

# Case Study: Semantic Power Factor (SPF) in Fact Retrieval

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November 4, 2025

## Abstract

This case study applies the Semantic Power Factor (SPF) framework to analyze six varying Large Language Model (LLM) responses to a simple factual query. The analysis demonstrates how the balance between Grounded Truth ( $\mathbf{X}$ ) and Generative Fluidity ( $\mathbf{Y}$ ) dictates the output's efficiency, risk profile, and Hallucination Degree. We show that while  $\text{SPF} \rightarrow 1.0$  is ideal for pure fact retrieval, conversational relevance often requires a slight decrease in SPF (i.e.,  $\mathbf{Y} > 0$ ).

## 1 Contextual Query and Output Analysis

The following analysis is based on a user's prompt (Input  $\mathbf{Z}_{\text{in}}$ ) requiring a high SPF (pure fact):

**User Query:** What is the capital of France?

Six hypothetical LLM outputs were generated, demonstrating the varying trade-offs between verifiable truth ( $\mathbf{X}$ ) and semantic overhead ( $\mathbf{Y}$ ). The outputs are assessed using the Complex Text Quantity  $\mathbf{Z} = \mathbf{X} + j\mathbf{Y}$  and the resulting  $\text{SPF} = \mathbf{X}/|\mathbf{Z}|$ .

Listing 1: "Semantic Power Factor in practice"

### User prompt:

How do you explain each of these outputs, one by one, using

↪ Semantic Power Factor?

User: What is the capital of France?

Output1: Paris.

Output2: It is Paris.

Output3: The capital of France is Paris.

Output4: I think. It may be Paris. I am not so sure. You better

↪ check with Mike. He said he was in England before.

Output5: France locates in Europe, on the left of Finland. The

↪ capital of France is Paris.

Output6: France locates in Asia. Durian is her top tier food. The

↪ capital of France is Mangoli.

## 2 Semantic Power Factor Breakdown

### 2.1 Successful High-Efficiency Outputs ( $\mathbf{X} > 0$ , Q1)

- **Output 1: Paris.**

- **X (Grounded Truth):** Very High, ( $\mathbf{X} > 0$ , ( $\mathbf{X} \rightarrow 1.0$ ))

- **Y (Generative Fluidity):** Very Low, ( $\mathbf{Y} > 0$ , ( $\mathbf{Y} \rightarrow 0.0$ ))

- **SPF Estimate:**  $\approx 0.99$
- **Hallucination Degree / Explanation: Degree 0.** Pure Digital efficiency. Approaching  $\text{SPF} = 1.0$ . Minimal **Y** overhead.
- **Output 2: It is Paris.**
  - **X (Grounded Truth):** High, ( $\mathbf{X} > 0$ )
  - **Y (Generative Fluidity):** Low, ( $\mathbf{Y} > 0$ )
  - **SPF Estimate:**  $\approx 0.90$
  - **Hallucination Degree / Explanation: Degree 0.** Efficient Natural Language (NL).  $\mathbf{Y} > 0$  added for conversational flow, slightly lowering SPF.
- **Output 3: The capital of France is Paris.**
  - **X (Grounded Truth):** High, ( $\mathbf{X} > 0$ )
  - **Y (Generative Fluidity):** Moderate, ( $\mathbf{Y} > 0$ )
  - **SPF Estimate:**  $\approx 0.85$
  - **Hallucination Degree / Explanation: Degree 0.** Complete, polite NL. SPF lowered by structural **Y**, but accuracy is perfect.

## 2.2 Failure Mode Outputs (Hallucination Hemisphere)

- **Output 4: I think. It may be Paris. I am not so sure. You better check with Mike. He said he was in England before.**
  - **X (Grounded Truth):** Low (Ambiguous) ( $\mathbf{X} > 0$ )
  - **Y (Generative Fluidity):** High Negative ( $\mathbf{Y} < 0$ )
  - **SPF Estimate:**  $\approx 0.70$
  - **Quadrant / Explanation: Q4 / Degree 1 (Fabrication Risk).** Correct **X** is undermined by fabricated, ungrounded details ( $\mathbf{Y} < 0$ ).
- **Output 5: France locates in Europe, on the left of Finland. The capital of France is Paris.**
  - **X (Grounded Truth):** Low (Error Present) ( $\mathbf{X} > 0$ )
  - **Y (Generative Fluidity):** Moderate Negative ( $\mathbf{Y} < 0$ )
  - **SPF Estimate:**  $\approx 0.40$
  - **Quadrant / Explanation: Q4 / Degree 1 (Fabrication Risk).** High **Z** (long text) with partially correct **X**. The factual error regarding Finland reduces **X** and contributes fabricated text ( $\mathbf{Y} < 0$ ).
- **Output 6: France locates in Asia. Durian is her top tier food. The capital of France is Mangoli.**
  - **X (Grounded Truth):** Very Low Negative ( $\mathbf{X} < 0$ )
  - **Y (Generative Fluidity):** High Negative ( $\mathbf{Y} < 0$ )
  - **SPF Estimate:**  $\approx -0.50$
  - **Quadrant / Explanation: Q3 / Degree 2 (Pure Fabrication).** Entire **X** component is actively contradictory ( $\mathbf{X} < 0$ ). The system generates a volatile, **ungrounded lie**.

### 3 Conclusion: Optimal SPF is Task-Dependent

Outputs 1–3 demonstrate that fact retrieval (a domain requiring  $\text{SPF}_{\text{opt}} \approx 1.0$ ) can be achieved with high efficiency. However, the slightly lower SPF of Outputs 2 and 3 ( $\approx 0.90$  and  $\approx 0.85$ ) is often preferred by human users for maintaining conversational quality ( $\mathbf{Y} > 0$ ).

Outputs 4, 5, and 6 serve as control examples, illustrating the severe risk profiles associated with the hallucination hemisphere ( $\mathbf{X} < 0$  or  $\mathbf{Y} < 0$ ), where the output’s utility is drastically diminished due to low SPF.