"PriorVAE: Encoding spatial priors with VAEs for small-area estimation"
Supplementary Materials

This manuscript was accepted for publication at the *Journal of the Royal Society Interface*.

Authors:

Elizaveta Semenova (University of Oxford),
Yidan Xu (University of Michigan),
Adam Howes (Imperial College London),
Theo Rashid (Imperial College London),
Samir Bhatt (Imperial College London, University of Copenhagen),
Swapnil Mishra (Imperial College London, University of Copenhagen),
Seth Flaxman (University of Oxford)

One-dimensional synthetic example: regular grid

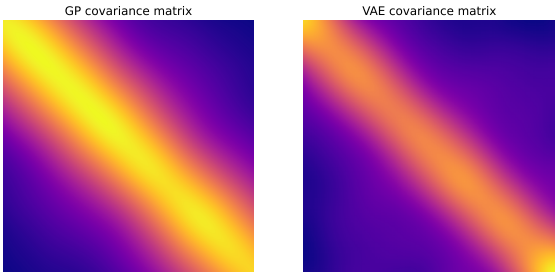


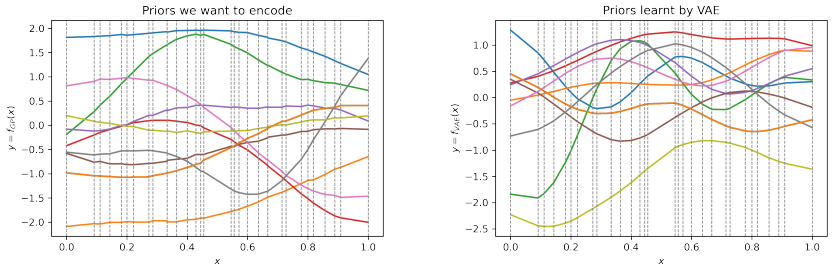
Figure 12: Covariance matrices computed empirically using 100 draws from the GP and VAE-GP priors.

One-dimensional synthetic example: irregular grid

Results of our approach on an irregular one-dimentional grid. Figure 13(a) shows samples of priors from the original GP, evaluated on the irregular grid, Figure 13(b) shows priors learnt by VAE and Figure 14 shows inference results for different number on observed datapoints.

Scottish lip cancer dataset

Here we present results concerned with the Scottish lip cancer dataset, produced by models with i.i.d., BYM and VAE-BYM random effects. Figure 15 shows posterior predictive distributions, produced by the three models when all of the available data was used to fit the models. We observe that the i.i.d. model already achieves a relatively good fit. BYM and VAE-BYM models are able to capture the remaming spatial dependence and are very similar between themselves.



(a) (b)

Figure 13: Learning one-dimensional GP-priors on an irregular grid with VAE: (a) prior samples from the original Gaussian process evaluations f_{GP}, (b) prior draws from f_{VAE} trained on f_{GP} draws

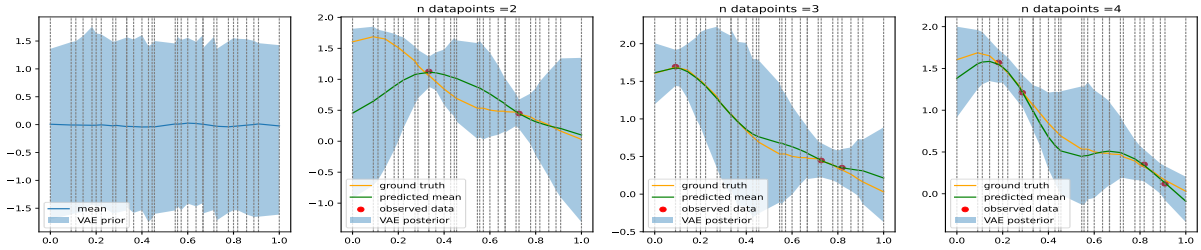
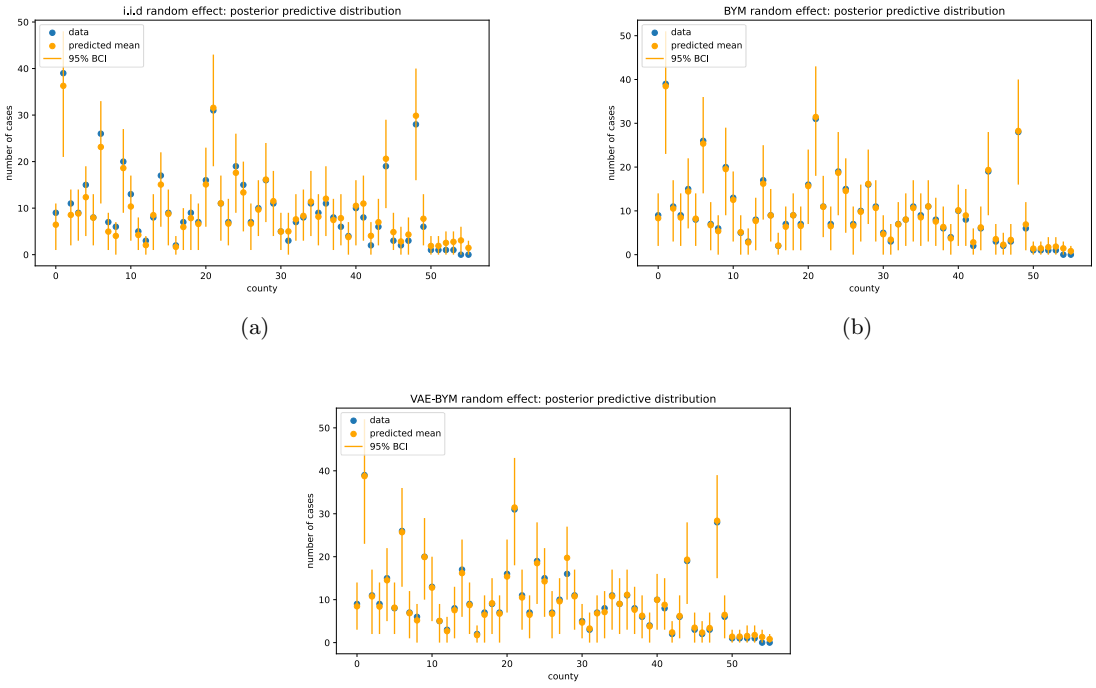


Figure 14: Results of MCMC inference on an irregular grid on noisy GP data by using the trained VAE priors f_{VAE}. The posterior mean of our model is shown in green with the 95% credible intervals shown in blue. Quality of the estimation improves with the growing number of data points.



(a) (b)

(c)

Figure 15: Posterior predictive distributions of the case count data for each out of 56 counties in Scotland. Each posterior distribution is represented by its point estimate (mean) and 95% Bayesian credible intervals

HIV prevalence in Zimbabwe dataset

Here we present results of the study using HIV prevalence data in Zimbabwe: posterior density of the spatial random effect for on earea during MCMC inference (Figure 16), autocorrelations of MCMC samples of a spatial random effect modelled via the VAE-CAR model (Figure 17) and autocorrelations of MCMC samples of a spatial random effect modelled via the CAR model (Figure 18).

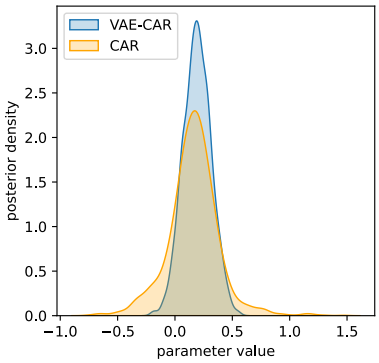


Figure 16: Posterior density of the spatial random effect for one area obtained during MCMC inference on HIV prevalence data from Zimbabwe.

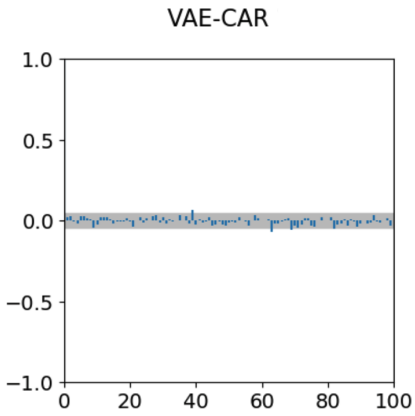


Figure 17: HIV Zimbabwe prevalence data. Autocorrelations of MCMC samples of a spatial random effect modelled via the VAE-CAR model.

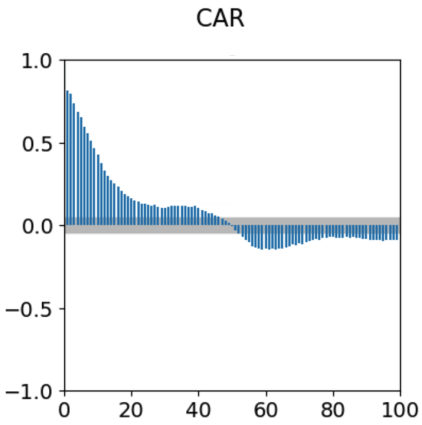


Figure 18: HIV Zimbabwe prevalence data. Autocorrelations of MCMC samples of a spatial random effect modelled via the CAR model.