AIME PREPARATION

1. WARMUP PROBLEMS

(1) A cubical box has edge length of 4cm. If the box is missing one of its sides what is its surface area?

(2) If a is an integer selected randomly between 1 and 10 and b is a randomly selected integer between -3 and 3, what is the probability that $\dfrac{b}{a}$ is an integer?

(3) A unit cube is dipped into a bucket of paint and then sliced into 125 congruent smaller cubes. For each integer $0 \leq k$, how many of the smaller cubes have exactly k painted sides?

(4) A certain right triangle has sides of length a, $a + b$ and $a - b$. What is $\dfrac{b}{a}$?

(5) How many pairs of square numbers less than 1000 differ by a multiple of 7?

(6) How many odd three digit integers have exactly two equal digits?

(7) A standard PIN for a debit card has 4 digits. Are there more PINs with all unique digits or with a single repeated digit?

(8) The math club at a certain school has 10 members, 4 of which are 7th graders. A 4 person math team is being formed for a competition that must include at least one 7th grader. How many possible 4 person teams exist?

(9) How many real solutions does $x^6 = (x + 1)^6$ have? Can you find all of them?

(10) At a large party each person shakes hands with at least one other person. Prove that the number of people who shook an odd number of hands is even.

(11) The school cafeteria serves chicken nuggets in packages of 4 and 7. What is the largest number of chicken nuggets that cannot be obtained by only taking whole packages of these sizes?

(12) If p and q are odd primes, how many integers less than pq are relatively prime to pq?

(13) What is 2^{6048} (mod 2017)