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Highlights

1. This paper analyzes the effects of four different imitation principles for updating strategies on the scope of disease transmission.
2. The spread size of infectious diseases is sensitive not only to imitation principles, but also to the structures of network topology and model parameters.
3. In networks with community structures, no matter which imitation principle individuals adopt, the counter-intuitive phenomenon exists.
4. In networks without community structures, the counter-intuitive phenomenon is influenced by imitation principles for updating strategies.

Effects of social network structures and behavioral responses on the spread of infectious diseases

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Abstract

Facing infectious diseases, the actions taken by individuals play an important role in the prevention and control of these diseases. Previous studies have shown that there is a counter-intuitive phenomenon in the spread of disease. A higher success rate of self-protection may not reduce the epidemic size. In this paper, the epidemic transmission process is studied using an evolutionary game model of complex networks. It is assumed that individuals choose their strategies by comparing their payoff with their neighbors' payoff in the process of disease transmission. We analyze the counter-intuitive phenomenon using four different imitation principles for updating strategies, and find that different imitation principles influence the scope and magnitude of the counter-intuitive phenomenon. In addition, we find that community structures also affect the counter-intuitive phenomenon. Specifically, the counter-intuitive phenomenon always exists in a network with community structures, but not necessarily in a network without community structures, where the existence of the counter-intuitive phenomenon is affected by imitation principles.

Keywords: Complex network, community structure, infectious disease, evolutionary game model, imitation principle, counter-intuitive phenomenon

2010 MSC: 00-01, 99-00

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1. Introduction

Infectious diseases have always posed a threat to human beings and have attracted widespread attention from scientists [1, 2, 3, 4]. According to the latest estimates from the United States Centers for Disease Control and Prevention, the World Health Organization and global health partners, the number of deaths caused by respiratory diseases due to seasonal influenza is about 650,000 per year[1]. Therefore, how to prevent infectious diseases has been the focus of researchers worldwide [3]. Preemptive vaccination is the most effective way to prevent and control infectious diseases. However, not all people involved in public health administration choose vaccination [5].

Whether people are vaccinated will be affected by their economic conditions, risks, living environment, and other factors [6]. In the case of infectious diseases with serious consequences, people usually adopt measures of vaccination. In the case of other infectious diseases, people may adopt self-protection methods such as paying attention to personal hygiene, and in the case of diseases with minor consequences, such as seasonal colds, most people adopt laissez-faire measures and may thus be protected from these diseases without incurring any costs [7]. The choice of a personal strategy is usually a trade-off between the cost of taking measures and the outcome of being infected [8]. On this basis, previous studies have input the strategies taken by individuals facing infectious diseases into a game model for analysis [9, 10, 11]. Game theory has been applied to predict the strategy of an individual from the costs and payoffs of actions and outcomes [3].

Many papers have been put forward to review and analyze the processes by which epidemics spread in complex networks [12, 13]. The majority of models of disease spread are based on disease statuses, such as the susceptible-infected (SI) model, the susceptible-infected-removed (SIR) model, and the susceptible-infected-susceptible (SIS) model [14]. With the development of research, a

[1] https://www.who.int/mediacentre/news/releases/2017/seasonal-flu/zh/

number of quantitative complex networks have been introduced to reveal the relationships between individuals [14, 15]. The structure of an interacting population can be represented by a complex network [16, 17]. Connected structures have been devised to demonstrate the framework [18]. The decisions of individuals are directly or indirectly influenced by their neighbors [19]. The behavior of an individual is affected by the course of an epidemic in a complex network [20].

Bauch et al. used a comprehensive model of game theory and infectious disease theory, which proved the conflict between individual utility and social utility as a whole [21]. A voluntary vaccination policy could not achieve the target of herd immunity. Bauch et al. pointed out that there was an interaction between the coverage of vaccination in social networks, the spread of epidemics, and the choice of individual immunization strategies [1]. Epidemics spread on the basis of the SIR model. A dynamic game model of the mechanism was established, and it was found that the scope of vaccination is affected not only by the epidemic scope of the disease but also by the degree of difficulty for individuals to imitate the immunization strategies of other individuals in the process of selecting their immunization strategy [1]. Wu et al. took into account the objective fact that vaccination could not achieve absolute immunity and used vaccination behavior and strategy selection as inputs of a game theory model [5]. In combination with the epidemic transmission process, it was found that improving the efficacy of vaccination could inhibit the spread of epidemics [5]. Zhang et al. believed that the measures taken by individuals in the face of infectious diseases included not only vaccination and negligence but also self-protection strategies [7]. They studied a dynamic game model of SIR epidemic transmission and found that self-protection occurred. After the success rate of an immunization strategy increased, the spread of epidemic disease increased, which resulted in a decline in the overall effectiveness of social networks. In other words, the Braess paradox appeared in the process of epidemic transmission, namely, the paradox that better conditions led to worse utility [7]. Ye et al. selected complex networks as spatial carriers according to the Parrondo paradox

[22]. In their study, a group game model was established, and the paradox of the influence of different network topologies of two-dimensional lattice networks, random networks, and scale-free networks on the Parrondo paradox was studied [22].

However, the existing literature does not take into account the effects of different imitation principles for updating strategies and network community structures on the scope of disease transmission. Many studies have shown that human behavior plays an important role in the spread of disease [23]. In real life, different groups adjust their strategies according to the payoffs of their neighbors and different strategy imitation principles. Studying the effects of different strategies on the spread of a disease can help people formulate better protective measures. Therefore, this study has refined the updating rules of strategies.

In this article, four different imitation principles for updating strategies are defined and analyzed on well-mixed (WM) networks, square-lattice (SL) networks, Erdös-Rényi (ER) networks, Barabási-Albert (BA) networks, Lancichinetti-Fortunato-Radicchi (LFR) networks, and Facebook networks. Not only are the effects of different imitation principles on disease transmission compared, but also the effects of community structures on disease transmission are studied. It is found that different strategies affect the scope and intensity of counter-intuitive phenomena.

This paper is basically an extension of the research [7]. Firstly, we study more imitation principles than reference [7]. We introduce different imitation principles into an epidemic process in complex networks. The three strategies of vaccination, self-protection, and laissez-faire are taken into account in the SIR model [7]. Moreover, we investigate the effects of the model parameters, network topologies, and imitation principles on the epidemic size of the disease. We employ more types of networks. It is shown that the spread sizes of infectious diseases in networks with community structures are more sensitive to imitation principles than that in networks without community structures. Furthermore, the influences of the same imitation principle on different networks are compared.

The effects of different imitation principles are also studied. In summary, we study more imitation principles and more types of networks, reaching more conclusions which are valuable. The rest of this paper is structured as follows. Section 2 introduces the process of building a dynamic game model based on the SIR epidemic transmission mechanism and four different imitation principles for updating strategies. Section 3 gives the experimental results for different network structures and imitation principles. Finally, Section 4 draws the conclusion of this study.

2. Model description

2.1. Epidemic propagation

During an epidemic spreading season, every individual can choose one of three strategies, namely, vaccination (Va), self-protection (Se) and laissez-faire (La). It is assumed that the individuals who adopt the vaccination strategy will never be infected by the disease during the epidemic season. The vaccination strategy is completely effective and provides perfect immunity. The individuals adopting the laissez-faire strategy will be susceptible to the epidemic disease. The individuals choosing the self-protection strategy will be divided into two groups. In one group, a fraction δ of individuals will never be infected. This means that they are successfully immune as a result of self-protection at a rate of δ. In the other group, a fraction (1-δ) of individuals will be susceptible. The overview of the epidemic propagation is shown in Fig.1.

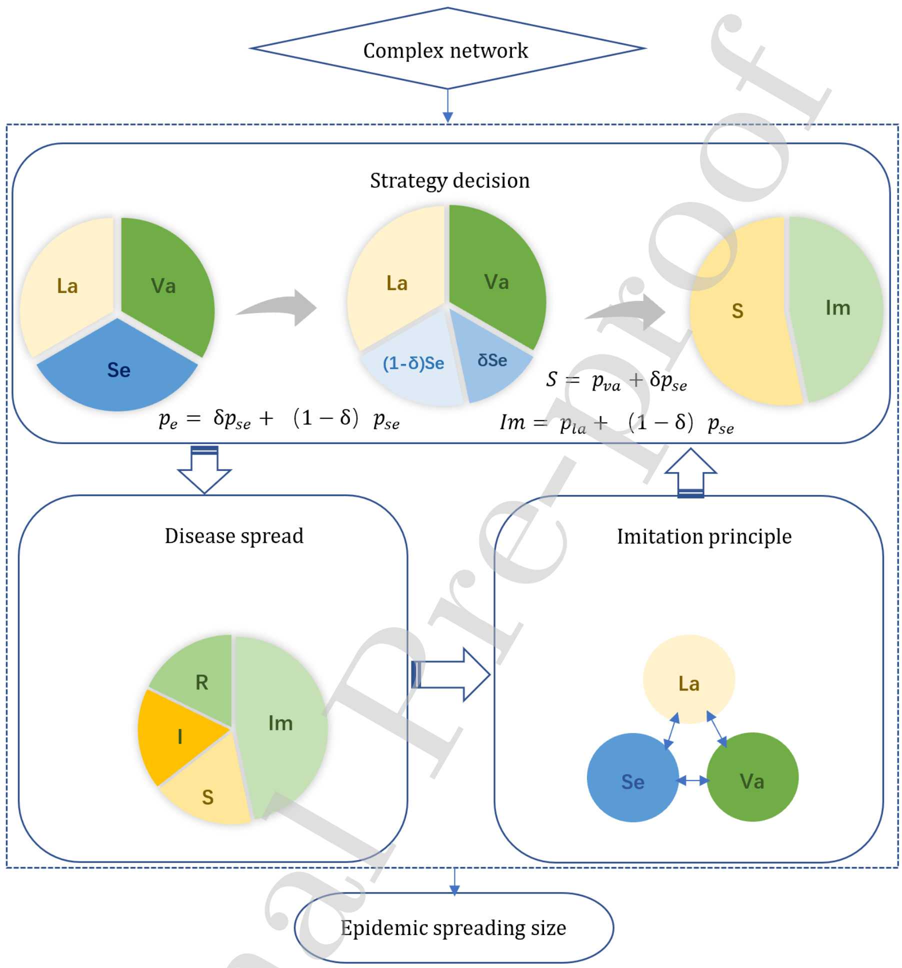


Figure 1: Overview of the epidemic propagation. Firstly, each individual initializes the strategy state into vaccination (Va), self-protection (Se) or laissez-faire (La) at the beginning of the epidemic spreading season. The individuals choosing the self-protection strategy are divided into two groups. The fraction (1-δ) of individuals adopting the self-protection strategy and all the individuals choosing the laissez-faire strategy are susceptible. The other individuals are immune. Secondly, the disease spread adopts the SIR model, in which the state of each individual is one of three possible states, namely, the susceptible state (S), the infected state (I) and the removed state (R). Thirdly, at the end of the epidemic spreading season each individual adjusts the strategy state by the imitation principle, and then a new epidemic season commences. Finally, after multiple epidemic seasons, the epidemic spreading size is calculated.

We demonstrate the process of epidemic propagation on complex networks. Here, p_{va}, p_{se} and p_{la} are respectively used to denote the proportions of individuals adopting vaccination, self-protection and laissez-faire strategies in a network. Obviously, $p_{va} + p_{se} + p_{la} = 1$. The costs of the three strategies are c_{va}, c_{se}, and c_{la}, respectively, and $0 = c_{la} < c_{se} < c_{va} < 1$. The cost of infection as an outcome is -1, and the cost of health as an outcome is 0. The payoffs for different strategies and outcomes are shown in Table 1.

Table 1: Payoffs for different strategies and outcomes.

Strategy	Cost	Outcome	Payoff
Vaccination	c_{va}	Health	$-c_{va}$
Self-protection	c_{se}	Health	$-c_{se}$
Self-protection	c_{se}	Infection	$-1 - c_{se}$
Laissez-faire	c_{la}	Health	0
Laissez-faire	c_{la}	Infection	-1

At the beginning of the epidemic season, each individual could select one strategy from the above three strategies. On the basis of the strategies, the individuals will be divided into two groups. One group is successfully immune (Im), including all the individuals choosing the vaccination strategy and the fraction δ of the individuals adopting the self-protection strategy. The other group is susceptible (S), including all the individuals choosing the laissez-faire strategy and the fraction $(1 - \delta)$ of the individuals adopting the self-protection strategy. (Table 2)

Table 2: Outcome states and proportions of individuals in each state.

State	Proportion
Immune (Im)	$p_{va} + \delta p_{se}$
Susceptible (S)	$p_{la} + (1 - \delta)p_{se}$

The transmission of the epidemic disease is described in an undirected network $G = (V, E)$, where V and E are the vertex and edge sets, respectively, and the vertex set V has N nodes [24, 25]. In the network, the adjacency matrix is A. The degree of a node is the number of edges connected to that node. In the networks, the epidemic propagation adopts the standard SIR model, which is one of the compartmental models of epidemics [14, 26]. In this model, the state of each individual is one of three possible states based on the epidemiological status [27]. The first is the susceptible state, in which the node is healthy but can be infected. The second is the infected state, in which the node has already been infected. The third is the removed state, in which the node has recovered or been removed. Nodes in the removed state can no longer be infected or infect other nodes. In the course of the disease transmission, the time unit is denoted as t. S, I and R stand for the percentages of nodes in the susceptible, infected and removed states, respectively. The state of the individual n at time t is denoted by $X_n \in \{S(t), I(t), R(t)\}$, $n \in \{1, ..., N\}$ [12]. Infected nodes infect their susceptible neighbors with a fixed probability of λ. Nodes are removed from the infected state to the removed state at a probability of μ. When no susceptible node can be infected and all of the infected nodes are removed, the transmission of the epidemic disease ends. The epidemic spreading season is defined as the period of the spreading process from the outbreak of the disease to its end. During the epidemic spreading season, the percentage of nodes in the removed state is termed the epidemic size (R_{ratio}). The SIR model is expressed by Equation 1 [7, 27],

$$\begin{cases} \dfrac{dS}{dt} = -\lambda IS, \\ \dfrac{dI}{dt} = \lambda IS - \mu I, \\ \dfrac{dR}{dt} = \mu I. \end{cases} \tag{1}$$

In accordance with the classical SIR model of epidemics, I_0 is defined as the number of initially infected individuals, which are randomly selected from the susceptible state. These infected individuals begin to infect their susceptible neighbors. At the end of the epidemic spreading season, we should calculate R_{ratio} for the network. Simultaneously, all the individuals could adjust their strategies according to their neighbors' strategies and payoffs. In this adjustment, they could follow different imitation principles. The four adjustment

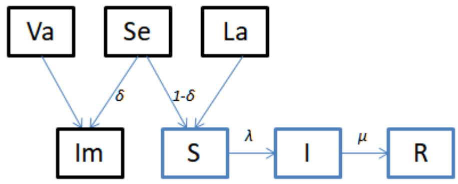


Figure 2: Transmission process of infectious disease. At the beginning of the diffusion stage, each individual could choose one of the three following strategies: vaccination, self-protection, or laissez-faire. Here, δ is the success rate of self-protection. When everyone has selected their strategy, all individuals can be divided into two classes: susceptible individuals and immune individuals. Then, the spreading dynamics follows the SIR model, with $N = Im + S + I + R$. Each infected individual will infect all their susceptible neighbors with a probability of λ and will then be removed with a probability of μ.

principles are described in the following subsections. The epidemic propagation is illustrated in Fig 2.

2.2. Four different imitation principles

2.2.1. Imitation principle 1

At the end of each epidemic season, individuals update their strategies according to their neighbors by an imitation principle. Zhang et al.suggested that an individual i randomly selects one neighbor j and then decides whether to adopt the neighbor j's strategy by the Fermi rule [7, 28, 29]. That is to say, the individual i adopts the strategy of her neighbor j with a Fermi probability of $w_{s_i \to s_j}$ such that

$$w_{s_i \to s_j} = \frac{1}{1 + exp[-\kappa(m_i - m_j)]} \tag{2}$$

The greater is the payoff for neighbor j than for individual i, the more likely it is that neighbor j's strategy will be adopted. Here, s_i is the strategy of individual i and m_i is individual i's payoff at the end of the epidemic season. The parameter κ is used to describe an individual's tendency to adopt their neighbor's strategy. The larger the value of κ is, the more the individual cares about the difference between their payoff and that of their neighbor. The framework of Imitation principle 1 is shown in Fig.3 and expressed by Equation3.

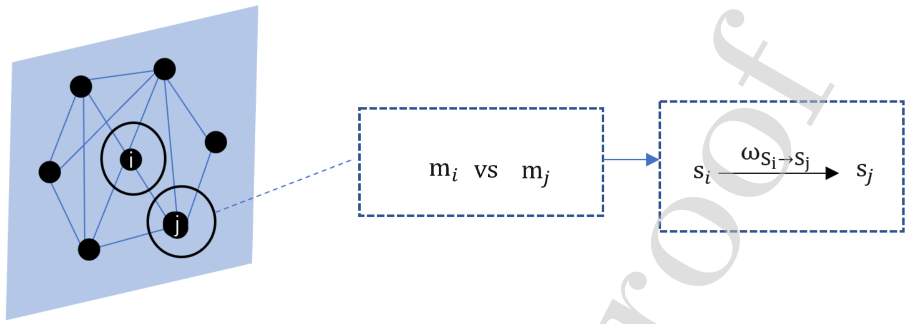


Figure 3: The framework of Imitation principle 1.

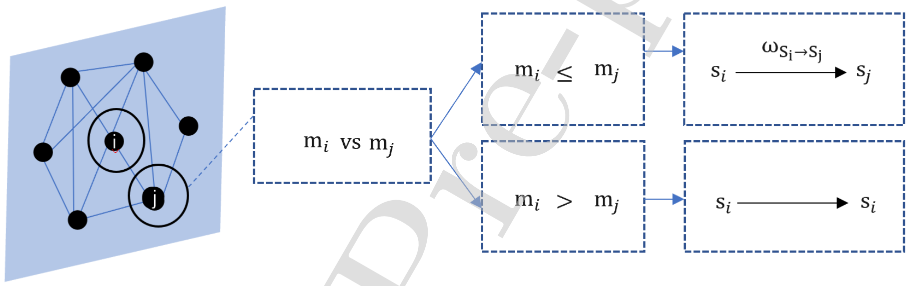


Figure 4: The framework of Imitation principle 2.

$$s_i^{\tau+1} = s_j^{\tau}, \text{ with the Fermi probability } w_{s_i \to s_j} \tag{3}$$

2.2.2. Imitation principle 2

175 After every epidemic season, an individual randomly selects one neighbor. If the payoff for this neighbor is greater than that for the individual, this individual will adopt the strategy of their neighbor with the Fermi probability. If the payoff for this neighbor is less than that for the individual, this individual will keep their own strategy. The framework of Imitation principle 2 is shown in Fig.4
180 and expressed by Equation4.

$$s_i^{\tau+1} = \begin{cases} s_j^{\tau}, \text{ with } w_{s_i^{\tau} \to s_j^{\tau}} \text{ for } m_i^{\tau} \leq m_j^{\tau}, \\ s_i^{\tau}, \text{ for } m_i^{\tau} > m_j^{\tau}. \end{cases} \tag{4}$$

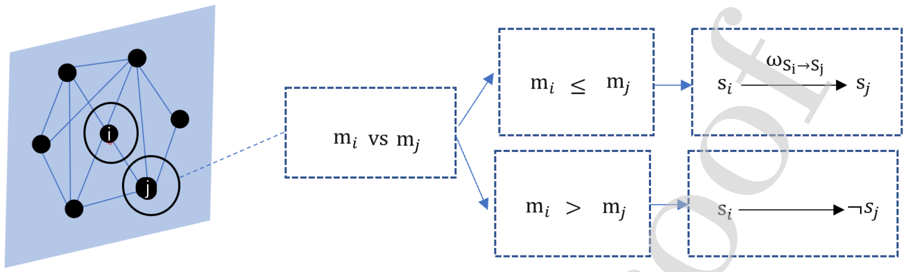


Figure 5: The framework of Imitation principle 3.

2.2.3. Imitation principle 3

After every epidemic season, an individual randomly selects one neighbor. If the payoff for this neighbor is greater than that for the individual, this individual will adopt the strategy of their neighbor with the Fermi probability. If the payoff for this neighbor is less than that for the individual, this individual will never adopt the strategy of their neighbor but will randomly choose one strategy from the other strategies. The framework of Imitation principle 3 is shown in Fig.5 and expressed by Equation5.

$$s_i^{\tau+1} = \begin{cases} s_j^{\tau}, \text{ with } w_{s_i^{\tau} \to s_j^{\tau}} \text{ for } m_i^{\tau} \leq m_j^{\tau}, \\ \neg s_j^{\tau}, \text{ for } m_i^{\tau} > m_j^{\tau}. \end{cases} \tag{5}$$

2.2.4. Imitation principle 4

After every epidemic season, an individual randomly selects one neighbor. If the payoff for this neighbor is greater than that for the individual, this individual will adopt the strategy of their neighbor with the Fermi probability. If the payoff for this neighbor is less than that for the individual and the strategy of this neighbor is the same as that of the individual, this individual will never adopt the strategy of their neighbor and will then select one strategy from the other two strategies. If the payoff for this neighbor is less than that for the individual and the strategy of this neighbor is different from their own strategy, this individual will keep their own strategy. The framework of Imitation principle 4 is shown in Fig.6 and expressed by Equation6.

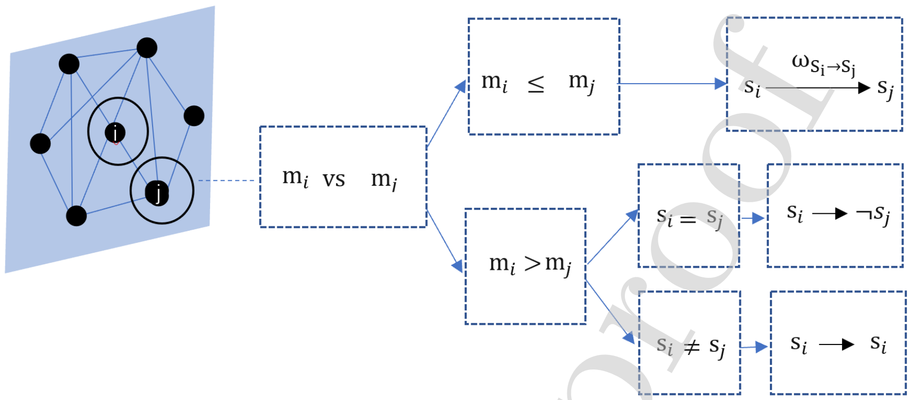


Figure 6: The framework of Imitation principle 4.

$$s_i^{\tau+1} = \begin{cases} s_j^\tau, \text{ with } w_{s_i^\tau \to s_j^\tau} \text{ for } m_i^\tau \leq m_j^\tau, \\ \neg s_j^\tau, \text{ for } m_i^\tau > m_j^\tau \text{ and } s_i^\tau = s_j^\tau, \\ s_i^\tau, \text{ for } m_i^\tau > m_j^\tau \text{ and } s_i^\tau \neq s_j^\tau. \end{cases} \tag{6}$$

In summary, at the end of each epidemic season, individuals update their strategies according to their neighbors by Fermi rule, which is called imitation principle 1. We extend the principle to three different imitation ones and name them principle 2, principle 3 and principle 4, respectively. Specifically, they are described as follows: (1) Principle 2. After every epidemic season, each individual randomly selects one neighbor. If the payoff for this neighbor is greater than that for the individual, this individual will adopt the strategy of this neighbor with Fermi rule. Otherwise, this individual will keep his/her own strategy. (2) Principle 3. In imitation principle 3, if the payoff for this neighbor is less than that for the individual, the individual will randomly choose one strategy from the strategies that the neighbor does not use. (3) Principle 4. In imitation principle 4, if the payoff for this neighbor is less than that for the individual and the strategy of this neighbor is different, this individual will keep his/her own strategy. After all individuals have adjusted their strategies, the proportions of individuals adopting the three strategies in the population are determined for the next epidemic season, and a new round of disease transmission will start. We measure the severity of the epidemic disease by the average disease transmission rate R_{ratio} for 1,000 seasons.

2.3. The network description

Previous studies of complex networks have provided a modeling technique for research on the spreading process of infectious diseases [12, 14]. Because the spreading process of an infectious disease is usually studied within a network, the topology of the network plays a crucial role in the spread of the disease. Many network structures are used to depict real-life situations. This section introduces several kinds of networks, namely, WM networks [30], BA networks [14, 30], ER networks [30, 31], SL networks [14], LFR networks [32], and Facebook networks [33].

A WM network is a regular network, where all nodes are connected to each other. This type of network is fully connected.

An SL network is a grid point network. The nodes in the network are the basic cell points, and the arrangement of grid points follows the same pattern. The neighbor relationship of lattice points is used to represent the connections between nodes. All the lattice points are identical from every direction.

An ER network is a random network, which was originally used to explain the real world. The network edges between the nodes are randomly generated. Each pair of nodes is connected with the same probability. However, real-life situations do not conform to this type of network.

A BA network is a scale-free network in which most nodes are connected to few other nodes. That is to say, richly connected nodes receive more connections [32]. The network generation process is based on a growth model and a preferential attachment model. The network begins on a small scale, and there is a gradual increase in the number of new nodes, which are attached to the original nodes with probabilities proportional to the degrees of the original nodes. The degree distribution follows a power law, as well as being scale-free, and is thus similar to situations in the actual world.

An LFR network is a network with community structures. Both of the degree distribution and community size distribution follow power laws with corresponding exponents γ and β respectively. The mixing parameter is α. Each point is connected to the nodes in the same community with a probability of $1 - \alpha$, and to the other nodes with a probability of α. The parameters also include average and maximum degree of nodes.

A Facebook network is a large scale real network. The Facebook dataset is a set of 10 different Facebook ego-networks. These 10 Facebook ego-networks are merged into one entire Facebook network, which has 4039 nodes and 88234 edges.

Zhang et al. studied the process of epidemic propagation in WM, BA, SL, and ER networks and found a counter-intuitive phenomenon. Within a certain range of success rates δ of the self-protection strategy, the spread of infectious diseases was not reduced but increased as δ rised [7]. We study the transmission process of infectious diseases not only in the above four types of networks without community structures but also in LFR networks and Facebook networks, which has community structures.

2.4. Mean epidemic spreading size

With respect to imitation dynamics, it is assumed that all nodes update their strategies at the end of the season by imitating their neighbors. After multiple epidemic seasons, the average spread size in the network is calculated. Algorithm 1 outlines the calculation of the mean value of R_{ratio}. The time complexity of obtaining the epidemic spreading size is $O(tMN)$, where M and N are the number of nodes and edges in the network, respectively [34, 35, 36].

Algorithm 1 Framework used for the calculation of the mean value of R_{ratio} using different imitation principles

Input: Adjacency matrix A of $G = (V, E)$

Output: Mean value of R_{ratio}

```
1:  Choose the strategy of each node to be va, se, or la.
2:  for τ = 0, 1, 2...1000 do
3:      Initialize the state of each node as immune or susceptible.
4:      Select I_0 susceptible nodes into I.
5:      for t = 0, 1, 2... do
6:          Nodes S → I. Infect S by I with a probability of λ.
7:          Nodes I → R. Remove I to R with a probability of μ.
8:          t ← t + 1
9:          Repeat steps 6 to 8.
10:         I = 0, break.
11:     end for
12:     Calculate R_ratio value of the network and payoff for each node.
13:     Adjust the strategy of each node by the imitation principle.
14:     τ ← τ + 1
15:     Repeat steps 3 to 14.
16: end for
17: Calculate the mean value of R_ratio.
```

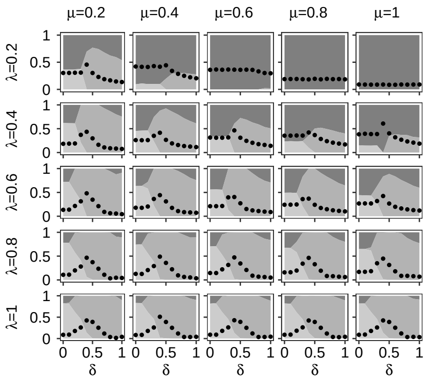
3. Numerical simulations and results

3.1. Results on synthetic datasets

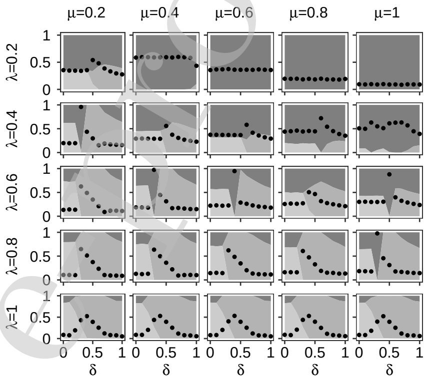
3.1.1. Effects of imitation principles

Simulations are carried out in WM, BA, ER, SL and LFR networks. Four imitation principles can be chosen in the strategy adjustment, and these have been described in section 2. The number of nodes in the WM, BA, ER and LFR networks is 1000. The number of nodes in the SL network is 2500 accordance with ref.[7]. The results for the epidemic size in the SL network with 2500 nodes are similar with those in the SL network with 1000 nodes. In the rest of this paper, the SL network with 2500 nodes is taken for comparison. In the BA, ER and SL networks, the mean degree is set to be 4. In LFR networks, average and maximum degree of nodes are set to be 20 and 50, respectively. Set $\gamma = 2$, $\beta = 1$ and $\alpha \sim [0.1, 0.9]$. The model parameters are denoted as $I_0 = 5$, $c_{va} = 0.4$, $c_{se} = 0.1$, $c_{la} = 0$, and $\kappa = 10$ in accordance with previous works [7]. When the counter-intuitive phenomenon occurs, a range of counter-intuition refers to the range of success rate δ of self-protection, and a magnitude of counter-intuition refers to the difference between the maximum and the minimum of the disease transmission size.

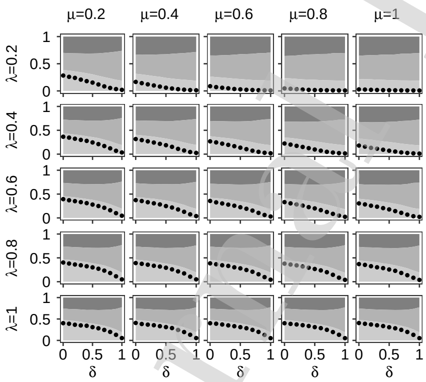
Fig.7 shows the results for the disease transmission range in the BA network using the four different imitation principles for updating strategies. As shown in Fig.7(a), when $\lambda = 0.2, \mu = 0.6, 0.8$, or 1,there are no significant counter-intuitive phenomena in terms of epidemic spread size in the case of imitation principle 1. The results according with imitation principle 1 are same with previous work [7]. As shown in Fig.7(b), when $\lambda = 0.2, \mu = 0.4, 0.6, 0.8$, or 1, there are also no significant counter-intuitive phenomena in the case of imitation principle 2. Otherwise, in the cases of imitation principles 1 and 2 there is an obvious counter-intuitive phenomenon with other pairs of λ, μ. Moreover, as λ increases the range of this counter-intuitive phenomenon increases. For fixed values of λ and μ, the range of the counter-intuitive phenomenon in the case of imitation principle 1 is larger than that for imitation principle 2. However, the magnitude of the counter-intuitive phenomenon in the case of imitation principle 1 is less than that for imitation principle 2. As shown in Fig.7(c), there are no counter-intuitive phenomena in the case of imitation principle 3. In other words, as δ increases,the disease transmission size remains steady or decreases. As shown in Fig.7(d), the disease transmission size in the case of imitation principle 4 becomes moderate. When $\lambda = 0.2$ and $\mu = 0.2$, the counter-intuitive phenomenon is weak, but when $\lambda = 0.2$ and μ is greater than 0.4, the counter-intuitive phenomenon disappears. When $\lambda = 0.4$ and $\mu = 0.2$, or 0.4, the counter-intuitive phenomenon is slight, but when $\lambda = 0.4$ and μ is greater than 0.6, the counter-intuitive phenomenon disappears. As λ increases from 0.6 to 1, there is always a counter-intuitive phenomenon. However, the magnitude of the counter-intuitive phenomenon in the case of imitation principle 4 is clearly less than those for imitation principles 1 and 2.



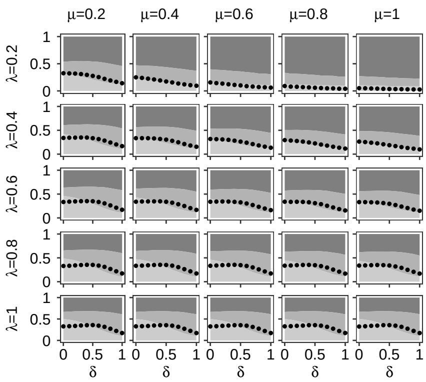
(a) Disease transmission on imitation principle 1.



(b) Disease transmission on imitation principle 2.

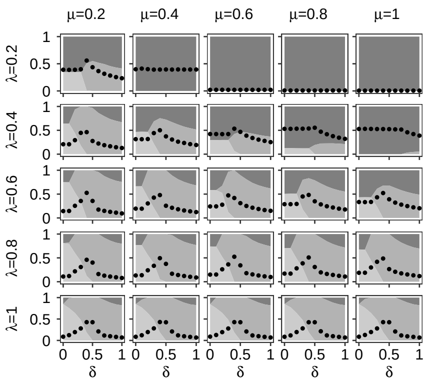


(c) Disease transmission on imitation principle 3.

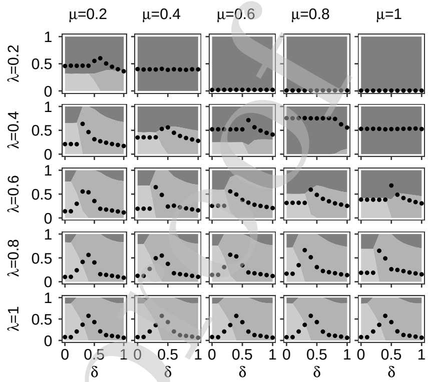


(d) Disease transmission on imitation principle 4.

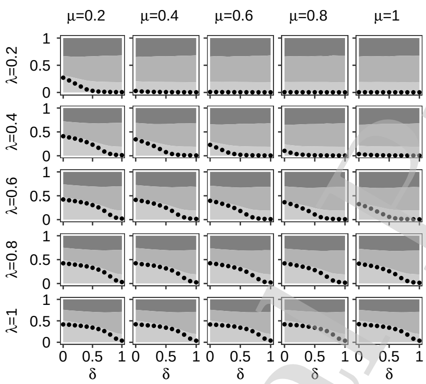
Figure 7: Results for the disease transmission range in the BA network using the four different imitation principles for updating strategies. The light gray, medium gray, and dark gray areas represent the proportions of individuals selecting vaccination (p_{va}), self-protection (p_{se}) and laissez-faire (p_{la}), respectively. The black points indicate the mean value of R_{ratio} for a given δ value. The four subgraphs represent the results for the four imitation principles, respectively, with given pairs of λ and μ.



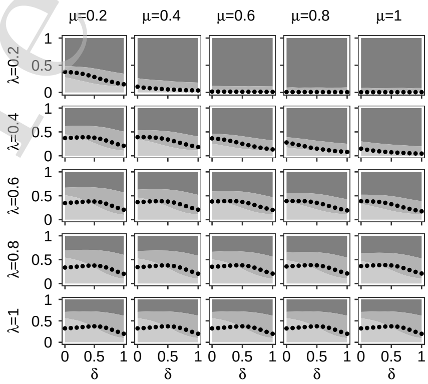
(a) Disease transmission on imitation principle 1.



(b) Disease transmission on imitation principle 2.

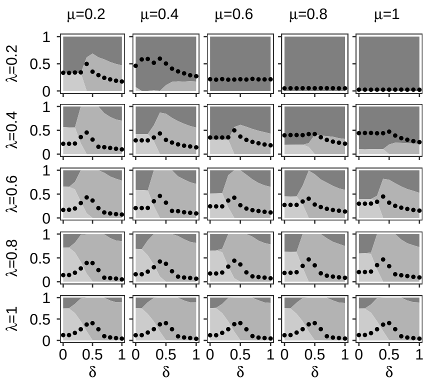


(c) Disease transmission on imitation principle 3.

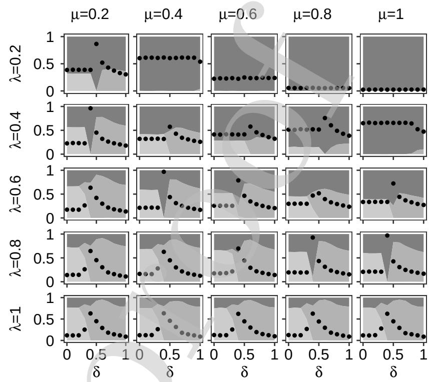


(d) Disease transmission on imitation principle 4.

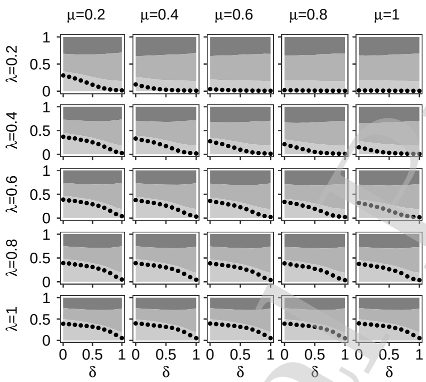
Figure 8: Results for the disease transmission range in the SL network with network size $N = 2500$ using the four different imitation principles for updating strategies. The light gray, medium gray, and dark gray areas represent the proportions of individuals selecting vaccination (p_{va}), self-protection (p_{se}) and laissez-faire (p_{la}), respectively. The black points indicate the mean value of R_{ratio} for a given δ value. The four subgraphs represent the results for the four imitation principles, respectively, with given pairs of λ and μ.



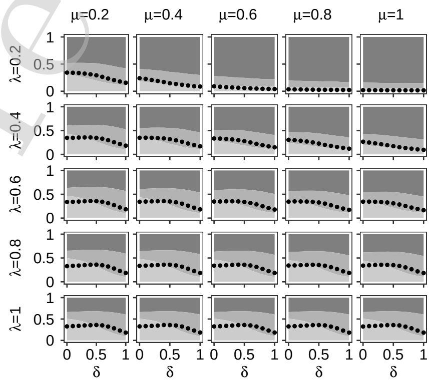
(a) Disease transmission on imitation principle 1.



(b) Disease transmission on imitation principle 2.



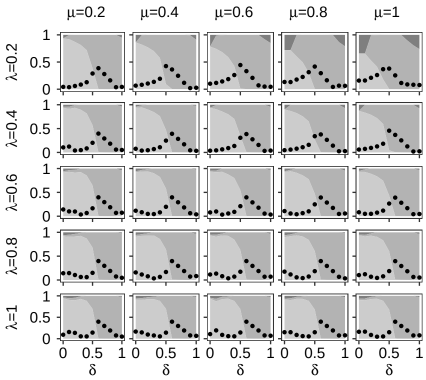
(c) Disease transmission on imitation principle 3.



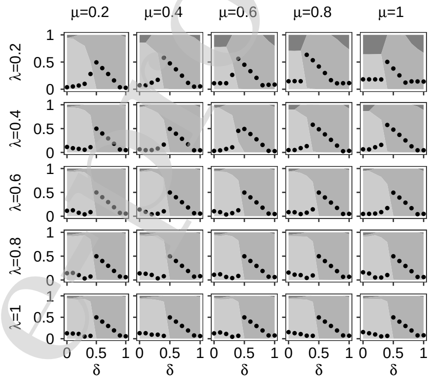
(d) Disease transmission on imitation principle 4.

Figure 9: Results for the disease transmission range in the ER network using the four different imitation principles for updating strategies. The light gray, medium gray, and dark gray areas represent the proportions of individuals selecting vaccination (p_{va}), self-protection (p_{se}) and laissez-faire (p_{la}), respectively. The black points indicate the mean value of R_{ratio} for a given δ value. The four subgraphs represent the results for the four imitation principles, respectively, with given pairs of λ and μ.

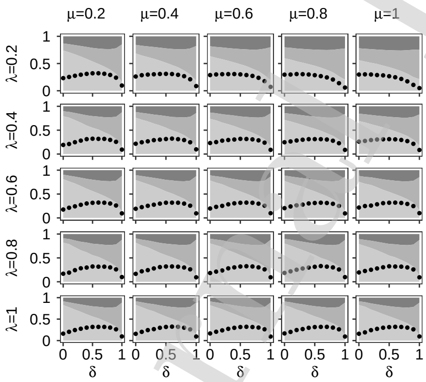
Fig.8 and Fig.9 show the results for the disease transmission range using the four different imitation principles for updating strategies in the SL network and ER network, respectively. The counter-intuitive phenomena in the SL and ER networks are similar to that in the BA network. There are counter-intuitive phenomena in the cases of imitation principles 1, 2, and 4. However, the range and magnitude of the counter-intuitive phenomenon in the case of imitation principle 4 are clearly less than in the cases of imitation principles 1 and 2. There are no counter-intuitive phenomena in the case of imitation principle 3.



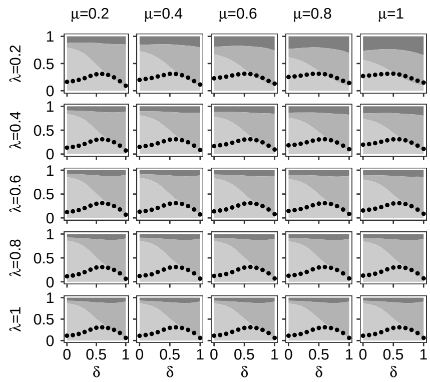
(a) Disease transmission on imitation principle 1.



(b) Disease transmission on imitation principle 2.



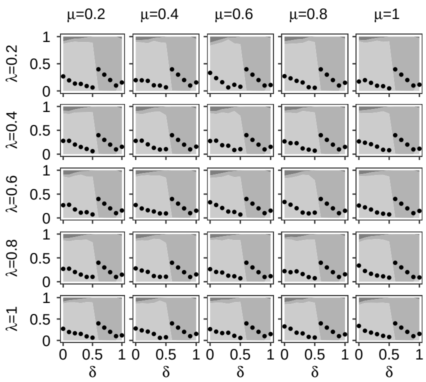
(c) Disease transmission on imitation principle3.



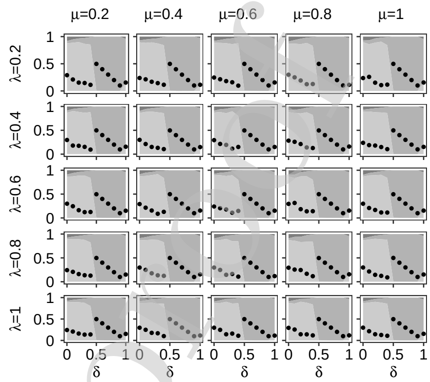
(d) Disease transmission on imitation principle 4.

Figure 10: Results for the disease transmission range in the LFR network with $\alpha = 0.6$ using the four different imitation principles for updating strategies. The light gray, medium gray, and dark gray areas represent the proportions of individuals selecting vaccination (p_{va}), self-protection (p_{se}) and laissez-faire (p_{la}), respectively. The black points indicate the mean value of R_{ratio} for a given δ value.The four subgraphs represent the results for the four imitation principles, respectively, with given pairs of λ and μ.

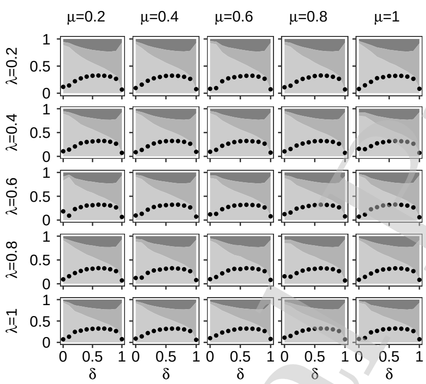
Fig.10 shows the results for the disease transmission range in the LFR network using the four different strategy updating principles. There are always counter-intuitive phenomena, as shown in the four subfigures, for any given combination of λ and μ. For fixed values of λ and μ, the ranges of the counter-intuitive phenomena in the cases of imitation principles 1 and 2 are smaller than those for imitation principles 3 and 4. However, the magnitudes of the counter-intuitive phenomena in the cases of imitation principles 1 and 2 are greater than those for the other two imitation principles. In other words, the ranges of the counter-intuitive phenomena in the cases of imitation principles 3 and 4 are larger, but the magnitudes are less.



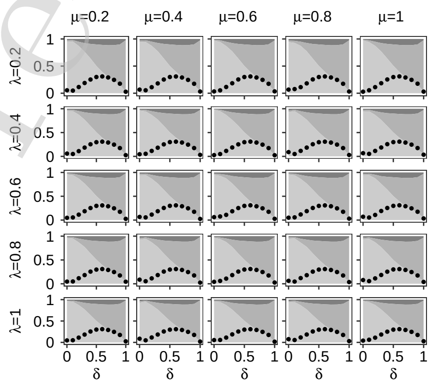
(a) Disease transmission on imitation principle 1.



(b) Disease transmission on imitation principle 2.

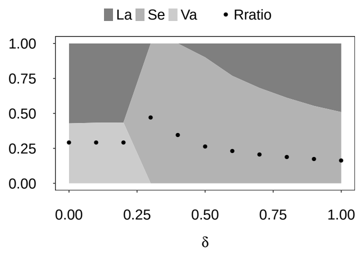


(c) Disease transmission on imitation principle 3.

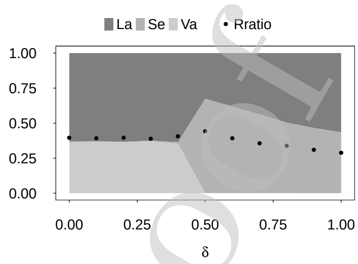


(d) Disease transmission on imitation principle 4.

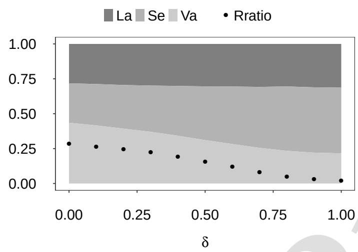
Figure 11: Results for the disease transmission range in the WM network using the four different imitation principles for updating strategies. The light gray, medium gray, and dark gray areas represent the proportions of individuals selecting vaccination (p_{va}), self-protection (p_{se}) and laissez-faire (p_{la}), respectively. The black points indicate the mean value of R_{ratio} for a given δ value. The four subgraphs represent the results of the four imitation principles, respectively, with given pairs of λ and μ.



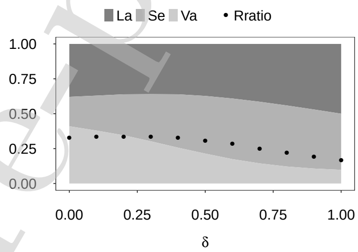
(a) Disease transmission on imitation principle 1.



(b) Disease transmission on imitation principle 2.



(c) Disease transmission on imitation principle 3.



(d) Disease transmission on imitation principle 4.

Figure 12: Results for epidemic propagation in the WM network using the four different imitation principles when $\lambda = 0.0025$ and $\mu = 1$. The light gray, medium gray, and dark gray areas represent the proportions of individuals adopting the strategies of vaccination (p_{va}), self-protection (p_{se}) and laissez-faire (p_{la}), respectively. The black points indicate the mean value of R_{ratio} for a given δ value. The four subgraphs represent the results for the four imitation principles, respectively, when $\lambda = 0.0025$ and $\mu = 1$.

Fig.11 shows the results for the disease transmission range in the WM network using the four different strategy updating principles. The four subgraphs demonstrate that the results for the WM network are similar to those for the LFR network. Moreover, the ranges of the counter-intuitive phenomenon in the cases of imitation principles 3 and 4 are larger than those for imitation principles 1 and 2, but the magnitudes are less. However, when λ is small enough, the counter-intuitive phenomenon disappears. As an example, Fig. 12 shows the results for the disease transmission range in the WM network when $\lambda = 0.0025$ and $\mu = 1$. In Fig. 12(c), we observe that there are no counter-intuitive phenomena in the case of imitation principle 3.

From the above, the epidemic spread size is affected by the imitation principle. Firstly, the results in terms of counter-intuitive phenomena are similar in the BA, ER, and SL networks. A counter-intuitive phenomenon is obvious in the cases of imitation principles 1 and 2, but there are no counter-intuitive phenomena in the case of imitation principle 3. In the case of imitation principle 4, the range of the counter-intuitive phenomenon is larger, but the magnitude is less. In contrast, in the LFR network there is always a counter-intuitive phenomenon in the cases of all four imitation principles. The ranges of the counter-intuitive phenomenon in the cases of imitation principles 1 and 2 are smaller than those for imitation principles 3 and 4. However, the magnitudes in the cases of principles 1 and 2 are greater than those for the other two imitation principles. In the WM network, when both of λ and μ are greater than 0.2 the results for epidemic propagation are similar to those in the LFR network. In the case when $\mu = 1$, and λ is small enough, the results are similar to those in the BA, ER and SL networks.

3.1.2. Effects of the topological structures of networks

The topological structures of the networks affect the counter-intuitive phenomenon in the epidemic spread size. The phenomenon is also sensitive to the community structure. For the same imitation principle, the phenomenon in the epidemic spread size tends to have a different outcome, depending on differences in the network structures. In the case of imitation principle 1, as shown in Fig.11(a) in the WM network as the δ value increases, the mean value of R_{ratio} exhibits an increase and then a decrease. As the success rate of the self-protection strategy increases, the spread size of the disease starts to increase significantly and then declines. This is the counter-intuitive phenomenon, and is the case in the BA, SL, ER, and LFR networks, as shown in Fig. 7(a), 8(a), 9(a) and 15(a), respectively [7]. However, when λ is small enough and μ is large enough i.e.,$\lambda = 0.2$ and $\mu = 1.0$, the counter-intuitive phenomenon vanishes for the BA, ER, and SL networks, but not for the LFR network.

As shown in Fig.7(b), 8(b), 9(b), 15(b), and 11(b), the phenomena for imitation principle 2 are similar to those for imitation principle 1. As the success rate of the self-protection strategy increases, the disease spread size does not always decrease in the BA, SL, and ER networks. Nevertheless, the counter-intuitive phenonema always exist in the LFR and WM networks.

In the case of imitation principle 3, as the success rate δ of the self-protection strategy rises, the mean value of R_{ratio} declines gradually, as shown in Fig.7(c), 8(c), and 9(c). The counter-intuitive phenomenon disappears in the BA, ER and SL networks. On the contrary, as shown in Fig.15(c), there are still counter-intuitive phenomena in the LFR network. In addition, as shown in the subgraphs of Fig.11(c) and 12(c) for the WM network with different pairs of λ and μ, the spread size R_{ratio} does not always exhibit a counter-intuitive phenomenon, but sometimes exhibits a decline, as δ rises.

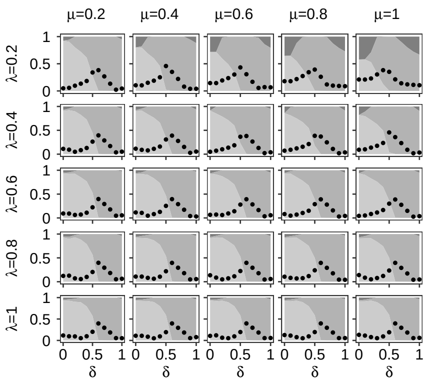
According to imitation principle 4, as the success rate δ of the self-protection strategy rises, the mean value of R_{ratio} exhibits an increase and then a decrease. There are counter-intuitive phenomena in Fig.7(d), 8(d), 9(d),15(d), and 11(d). However, the phenomena shown in the BA, ER, and SL networks are not as obvious as those shown in the WM and LFR networks. Moreover, the magnitudes of counter-intuition for the WM and LFR networks are larger than another three kinds of networks.

3.1.3. Effects of parameters

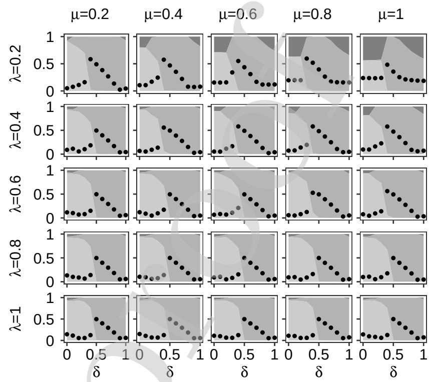
The counter-intuitive phenomenon in the epidemic spread size is sensitive to the model parameters, such as λ and μ. In the cases of imitation principles 1 and 2, it is demonstrated that the model parameters affect the mean value of R_{ratio} in the BA, ER, and SL networks. As shown in Fig.7, 8, and 9, Sometimes as the value of λ decreases, the phenomenon gradually disappears. As shown in the respective subgraphs for the BA, ER, and SL networks, as λ decreases from 0.6 to 0.2 for a given value of μ ($\sim [0.4, 1.0]$), the counter-intuitive phenomenon gradually peters out. The same behavior is observed as the value of μ increases at $\lambda = 0.2$ or 0.4. In the case with other pairs of λ and μ, in particular,

$\lambda \sim [0.6, 1.0]$ and $\mu \sim [0.2, 1.0]$, the counter-intuitive phenomenon is clear. The main reason for the different phenomena is that the parameters have significant effects on the results for the disease transmission size. As shown in Fig.12(c), in the WM network with $\lambda = 0.0025$ and $\mu = 1$, there is no counter-intuitive phenomenon for imitation principle 3. However, it is shown that with other combination of λ and μ, the counter-intuitive phenomenon is obvious for the WM network in Fig.11.

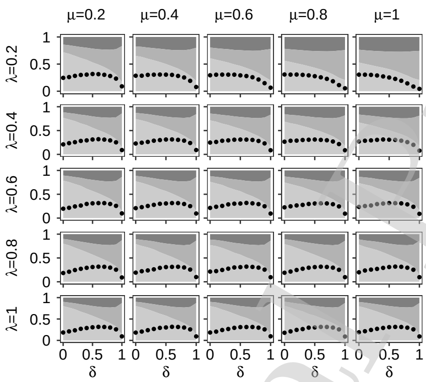
The results for the disease transmission range are insensetive to the network parameters, such as the mixing parameter α in LFR networks. With the increase of the mixing parameter α, there are always the counter-intuitive phenomenon. We test the results with different values $\alpha \sim [0.1, 0.9]$. In the paper, we take LFR networks with $\alpha = 0.8$, $\alpha = 0.5$ and $\alpha = 0.2$ for comparison, which are shown in Fig.13, 14 and 15. The clearness of the community structure has no influence on the counter-intuitive phenomenon.



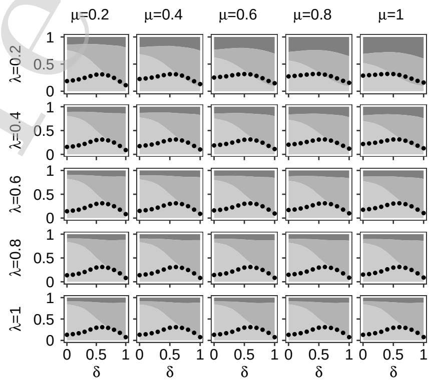
(a) Disease transmission on imitation principle 1.



(b) Disease transmission on imitation principle 2.

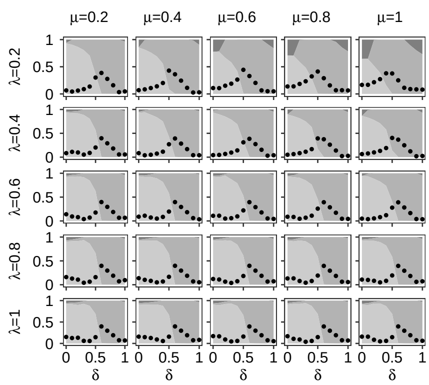


(c) Disease transmission on imitation principle3.

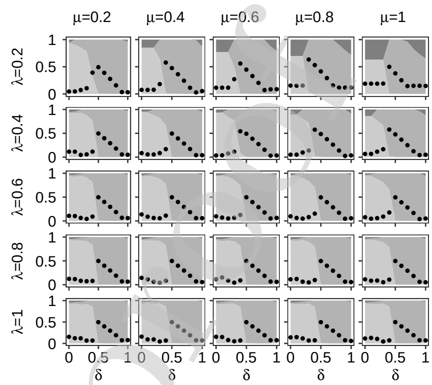


(d) Disease transmission on imitation principle 4.

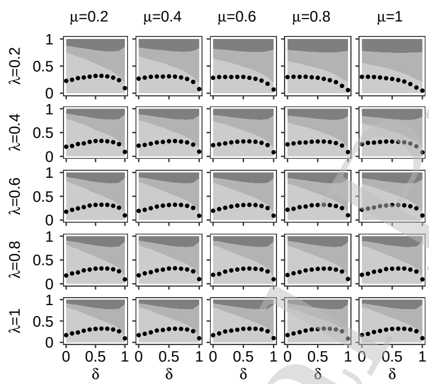
Figure 13: Results for the disease transmission range in the LFR network with $\alpha = 0.2$. The light gray, medium gray, and dark gray areas represent the proportions of individuals selecting vaccination (p_{va}), self-protection (p_{se}) and laissez-faire (p_{la}), respectively. The black points indicate the mean value of R_{ratio} for a given δ value.The four subgraphs represent the results for the four imitation principles with given pairs of λ and μ.



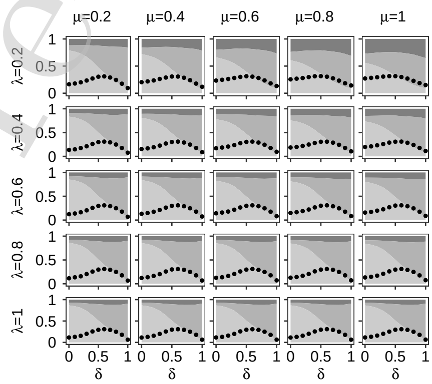
(a) Disease transmission on imitation principle 1.



(b) Disease transmission on imitation principle 2.

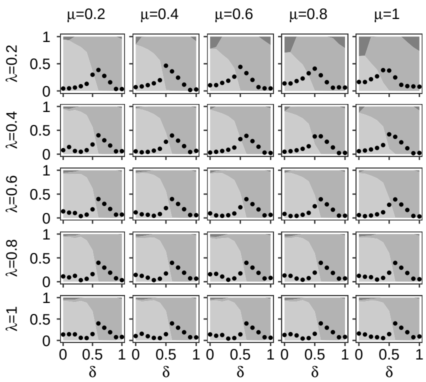


(c) Disease transmission on imitation principle3.

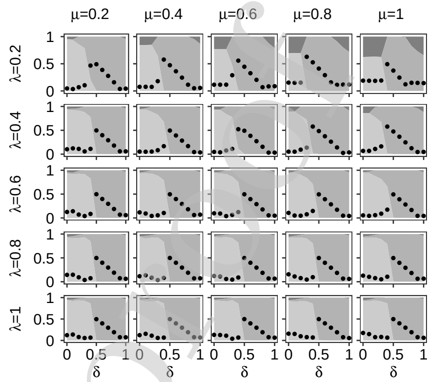


(d) Disease transmission on imitation principle 4.

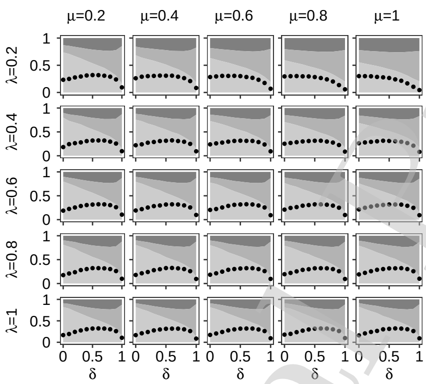
Figure 14: Results for the disease transmission range in the LFR network with $\alpha = 0.5$. The light gray, medium gray, and dark gray areas represent the proportions of individuals selecting vaccination (p_{va}), self-protection (p_{se}) and laissez-faire (p_{la}), respectively. The black points indicate the mean value of R_{ratio} for a given δ value.The four subgraphs represent the results for the four imitation principles, respectively, with given pairs of λ and μ.



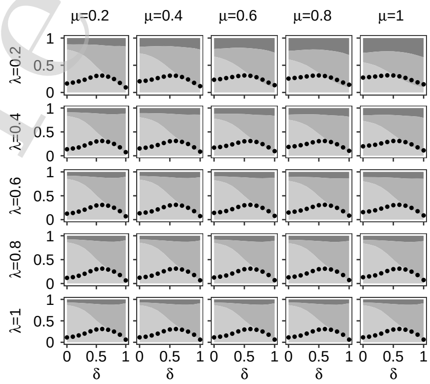
(a) Disease transmission on imitation principle 1.



(b) Disease transmission on imitation principle 2.



(c) Disease transmission on imitation principle3.

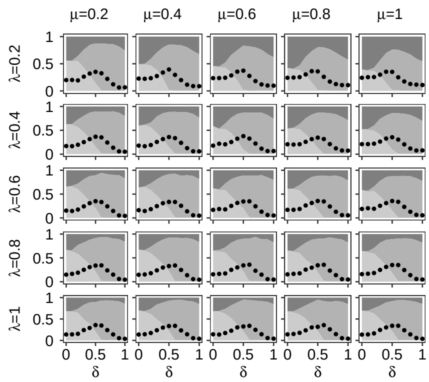


(d) Disease transmission on imitation principle 4.

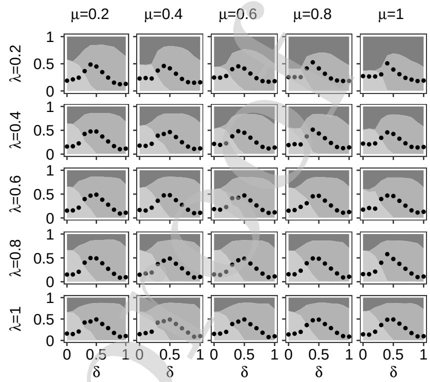
Figure 15: Results for the disease transmission range in the LFR network with $\alpha = 0.8$. The light gray, medium gray, and dark gray areas represent the proportions of individuals selecting vaccination (p_{va}), self-protection (p_{se}) and laissez-faire (p_{la}), respectively. The black points indicate the mean value of R_{ratio} for a given δ value.The four subgraphs represent the results for the four imitation principles, respectively, with given pairs of λ and μ.

3.2. Results on real scenarios

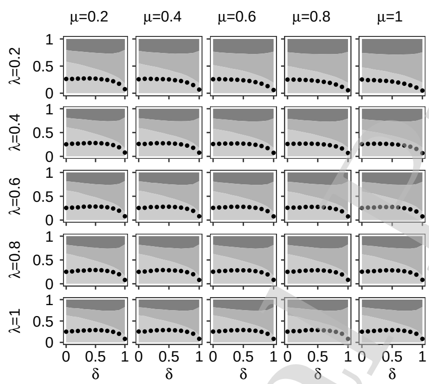
The proposed method can be applied to large scale real networks. We have conducted more experiments on one Facebook network with 4039 nodes and 88234 edges. The numerical results of the disease transmission range are given in Fig.16.



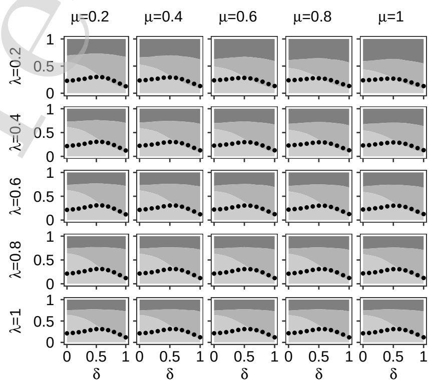
(a) Disease transmission on imitation principle 1.



(b) Disease transmission on imitation principle 2.



(c) Disease transmission on imitation principle3.



(d) Disease transmission on imitation principle 4.

Figure 16: Results for the disease transmission range in the Facebook network. The light gray, medium gray, and dark gray areas represent the proportions of individuals selecting vaccination (p_{va}), self-protection (p_{se}) and laissez-faire (p_{la}), respectively. The black points indicate the mean value of R_{ratio} for a given δ value.The four subgraphs represent the results for the four imitation principles with given pairs of λ and μ.

Fig.16 shows the results for the disease transmission range in the Facebook network using the four different imitation principles for updating strategies. In the existed imitation principle there are always counter-intuitive phenomena, which are similar with the results in the case of imitation principle 2 but are different from other extended imitation principles. Moreover, in the cases of imitation principles 1 and 2 there is an obvious counter-intuitive phenomenon with any pair of λ, μ. In the cases of imitation principle 3, as shown in Fig.16(c), when $\lambda = 0.2, 0.4$, there are no significant counter-intuitive phenomena in terms of epidemic spread size. As δ increases, the disease transmission size remains steady or decreases. Moreover, as λ increases, the range of this counter-intuitive phenomenon increases. However, the magnitude of the counter-intuitive phenomenon is slight. In the cases of imitation principle 4, as shown in Fig.16(d), the disease transmission size in the case of imitation principle 4 becomes moderate. The range and the magnitude of the counter-intuitive phenomenon are clearly less than those for imitation principles 1 and 2.

4. Conclusions and future work

The prevention and control of infectious diseases have attracted much attention from human beings. Human behavior plays an important role in the propagation of epidemics. This paper discusses the process of epidemic propagation in the cases of four different imitation principles and six kinds of network topology. In the process of epidemic propagation, there is a kind of counter-intuitive phenomenon. As the success rate δ of the self-protection strategy rises, the fraction of individuals in the removed state increases at first and then decreases. It has been shown that the counter-intuitive phenomenon in the process of epidemic propagation is affected not only by human reactions and imitation principles but also by the network topology. In networks with community structure, no matter which imitation principle individuals adopt, the counter-intuitive phenomenon exists. This suggests that in a social community with close ties the risk of infection is not reduced as the success rate of the self-protection strategy increases. At this point, an individual should choose the vaccination strategy or reduce contact with others. However, if individuals are in a network without community structure, the counter-intuitive phenomenon is influenced by imitation principles for updating strategies, and hence they can choose different imitation principles to reduce the risk of infection.

In real life, the actual activities of human beings are often more complex than is assumed. Their behavioral responses may be influenced not only by the structures of social networks and imitation principles, but also by factors such as the environment and policies. In the future, we should pay more attention to proposing models that integrate multiple factors, such as the influence between the epidemic spreading and awareness diffusion regarding epidemics in multiplex networks [23, 34, 37, 38], so as to provide more reliable suggestions and opinions.

References

[1] C. T. Bauch, Imitation dynamics predict vaccinating behaviour., Proceedings Biological Sciences 272 (1573) (2005) 1669–1675.

[2] P. Poletti, B. Caprile, M. Ajelli, A. Pugliese, S. Merler, Spontaneous behavioural changes in response to epidemics., Journal of Theoretical Biology 260 (1) (2009) 31.

[3] C. T. Bauch, M. R. Hilleman, Vaccination and the theory of games, Proceedings of the National Academy of Sciences of the United States of America 101 (36) (2004) 13391.

[4] C. Buono, L. G. Alvarezzuzek, P. A. Macri, L. A. Braunstein, Epidemics in partially overlapped multiplex networks., Plos One 9 (3) (2014) e92200.

[5] B. Wu, F. Fu, L. Wang, Imperfect vaccine aggravates the long-standing dilemma of voluntary vaccination, Plos One 6 (6) (2011) e20577.

[6] H. F. Zhang, P. P. Shu, Z. Wang, M. Tang, M. Small, Preferential imitation can invalidate targeted subsidy policies on seasonal-influenza diseases, Applied Mathematics & Computation 294 (2017) 332–342.

[7] H. F. Zhang, Z. Yang, Z. X. Wu, B. H. Wang, T. Zhou, Braess's paradox in epidemic game: Better condition results in less payoff, Scientific Reports 3 (3) (2013) 3292.

[8] T. Oraby, V. Thampi, C. T. Bauch, The influence of social norms on the dynamics of vaccinating behaviour for paediatric infectious diseases., Proceedings Biological Sciences 281 (1780) (2014) 20133172.

[9] B. Shi, W. Wang, H. Qiu, Y. W. Chen, S. Peng, Exploring voluntary vaccinating behaviors using evolutionary n-person threshold games, Scientific Reports 7 (1).

[10] Z. Bu, H. Li, J. Cao, Z. Wang, G. Gao, Dynamic cluster formation game for attributed graph clustering, IEEE Transactions on Cybernetics 1-14.

[11] Z. Bu, J. Cao, H. Li, G. Gao, H. Tao, Gleam: a graph clustering framework based on potential game optimization for large-scale social networks, Knowledge and Information Systems.

[12] C. Nowzari, V. M. Preciado, G. J. Pappas, Analysis and control of epidemics: A survey of spreading processes on complex networks, IEEE Control Systems 36 (1) (2015) 26–46.

[13] H. Ohtsuki, C. Hauert, E. Lieberman, M. A. Nowak, A simple rule for the evolution of cooperation on graphs and social networks, Nature 441 (7092) (2006) 502.

[14] M. J. Keeling, K. T. Eames, Networks and epidemic models, Journal of the Royal Society Interface 2 (4) (2005) 295.

[15] X. Ma, L. Gao, X. Yong, L. Fu, Semi-supervised clustering algorithm for community structure detection in complex networks, Physica A-statistical Mechanics and Its Applications 389 (1) (2010) 187–197.

[16] M. Nielsen, Neural Networks and Deep Learning, http://neuralnetworksanddeeplearning.com, 2016.

[17] X. Ma, D. Dong, Evolutionary nonnegative matrix factorization algorithms for community detection in dynamic networks, IEEE Transactions on Knowledge and Data Engineering 29 (5) (2017) 1045–1058. doi:10.1109/TKDE.2017.2657752.

[18] X. Ma, P. Sun, G. Qin, Nonnegative matrix factorization algorithms for link prediction in temporal networks using graph communicability, Pattern Recognition 71. doi:10.1016/j.patcog.2017.06.025.

[19] G. L. Ma, X., Predicting protein complexes in protein interaction networks using a core-attachment algorithm based on graph communicability, Information Sciences 189 (2012) 233–254.

[20] G. P. Harmer, D. Abbott, Game theory: Losing strategies can win by parrondo's paradox, Nature 402 (402) (1999) 864.

[21] C. T. Bauch, A. P. Galvani, D. J. Earn, Group interest versus self-interest in smallpox vaccination policy., Proceedings of the National Academy of Sciences of the United States of America 100 (18) (2003) 10564–7.

[22] Y. Ye, H. C. Kang, Y. Cen, N. Xie, Effects of behavioral patterns and network topology structures on parrondo's paradox, Scientific Reports 6 (2016) 37028.

[23] J. Q. Kan, H. F. Zhang, Effects of awareness diffusion and self-initiated awareness behavior on epidemic spreading - an approach based on multiplex networks, Communications in Nonlinear Science & Numerical Simulation 44 (2017) 193–203.

[24] Fortunato, Santo, Community detection in graphs, Physics Reports 486 (3–5) (2010) 75–174.

[25] P. C. Pinto, P. Thiran, M. Vetterli, Locating the source of diffusion in large-scale networks, Physical Review Letters 109 (6) (2012) 068702.

[26] K. Zhu, L. Ying, Information source detection in the sir model: A sample-path-based approach, IEEE/ACM Transactions on Networking 24 (1) (2016) 408–421.

[27] M. Newman, Networks: An Introduction, Oxford University Press, Inc., 2010.

[28] A. Traulsen, M. A. Nowak, J. M. Pacheco, Stochastic dynamics of invasion and fixation, Physical Review E Statistical Nonlinear & Soft Matter Physics 74 (1 Pt 1) (2006) 011909.

[29] M. Perc, A. Szolnoki, Coevolutionary games–a mini review, Biosystems 99 (2) (2009) 109–125.

[30] S. H. Strogatz, Exploring complex networks., Nature 410 (6825) (2001) 268.

[31] P. Erdős, A. Rényi, On random graphs i, Publicationes Mathematicae 6 (1959) 290–297.

[32] A. Lancichinetti, S. Fortunato, F. Radicchi, Benchmark graphs for testing community detection algorithms., Physical Review E Statistical Nonlinear & Soft Matter Physics 78 (2) (2008) 046110.

[33] J. Mcauley, J. Leskovec, Learning to discover social circles in ego networks, NIPS 1 (2012) 539–547.

[34] H. J. Li, B. Zhan, A. Li, Z. Liu, Y. Shi, Fast and accurate mining the community structure: integrating center locating and membership optimization, IEEE Transactions on Knowledge and Data Engineering 1-1.

[35] H. J. Li, Z. Bu, Z. Wang, J. Cao, Y. Shi, Enhance the performance of network computation by a tunable weighting strategy, IEEE Transactions on Emerging Topics in Computing 2(3) (2018) 214–223.

[36] H.-J. Li, Z. Bu, Y. Li, Z. Zhang, Y. Chu, G. Li, J. Cao, Evolving the attribute flow for dynamical clustering in signed networks, Chaos, Solitons & Fractals 110 (2018) 20 – 27. doi:https://doi.org/10.1016/j.chaos.2018.02.009.
URL http://www.sciencedirect.com/science/article/pii/S0960077918300559

[37] X. Ma, P. Sun, Fusing heterogeneous genomic data to discover cancer progression related dynamic modules, in: 2018 IEEE International Con-

ference on Bioinformatics and Biomedicine (BIBM), 2018, pp. 114–121. doi:10.1109/BIBM.2018.8621165.

[38] H. J. Li, L. Wang, Multi-scale asynchronous belief percolation model on multiplex networks, New Journal of Physics 21(1).