# Problem Solving

Artificial Intelligence

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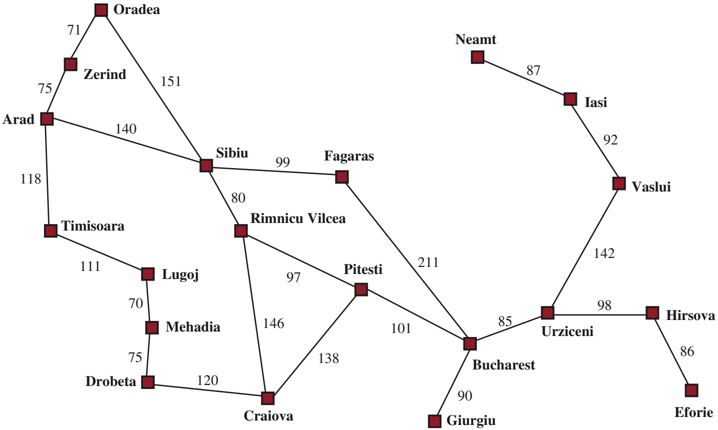
# Problem-Solving Agents

Consider a road map of Romanian cities:




- In this lesson we consider a *state* to be our location in one of these cities.
- A *goal* is a state in which we are located in a particular city.

This is the essence of problem solving: transforming a current state into a goal state. The first family of algorithms we'll study for problem solving are *search* algorithms.

# Problem Solving Process

To solve a problem, we

- Formulate a **goal**, e.g., "reach Bucharest"
- Formulate the **problem** as a set of states and actions that move us from one state to another.
  - Problem is a **model** – an *abstract* mathematical description.
  - Abstraction is essence and ignorance.
  - Key skill in problem formulation is finding the right **level of abstraction**.
- **Search** the possible sequences of action in our problem model that transforms our state from the current state to the goal state. A sequence of actions that gets us to the goal state is called a *solution*. May be many; pick one.
- **Execute** the actions in the solution.

# Open-Loop vs. Closed-Loop

- In an **open-loop** system the agent gets no feedback, i.e., sensor input, after executing an action.
  - If the agent's model is perfect and actions are deterministic, then the agent can operate in an open-loop fashion, simply executing the actions in the solution one after the other.
- In a **closed-loop** system gets sensory feedback after every action, so it can check whether the action had the expected effect.
  - If the environment is partially observable or actions are nondeterministic, closed-loop control is necessary.
  - Say the agent executes to `ToSibiu` action but ends up in `Zerind`. Closed-loop feedback will alert the agent to this fact so it can re-plan.

# Search Problems and Solutions

A search problem consists of:

- A set of **states**, which we call a **state space**.
- **Initial state**
- A set of **goal states**.
  - Typically use an `IS-GOAL(s)` predicate function to identify goal states.
- Sets of **actions** available in each state, `ACTION(s)`
  - `ACTION(Arad)= {ToSibiu, ToTimisoara, ToZerind}`
- A **transition model**, `RESULT(s, a)`
  - `RESULT(Arad, ToZerind)= Zerind`
- An **action cost function**, `ACTION-COST(s, a, s')` or $c(s, a, s')$ which returns the cost of executing action $a$ in state $s$ and reaching state $s'$.

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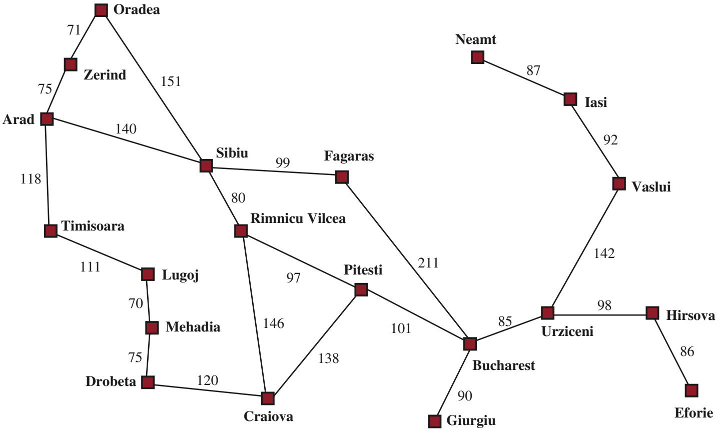
# Solution

- A solution is a path from the start state to the a goal state.
- An optimal solution is a solution with lowest cost among all solutions.




- How many paths are there from `Arad` to `Bucharest`?
- What is/are the solutions to the `Arad-to-Bucharest` problem (assume perfect information – fully observable, known dynamics, and deterministic actions)?

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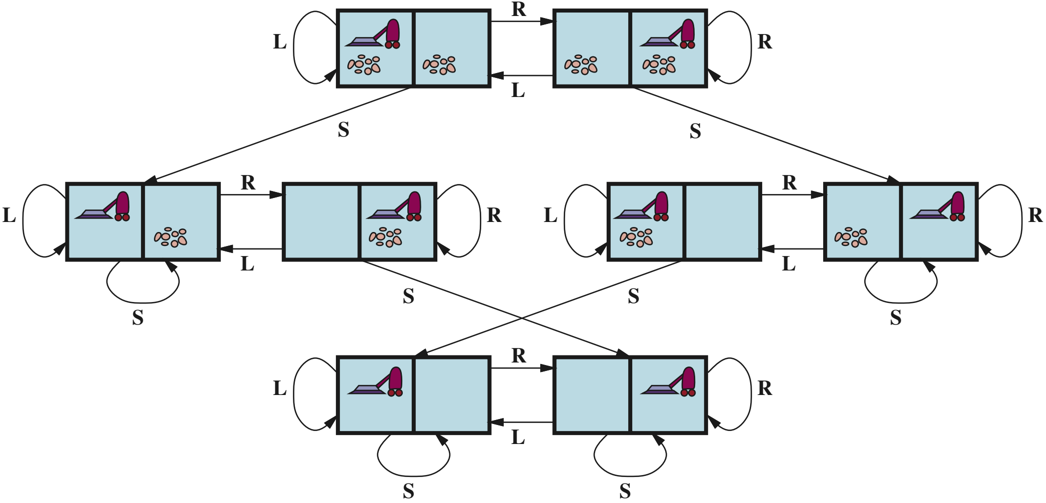
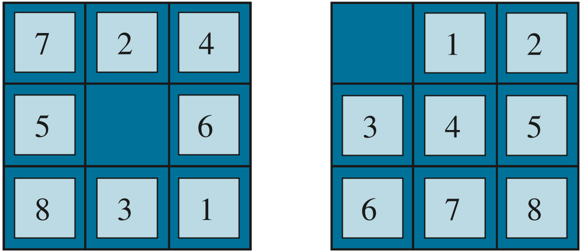
# Vacuum State Space Graph



# Agents

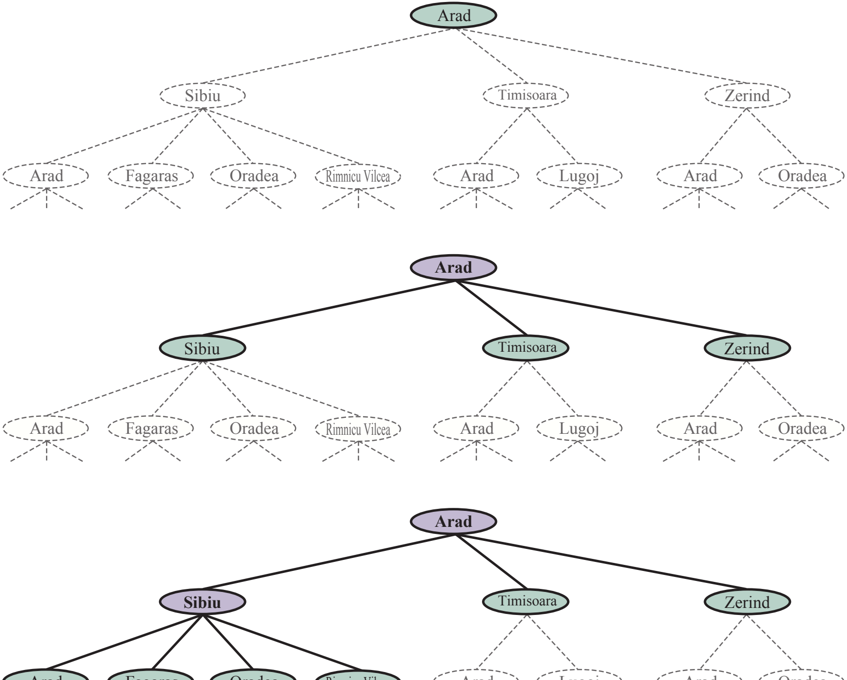



Start State Goal State

# Search Algorithms

- Search tree

# Searching State Space

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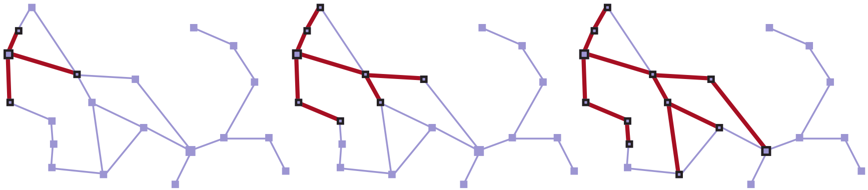
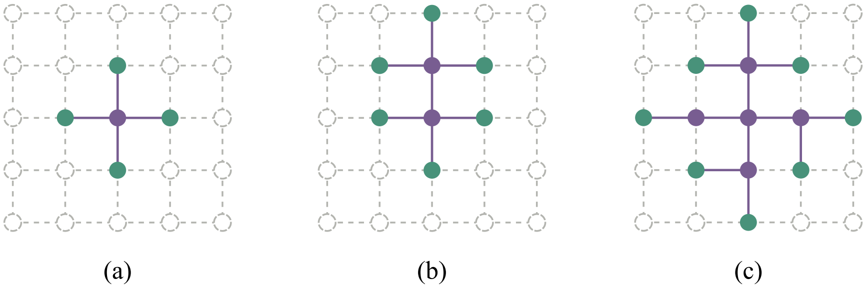
# Search Tree Expansion



# Separation Property of Graph Search




- (a) Only root expanded.
- (b) Top frontier node expanded.
- (c) Remaining successors of root expanded in clockwise order.

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# Best-First Search Algorithm

```
function BEST-FIRST-SEARCH(problem, f) returns a solution node or failure
  node ← NODE(STATE=problem.INITIAL)
  frontier ← a priority queue ordered by f, with node as an element
  reached ← a lookup table, with one entry with key problem.INITIAL and value node
  while not IS-EMPTY(frontier) do
    node ← POP(frontier)
    if problem.IS-GOAL(node.STATE) then return node
    for each child in EXPAND(problem, node) do
      s ← child.STATE
      if s is not in reached or child.PATH-COST < reached[s].PATH-COST then
        reached[s] ← child
        add child to frontier
  return failure

function EXPAND(problem, node) yields nodes
  s ← node.STATE
  for each action in problem.ACTIONS(s) do
    s′ ← problem.RESULT(s, action)
    cost ← node.PATH-COST + problem.ACTION-COST(s, action, s′)
    yield NODE(STATE=s′, PARENT=node, ACTION=action, PATH-COST=cost)
```

# Search Data Structures

Node:

- `node.STATE`: the state to which the node corresponds;
- `node.PARENT`: the node in the tree that generated this node;
- `node.ACTION`: the action that was applied to the parent's state to generate this node;
- `node.PATH-COST`: the total cost of the path from the initial state to this node. In mathematical formulas, we use g(node) as a synonym for PATH-COST.

Frontier:

- `IS-EMPTY(frontier)` returns true only if there are no nodes in the frontier.
- `POP(frontier)` removes the top node from the frontier and returns it.
- `TOP(frontier)` returns (but does not remove) the top node of the frontier.
- `ADD(node, frontier)` inserts node into its proper place in the queue.

Queues used in search algorithms:

- A **priority queue** first pops the node with the minimum cost according to some evaluation function, f . It is used in best-first search.
- A **FIFO queue** or first-in-first-out queue first pops the node that was added to the queue first; we shall see it is used in breadth-first search.
- A **LIFO queue** or last-in-first-out queue (also known as a stack) pops first the most recently added node; we shall see it is used in depth-first search.

# Redundant Paths

Repeated states

cycles

redundant paths

graph search

tree-like search

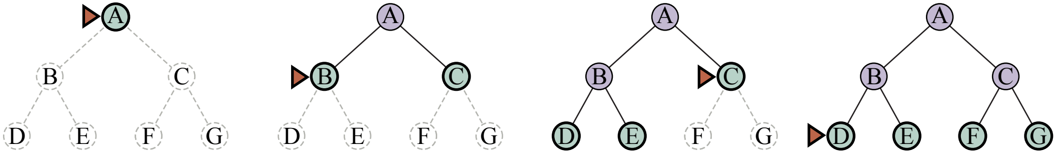
# Measuring Problem-Solving Performance

- Completeness: Is the algorithm guaranteed to find a solution when there is one, and to correctly report failure when there is not?
- Cost optimality: Does it find a solution with the lowest path cost of all solutions?
- Time complexity: How long does it take to find a solution? This can be measured in seconds, or more abstractly by the number of states and actions considered.
- Space complexity: How much memory is needed to perform the search?

# Uninformed Search Strategies

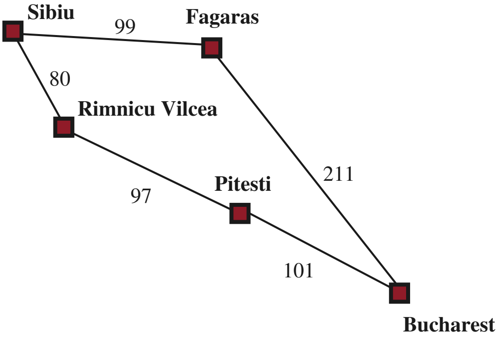
Strategy:

# Breadth-First Search

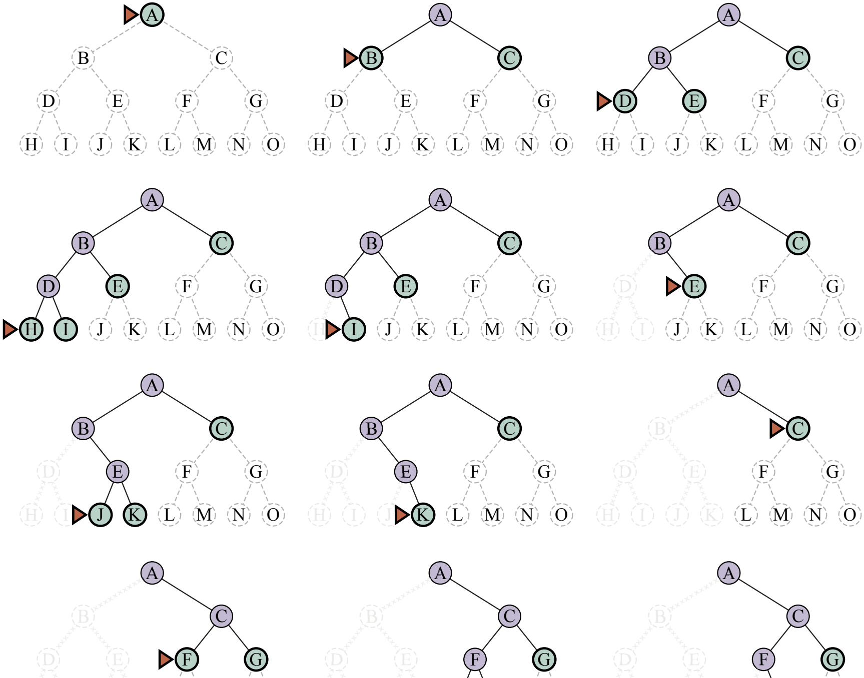



```
function BREADTH-FIRST-SEARCH(problem) returns a solution node or failure
  node ← NODE(problem.INITIAL)
  if problem.IS-GOAL(node.STATE) then return node
  frontier ← a FIFO queue, with node as an element
  reached ← {problem.INITIAL}
  while not IS-EMPTY(frontier) do
    node ← POP(frontier)
    for each child in EXPAND(problem, node) do
      s ← child.STATE
      if problem.IS-GOAL(s) then return child
      if s is not in reached then
        add s to reached
        add child to frontier
  return failure
```

# Dijkstra's Algorithm



# Depth-First Search



# Depth-Limited Search and Iterative Deepening Search

**function** ITERATIVE-DEEPENING-SEARCH(*problem*) **returns** a solution node or *failure*
  **for** *depth* = 0 **to** $\infty$ **do**
    *result* ← DEPTH-LIMITED-SEARCH(*problem*, *depth*)
    **if** *result* $\neq$ *cutoff* **then return** *result*

**function** DEPTH-LIMITED-SEARCH(*problem*, $\ell$) **returns** a node or *failure* or *cutoff*
  *frontier* ← a LIFO queue (stack) with NODE(*problem*.INITIAL) as an element
  *result* ← *failure*
  **while not** IS-EMPTY(*frontier*) **do**
    *node* ← POP(*frontier*)
    **if** *problem*.IS-GOAL(*node*.STATE) **then return** *node*
    **if** DEPTH(*node*) $> \ell$ **then**
      *result* ← *cutoff*
    **else if not** IS-CYCLE(*node*) **do**
      **for each** *child* **in** EXPAND(*problem*, *node*) **do**
        add *child* to *frontier*
  **return** *result*

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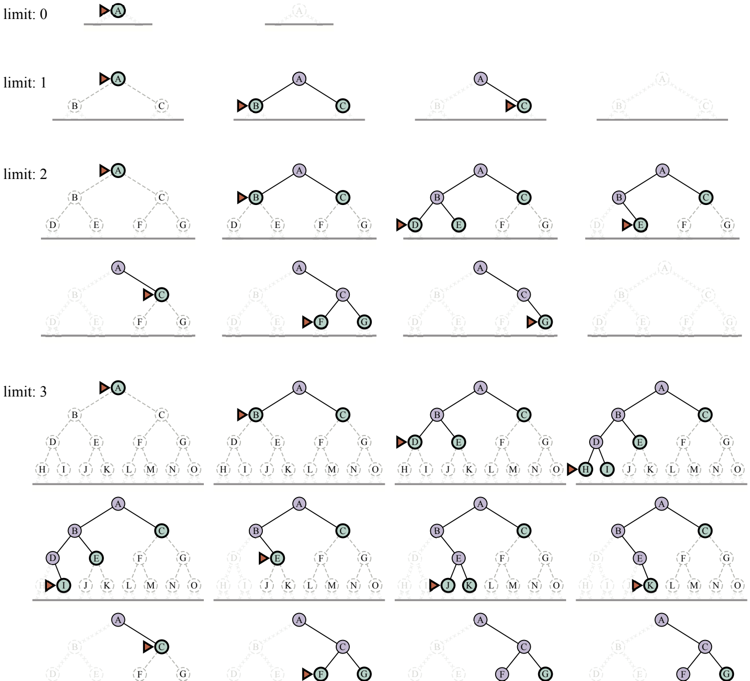
# Progression of Iterative Deepening Search



# Bidirectional Best-First Search

```
function BIBF-SEARCH(problem_F, f_F, problem_B, f_B) returns a solution node, or failure
  node_F ← NODE(problem_F.INITIAL)                          // Node for a start state
  node_B ← NODE(problem_B.INITIAL)                          // Node for a goal state
  frontier_F ← a priority queue ordered by f_F, with node_F as an element
  frontier_B ← a priority queue ordered by f_B, with node_B as an element
  reached_F ← a lookup table, with one key node_F.STATE and value node_F
  reached_B ← a lookup table, with one key node_B.STATE and value node_B
  solution ← failure
  while not TERMINATED(solution, frontier_F, frontier_B) do
    if f_F(TOP(frontier_F)) < f_B(TOP(frontier_B)) then
      solution ← PROCEED(F, problem_F, frontier_F, reached_F, reached_B, solution)
    else solution ← PROCEED(B, problem_B, frontier_B, reached_B, reached_F, solution)
  return solution

function PROCEED(dir, problem, frontier, reached, reached_2, solution) returns a solution
        // Expand node on frontier; check against the other frontier in reached_2.
        // The variable "dir" is the direction: either F for forward or B for backward.
  node ← POP(frontier)
  for each child in EXPAND(problem, node) do
    s ← child.STATE
    if s not in reached or PATH-COST(child) < PATH-COST(reached[s]) then
      reached[s] ← child
      add child to frontier
      if s is in reached_2 then
        solution_2 ← JOIN-NODES(dir, child, reached_2[s]))
        if PATH-COST(solution_2) < PATH-COST(solution) then
```

# Comparing Uninformed Search Algorithms

| Criterion | Breadth-First | Uniform-Cost | Depth-First | Depth-Limited | Iterative Deepening | Bidirectional (if applicable) |
|---|---|---|---|---|---|---|
| Complete? | Yes[1] | Yes[1,2] | No | No | Yes[1] | Yes[1,4] |
| Optimal cost? | Yes[3] | Yes | No | No | Yes[3] | Yes[3,4] |
| Time | $O(b^d)$ | $O(b^{1+\lfloor C^*/\epsilon \rfloor})$ | $O(b^m)$ | $O(b^\ell)$ | $O(b^d)$ | $O(b^{d/2})$ |
| Space | $O(b^d)$ | $O(b^{1+\lfloor C^*/\epsilon \rfloor})$ | $O(bm)$ | $O(b\ell)$ | $O(bd)$ | $O(b^{d/2})$ |