

**function** FIXED-LAG-SMOOTHING( $e_t, hmm, d$ ) **returns** a distribution over  $\mathbf{X}_{t-d}$

**inputs:**  $e_t$ , the current evidence for time step  $t$

$hmm$ , a hidden Markov model with  $S \times S$  transition matrix  $\mathbf{T}$

$d$ , the length of the lag for smoothing

**persistent:**  $t$ , the current time, initially 1

$\mathbf{f}$ , the forward message  $\mathbf{P}(X_t | e_{1:t})$ , initially  $hmm.\text{PRIOR}$

$\mathbf{B}$ , the  $d$ -step backward transformation matrix, initially the identity matrix

$e_{t-d:t}$ , double-ended list of evidence from  $t - d$  to  $t$ , initially empty

**local variables:**  $\mathbf{O}_{t-d}, \mathbf{O}_t$ , diagonal matrices containing the sensor model information

add  $e_t$  to the end of  $e_{t-d:t}$

$\mathbf{O}_t \leftarrow$  diagonal matrix containing  $\mathbf{P}(e_t | X_t)$

**if**  $t > d$  **then**

$\mathbf{f} \leftarrow \text{FORWARD}(\mathbf{f}, e_{t-d})$

remove  $e_{t-d-1}$  from the beginning of  $e_{t-d:t}$

$\mathbf{O}_{t-d} \leftarrow$  diagonal matrix containing  $\mathbf{P}(e_{t-d} | X_{t-d})$

$\mathbf{B} \leftarrow \mathbf{O}_{t-d}^{-1} \mathbf{T}^{-1} \mathbf{B} \mathbf{O}_t$

**else**  $\mathbf{B} \leftarrow \mathbf{B} \mathbf{O}_t$

$t \leftarrow t + 1$

**if**  $t > d + 1$  **then return** NORMALIZE( $\mathbf{f} \times \mathbf{B} \mathbf{1}$ ) **else return** null