Reinforcement Learning

Temporal-Difference Learning (RLbook 6)

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TD Prediction

$$V(S_t) \leftarrow V(S_t) + \alpha\left[G_t - V(S_t)\right] \tag{6.1}$$

$$V(S_t) \leftarrow V(S_t) + \alpha\left[T_{t+1} + V(S_{t+1}) - \gamma V(S_t)\right]$$

Tabular TD(0) Algorithm

Tabular TD(0) for estimating v_π

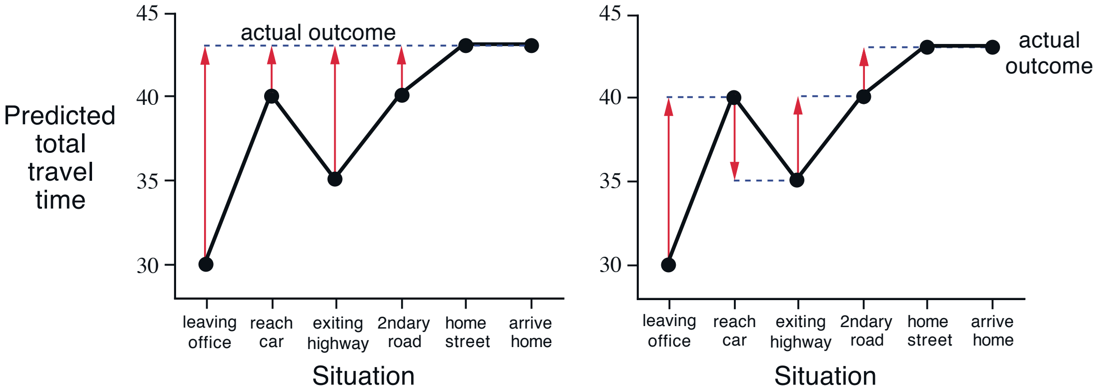
Input: the policy π to be evaluated
Algorithm parameter: step size $\alpha \in (0, 1]$
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(\textit{terminal}) = 0$

Loop for each episode:
  Initialize S
  Loop for each step of episode:
    $A \leftarrow$ action given by π for S
    Take action A, observe R, S'
    $V(S) \leftarrow V(S) + \alpha\big[R + \gamma V(S') - V(S)\big]$
    $S \leftarrow S'$
  until S is terminal

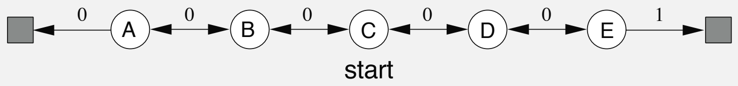
TD(0) Backup

TD(0)

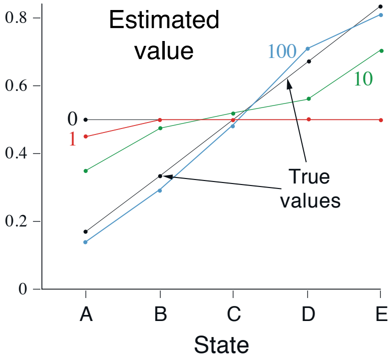
Example: Driving Home



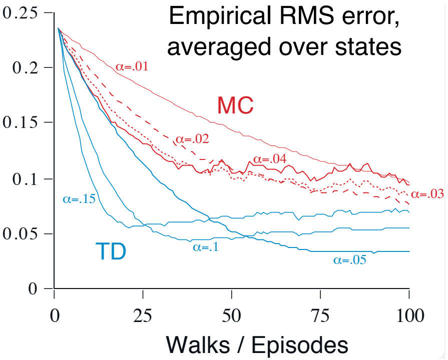
Example: Random Walk



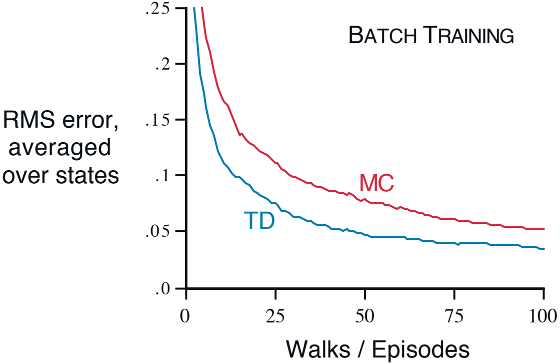
Random Walk State Values



Random Walk Error Rates



TD vs MC Performance

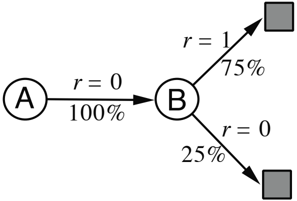


Example: Predicting Returns

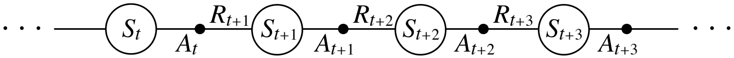
Given these 8 episodes:

- $A, 0, B, 0;\ B, 1;\ B, 1;\ B, 1;\ B, 1;\ B, 1;\ B, 1;\ B, 1;$

What are the value estimates for A and B?



Sarsa: On-policy TD Control



Sarsa update:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[R_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(S_t, A_t)\right]$$

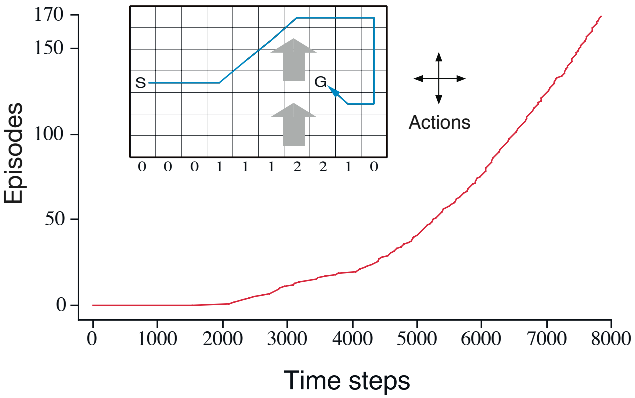
Sarsa Algorithm

Sarsa (on-policy TD control) for estimating $Q \approx q_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(\mathit{terminal}, \cdot) = 0$

Loop for each episode:
  Initialize S
  Choose A from S using policy derived from Q (e.g., ε-greedy)
  Loop for each step of episode:
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ε-greedy)
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma Q(S', A') - Q(S, A)\big]$
    $S \leftarrow S'$; $A \leftarrow A'$;
  until S is terminal

Example: Windy Grid World



Q-Learning: Off-policy TD Control

Sarsa update:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[R_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(S_t, A_t)\right]$$

Q-learning update:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[R_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(S_t, A_t)\right]$$

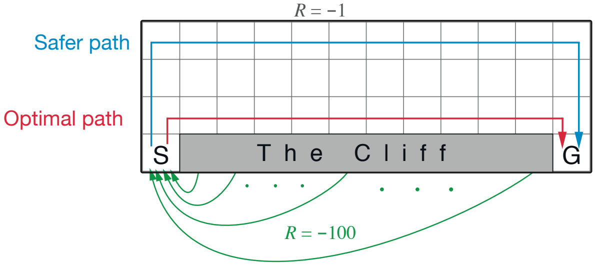
Q-Learning Algorithm

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

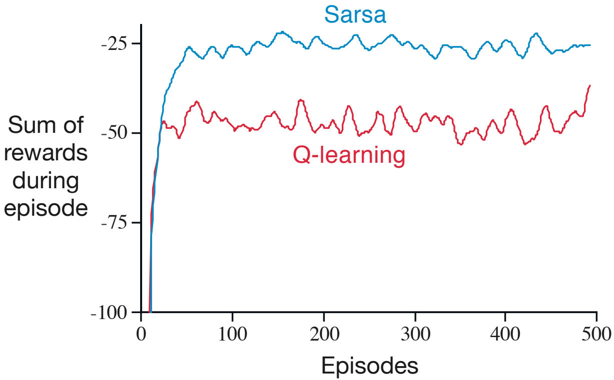
Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(\mathit{terminal}, \cdot) = 0$

Loop for each episode:
    Initialize S
    Loop for each step of episode:
        Choose A from S using policy derived from Q (e.g., ε-greedy)
        Take action A, observe R, S'
        $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
        $S \leftarrow S'$
    until S is terminal

Example: Cliff Walking



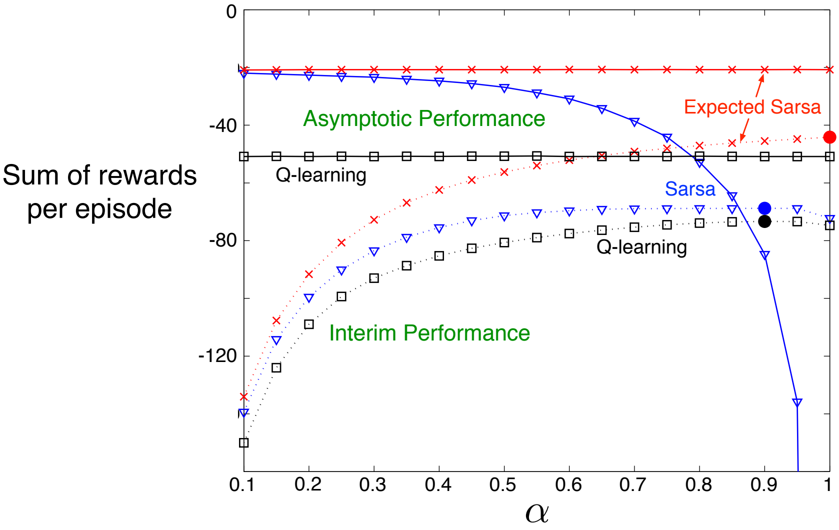
Sarsa vs Q-learning in Cliff Walking



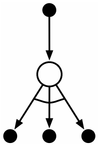
Expected Sarsa

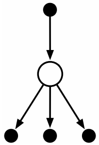
$$
\begin{aligned}
Q(S_t, A_t) &\leftarrow Q(s_t, A_t) + \alpha \left[R_{t+1} + \gamma \mathbb{E}_\pi \left[Q(S_{t+1}, A_{t+1}) \mid s_{t+1}\right] - Q(S_t, A_t)\right] \\
&= Q(s_t, A_t) + \alpha \left[R_{t+1} + \gamma \sum_a \pi(a \mid S_{t+1}) Q(S_{t+1}, a) - Q(S_t, A_t)\right] \qquad (6.9)
\end{aligned}
$$

Asymptotic Performance of TD Control Methods

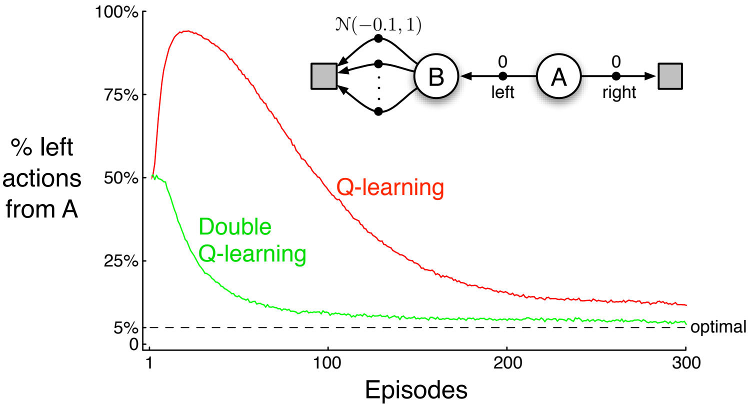


Q-learning vs Expected Sarsa Backup





Double Q-learning Performance



Double Q-learning Algorithm

Double Q-learning, for estimating $Q_1 \approx Q_2 \approx q_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q_1(s, a)$ and $Q_2(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, such that $Q(\mathit{terminal}, \cdot) = 0$

Loop for each episode:
    Initialize S
    Loop for each step of episode:
        Choose A from S using the policy ε-greedy in $Q_1 + Q_2$
        Take action A, observe R, S'
        With 0.5 probabilility:
            $Q_1(S, A) \leftarrow Q_1(S, A) + \alpha\Big(R + \gamma Q_2\big(S', \arg\max_a Q_1(S', a)\big) - Q_1(S, A)\Big)$
        else:
            $Q_2(S, A) \leftarrow Q_2(S, A) + \alpha\Big(R + \gamma Q_1\big(S', \arg\max_a Q_2(S', a)\big) - Q_2(S, A)\Big)$
        $S \leftarrow S'$
    until S is terminal

Games and Afterstates