Physics (I) Exam#1 (Chs.2,3,4,5,6) 2024/09/27

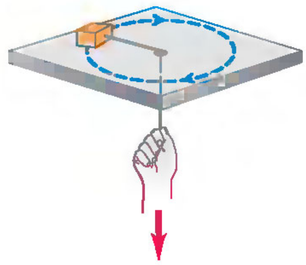
Calculator, dictionary, mobile phone, and scratch paper are NOT allowed. Cheating will be punished by the University regulations.

1. According to a Scientific American article, current freeways can sustain about M vehicles per lane per hour in smooth traffic flow at v (km/h). With more vehicles the traffic flow becomes "turbulent" (stop-and-go).
(a)(10%) If a vehicle is L meters long on the average, what is the average spacing between vehicles at the above traffic density?
(b)(10%) Collision-avoidance automated control systems, which operate by bouncing radar or sonar signals off surrounding vehicles and then accelerate or brake the car when necessary, could greatly reduce the required spacing between vehicles. If the average spacing is d meters, how many vehicles per hour can a lane of traffic carry at v (km/h)?
Ans: (P2.58)
(a) $\dfrac{v\ (\text{km/h})}{M\ (\text{h}^{-1})} - L\ (\text{m}) = \dfrac{v}{M}10^3 - L\ (\text{m})$
(b) $\dfrac{v\ (\text{km/h})}{(L+d)\ (\text{m})} = \dfrac{v}{L+d}10^3\ \ (\text{h}^{-1})$

2. A river flows due south with a speed of v_R (m/s). A man steers a motorboat across the river; his velocity relative to the water is v_B (m/s) due east. The river is w (m) wide.
(a)(4%) What is his velocity (magnitude and direction) relative to the earth?
(b)(4%) How much time is required to cross the river?
(c)(4%) How far south of his starting point will he reach the opposite bank?
(d)(4%) In which direction should the motorboat head in order to reach a point on the opposite bank directly east from the starting point?
(e)(4%) What is the velocity of the boat relative to the earth?
Ans: (P3.41, P3.42)

3. A large crate with mass m rests on a horizontal floor. The coefficients of friction between the crate and the floor are μ_s and μ_k. A woman pushes downward at an angle θ below the horizontal on the crate with a force $\boldsymbol{F}$.
(a)(10%) What magnitude of force F is required to keep the crate moving at constant velocity?
(b)(10%) If μ_s is greater than some critical value, the woman cannot start the crate moving no matter how hard she pushes. Calculate this critical value of μ_s.
Ans: (P.5.43)

4. A small block with a mass m (kg) is attached to a cord passing through a hole in a frictionless, horizontal surface (Fig.). The block is originally revolving at a distance of r_1 (m) from the hole with a speed of v_1 (m/s). The cord is then pulled from below, shortening the radius of the circle in which the block revolves to r_2 (m). At this new distance, the speed of the block is observed to be v_2 (m/s).



(a)(5%) What is the tension in the cord in the original situation when the block has speed v_1 (m/s).

(b)(5%) How much work was done by the person who pulled on the cord?

(c)(10%) If the person pulls the cord by a force F and the cord moves at a constant speed v_0, what will be the tangential acceleration of the block when its instantaneous speed is v?

Ans: (P6.69)

(c)

$v^2 = v_t^2 + v_0^2$, v_t : tangential velocity

$$Fv_0 = \frac{d}{dt}(\frac{1}{2}mv^2) = \frac{d}{dt}(\frac{1}{2}mv_t^2) + \underbrace{\frac{d}{dt}(\frac{1}{2}mv_0^2)}_{=0} = mv_t \underbrace{\frac{dv_t}{dt}}_{=a_t}$$

$$\Rightarrow \quad a_t = \frac{Fv_0}{mv_t} = \frac{Fv_0}{m\sqrt{v^2 - v_0^2}}$$

5. (15%) Jenny rushes down onto a subway platform to find her train already departing. She stops and watches the cars go by. Each car is L long. The first moves past her in t_1 and the second in t_2. Find the constant acceleration of the train.

Ans:

The average speed of every point on the train as the first car passes Liz is given by:

$$v_1 = \frac{\Delta x}{\Delta t} = \frac{L}{t_1}.$$

The train has this as its instantaneous speed halfway through the t_1 time.

Similarly, halfway through the next t_2 , the speed of the train is $v_2 = \frac{L}{t_2}$.

The time required for the speed to change from v_1 to v_2 is

$$\frac{1}{2}(t_1) + \frac{1}{2}(t_2) = \frac{1}{2}(t_1 + t_2)$$

so the acceleration is $a_x = \frac{\Delta \upsilon_x}{\Delta t} = \frac{v_2 - v_1}{\frac{1}{2}(t_1 + t_2)} = \frac{2}{t_1 + t_2}\left(\frac{L}{t_2} - \frac{L}{t_1}\right) = \frac{2L(t_1 - t_2)}{t_1 t_2 (t_1 + t_2)}$.

6. In most problems, the ropes, cords, or cables have so little mass compared to other objects in the problem that you can safely ignore their mass. But if the rope is the only object in the problem, then clearly you cannot ignore its mass. For example, suppose we have a clothesline attached to two poles (Fig.). The clothesline has a mass M, and each end makes an angle θ with the horizontal. What are

(a)(5%) the tension at the ends of the clothesline and
(b)(5%) the tension at the lowest point?
(c)(5%) Why can't we have θ = 0?

Ans: (P5.63)

(a) $T_{\text{end}} \sin\theta = Mg/2$, so $T_{\text{end}} = Mg/(2\sin\theta)$.

(b) $T_{\text{end}} \cos\theta = T_{\text{middle}}$, so $T_{\text{middle}} = Mg\cos\theta/(2\sin\theta) = Mg/(2\tan\theta)$.

(c) Because this would cause a division by zero in the equation for T_{end} or T_{middle}.

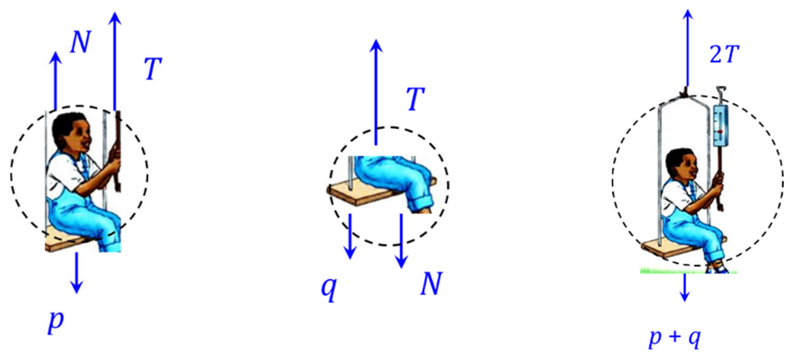
7. An inventive child named Nick wants to reach an apple in a tree without climbing the tree. Sitting in a chair connected to a rope that passes over a frictionless pulley (Fig.), Nick pulls on the loose end of the rope with such a force that the spring scale reads T. Nick's true weight is p, and the chair weighs q. Nick's feet are not touching the ground.

(a)(10%) Draw one pair of diagrams showing the forces for Nick and the chair considered as separate systems and another diagram for Nick and the chair considered as one system.

If he can be lifted up,
(b)(5%) what is the Nick's acceleration?
(c)(5%) find the force Nick exerts on the chair.

Ans:

(a)



(b) $2T-(p+q)=ma=\dfrac{p+q}{g}a \quad\Rightarrow\quad a=\left[\dfrac{2T-(p+q)}{p+q}\right]g$

(c) $T-q-N=\dfrac{q}{g}a \quad\Rightarrow\quad N=(\dfrac{p-q}{p+q})T$