3 Extra Credit Problem

2. In this problem you will find a tight bound on this sum

$$\sum_{i=2}^{n} \frac{i}{\log_2 i}$$

using any technique.

a) State a simplified upper bound on the sum. Show your work.

$$\sum_{i=2}^{n} \frac{i}{\log_2 i} \to \sum_{i=2}^{n} \frac{n}{\log_2 n} \to \frac{n}{\log_2 n} * (n-2+1)$$

$$\frac{n}{\log_2 n} * (n-2+1) \to \frac{n(n-1)}{\log_2 n} \to \frac{n^2-n}{\log_2 n} \le \frac{n^2}{\log_2 n} \in O\left(\frac{n^2}{\log_2 n}\right)$$

b) State a simplified lower bound on the sum. Show your work.

$$\sum_{i=2}^{n} \frac{i}{\log_2 i} \ge \sum_{i=\lfloor \frac{n}{2} \rfloor+1}^{n} \frac{n}{\log_2 n} \to \sum_{i=\lfloor \frac{n}{2} \rfloor+1}^{n} \frac{\lfloor \frac{n}{2} \rfloor + 1}{\log_2 \lfloor \frac{n}{2} \rfloor + 1} \to \frac{\lfloor \frac{n}{2} \rfloor + 1}{\log_2 \lfloor \frac{n}{2} \rfloor + 1} \sum_{i=\lfloor \frac{n}{2} \rfloor+1}^{n} 1$$

$$\sum_{i=\lfloor \frac{n}{2} \rfloor+1}^{n} 1 \to \left\lceil \frac{n}{2} \right\rceil$$

$$\frac{\lfloor \frac{n}{2} \rfloor + 1}{\log_2 \lfloor \frac{n}{2} \rfloor + 1} * \left\lceil \frac{n}{2} \right\rceil$$

$$\frac{\lfloor \frac{n}{2} \rfloor + 1}{\log_2 \lfloor \frac{n}{2} \rfloor + 1} * \left\lceil \frac{n}{2} \right\rceil \ge \frac{\frac{n}{2}+1}{log_2 \frac{n}{2}+1} * \frac{n}{2} = \frac{n}{2} * \frac{\frac{n}{2}+1}{log_2 n - log_2 2} = \frac{\frac{n}{2}+1}{log_2 n - 1} * \frac{n}{2} = \frac{1}{4} * \frac{n^2}{log_2 n - 1}$$

$$so,\ \frac{1}{4} * \frac{n^2}{log_2 n - 1} \in \Omega\left(\frac{n^2}{log_2 n}\right) \blacksquare$$