EXPECTED VALUE PROBLEMS

NEW HAMPSHIRE STATE TEAM
NATIONAL MATHCOUNTS PREPARATION

(1) What is the expected value of the product of a randomly chosen row or column on a telephone keypad?

(2) What is your expected individual score at a Mathcounts competition if your probability of answering any particular sprint problem correctly is $\frac{2}{3}$ and your probability of answering each target question correctly is $\frac{3}{4}$.

(3) You have two pennies, one nickle, three dimes, and four quarters in your pocket. If you take two coins at random from your pocket, what is the expected value of those coins?

(4) Joe picks a random two digit integer. What is the expected value of the sum of the digits of his number?

(5) If you roll two standard dice what is the expected value of the absolute value of the difference between the faces? What if we roll a blue die and a red die and always subtract the red value from the blue value (even if it is larger)?

(6) On a ten question multiple choice exam, each question has the same number of choices as the problem number (e.g. problem #7 has options a,b,c,d,e,f, and g) and a correct answer is worth the square of the problem number points (so answering number 4 correctly is worth 16 points). If you randomly guess on each problem how many points do you expect to score? What if wrong answers subtract the problem number from your score?

(7) What is the expected value of the number of intersections of a line that passes through the point $(0, 2)$ and $(x, 0$ and the circle of radius 1 centered at the origin if $-5 < x < 5$?

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(8) Consider the number of ways to tile a 1×8 board with 1×1 squares and 1×2 dominoes. What is the expected number of dominoes in such a tiling?

(9) Sally picks a random integer n between 2 and 100. What is the expected value of the number of primes that divide n?

(10) A king is placed at random on a standard 8×8 chessboard. What is the expected value of the number of squares that it can move to in one move?

(11) If there are 100 people at the party and each pair of people has probability .25 of being friends, what is the expected number of friendship pairs in the room?

(12) What is the expected value of the number of coin flips needed to get both at least one heads and one tails? What is the expected value of the number of rolls of a standard die necessary to obtain all 6 faces.

(13) If 200 students take a 6-question exam and each question is answered correctly by at least 120 students must there be a pair of students such that each question was answered correctly by at least one of them?