reader says: This is an application of the lone-chooser method for 4 players! The spare tire acts as the lone-chooser!

Developing an MLA course

If a course already exists at your institution,

- ask your colleagues how flexible the format is (so that you can boldly experiment with content and pedagogy).
- it can often be easy to 'take leadership' in making changes to this course.
- find out what you can about your audience prior to teaching the course (majors, preparedness, etc.).
- use the efforts of others in developing the curriculum (either on your campus or elsewhere). That is, don't reinvent the wheel unless and until you want to.

Your Turn

A) Suppose an MLA course does not exist at your institution. How might you convince your colleagues of the value in introducing one?
B) Suppose an MLA course does exist at your institution. What would you change about it?

Wrap-up

Questions?

Email

drbuske@stcloudstate.edu