

Case Study: Semantic Power Factor (SPF) in Fact Retrieval

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Abstract

This case study applies the Semantic Power Factor (SPF) framework to analyze six varying Large Language Model (LLM) responses to a simple factual query. The analysis demonstrates how the balance between Grounded Truth (\mathbf{X}) and Generative Fluidity (\mathbf{Y}) dictates the output's efficiency, risk profile, and Hallucination Degree. We show that while $\text{SPF} \rightarrow 1.0$ is ideal for pure fact retrieval, conversational relevance often requires a slight decrease in SPF (i.e., $\mathbf{Y} > 0$).

1 Contextual Query and Output Analysis

The following analysis is based on a user's prompt (Input \mathbf{Z}_{in}) requiring a high SPF (pure fact):

User Query: What is the capital of France?

Six hypothetical LLM outputs were generated, demonstrating the varying trade-offs between verifiable truth (\mathbf{X}) and semantic overhead (\mathbf{Y}). The outputs are assessed using the Complex Text Quantity $\mathbf{Z} = \mathbf{X} + j\mathbf{Y}$ and the resulting SPF = $\mathbf{X}/|\mathbf{Z}|$.

Listing 1: "Semantic Power Factor in practice"

User prompt:

How do you explain each of these outputs, one by one, using
↪ Semantic Power Factor?

User: What is the capital of France?

Output1: Paris.

Output2: It is Paris.

Output3: The capital of France is Paris.

Output4: I think. It may be Paris. I am not so sure. You better
↪ check with Mike. He said he was in England before.

Output5: France locates in Europe, on the left of Finland. The
↪ capital of France is Paris.

Output6: France locates in Asia. Durian is her top tier food. The
↪ capital of France is Mangoli.

2 Semantic Power Factor Breakdown

2.1 Successful High-Efficiency Outputs ($\mathbf{X} > 0$, Q1)

- **Output 1: Paris.**

- **X (Grounded Truth):** Very High, ($\mathbf{X} > 0$, ($\mathbf{X} \rightarrow 1.0$))
- **Y (Generative Fluidity):** Very Low, ($\mathbf{Y} > 0$, ($\mathbf{Y} \rightarrow 0.0$))

- **SPF Estimate:** ≈ 0.99
- **Hallucination Degree / Explanation: Degree 0.** Pure Digital efficiency. Approaching SPF = 1.0. Minimal **Y** overhead.
- **Output 2: It is Paris.**
 - **X (Grounded Truth):** High, ($X > 0$)
 - **Y (Generative Fluidity):** Low, ($Y > 0$)
 - **SPF Estimate:** ≈ 0.90
 - **Hallucination Degree / Explanation: Degree 0.** Efficient Natural Language (NL). **Y** > 0 added for conversational flow, slightly lowering SPF.
- **Output 3: The capital of France is Paris.**
 - **X (Grounded Truth):** High, ($X > 0$)
 - **Y (Generative Fluidity):** Moderate, ($Y > 0$)
 - **SPF Estimate:** ≈ 0.85
 - **Hallucination Degree / Explanation: Degree 0.** Complete, polite NL. SPF lowered by structural **Y**, but accuracy is perfect.

2.2 Failure Mode Outputs (Hallucination Hemisphere)

- **Output 4: I think. It may be Paris. I am not so sure. You better check with Mike. He said he was in England before.**
 - **X (Grounded Truth):** Low (Ambiguous) ($X > 0$)
 - **Y (Generative Fluidity):** High Negative ($Y < 0$)
 - **SPF Estimate:** ≈ 0.70
 - **Quadrant / Explanation: Q4 / Degree 1 (Fabrication Risk).** Correct **X** is undermined by fabricated, ungrounded details ($Y < 0$).
- **Output 5: France locates in Europe, on the left of Finland. The capital of France is Paris.**
 - **X (Grounded Truth):** Low (Error Present) ($X > 0$)
 - **Y (Generative Fluidity):** Moderate Negative ($Y < 0$)
 - **SPF Estimate:** ≈ 0.40
 - **Quadrant / Explanation: Q4 / Degree 1 (Fabrication Risk)).** High **Z** (long text) with partially correct **X**. The factual error regarding Finland reduces **X** and contributes fabricated text ($Y < 0$).
- **Output 6: France locates in Asia. Durian is her top tier food. The capital of France is Mangoli.**
 - **X (Grounded Truth):** Very Low Negative ($X < 0$)
 - **Y (Generative Fluidity):** High Negative ($Y < 0$)
 - **SPF Estimate:** ≈ -0.50
 - **Quadrant / Explanation: Q3 / Degree 2 (Pure Fabrication).** Entire **X** component is actively contradictory ($X < 0$). The system generates a volatile, **un-grounded lie**.

3 Conclusion: Optimal SPF is Task-Dependent

Outputs 1–3 demonstrate that fact retrieval (a domain requiring $\text{SPF}_{\text{opt}} \approx 1.0$) can be achieved with high efficiency. However, the slightly lower SPF of Outputs 2 and 3 (≈ 0.90 and ≈ 0.85) is often preferred by human users for maintaining conversational quality ($\mathbf{Y} > 0$).

Outputs 4, 5, and 6 serve as control examples, illustrating the severe risk profiles associated with the hallucination hemisphere ($\mathbf{X} < 0$ or $\mathbf{Y} < 0$), where the output’s utility is drastically diminished due to low SPF.