



Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Lecture with Computer Exercises: Modelling and Simulating Social Systems with MATLAB

Project Report

Self-Organized Criticality in Sandpile Models



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Abstract

blah blah

Acknowledgements

blah blah

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Chapter 1

to do list, beta texts

1.1 Individual contributions

1.2 Introduction and Motivations

1.3 Description of the Model

1.4 Implementation

1.5 Simulation Results and Discussion

1.6 Summary and Outlook

1.7 Sandpile model

1.7.1 Bak-Tang-Wiesenfeld sandpile model

The classical sandpile model represents a cellular automation describing a dynamical system following certain rules that can be described as follows.

The field/lattice, which we choose to be two-dimensional, represents a sandpile. Each site on the lattice has a certain value z that intuitively represents the height or slope of the sandpile at certain position described with the coordinates x and y . At each time step, a number of grains of sand is placed on top of a random site, which increases its value by a given value, e.g. one. If the value of the site exceeds a critical value z_c (e.g. three), the site collapses/topples and its grains are evenly distributed to its neighbours.

In certain cases some of the adjacent sites will exceed the critical value too and the toppling process will continue until an equilibrium

state is again reached. This series of collapsing sites is classically described as an avalanche. The next grain is not placed until the equilibrium state is reached, meaning that the time scale of the random grain placement and of the development of avalanches are decoupled.

The classical model description can mathematically be represented as follows.

Initially, the lattice is empty:

$$z(x, y) = 0 \quad \forall x, y$$

Then, the value of a random site x, y is increased:

$$z(x, y) \rightarrow z(x, y) + 1$$

If its value exceeds the critical value $z_c = 3$, then it topples and distributes its grains to its neighbours:

$$\begin{aligned} z(x, y) > 3 &\stackrel{?}{\Rightarrow} z(x, y) \rightarrow z(x, y) - 4 \\ z(x \pm 1, y) &\rightarrow z(x \pm 1, y) + 1 \\ z(x, y \pm 1) &\rightarrow z(x, y \pm 1) + 1 \end{aligned}$$

Clearly, many variations of the described model can be considered and can produce different results. The classical sandpile model, as originally described by Per Bak, Chao Tang and Kurt Wiesenfeld, represents the starting point of any further investigations considered in this paper.

1.7.2 Abelian sandpile model

Bibliography

- [1] Alessandro Vespignani Alain Barrat and Stefano Zapperi.
Fluctuations and correlations in sandpile models. September 1999.
- [2] Michael Creutz. Cellular automata and self-organized criticality.
November 1996.
- [3] Alessandro Vespignani and Alessandro Vespignani. How
self-organized criticality works: A unified mean-field picture.
June 1998.

Appendix A

MATLAB-Code

