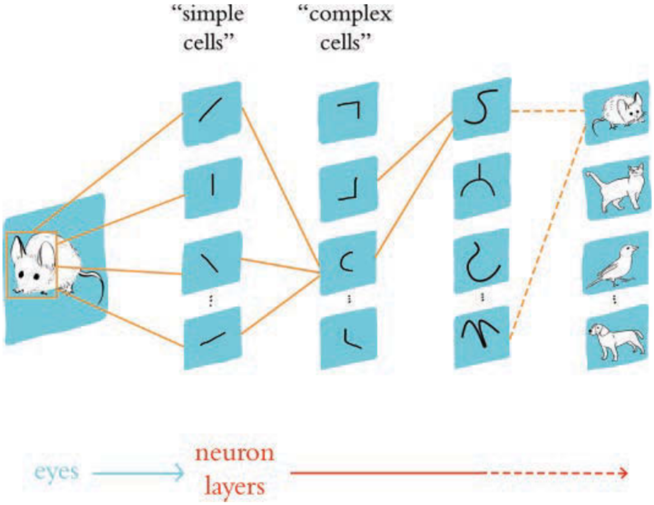
Deep Networks

CS 4277 Deep Learning

Kennesaw State University

Biological Vision

In the 1950s, Hubel and Wiesel at Johns Hopkins, experimenting on cats, discovered the hierarchical nature of neurons in the visual cortex.

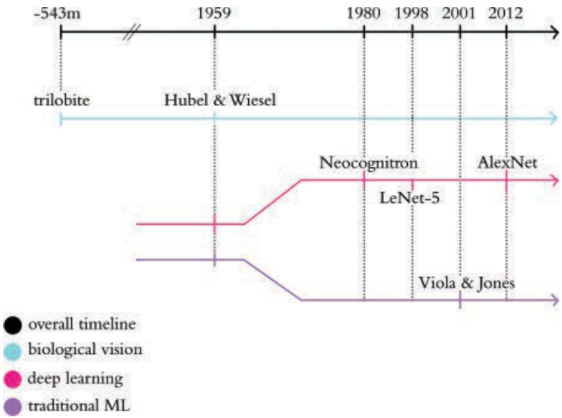


[1]

[1] https://www.deeplearningillustrated.com

Machine Vision

In 1980 Kunihiko Fukushima proposed the *Neocognitron* architecture explicitly based on neuron layers in biological vision.



[2]

It took the success of LeCun and Bengio's *LeNet-5*, and later Krizhevsky and Stuskever's *AlexNet* to realize the full potential of a deeply layered machine vision model and firmly establish the supremacy of Deep Learning for machine vision.

[2] https://www.deeplearningillustrated.com

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Shallow Networks

Recall the grapphical depiction of a single input/output network:

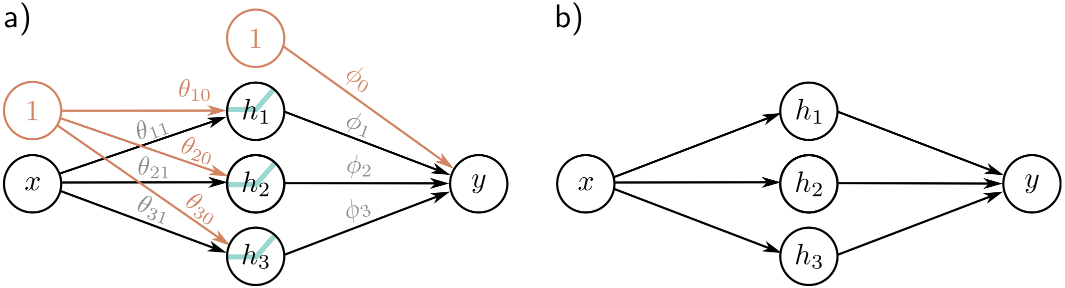


Figure 1: Fig 3.4

Note the

- θs for weights on preactivations,
- ϕs for weights on activations, and
- bias terms indicated with a "from" index of 0.

Composing Shallow Networks

Now recall the general formulation of a single input/output shallow network (left half of figure on right):

$$h_d = a(\theta_{d0} + \theta_{d1} x)$$

$$y = \phi_0 + \sum_{d=1}^{D} \phi_d h_d$$

You could concatenate this with another shallow network with the same architecture (right half of figure on right) that takes the first network's output as its input:

$$h'_d = a(\theta'_{d0} + \theta'_{d1} y)$$

$$y' = \phi'_0 + \sum_{d=1}^{D} \phi'_d h'_d$$

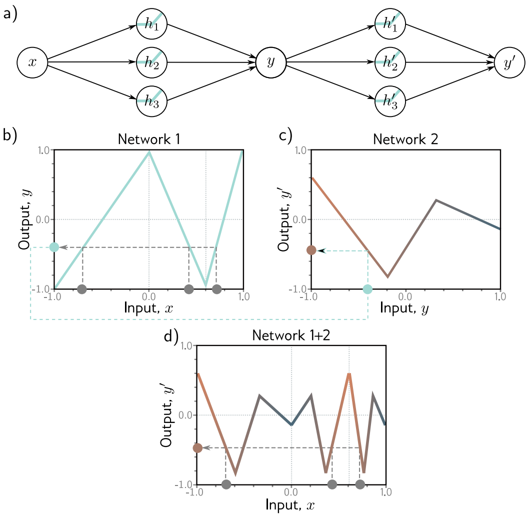


Figure 2: Fig 4.1

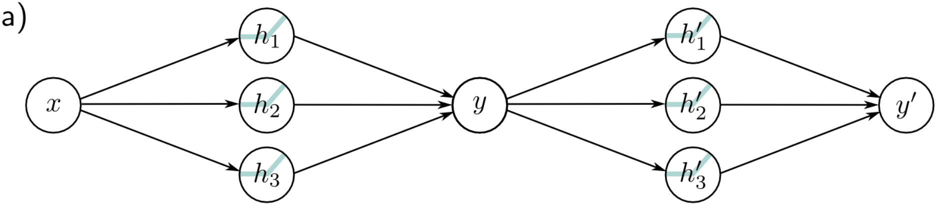
Composed 1D Shallow Networks

If we substitute the expression for y (notice I'm using i to index over units in the first hidden layer, and d for the second, where there is the same number D of hidden units in each layer):

$$y = \phi_0 + \sum_{i=1}^{D} \phi_i h_i$$

into the formulas for the hidden units in the second shallow network we get:

$$h'_d = a(\theta'_{d0} + \theta'_{d1} y) = a(\theta'_{d0} + \theta'_{d1}\phi_0 + \sum_{i=1}^{D} \theta'_{d1}\phi_i h_i)$$



From Composed 1D Shallow Nets to 1D Deep Net

If we then let $\psi_{d0} = \theta'_{d0} + \theta'_{d1}\phi_0$ and $\psi_{di} = \theta'_{d1}\phi_i$ we get:

$$h'_d = a(\psi_{d0} + \sum_{i=1}^{D} \psi_{di} h_i)$$

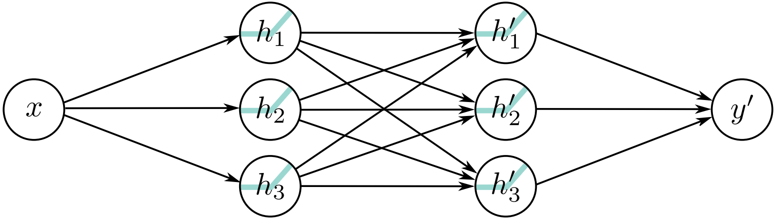
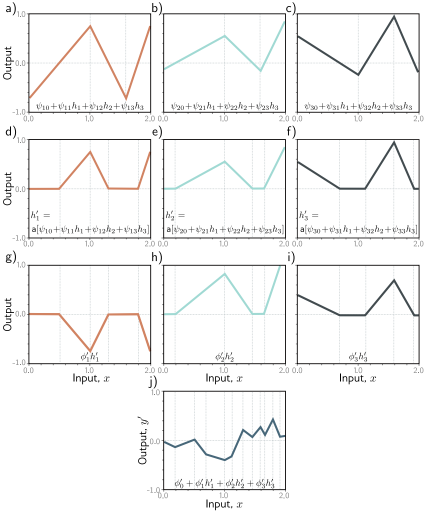


Figure 3: Fig 4.4

Deep Neural Network Signal Flow

4.3 (Fig 4.5)



Hyperparameters

The hyperparameters of a network are fixed quantities describing the architecure of the network. They include:

- *Width*, D: number of hidden units in each layer
- *Depth*, K: number of hidden layers

The dimensionality of the input and output are also fixed.

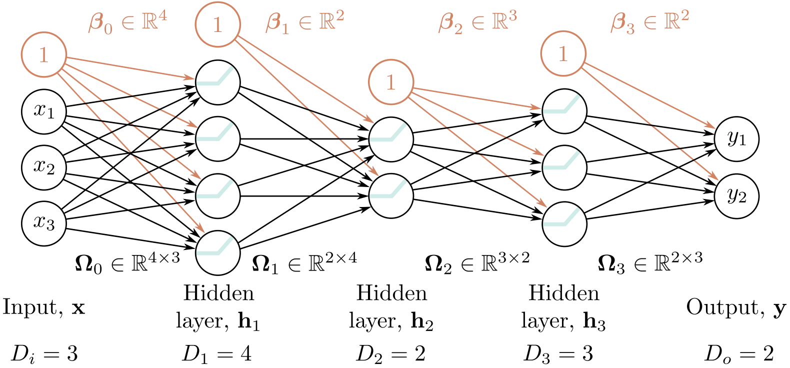


Figure 4: Fig 4.6

The parameters, biases and weights, are adjusted during training and denoted with greek letters.

Linear Algebra Interlude

- Vector Addition
- Scalar Multiplication
- $\boldsymbol{A}\vec{x}$

Matrix Network Notation

We can take the previous 1D deep net formulated with sums and formulate it with matrices:

$$\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = a \left[\begin{bmatrix} \theta_{10} \\ \theta_{20} \\ \theta_{30} \end{bmatrix} + \begin{bmatrix} \theta_{11} \\ \theta_{21} \\ \theta_{31} \end{bmatrix} x \right]$$

$$\begin{bmatrix} h'_1 \\ h'_2 \\ h'_3 \end{bmatrix} = a \left[\begin{bmatrix} \psi_{10} \\ \psi_{20} \\ \psi_{30} \end{bmatrix} + \begin{bmatrix} \psi_{11} & \psi_{12} & \psi_{13} \\ \psi_{21} & \psi_{22} & \psi_{23} \\ \psi_{31} & \psi_{23} & \psi_{33} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} \right]$$

$$y' = \phi'_0 + \begin{bmatrix} \phi'_1 & \phi'_2 & \phi'_3 \end{bmatrix} \begin{bmatrix} h'_1 \\ h'_2 \\ h'_3 \end{bmatrix}$$

Or, collapsing the vectors and matrices:

$$\boldsymbol{h} = a(\boldsymbol{\theta}_0 + \boldsymbol{\theta} x)$$

$$\boldsymbol{h}' = a(\boldsymbol{\psi}_0 + \boldsymbol{\Psi}\boldsymbol{h})$$

$$y' = \phi'_0 + \boldsymbol{\phi}'^T \boldsymbol{h'}$$

General Matrix Formulation

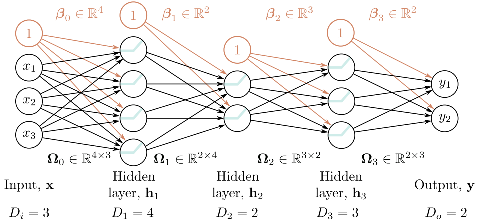
A general formulation of a network $\boldsymbol{y} = f(\vec{x}, \vec{\phi})$ where

- K is the number of layers,
- $\vec{\phi}$ refers to all the learned parameters $\{\boldsymbol{\beta}_k, \boldsymbol{\Omega}_k\}_{k=0}^{K}$,
- $\boldsymbol{\beta}_k$ are the biases in layer k and $\boldsymbol{\Omega}_k$ are the weights in layer k (replacing the θs and ϕs from before).

$$\begin{aligned} \boldsymbol{h}_K &= a\left(\boldsymbol{\beta}_{K-1} + \boldsymbol{\Omega}_{K-1}\boldsymbol{h}\right)_{K-1}) \\ \boldsymbol{y} &= \boldsymbol{\beta}_K + \boldsymbol{\Omega}_K \boldsymbol{h}_K \end{aligned} \tag{4.15}$$

We can write the whole network as:

$$\boldsymbol{y} = \boldsymbol{\beta}_K + \boldsymbol{\Omega}_K a\left(\boldsymbol{\beta}_{K-1} + \boldsymbol{\Omega}_{K-1} a(\ldots \boldsymbol{\beta}_1 + \boldsymbol{\Omega}_1 a(\boldsymbol{\beta}_0 + \boldsymbol{\Omega}_0 a))\right) \tag{4.16}$$



Capacity of Shallow vs. Deep Neural Networks

Considering 1D networks:

- A shallow network with $D > 2$ hidden units can create up to $D + 1$ linear regions.
- A deep network with K layers of $D > 2$ hidden units can create up to $(D + 1)^K$ linear regions.

Some functions require exponentially more hidden units than an equivalent deep network, a phenomenon known as *depth efficiency*

Deep nets seem to generalize better than shallow nets but require more training.

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Closing Thoughts

We now have the terminology and knowledge of the feed-forward operation of deep neural networks. For the rest of the course we will

- learn how deep networks are trained, and
- survey the major deep network architectures for a range of applications.