Reversible moist processes and CM1 microphysics

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The Goddard microphysics scheme in CM1 has separate functions for the saturation vapor pressure over liquid and ice. These can be derived from the Clausius Clapeyron equation,

$$\frac{de_{s\alpha}}{dT} = \frac{e_{s\alpha}L_\alpha}{R_v T^2},$$

where α represents either ice or liquid water. The definition is completed with the added constraint that $e_{si}(T_0) = e_{sl}(T_0)$ for $T_0 = 273.16$ being the triple point of water. When air is super-saturated the quantity of condensate is divided into liquid and solid depending on the temperature so that the fraction of ice is given by,

$$f_i = \begin{cases} 1, & T < T_{00}, \\ 1 - \dfrac{T - T_{00}}{T_0 - T_{00}}, & T_{00} \leq T < T_0, \\ 0, & T \geq T_0. \end{cases}$$

The total amount of condensate is calculated by defining a saturation vapor pressure for the mixture e_{sm} so that,

$$e_{sm} = f_i e_{si} + f_l e_{sl}.$$

We shall see that these choices have consequences for the form of a reversibly defined moist adiabat.

Consider the thermodynamic balance for a parcel of air per unit mass of dry air,

$$dQ = (c_{pd} + q_v c_{pv} + q_l c_l + q_i c_i)dT - \frac{1}{\rho}dp - L_v dr_l - L_s dr_i,$$

where q_α is the mixing ratio of species α. With a little rearrangement this may be written,

$$\frac{dQ}{T} = d\left\{ (c_{pd} + q_t c_l)\ln(T) - R_d \ln(p_d) - \frac{q_l L_v}{T} - \frac{q_i L_s}{T} - R_v\left(q_l \ln(e_{sl}) + q_i \ln(e_{si}) + q_v \ln(e)\right) \right\}$$
$$- R_v\left\{ \ln(e_{sl})dq_l + \ln(e_{si})dq_i + \ln(e)dq_v \right\}.$$

The first term is the entropy of the mixture, while the second represents entropy sources due to evaporation of unsaturated air, or condensation into super-saturated air. To demonstrate this, consider reversible transformations in the simple case where all the condensate is either liquid or solid depending
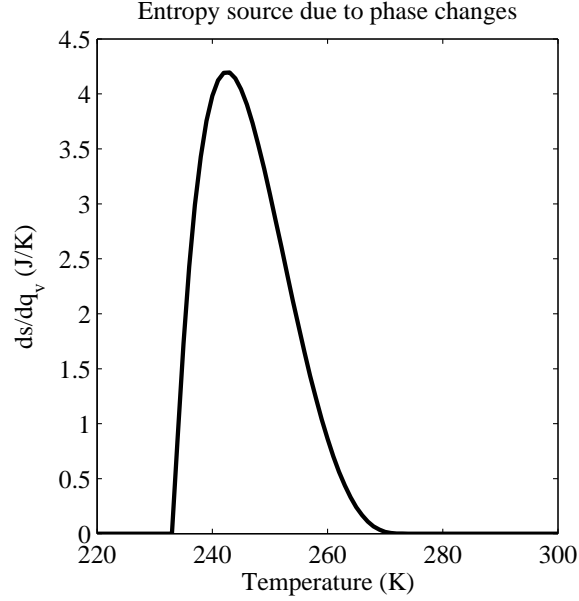
Figure 1: Co-efficient of the differential of the right hand side of (1).

on whether the temperature is above or below T_0. When the air is sub-saturated $dq_v = dq_l = dq_i = 0$, and hence the entropy s is conserved absent any external heating. When the air is saturated and above freezing $e = e_{sl}$ and $dq_l = -dq_v$, again giving conservation of entropy. Similar arguments can be made for the below freezing case.

The case is slightly more interesting when solid and liquid water are allowed to co-exist. For the case of the Goddard microphysics scheme, if the total water content of a parcel is conserved,

$$dq_l = -(1 - f_i)dq_v$$
$$dq_i = -f_i dq_v$$

Absent any heating we thus have that,

$$ds = R_v \ln \left\{ \frac{e}{e_{sl}^{(1-f_i)} e_{si}^{f_i}} \right\} dq_v$$

Thus, for a process to be truly reversible, the correct form of e_{sm} would be given by the denominator in the equation above. Since this is not true in the Goddard scheme, adiabatic parcel ascents have an entropy source given by,

$$ds = R_v \ln \left\{ \frac{(1 - f_i)e_{sl} + f_i e_{si}}{e_{sl}^{(1-f_i)} e_{si}^{f_i}} \right\} dq_v \tag{1}$$

The co-efficient of the differential on the right hand side is only dependent on temperature, and is shown in figure 1. An upper bound on the effect that this might have on a moist adiabat can be obtained by multiplying a typical surface mixing ratio of water vapor by the maximum value reached in figure 1. The change in entropy dues to phase changes is thus $O(0.1 \text{ J/K})$. At a pressure of 100 hPa
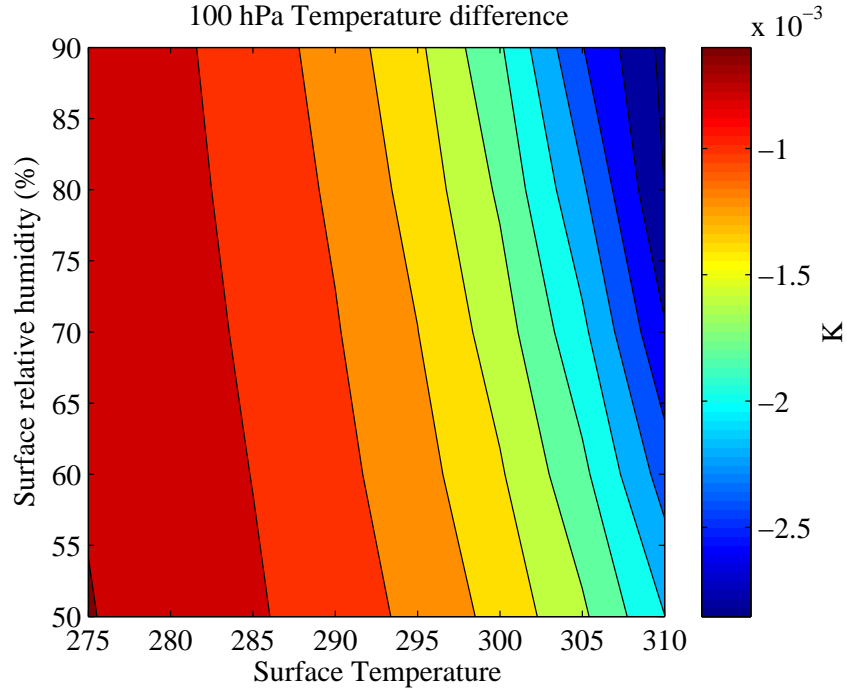
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Figure 2: Temperature difference due to entropy of phase changes at 100 hPa for parcels initialized at 1000 hPa with different temperatures and relative humidities.

and a water content of 30 g/kg this corresponds to a temperature difference of approximately $O(0.01)$ K.

   More quantitatively, we can assess the effect of the entropy source due to phase changes directly using parcel ascents. A number of parcel ascents were constructed starting at 1000 hPa with different temperatures and humidities, and lifting to 100 hPa. Entropy was conserved along the path, although the Goddard thermodynamics were used and thus the ascents are not truly adiabatic. The ascents were then used to estimate the entropy that should have been gained by integrating equation (1) up to 100 hPa. Finally, the corrected temperature at 100 hPa was calculated using the corrected value of the entropy. The difference in temperature that the entropy source produced is plotted in figure 2. This confirms the smallness of the initial estimate.