



Instruments Designed for Teaching

# **MAGNETIC TORQUE\***

## **STUDENT MANUAL**

A PRODUCT OF TEACHSPIN, INC.

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# MAGNETIC TORQUE

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# Magnetic Torque Student Manual

## Chapter 1. Introduction

TeachSpin designed this instrument, “Magnetic Torque” M $\tau$ 1-A, to provide students with a hands-on apparatus that will demonstrate the physics of the interaction of a magnetic moment with magnetic fields. You will discover, experimentally, the effects of magnetic torque and force on a magnetic dipole.

In the Experiments section, we describe *five* independent ways that students can measure the magnetic moment embedded in the ball. Because they are independent, instructors can choose the ones best suited to their goals. We hope you will do at least three or four. ***TeachSpin believes that doing multiple experiments, such as these, on one instrument allows students to develop a comfort and expertise with the apparatus that frees them to turn their full attention both to acquiring excellent data and to understanding the physics both employed and revealed.***

Using the manually-rotated horizontal magnetic field, students are also able to perform a classical analog of the Pulsed NMR spin flip. In this analog, the rate at which the horizontal field is rotated represents the frequency of the radio frequency (RF) pulse used to tip the nuclear spins, while the time of rotation represents the pulse length.

The Magnetic Torque instrument is subtitled “A New Classic” because, in 1997, it was a new instrument for teaching classic physics principles. We are very proud that the experiments involved with this instrument have become part of the “classic” repertoire in physics teaching labs all over the world.

This manual is a comprehensive introduction and description of both the Magnetic Torque instrument and the student experiments for which it was designed. We have tried to include all of the information about the instrument that may be helpful. The manual discusses techniques for doing each experiment. Your instructor will choose those experiments best suited to your physics program.

## Chapter 2. The Instrument

- A. The Magnet** – What we refer to as “the magnet” includes the two co-axial coils, the air bearing and the strobe light all mounted on a wooden base.

### 1. Coils

The coils are composed of #18 copper wire that is wound on phenolic bobbins. **Each coil has 195 turns.** The two coils are always connected in series so that the same current flows through each turn. This current is displayed on the analog ammeter. It is important to note that the coils have some resistance, and that resistance is temperature-dependent. If significant current (~3-4 amps), is allowed to flow through the coils for a long time, the coils' temperature begins to rise. You can feel the increase in temperature. As the copper heats up, its resistance increases, and since the power supply is **not** current-regulated, the current decreases. It is therefore a good idea to have the students turn the current down to zero when not making observations or measurements. They should also avoid using high currents for any appreciable length of time. The instrument is designed to sustain the full output power of the supply without any danger. However, since the maximum current will be decreased as the temperature of the coils increases, students may not be able to obtain the highest fields if they allow the coils to get too hot.

A FIELD GRADIENT ON/OFF switch determines whether the current in the coils is moving in the same direction giving a uniform field, or in opposite directions creating a uniform gradient across the center of the sphere.

All experiments are done at the center of the coils and students will need a calibration for both the magnetic field (tesla/ampere) and the magnetic field gradient (tesla/meter/ampere) at that location.

- a. To calculate the magnetic field** at the center of the apparatus, one needs to perform an integral, since each of the turns has a different radius and a different distance from the center of the pair. Such a calculation may be a bit tedious for students, so we have calculated an equivalent ‘radius’ and equivalent ‘distance between the coils’ so that students can represent each coil by a single turn.

Using this representation, students can use the Biot-Savart law to find an equation for the magnitude of the magnetic field at the center of the system, which is where the magnetic dipole will be located in all of these experiments. *Therefore, it is only the magnetic field in the central region of the instrument that is important.*

By evaluating the magnitude of the field with one ampere of current flowing in the same direction in each of the coils, they will be able to find a theoretical calibration constant for the magnetic field.

$$\text{equivalent radius: } [r] = 0.109 \text{ m}$$

$$\text{equivalent separation between coils: } [d] = 0.138 \text{ m}$$

$$B = (1.36 \pm 0.03) \times 10^{-3} \text{ tesla/ampere (I in amperes)}$$

- b. The magnetic field gradient** at the center of the apparatus must also be determined. The field gradient can be calculated by differentiating the expression of the magnetic field with respect to  $z$ , where  $z$  is the center axis of the coils.

The expression for the on-axis magnetic field gradient due to one current loop is:

$$\frac{dB}{dz} = -\frac{3}{2} \mu_0 I r^2 \frac{z}{(r^2 + z^2)^{5/2}} \quad (\text{Field Gradient/loop of wire})$$

Using the equivalent radius, the value of  $\frac{dB}{dz}$  for both a single turn and at the central region of Mτ1-A can be calculated.

$$\frac{dB}{dz} = 4.33 \times 10^{-5} I \text{ tesla/ampere} \cdot \text{meter} \quad (\text{per loop of wire})$$

For both coils together, the  $z$ -axis field gradient of the entire magnet is:

$$\frac{dB}{dz} = (1.69 \pm 0.04) \times 10^{-2} I \text{ tesla/ampere} \cdot \text{meter} \quad (\text{for magnet})$$

Note: Because current will be in amperes, the units of  $dB/dz$  will be tesla/meter.

## 2. Air bearing

The air bearing is a spherical hollow in the brass cylinder that sits on the base of bottom coil form. The bearing has a narrow opening in its base through which air can be pumped into the spherical hollow. The ball sits in this hollow and floats on a layer of air that provides support with minimal friction. A vinyl hose connects an air pump housed inside the Magnetic Torque Controller to the under-side of the air bearing. Be careful not to restrict the air-flow by accidentally “kinking” the hose.

## 3. Strobe light

The strobe light sits in an insulated housing mounted on the upper coil. A knob on the Controller is used to vary the flash rate. The frequency is automatically measured and read out to two significant figures on the front panel of the Controller.

## 4. Bulls-eye level

If the air bearing is not level, an additional torque due to unequal air flow can result. Such a torque can produce erroneous data. Leveling can easily be accomplished by placing shims underneath the rubber feet below the magnet

## B. The Controller

The Controller houses the air pump, the electronic controls for the strobe light and a voltage controlled power supply.

1. **MAGNET CURRENT:** Analog ammeter displays the current passing through the coils which is controlled by the knob below. (Note: the coils are connected in series).
2. **FIELD DIRECTION DOWN/UP:** Controls whether the magnetic field at the center of the coils is up or down by reversing the current in both coils simultaneously.
3. **FIELD GRADIENT switch:** Toggles between uniform field and field gradient. When in OFF position, the current in both coils is in the same direction and the field is uniform. Switching to ON will reverse the current in the upper coil only to create a field gradient. In this condition, the magnitude of the field itself at the center of the system is zero.
4. **STROBE FREQUENCY:** LED displays the frequency of the strobe light in hertz. With, MT1, you must wait for the instrument to count up to the actual frequency. With an upgraded or MT2 instrument, strobe results are almost immediate.
5. **STROBE ADJUST:** Knob allows continuous adjustment of strobe frequency.
6. **STROBE OFF/ON:** Turns the strobe on and off.
5. **AIR OFF/ON:** Turns the air pump on and off.
6. **Pilot light:** Indicates when the ac power is on for the entire system
7. **Back Panel**
  - On/off ac switch for all of the components inside
  - power-entry socket for the cord to connect the system to ac line power
  - Cinch-Jones connector for connecting the Controller to the magnet
  - air hose connector



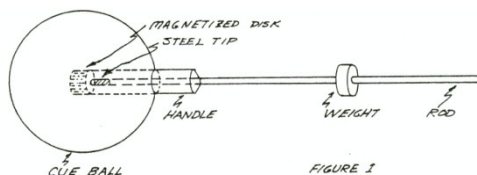
## C. The Accessories

### 1. Cue Ball

The cue ball is an Aramith snooker ball with a small cylindrical permanent magnet at its center. The magnet acts as a magnetic dipole. The “handle” is embedded in the ball with its axis along the magnetic moment vector of the dipole.

The size and location of the handle have been chosen so that the center-of-mass of the sphere/magnet/handle of the system can be considered to be the center of the ball.

A small axial hole drilled in the handle can hold a thin aluminum tube. The rod has a steel-tipped end that holds fast to the magnet inside of the ball. A small, clear plastic cylinder that slides along the length of the tube is used to vary the gravitational torque on the ball/handle system.



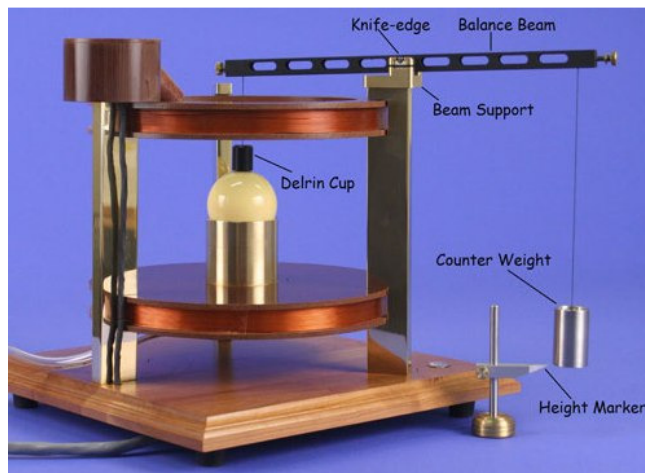
By balancing the gravitational torque with a magnetic torque produced by the uniform field of the coils, students can make a straightforward measurement of the magnetic moment of the dipole.

### 2. Magnetic Force Balance Kit

With the Magnetic Force Balance, students use the magnetic force due to a field gradient to measure the magnetic moment of the dipole embedded in the ball.

To use the balance, a V-groove agate bearing block is installed on top of a brass magnet support. The knife edge at the center of the balance beam is inserted into the V- groove

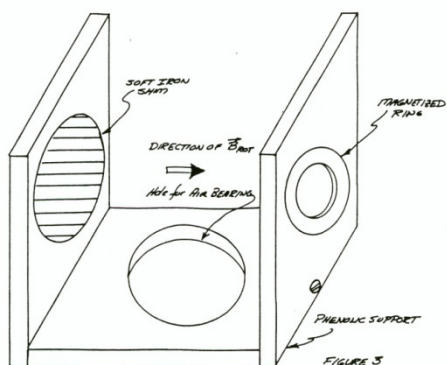
A brass counterweight hangs from one end of the non-magnetic beam. The handle of the ball is inserted into a Delrin cup hanging from the other end. The ball is held by static friction of the O-ring which is housed inside the Delrin cup.



Both position of the beam and the length of the string holding the ball can be adjusted so that the ball is centered above the air bearing.

The adjustable height marker is set to indicate the location of the bottom of the counterweight in the balanced condition. Non-magnetic ball bearings can be inserted into the top of the counter weight to create additional gravitational forces which are then “balanced” by magnetic force. Two sets of ball bearings are provided.

### 3. Rotating Magnetic Field



The sides of the rotating magnetic field assembly hold a set of permanent magnets and soft iron shims specially configured to provide a uniform horizontal magnetic field ( $\sim 1.0$  mT). Using a hole in its base, the assembly slips over the air bearing. The ball is then centered in the uniform field.

Using the air bearing as its axis of rotation, the assembly can be manually rotated to demonstrate the classical analog of nuclear magnetic resonance. (It's the lowest frequency NMR ever!)

### 4. Hall Effect Probe (Recommended additional accessory)

A Hall effect probe is recommended both to verify the theoretical calibrations of the magnetic field and field gradients as well as for finding the magnetic moment of the cue ball from the  $1/r^3$  dependence of the axial dipole field. The magnetic fields will be on the order of  $10^{-3}$  tesla and the gradients on the order of  $10^{-3}$  T/m. Therefore the probe must be able to measure with **resolution  $10^{-5}$  T** to detect this gradient which is  $10^{-5}$  T/cm.

## Chapter 3. Qualitative Introductory Experiment

### A. Torque vs. Force on a dipole – a dramatic qualitative introduction.

#### 1. Objectives

This experiment uses the Tower Kit for a dramatic qualitative demonstration that compares the response of a dipole to a uniform magnetic field to its behavior in a field gradient.

#### 2. Equipment

Magnet, Controller, tower, calibrated spring connected to a brass rod held in position by pressure fit through the tower cap, permanent magnet disk mounted in a gimbal that is connected to the bottom of the spring.

#### 3. Theory

In a uniform magnetic field, there is no net force on a magnetic dipole. There is, however, a net torque which is given by  $\boldsymbol{\mu} \times \mathbf{B}$ . This torque becomes apparent when the dipole moment is not parallel to the applied field.

By contrast, in a magnetic field gradient, the dipole, once aligned either parallel or anti-parallel to the net local field, experiences a net force directly proportional to the magnitude of the gradient.

A detailed theoretical development will be in the section on the Magnetic Force Balance experiment.

#### 4. Procedure

This experiment is best done as a demonstration by the instructor or a series of question-driven observations.

- a. Make sure toggles are set with AIR = OFF; FIELD DIRECTION = UP, FIELD GRADIENT = OFF and the MAGNET CURRENT set at zero.
- b. Place the clear plastic tower on top of the air-bearing.
- c. If necessary, release the thumb-screw in the tower cap and insert the rod with the spring and suspended magnet through the hole in the cap. Tighten the screw to hold the rod in place and put the cap onto the tower.
- d. With the cap in place, loosen the thumb-screw and adjust the brass rod until the dipole is at the center of coils – where the center of the ball will be when it is floating in the air bearing. Note the position reading of some reference spot on the gimbal.

**Observation 1:**

Look carefully at the magnet/gimbal assembly. An arrow shows the direction of the 'north' end of the dipole. Which way is the 'north' end of the dipole facing?

(In the northern hemisphere the magnet points almost directly down, aligned along the magnetic field lines of the Earth's magnetic field.)

Because the field toggle is on UP, any current in the coils, no matter how small, will produce a field pointing UP.

- e. Watching the gimbal carefully, turn on a small amount of current. Around  $\frac{1}{2}$  amp should work fine.

**Question 1:** What has happened to the gimbal/dipole?

(If for some reason, nothing seems to have happened, give the tower a little shake. You may have a gimbal with a lot of friction.) Why does what happened make sense? How is this similar to what happens with a compass?

(When the current is turned on in this configuration a torque causes the dipole to rotate, then oscillate a few moments before settling down with the dipole moment of the magnetic pointing up. Similarly, a compass orients along Earth's field lines.)

**Make a Prediction:**

In a few moments you are going to increase the current in the coils. What will happen to the magnitude of the magnetic field? What do you think will happen to the dipole? Will it be pulled up in the direction of the field or pushed down away from the field or something else?

(Most groups of students end up arguing up vs. down.)

- f. Once everyone has expressed an opinion, watch the gimbal carefully as you increase the current first to one and then to two amperes. Note the position reading of the gimbal each time.

**Question 2:** Any ideas about what is happening?

(Help students understand that this is a dipole not a monopole so there is no net force. Although it may oscillate, there is no net force to move it up or down.)

**Now let's see what happens if we create a *field gradient*.**

**g. Set the current back to zero, then flip the FIELD GRADIENT toggle to ON.**

Because of their configuration, the field gradient along the axis at the center of the coils is uniform for several centimeters,.

- h. Again, note the position of the gimbal. Then, increase the current in  $\frac{1}{2}$  ampere steps and notice what happens.

**Question 3:** From the Biot-Savart Law we know that the field gradient increases linearly with current. Based on your observations, how does the magnitude of the force on the dipole depend on the size of the gradient?

How does this explain why permanent magnets 'pull' on each other?

i. OPTIONAL:

Remove the magnet/spring assembly from the tower. Rotate the magnet within the gimbal so that the arrow showing the magnet's field is pointing "down", opposite its usual direction. Use the threaded eye-hook to secure the magnet and keep it from rotating. Return the magnet/spring assembly to the tower and repeat the series of observations from e, f, g and h.

Set the tower in position and turn on a small amount of current. Note what happens.

Increase the current – again note what happens

Set the current at one ampere and turn on the field gradient. Note the result.

Increase the current and notice what happens.

## Chapter 4. Measuring the Magnetic Moment of the Embedded Dipole Three Different Ways

The following experiments use both uniform magnetic fields and field gradients to measure the magnetic moment of the dipole embedded in the ball three different, and independent, ways.

Each experiment uses different physics principles, uses different measuring techniques, and makes different assumptions, each of which can create a unique set of systematic errors. While some systematic errors can be minimized, others come with the territory, just as in any research project. It will be up to the students to decide which experiment has probably given the best measure of the magnetic moment.

*Each of these experiments stands on its own so the instructor can pick and choose which experiments are most appropriate for the particular course.*

*We at TeachSpin, however, believe that doing multiple experiments, such as these, on one instrument allows students to develop a comfort and expertise with the apparatus that frees them to turn their full attention to acquiring excellent data as well as to understanding the physics both employed and revealed.*

### A. Static Experiment: Balancing magnetic torque and gravitational torque

#### 1. Objectives

Balancing an applied gravitation torque with an equal and opposite magnetic torque, we will use the relationship  $|\boldsymbol{\mu} \times \mathbf{B}| = |\mathbf{r} \times m\mathbf{g}|$  to find magnetic moment of the embedded dipole. In the process, we will also learn something about when an extra unknown does not affect a measurement, and grapple with the units of magnetic moment.

#### 2. Equipment

Magnet, Controller, air bearing, cue ball, aluminum rod with a steel end, weight, ruler, balance, calipers

#### 3. Theory

The students will likely know from their electricity and magnetism course that a loop of continuous current is referred to as a magnetic dipole. The neodymium iron boron disk magnet inside the ball is not a loop of current. In fact it is a .375 inch diameter, .25 inch thick disk magnetized along the axis of the disk. But its magnetic field is such that it acts as if it were a magnetic dipole. In a uniform magnetic field (which is the case at the center of the two-coil configuration), a magnetic dipole experiences a magnetic torque that is given by the expression:

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

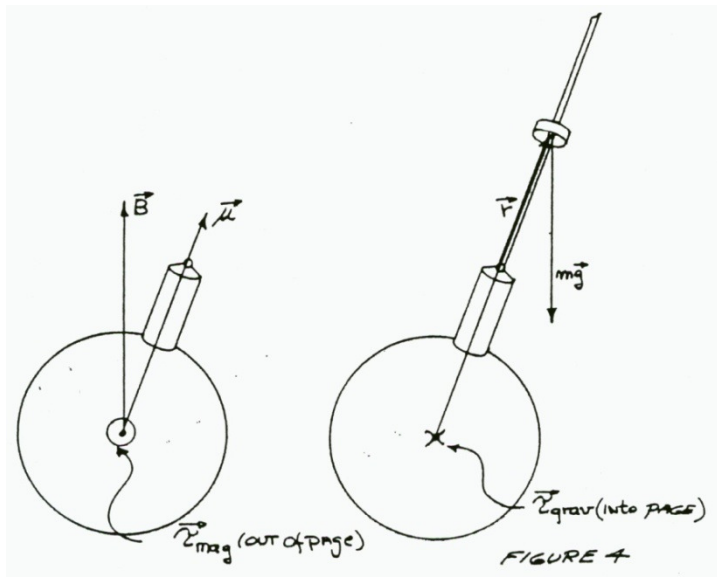
The magnet's dipole moment is aligned parallel to the handle on the ball. The magnetic field produced by the coils is vertical, either up or down. If the magnetic field points up, and the magnetic moment is aligned at some angle  $\theta$  away from the direction of the magnetic field, the ball will experience a torque that will tend to rotate it so that the handle of the ball points upward.

But, if the aluminum rod is placed in the handle of the ball, there is now another torque due to the earth's gravitational field. The expression for this torque is:

$$\tau = \mathbf{r} \times m\mathbf{g}$$

The gravitational torque tends to cause the ball to rotate so that the ball's handle points downward (Figure 4). Since a net torque causes a change in angular momentum, the ball will rotate if the gravitational torque is larger than the magnetic torque, or vice versa.

But, when the magnetic torque is equal to the gravitational torque, the stationary ball will not rotate, since the net torque on the ball is zero.



This “balanced” configuration is described mathematically by:

$$\mu B \sin \theta = r m g \sin \theta$$

From the diagram, you can see that the angle,  $\theta$  between  $\mu$  and  $B$  is the same as that between  $r$  and  $g$ . So:

$$\mu B = r m g.$$

Simplified this just becomes:

$$r = \frac{\mu}{mg} B$$

If we measure  $r$  for various magnetic fields, the functional dependence of  $r$  to  $B$  should be a straight line with the slope being an expression that contains  $\mu$ .

Notice that  $\sin \theta$  cancels out in this expression. That is equivalent to saying that if  $r$  and  $B$  are adjusted to the balance condition, then the ball will balance at any orientation! So you don't have to get the torque-arm perfectly horizontal. It just has to be stable.

In this experiment, the magnetic moment,  $\mu$ , is an unknown constant that we want to determine. The variables are the  $\mathbf{B}$ -field at the center of the instrument (which can be calculated from the current  $I$ ), and  $\mathbf{r}$ , the displacement of the center of mass of the weight from the center of the ball.

Because of the way the equation is written, it will be easiest to find  $\mu$  from the slope of a graph with  $B$  (calculated from  $I$ ) on the x-axis.

For making measurements, it makes more sense to set the location of the sliding mass first and then adjust the current until the torques are balanced.

#### 4. Procedure

- Measure all constants involved in the experiment. Use a digital scale to determine the value of the weight. Use calipers to measure the diameter of the ball, and a ruler to measure the length of the ball's handle. These measurements will be needed to calculate the position,  $r$ , of the weight. *Make sure to keep all measurements in SI units, specifically keeping the magnetic field in teslas.*
- Turn on the power supply and the air. Keep the field gradient and the strobe light off. Set the direction of the magnetic field on 'up' so that the handle of the ball would point upward when the ball rests on the air bearing. Set and record the location of the sliding weight – the distance from the end of the handle to the middle of the weight. Add this value to the length from the center of the ball to the end of the handle, and the resulting value is the  $r$  of the weight
- Adjust the current until the aluminum tube remains at about 90 degrees with respect to the vertical. You might have to steady the tube, weight, and ball with your hand, because the system tends to oscillate and drift due, in part, to the Earth's magnetic field.
- Measure a location and current for at least six different positions of the weight.

A data table such as the one below is a good way to organize the measurements.

Measurements	Raw Data		Calculated Information	
Diameter of Sphere:	Distance center of slider to end of handle	Current	$r$ (m) Slider to center of sphere	$B$ (T) Magnetic Field
Length of handle:				
Mass of slider:				

- OPTIONAL: Repeat a few measurements for some angle of the torque-arm to see if it is true that the torque-arm does not have to be horizontal.

## B. Harmonic oscillation of a spherical pendulum

### 1. Objectives

The primary objectives of this experiment are to determine the dipole moment of the magnet inside of the ball, and to study the behavior of a physical pendulum's small-amplitude oscillation.

### 2. Equipment

Magnet, Controller, air bearing, cue ball, stopwatch, calipers, and balance.

### 3. Theory

This experiment involves *dynamics* rather than statics principles. From classical mechanics, the students should know that for rotational motion, the net torque on an object causes a change in that object's angular momentum, given by the expression:

$$\sum \tau = \frac{d\mathbf{L}}{dt}$$

For our particular system, if the cue ball is placed in the air bearing with a uniform magnetic field present, the ball will experience a net torque, and will change its angular momentum. However, it's important to note the direction of the magnetic moment relative to the magnetic field.

If the magnetic moment in the ball is displaced an angle  $\theta$  from the axis of the coils (the direction of the field), it experiences a restoring torque that acts against the angular displacement of  $\mu$ . The angular momentum for a rigid sphere is  $\mathbf{L} = I\omega$  where  $I$  is the moment of inertia of a sphere and  $\omega$  is the angular velocity. Substituting in the above master equation

$$\mu \times \mathbf{B} = \frac{d}{dt}(I\omega) \quad \text{but} \quad \omega = \frac{d\theta}{dt}$$

Thus, the differential equation of motion becomes:

$$-|\mu \times \mathbf{B}| = I \frac{d^2\theta}{dt^2}$$

Here,  $\theta$  is the angular displacement of  $\mu$  from the direction of  $\mathbf{B}$ . The minus sign indicates that the torque is restoring in nature. In scalar form we have:

$$-\mu B \sin \theta = I \frac{d^2\theta}{dt^2}$$

But for small-angle displacements in radian measure,  $\sin \theta \approx \theta$ , so

$$-\mu B \theta = I \frac{d^2\theta}{dt^2}$$

We'll guess the solution of this equation to be  $\theta(t) = A \cos \omega t$ , where  $\omega$  and  $A$  are constants. Substituting into the differential equation we have:

$$-\mu B A \cos \omega t = -I A \omega^2 \cos \omega t$$

Rearranging, we find that

$$\omega^2 = \frac{\mu}{I} B$$

The period of oscillation,  $T$  is related to the angular velocity as:

$$T = \frac{2\pi}{\omega}$$

Squaring the equation for  $T$  and substituting our expression for  $\omega$  gives:

$$T^2 = \frac{4\pi^2 I}{\mu B}$$

Remember that this expression is only applicable for a small angle displacement. The rotational inertia of the ball,  $I$ , can be well approximated to be the moment of inertia of a uniform solid sphere, namely:

$$I = \frac{2}{5} MR^2$$

In this case  $M$  is the mass of the ball and  $R$  is the ball's radius.  $B$  is again the independent variable, and  $T$  can be measured using a stopwatch. A graph of  $T^2$  vs.  $\frac{1}{B}$  should give a straight line. The magnetic moment can be calculated from the slope.

#### 4. Procedure

- First, make the measurements needed to determine the moment of inertia of the sphere, its mass and its radius. The mass can be determined using a balance, and the radius can be determined using the calipers.
- For this experiment, the field gradient should be OFF, the strobe light should be OFF, the air should be ON, and the field direction should be UP.

Because the magnetic torque is the only torque involved in this experiment, the experiment can be performed at low currents (and thus small magnetic field,  $B$ ).

- Place the cue ball on the air bearing and set the current at or near one amp. Give the handle of the ball a small angular displacement from the vertical. Release the ball from rest, and it will oscillate.
- Measure the amount of time it takes the ball to complete twenty (20) full cycles of motion with a stopwatch. Repeat this for currents up to 4 amps.

**Make sure to start your stopwatch at a count of '0' so that you count the first full cycle.**

This measured time, divided by twenty, will be the period of oscillation for the ball at that particular applied magnetic field.

HINT: Because of the  $1/B$  independent variable that will be graphed, it's a good idea to obtain data for quite a few different lower currents in order to obtain an even distribution of data points on the graph of  $T^2$  vs.  $1/B$ .

## C. Precessional motion of a spinning sphere

### 1. Objectives

The primary objective again is to measure the dipole moment of a permanent magnet inside the cue ball. A secondary objective is to observe and quantify the motion of a spinning sphere subject to an external torque.

### 2. Equipment

Magnet, Controller, air bearing, cue ball, strobe light, stopwatch, calipers, balance

### 3. Theory

When the magnetic moment is displaced at some angle from the direction of the magnetic field, the magnetic dipole (and subsequently the ball) experiences a torque that causes a change in the ball's angular momentum in the direction of the torque.

This is the central principle of this experiment. The ball is displaced from the vertical position and *set spinning*, with its spin-axis running through the handle of the ball. This creates a large spin angular momentum. The spin axis will remain in a fixed position until a uniform magnetic field is turned on.

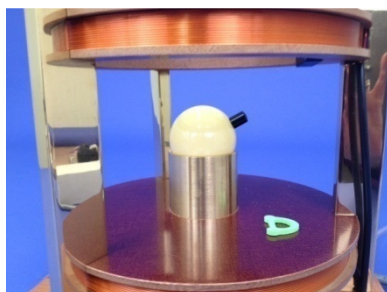
When a uniform magnetic field is present, the magnetic dipole will experience a torque,  $\boldsymbol{\mu} \times \mathbf{B}$ . This torque will cause a change of angular momentum in the direction of the torque. However, because the ball already has a large spin angular momentum ( $\mathbf{L}_s$ ), it will precess rather than just turning to point along the B field of the coils. (This motion is similar to that of the spinning gyroscope in the earth's gravitational field. Students may be familiar with gyroscopic motion from their mechanics course.)

For this experiment:

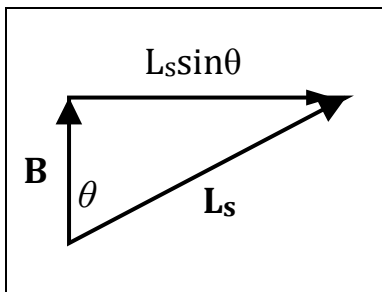
$$\text{Magnetic Torque} = \text{Rate of Change of Angular Momentum}$$

The differential equation for the motion of the ball is

$$\boldsymbol{\mu} \times \mathbf{B} = \frac{d}{dt} \mathbf{L}$$



Side View

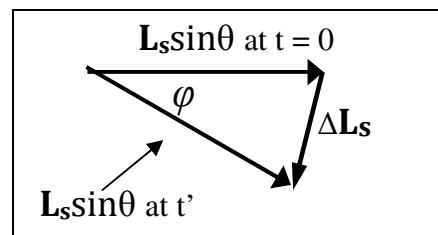


The Vectors

This 'side view' shows the sphere sitting in the air bearing with its handle at an angle to the horizontal. The ball has been spun so that the spin angular momentum vector is as shown.

Because the ball is spinning around the handle, there is a spin angular momentum vector, which we will call  $\mathbf{L}_s$ , pointing along the handle of the ball. Because the magnetic moment vector and the spin angular momentum vector are parallel, the magnetic torque vector and thus the precession direction will be out of the page – towards you.

If we *look down from above* at the change of angular momentum for a short time  $\Delta t$ , the pictures and vectors looks like the ones shown below.

Top View at time  $t = 0$ Top View at time  $t'$ 

The Vectors

From the definition of radian measure,  $s = r\phi$ , therefore, for small  $\Delta\phi$  :

$$\Delta L_s = \Delta\phi L_s \sin\theta$$

$$\frac{\Delta L_s}{\Delta t} = \frac{\Delta\phi}{\Delta t} L_s \sin\theta$$

As  $\Delta t \rightarrow 0$ ,  $\Delta\phi/\Delta t$  gives the rate of change of  $\phi$  which is the precession rate  $\Omega_p$ :

$$\frac{dL_s}{dt} = \Omega_p L_s \sin\theta$$

Now,  $dL_s/dt$  is given by the torque, so

$$\frac{dL_s}{dt} = \mu B \sin\theta$$

So  $\mu B \sin\theta = \Omega_p L_s \sin\theta$

and  $\mu B = \Omega_p L_s$

or  $\Omega_p = \frac{\mu}{L_s} B$

Interestingly, this equation predicts that the precession frequency,  $\Omega_p$ , does *not* depend on the angle  $\theta$ .

In this algebraic format, the precession frequency,  $\Omega_p$ , (in radians/second) is the dependent variable – the one on the ‘y’ axis. This frequency can be determined by measuring the time needed for the handle of the ball to precess through a full circle,  $2\pi$  radians.  $\Omega_p$  can then be calculated as  $2\pi/T$ . The magnetic field is the independent variable which we will plot on the ‘x’ axis.

The magnitude of the angular momentum  $L$  can be calculated from the moment of inertia and the spin frequency,  $\omega$ . That frequency can be measured using the strobe light. The handle of the ball has a white dot on its top. As the ball spins the strobe light reflects off of this white dot. When the strobe light frequency matches the spin rate of the ball, the dot will appear stationary. Or you can measure the spinning frequency directly by a non-contact tachometer. Stick a small piece of silver tape on the cue ball to reflect a laser source from the tachometer, and the spinning frequency will be calculated automatically in rpm. If that angular momentum can be held constant, the graph of  $\Omega_p$  vs.  $B$  will yield a straight line. From the slope of this line  $\mu$  can be determined.

#### 4. Procedure

Overview: In order to keep the angular momentum,  $I\omega$ , constant, we must find a way not only to measure  $I$  and  $\omega$ , but also to have  $\omega$  be the same for every measurement. To do that, you will choose and set the desired frequency of the strobe light, get the ball spinning slightly faster than the desired frequency and make your measurements when the ball has slowed down to match the preset frequency. A frequency between 4.5 and 6 Hertz works well. Set the strobe light at a frequency in that range and check the ball throughout the experiment. The room does not have to be completely dark for the strobe light to illuminate the white dot sufficiently. You will be able to record data easily.

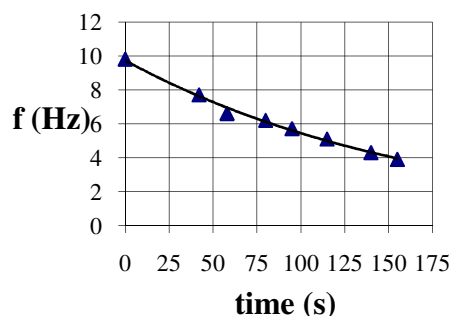
- a. Assume the ball to be a solid sphere and measure the physical dimensions needed to find its moment of inertia. Measure the mass of the ball and its radius with the balance and the calipers, respectively.

***Before you begin making measurements, practice spinning the ball. Do this with the air on and the current off.***

- b. Orient the ball so that the handle faces the strobe. A good technique is to spin the ball (give it a good, fast spin!) and then **use the tip of your fingernail to ‘settle’ it so it spins smoothly about the handle’s axis.**

Notice that the frequency of the ball’s spin *does change with time.*

A graph of frequency vs time spinning will approximate an exponential decay. The 1997 graph is shown here. With the new pumps it should be significantly flatter.



If you make a similar graph for your apparatus, you will be able to choose a good frequency range for your experiment. Find a range where the rotational frequency does not change significantly during the time it takes the ball to precess through one period

**NOW IT IS TIME TO MEASURE THE PRECESSION FREQUENCY** as a function of the magnetic field. We will not measure this frequency directly but rather calculate it from the period of precession, the time for the axis of rotation to make one full revolution in space.

- c. An effective technique for making excellent measurements of the precession period

In a uniform magnetic field, a dipole experiences a torque, but in a field gradient, there is only a force. This means that if we spin the ball with its embedded dipole in a uniform field, the ball will precess as the torque changes the direction of the angular momentum vector. BUT, *if we spin the ball in a field gradient, the ball will just sit there.*

**SO – here’s what we will do:** turn on the air, flip the field toggle to Field Gradient ON, and spin up the ball. When ready to make measurements, we will first set the current to the desired value and, then start the stopwatch as we flip the toggle to Gradient OFF. This simultaneously changes the B field from 0 to the desired value and starts the precession.

- d. Taking data for period of precession as a function of current.
- Set the strobe light to the desired frequency and record that number.
  - Set: CURRENT = OFF; AIR = ON; FIELD DIRECTION = UP; FIELD GRADIENT = ON.
  - Spin up the ball with the handle facing the strobe light and set a marker just below the handle. Now, watch the white dot. At first, it will seem to be all around the handle, but as the ball slows down, the white dot will begin to have a regular rotation of its own. This rotation will slow down until the white dot 'stops'.
  - As soon as the white dot stops moving, set the current at your desired level.
  - Next, flip the toggle to field GRADIENT = OFF, and time a period of the ball's precession. (That's the time it takes the handle to make a complete circuit back to your marker.)
  - Record this time for that current, turn the current off, and spin the ball up again.
  - Repeat the same procedure, with a new current value. Continue on in 0.5 A steps until the current reaches 4 amperes. This should give you enough data. Record your data in an appropriate table.

(You are turning the current off between precession measurements to keep the coils as cool as possible. Why?)