Deep Networks

CS 4277 Deep Learning

Kennesaw State University

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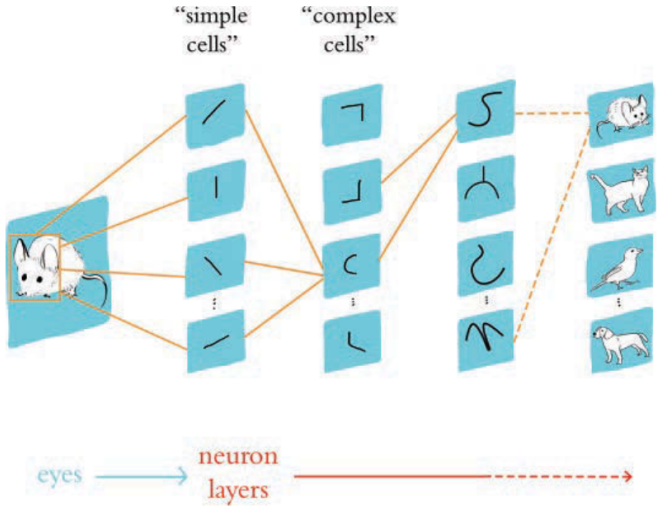
Biological Vision

In the 1950s, Hubel and Wiesel at Johns Hopkins, experimenting on cats, discovered the hierarchical nature of neurons in the visual cortex.



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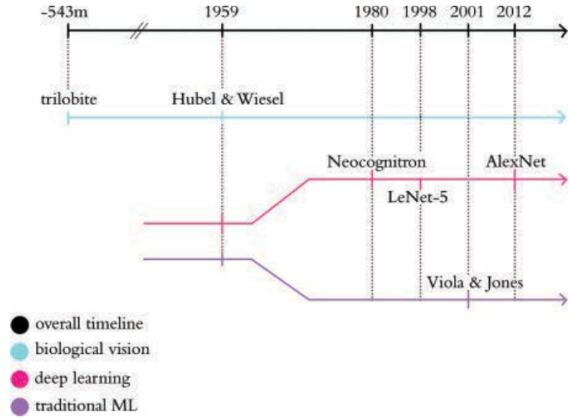
[1] https://www.deeplearningillustrated.com

Machine Vision

In 1980 Kunihiko Fukushima proposed the *Neocognitron* architecture explicitly based on neuron layers in biological vision.



[2]

It took the success of LeCun and Bengio's *LeNet-5*, and later Krizhevsky and Stuskever's *AlexNet* to realize the full potential of a deeply layered machine vision model and firmly establish the supremacy of Deep Learning for machine vision.

[2] https://www.deeplearningillustrated.com

Shallow Networks

Recall the grapphical depiction of a single input/output network:

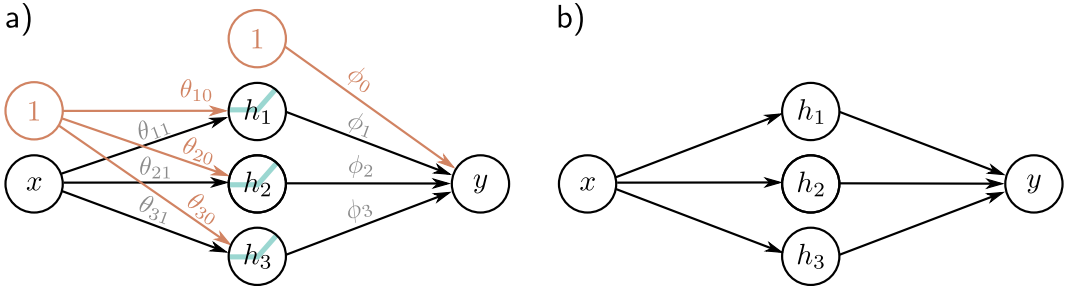


Figure 1: Fig 3.4

Note the

- θs for weights on preactivations,
- ϕs for weights on activations, and
- bias terms indicated with a "from" index of 0.

Composing Shallow Networks

Now recall the general formulation of a single input/output shallow network (left half of figure on right):

$$h_d = a(\theta_{d0} + \theta_{d1}x)$$

$$y = \phi_0 + \sum_{d=1}^{D} \phi_d h_d$$

You could concatenate this with another shallow network with the same architecture (right half of figure on right) that takes the first network's output as its input:

$$h'_d = a(\theta'_{d0} + \theta'_{d1}y)$$

$$y' = \phi'_0 + \sum_{d=1}^{D} \phi'_d h'_d$$

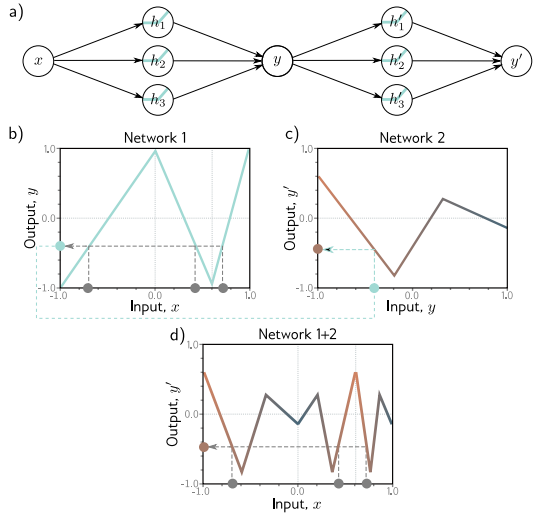


Figure 2: Fig 4.1

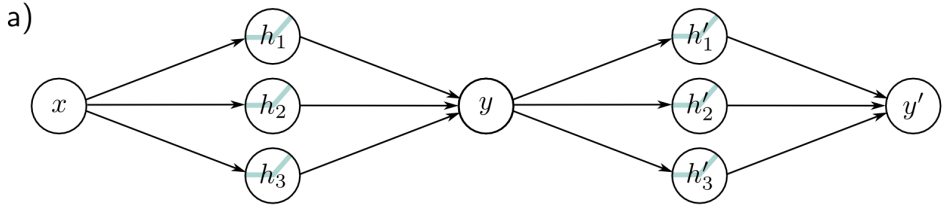
Composed 1D Shallow Networks

If we substitute the expression for y (notice I'm using i to index over units in the first hidden layer, and d for the second, where there is the same number D of hidden units in each layer):

$$y = \phi_0 + \sum_{i=1}^{D} \phi_i h_i$$

into the formulas for the hidden units in the second shallow network we get:

$$h'_d = a(\theta'_{d0} + \theta'_{d1} y) = a(\theta'_{d0} + \theta'_{d1}\phi_0 + \sum_{i=1}^{D} \theta'_{d1}\phi_i h_i)$$



From Composed 1D Shallow Nets to 1D Deep Net

If we then let $\psi_{d0} = \theta'_{d0} + \theta'_{d1}\phi_0$ and $\psi_{di} = \theta'_{d1}\phi_i$ we get:

$$h'_d = a(\psi_{d0} + \sum_{i=1}^{D} \psi_{di} h_i)$$

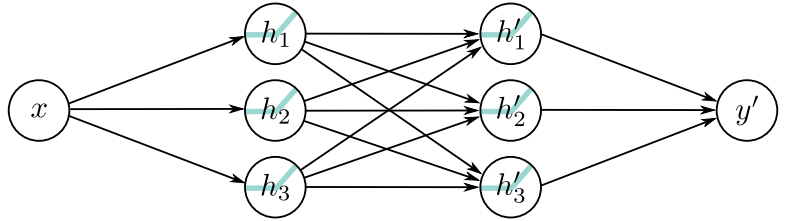
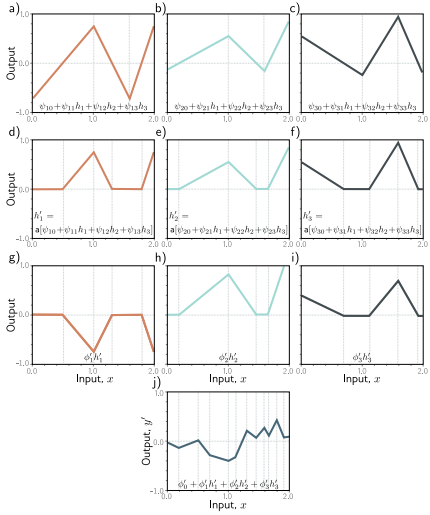


Figure 3: Fig 4.4

Deep Neural Network Signal Flow

4.3 (Fig 4.5)



Hyperparameters

The hyperparameters of a network are fixed quantities describing the architecure of the network. They include:

- *Width*, D: number of hidden units in each layer
- *Depth*, K: number of hidden layers

The dimensionality of the input and output are also fixed.

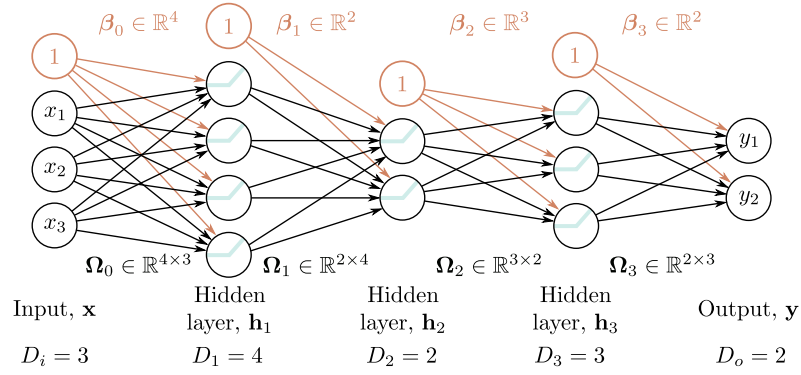


Figure 4: Fig 4.6

The parameters, biases and weights, are adjusted during training and denoted with greek letters.

Linear Algebra Interlude

- Vector Addition
- Scalar Multiplication
- $\boldsymbol{A}\vec{x}$

Matrix Network Notation

We can take the previous 1D deep net formulated with sums and formulate it with matrices:

$$\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = a\left[\begin{bmatrix} \theta_{10} \\ \theta_{20} \\ \theta_{30} \end{bmatrix} + \begin{bmatrix} \theta_{11} \\ \theta_{21} \\ \theta_{31} \end{bmatrix} x\right]$$

$$\begin{bmatrix} h'_1 \\ h'_2 \\ h'_3 \end{bmatrix} = a\left[\begin{bmatrix} \psi_{10} \\ \psi_{20} \\ \psi_{30} \end{bmatrix} + \begin{bmatrix} \psi_{11} & \psi_{12} & \psi_{13} \\ \psi_{21} & \psi_{22} & \psi_{23} \\ \psi_{31} & \psi_{23} & \psi_{33} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}\right]$$

$$y' = \phi'_0 + \begin{bmatrix} \phi'_1 & \phi'_2 & \phi'_3 \end{bmatrix} \begin{bmatrix} h'_1 \\ h'_2 \\ h'_3 \end{bmatrix}$$

Or, collapsing the vectors and matrices:

$$\boldsymbol{h} = a(\boldsymbol{\theta}_0 + \boldsymbol{\theta} x)$$
$$\boldsymbol{h}' = a(\boldsymbol{\psi}_0 + \boldsymbol{\Psi}\boldsymbol{h})$$
$$y' = \phi'_0 + \boldsymbol{\phi}'^T \boldsymbol{h}'$$

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General Matrix Formulation

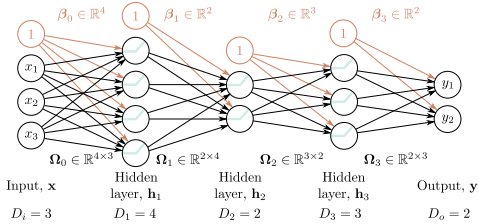
A general formulation of a network $\boldsymbol{y} = f(\vec{x}, \vec{\phi})$ where

- K is the number of layers,
- $\vec{\phi}$ refers to all the learned parameters $\{\boldsymbol{\beta}_k, \boldsymbol{\Omega}_k\}_{k=0}^{K}$,
- $\boldsymbol{\beta}_k$ are the biases in layer k and $\boldsymbol{\Omega}_k$ are the weights in layer k (replacing the θs and ϕs from before).

$$\begin{aligned} \boldsymbol{h}_K &= a\left(\boldsymbol{\beta}_{K-1} + \boldsymbol{\Omega}_{K-1}\boldsymbol{h}\right)_{K-1}) \\ \boldsymbol{y} &= \boldsymbol{\beta}_K + \boldsymbol{\Omega}_K \boldsymbol{h}_K \end{aligned} \tag{4.15}$$

We can write the whole network as:

$$\boldsymbol{y} = \boldsymbol{\beta}_K + \boldsymbol{\Omega}_K a\left(\boldsymbol{\beta}_{K-1} + \boldsymbol{\Omega}_{K-1} a(...\boldsymbol{\beta}_1 + \boldsymbol{\Omega}_1 a(\boldsymbol{\beta}_0 + \boldsymbol{\Omega}_0 a))\right) \tag{4.16}$$



Capacity of Shallow vs. Deep Neural Networks

Considering 1D networks:

- A shallow network with $D > 2$ hidden units can create up to $D + 1$ linear regions.
- A deep network with K layers of $D > 2$ hidden units can create up to $(D + 1)^K$ linear regions.

Some functions require exponentially more hidden units than an equivalent deep network, a phenomenon known as *depth efficiency*

Deep nets seem to generalize better than shallow nets but require more training.

Closing Thoughts

We now have the terminology and knowledge of the feed-forward operation of deep neural networks. FOr the rest of the course we will

- learn how deep networks are trained, and
- survey the major deep network architectures for a range of applications.