

Artificial Intelligence

Simple Decisions (AIMA 16)

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Making Simple Decisions

Decision Network

Influence diagram

Combining Beliefs and Desires under Uncertainty

$$Pr(RESULT(a) = s') = \sum_s Pr(s)Pr(s' | s, a)$$

Decision Theory

Expected utility:

$$EU(a) = \sum_{s'} Pr(RESULT(a) = s')U(s') \quad (15.1)$$

Principle of **maximum expected utility** (MEU):

$$action = \operatorname{argmax}_a EU(a)$$

If an agent acts so as to maximize a utility function that correctly reflects the performance measure, then the agent will achieve the highest possible performance score (averaged over all the possible environments).

Basis of Utility Theory

Notation:

- ▶ $A \succ B$: the agent prefers A over B .
- ▶ $A \sim B$: the agent is indifferent between A and B .
- ▶ $A \succsim B$: the agent prefers A over B or is indifferent between them.

Lottery L with outcomes S_1, \dots, S_n that occur with probabilities p_1, \dots, p_n :

$$L = [p_1, S_1; p_2, S_2; \dots; p_n, S_n]$$

Axioms of Utility Theory (1/2)

Six constraints that we require any reasonable preference relation to obey:

- ▶ **Orderability:** Given any two lotteries, a rational agent must either prefer one or else rate them as equally preferable. That is, the agent cannot avoid deciding. Refusing to bet is like refusing to allow time to pass.
 - ▶ Exactly one of $(A \succ B)$, $(B \succ A)$, or $(A \sim B)$ holds.
- ▶ **Transitivity:** Given any three lotteries, if an agent prefers A to B and prefers B to C , then the agent must prefer A to C .
 - ▶ $(A \succ B) \wedge (B \succ C) \implies (A \succ C).$
- ▶ **Continuity:** If some lottery B is between A and C in preference, then there is some probability p for which the rational agent will be indifferent between getting B for sure and the lottery that yields A with probability p and C with probability $1 - p$.
 - ▶ $A \succ B \succ C \implies \exists p[p, A; 1 - p, C] \sim B.$
- ▶ **Substitutability:** If an agent is indifferent between two lotteries A and B , then the agent is indifferent between two more complex lotteries that are the same except that B is substituted for A in one of them. This holds regardless of the probabilities and the other outcome(s) in the lotteries.
 - ▶ $A \sim B \implies [p, A; 1 - p, C] \sim [p, B; 1 - p, C].$
 - ▶ This also holds if we substitute \succ for \sim in this axiom.

Axioms of Utility Theory (2/2)

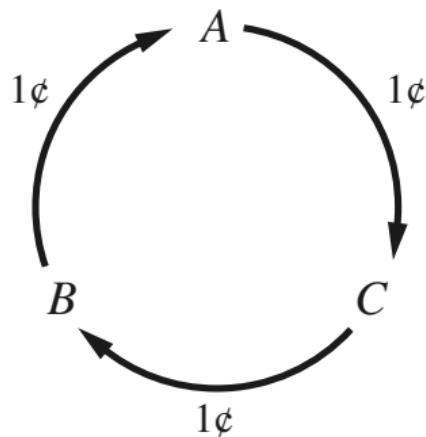
- ▶ **Monotonicity:** Suppose two lotteries have the same two possible outcomes, A and B . If an agent prefers A to B , then the agent must prefer the lottery that has a higher probability for A (and vice versa).
 - ▶ $A \succ B \implies (p > q \iff [p, A; 1 - p, B] \succ [q, A; 1 - q, B]).$
- ▶ **Decomposability:** Compound lotteries can be reduced to simpler ones using the laws of probability. This has been called the “no fun in gambling” rule: as Figure 15.1(b) shows, it compresses two consecutive lotteries into a single equivalent lottery.
 - ▶ $[p, A; 1 - p, [q, B; 1 - q, C]] \sim [p, A; (1 - p)q, B; (1 - p)(1 - q), C].$

From Rational Preferences to Utilities

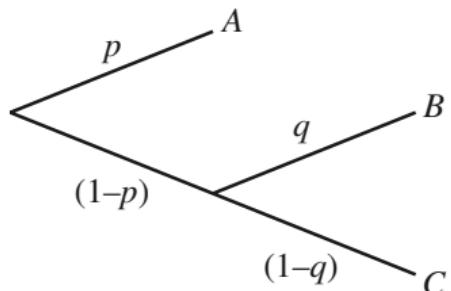
- ▶ **Existence of Utility Function:** If an agent's preferences obey the axioms of utility, then there exists a function U such that $U(A) > U(B)$ if and only if A is preferred to B , and $U(A) = U(B)$ if and only if the agent is indifferent between A and B . That is,
 - ▶ $U(A) > U(B) \iff A \succ B$ and $U(A) = U(B) \iff A \sim B$.
- ▶ **Expected Utility of a Lottery:** The utility of a lottery is the sum of the probability of each outcome times the utility of that outcome.
 - ▶ $U([p_1, S_1; \dots; p_n, S_n]) = \sum_i p_i U(S_i)$.

Value function or ordinal utility function: an agent needs only a preference ranking on states – the numbers don't matter

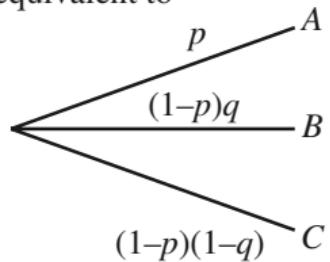
Nontransitive Preferences



Decomposability Axiom



is equivalent to



Utility of Money Example

Utility of Money in General

Optimism Error

Deterministic Dominace

Dominance under Uncertainty

Stochastic Dominance

Decision Networks

- ▶ Chance nodes (ovals) represent random variables, just as they do in Bayesian networks. The agent could be uncertain about the construction cost, the level of air traffic and the potential for litigation, and the Safety, Quietness, and total Frugality variables, each of which also depends on the site chosen. Each chance node has associated with it a conditional distribution that is indexed by the state of the parent nodes. In decision networks, the parent nodes can include decision nodes as well as chance nodes. Note that each of the current-state chance nodes could be part of a large Bayesian network for assessing construction costs, air traffic levels, or litigation potentials.
- ▶ Decision nodes (rectangles) represent points where the decision maker has a choice of actions. In this case, the AirportSite action can take on a different value for each site under consideration. The choice influences the safety, quietness, and frugality of the solution. In this chapter, we assume that we are dealing with a single decision node. Chapter 16 deals with cases in which more than one decision must be made.
- ▶ Utility nodes (diamonds) represent the agent's utility function. The utility node has as parents all variables describing the outcomes that directly affect utility. Associated with the utility node is a description of the agent's utility as a function of the parent attributes. The description could be just a tabulation of the function, or it might be a parameterized additive or linear function of the attribute values. For now, we will assume that the function is deterministic; that is, given the values of its parent variables, the value of the utility node is fully determined.

Decision Network for Airport Siting

Simplified Decision Network for Airport Siting

Value of Information

Information Gathering Agent

Decision Network for Ice Cream

Off Switch Game

