

## DICE PROBLEMS

CROSSROADS ACADEMY  
AMC PREPARATION

### 1. DICE WARMUP

- (1) What is the probability of rolling a 6 sided die and getting a 4?
- (2) What is the probability of rolling a 20 sided die and getting a prime?
- (3) What is the probability of rolling a 101 sided die and getting a perfect square? How about a perfect cube?
- (4) What is the probability of rolling a 6 sided die and getting a even number or a three?
- (5) What is the probability of rolling a 8 sided die and getting a prime and an odd number?
- (6) What is the probability of rolling two 6 sided dice and getting numbers whose product is twelve?
- (7) What is the probability of rolling two 6 sided dice and getting numbers whose sum is greater than 8? What about less than 3?
- (8) What is the mean value of a roll of a 7 sided die?
- (9) What is the median value of a roll of a 13 sided die?

## 2. Two DICE AT ONCE

- (1) Compute all of the possible sums of rolling two dice:

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

- (2) Compute all of the possible products of rolling two dice:

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

- (3) What is the expected value of the sum of two 6 sided dice?

- (4) What is the expected value of the product of two 6 sided dice?

### 3. MORE FUN WITH DICE

- (1) How many rolls of a standard six sided dice do you think it will take until you have rolled every number at least once? Let's try it out: Roll your dice and record the number that you rolled below. Keep rolling and recording until you have seen each value (1,2,3,4,5,6). How many rolls did it take? If you are surprised by the number, try it again. Can you compute the expected number of rolls?
- (2) In problem 1 of part 2 you computed a table of all the possible ways to get various integers between 2 and 12 as the sum of two die rolls. If we relabel the dice so that one of them has numbers (0,1,2,3,4,5) and the other has numbers (2,3,4,5,6,7) what does the table look like?

+	0	1	2	3	4	5
2						
3						
4						
5						
6						
7						

Can you come up with another way to relabel the faces of the dice with positive integers so that the distribution of sums remains the same?

- (3) Consider four dice labeled with the following face values: Dice 1 (1,1,7,7,7,7), Dice 2 (5,5,5,5,5,5), Dice 3 (3,3,3,3,9,9), and Dice 4 (2,2,2,8,8,8). What is the expected value for rolling each die separately? If we were going to play a game where we each could pick one of the die to roll and the highest roll wins, which die should you pick? Can you create another set of face values for 3 or 4 dice that have this property? Here is another set of three dice: Dice 1 (2,2,4,4,9,9), Dice 2 (1,1,6,6,8,8), and Dice 3 (3,3,5,5,7,7).