Artificial Intelligence

Probabilistic Inference

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Exact Inference in Bayesian Networks

AIMA

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Enumeration Algorithm

function ENUMERATION-ASK(X, $\mathbf{e}$, bn) **returns** a distribution over X
  inputs: X, the query variable
    $\mathbf{e}$, observed values for variables $\mathbf{E}$
    bn, a Bayes net with variables $vars$

  $\mathbf{Q}(X) \leftarrow$ a distribution over X, initially empty
  for each value x_i of X **do**
    $\mathbf{Q}(x_i) \leftarrow$ ENUMERATE-ALL($vars$, $\mathbf{e}_{x_i}$)
      where $\mathbf{e}_{x_i}$ is $\mathbf{e}$ extended with $X = x_i$
  return NORMALIZE($\mathbf{Q}(X)$)

function ENUMERATE-ALL($vars$, $\mathbf{e}$) **returns** a real number
  if EMPTY?($vars$) **then return** 1.0
  $V \leftarrow$ FIRST($vars$)
  if V is an evidence variable with value v in $\mathbf{e}$
    then return $P(v \mid parents(V)) \times$ ENUMERATE-ALL(REST($vars$), $\mathbf{e}$)
    else return $\sum_v P(v \mid parents(V)) \times$ ENUMERATE-ALL(REST($vars$), $\mathbf{e}_v$)
      where $\mathbf{e}_v$ is $\mathbf{e}$ extended with $V = v$

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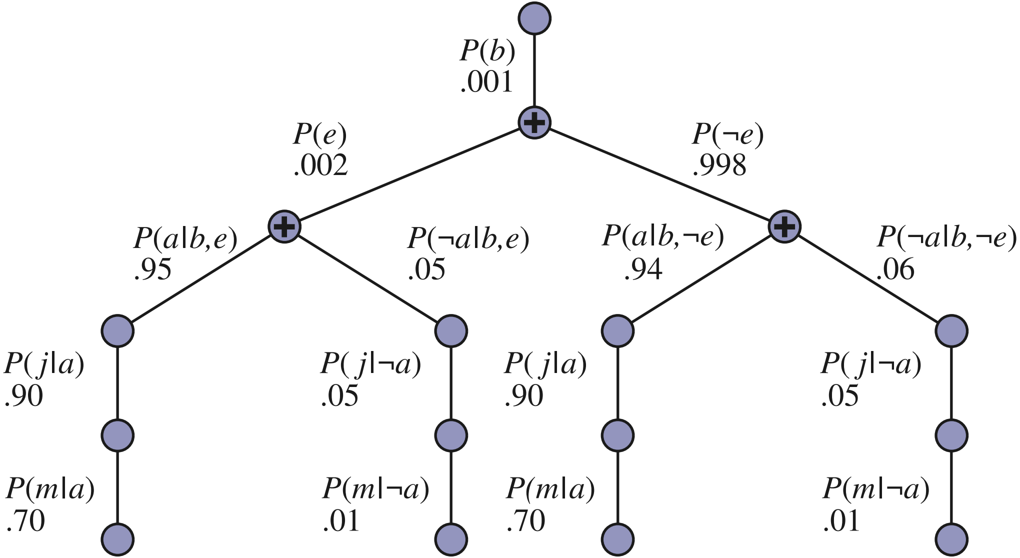
Repeated Calculations



Pointwise Products

X	Y	$\mathbf{f}(X,Y)$	Y	Z	$\mathbf{g}(Y,Z)$	X	Y	Z	$\mathbf{h}(X,Y,Z)$
t	t	.3	t	t	.2	t	t	t	$.3\times .2 = .06$
t	f	.7	t	f	.8	t	t	f	$.3\times .8 = .24$
f	t	.9	f	t	.6	t	f	t	$.7\times .6 = .42$
f	f	.1	f	f	.4	t	f	f	$.7\times .4 = .28$
						f	t	t	$.9\times .2 = .18$
						f	t	f	$.9\times .8 = .72$
						f	f	t	$.1\times .6 = .06$
						f	f	f	$.1\times .4 = .04$

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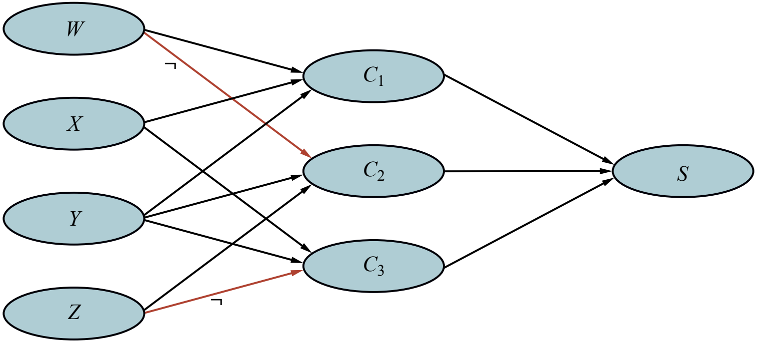
Variable Elimination Algorithm

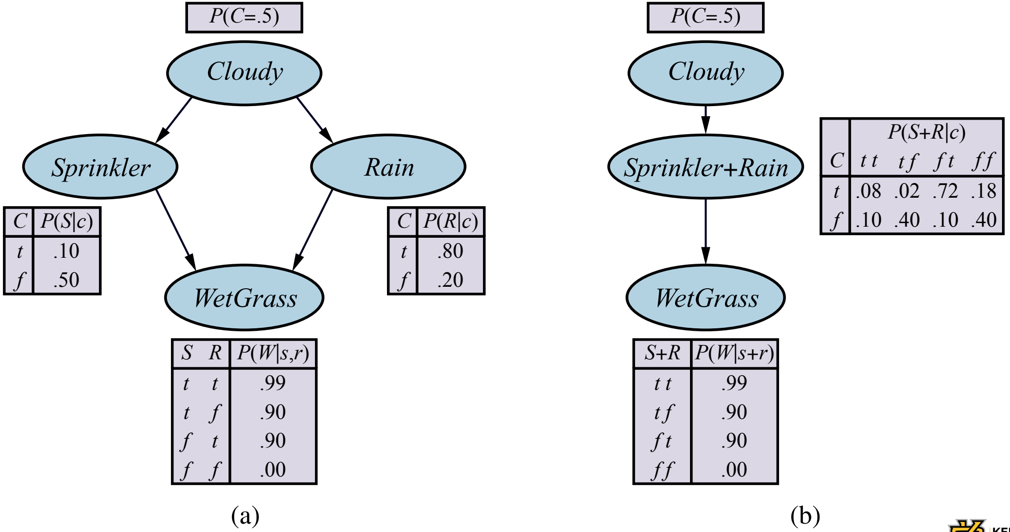
```
function ELIMINATION-ASK(X, e, bn) returns a distribution over X
  inputs: X, the query variable
          e, observed values for variables E
          bn, a Bayesian network with variables vars

  factors ← [ ]
  for each V in ORDER(vars) do
    factors ← [MAKE-FACTOR(V, e)] + factors
    if V is a hidden variable then factors ← SUM-OUT(V, factors)
  return NORMALIZE(POINTWISE-PRODUCT(factors))
```

Complexity of Exact Inference



Clustering Algorithms

aka joint trees.



Direct Sampling Methods

Prior Sampling

function PRIOR-SAMPLE(*bn*) **returns** an event sampled from the prior specified by *bn*
inputs: *bn*, a Bayesian network specifying joint distribution $\mathbf{P}(X_1, \ldots, X_n)$

$\mathbf{x} \leftarrow$ an event with n elements
for each variable X_i **in** $X_1, \ldots, X_n$ **do**
  $\mathbf{x}[i] \leftarrow$ a random sample from $\mathbf{P}(X_i \mid parents(X_i))$
return x

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Rejection Sampling

function REJECTION-SAMPLING($X, \mathbf{e}, bn, N$) **returns** an estimate of $\mathbf{P}(X \mid \mathbf{e})$
  inputs: X, the query variable
        $\mathbf{e}$, observed values for variables $\mathbf{E}$
        bn, a Bayesian network
        N, the total number of samples to be generated
  local variables: $\mathbf{C}$, a vector of counts for each value of X, initially zero

  for $j = 1$ **to** N **do**
    $\mathbf{x} \leftarrow$ PRIOR-SAMPLE(bn)
    if $\mathbf{x}$ is consistent with $\mathbf{e}$ **then**
      $\mathbf{C}[j] \leftarrow \mathbf{C}[j]+1$ where x_j is the value of X in $\mathbf{x}$
  return NORMALIZE($\mathbf{C}$)

Importance Sampling

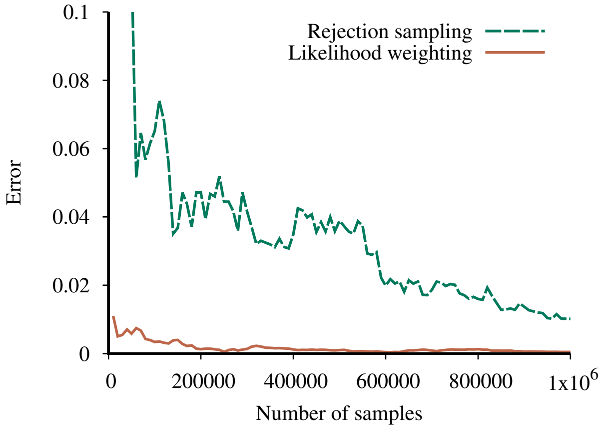
function LIKELIHOOD-WEIGHTING($X, \mathbf{e}, bn, N$) **returns** an estimate of $\mathbf{P}(X \mid \mathbf{e})$
  inputs: X, the query variable
    $\mathbf{e}$, observed values for variables $\mathbf{E}$
    bn, a Bayesian network specifying joint distribution $\mathbf{P}(X_1, \ldots, X_n)$
    N, the total number of samples to be generated
  local variables: $\mathbf{W}$, a vector of weighted counts for each value of X, initially zero

  for $j = 1$ **to** N **do**
    $\mathbf{x}, w \leftarrow$ WEIGHTED-SAMPLE($bn, \mathbf{e}$)
    $\mathbf{W}[j] \leftarrow \mathbf{W}[j] + w$ where x_j is the value of X in $\mathbf{x}$
  return NORMALIZE($\mathbf{W}$)

function WEIGHTED-SAMPLE($bn, \mathbf{e}$) **returns** an event and a weight

  $w \leftarrow 1$; $\mathbf{x} \leftarrow$ an event with n elements, with values fixed from $\mathbf{e}$
  for $i = 1$ **to** n **do**
    if X_i is an evidence variable with value x_{ij} in $\mathbf{e}$
      then $w \leftarrow w \times P(X_i = x_{ij} \mid parents(X_i))$
      else $\mathbf{x}[i] \leftarrow$ a random sample from $\mathbf{P}(X_i \mid parents(X_i))$
  return $\mathbf{x}$, w

Rejection vs. Importance Sampling



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Markov Chain Monte Carlo (MCMC) Algorithms

Instead of generating each sample from scratch, MCMC algorithms generate a sample by making a random change to the preceding sample. Think of an MCMC algorithm as being in a particular current state that specifies a value for every variable and generating a next state by making random changes to the current state.

Gibbs Sampling

function GIBBS-ASK($X, \mathbf{e}, bn, N$) **returns** an estimate of $\mathbf{P}(X \mid \mathbf{e})$
  local variables: $\mathbf{C}$, a vector of counts for each value of X, initially zero
    $\mathbf{Z}$, the nonevidence variables in bn
    $\mathbf{x}$, the current state of the network, initialized from $\mathbf{e}$

  initialize $\mathbf{x}$ with random values for the variables in $\mathbf{Z}$
  for $k = 1$ **to** N **do**
    choose any variable Z_i from $\mathbf{Z}$ according to any distribution $\rho(i)$
    set the value of Z_i in $\mathbf{x}$ by sampling from $\mathbf{P}(Z_i \mid mb(Z_i))$
    $\mathbf{C}[j] \leftarrow \mathbf{C}[j] + 1$ where x_j is the value of X in $\mathbf{x}$
  return NORMALIZE($\mathbf{C}$)

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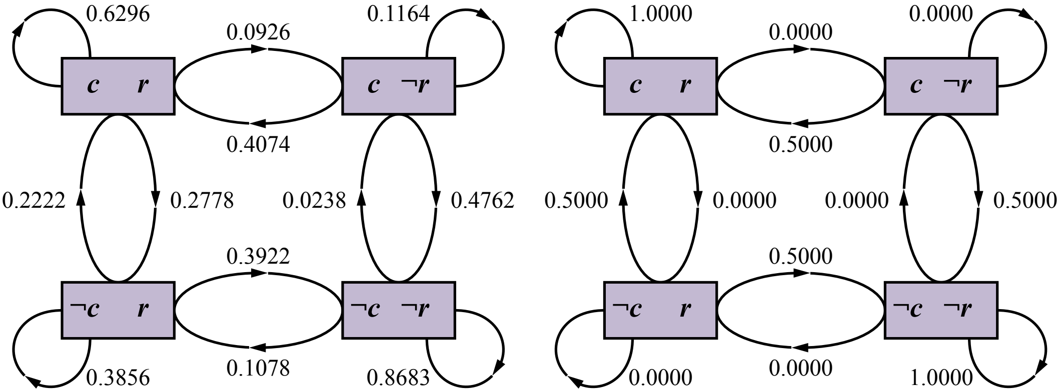
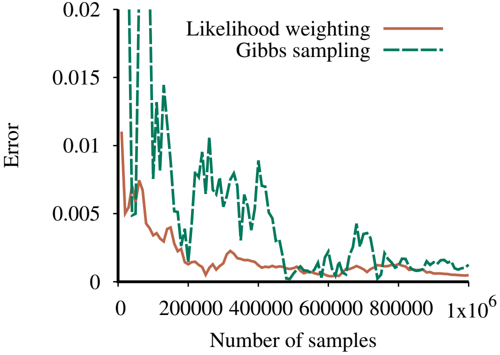
Markov Chains

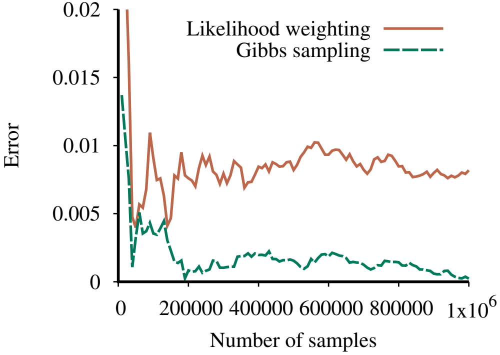


(a) (b)

Gibbs Sampling vs. Importance Sampling



(a)



(b)