4.2: 1d, 26, 3d
4.3: 2df, 4*
4.4: 2h, 8

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CS 250
4.2 - 4.4

1d. $x \sim y$ iff $x-y$ is an Integer, over $\mathbb{Q}$

iff $\frac{a}{b} - \frac{c}{d}$ is an integer

iff $\frac{a}{b} - \frac{nc}{nd}$ is an integer, $nd = b$

iff $\frac{a}{b} - \frac{nc}{nd}$ ~~is an integer,~~ $nd = b$, ~~[illegible]~~ $b \mid (a - nc)$

reflexive: $x - x = 0$, $0 \in \mathbb{Z}$

symmetric: $x - y \in \mathbb{Z}$, then $y - x \in \mathbb{Z}$ because $\text{abs}(x-y) = \text{abs}(y-x)$

transitive: if $x-y \in \mathbb{Z}$, and $y - z \in \mathbb{Z}$, then $x - z \in \mathbb{Z}$

because $(x-y) + (y-z) \in \mathbb{Z}$ (any 2 integers added $\in \mathbb{Z}$)

$= (x - z) \in \mathbb{Z}$, QED.

$x \sim y$ iff $x-y$ is an integer over $\mathbb{Q}$ is an eq. rel because all three properties have been shown.

26. aRb iff $a/b \in \mathbb{Z}$, over $\mathbb{Q} - \{0\}$

reflexive: $x/x \in \mathbb{Z}$, satisfied

⇒ symmetric: $x/y \in \mathbb{Z}$, $y/x \notin \mathbb{Z}$: example: $\frac{2}{1} \in \mathbb{Z}$, $\frac{1}{2} \notin \mathbb{Z}$, not satisfied

transitive: $x/y \in \mathbb{Z}$, $y/z \in \mathbb{Z}$, $\Rightarrow$ $x/z \in \mathbb{Z}$ if $y \mid x$, and $z \mid y$, then $z \mid x$

or ~~[illegible]~~ $x/z \in \mathbb{Z}$, satisfied

3d. describe eq. classes for $x \sim y$ iff $f(x) = f(y)$; $F(x) = \text{floor}(x/3)$

$[0] = \{0, 1, 2\}$

$[1] = \{3, 4, 5\}$

$[2] = \{6, 7, 8\}$

$[n] = \{3n, 3n+1, 3n+2\}$ $\forall n \in \mathbb{Z}$

4.3

2 is R a partial order?

d) R = is Sister Of: no because it is not anti symmetric

f) $R = \{(2,1), (1,3), (2,3)\}$ $2R1$, $1R3$, $2R3$

R is antisymmetric,

R is transitive ($2R1$, $1R3 \Rightarrow 2R3$)

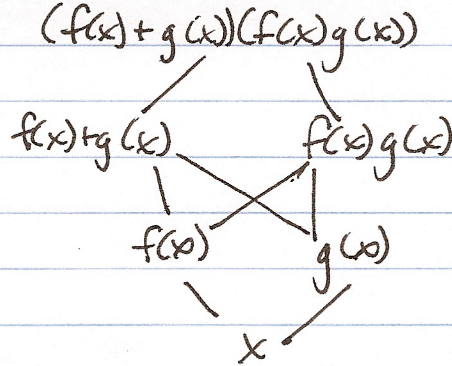
R is irreflexive ✓ partial order.

4.3 d) How to evaluate following as set of time-oriented tasks?

$$(f(x) + g(x))(f(x)\,g(x))$$

Subexpressions: $\{x, f(x), g(x), f(x)+g(x), f(x)g(x), (f(x)+g(x))(f(x)g(x))\}$

Poset diagram:



~~Minimal elements = least elements~~ =

Minimal elements: $\{x\}$

Maximal elements: $\{(f(x)+g(x))(f(x)g(x))$

Least element: x

Greatest element: $(f(x)+g(x))(f(x)g(x))$

4.4 2h Inductively prove $2+6+12+20+\cdots+n(n+1) = \frac{n(n+1)(n+2)}{3}$

Basis: $P(1) = \frac{1(1+1)(1+2)}{3} = \frac{6}{3} = 2$, $1(1+1) = 2$ ✓

Induction: assume $P(n) = \frac{n(n+1)(n+2)}{3} = 2+6+12+20+\cdots+n(n+1)$ is true

show that $P(n+1)$ is true

$$\frac{n(n+1)(n+2)}{3} + (n+1)(n+2) = 2+6+12+20+\cdots+n(n+1)+(n+1)(n+2)$$

$$\frac{n(n+1)(n+2)+3(n+1)(n+2)}{3} = 2+6+12+20+\cdots+n(n+1)+(n+1)(n+2)$$

$$\frac{(n+3)(n+1)(n+2)}{3} = 2+6+12+20+\cdots+n(n+1)+(n+1)(n+2)$$

$$= P(n+1) = \frac{(n+1)(n+2)(n+3)}{3} \checkmark$$

QED.

8. Use induction to prove a finite set w/ n elements contains 2^n subsets. $P(S_n) = |Power(S_n)|$

Basis: Show $P(1)$ is true: $S = \{a\}$, $Power(S) = \{\emptyset, a\}$, $P(1) = 2 = 2^1$ ✓

Induction: Assume $P(n)$ is true, ie $|Power(S_n)| = 2^n$

Show $P(n+1)$ is true, that is $|Power(S_{n+1})| = 2^{n+1}$

if $S_n = \{a_1, a_2, a_3, \ldots, a_n\}$

then we can express S_{n+1} as $S_n \cup \{a_{n+1}\}$

~~$|Power(S \cup \{a_{n+1}\})| = |Power(S)| +$ all new subsets possible w/ addition of a_{n+1}~~

$S_{n+1} = \{a_1, a_2, a_3, \ldots, a_n, a_{n+1}\}$

$\cancel{|Power(\{a_1, a_2, a_3, \ldots, a_n, a_{n+1}\})|} = 2^n + k$

Since $|Power(S_n)| = 2^n$, and $S_{n+1} = S \cup \{a_{n+1}\}$, we know that $|Power(S_{n+1})| = 2^n + k$, where k is the number of subsets in $Power(S_{n+1})$ that are not in $Power(S_n)$. ~~These~~ new subsets can only be made from existing subsets by combining a_{n+1} with each of them, resulting in 2^n new subsets. Thus, $Power(S_{n+1}) = 2^n + 2^n = 2^{n+1}$. ~~QED~~ $P(n+1) = 2^{n+1}$. QED.