Probability
Lecture 7

Lecture objectives

- Determine sample spaces and find the probability of an event, using classical probability or empirical probability.
- Find the probability of compound events, using the addition rules.

Introduction to the Probability

Probability as a general concept can be defined as the chance of an event occurring. Many people are familiar with probability from observing or playing games of chance, such as card games, slot machines, or lotteries. In addition to being used in games of chance, probability theory is used in the fields of insurance, investments, and weather forecasting and in various other areas.

Definitions

A probability experiment is a chance process that leads to well-defined results called outcomes.
An outcome is the result of a single trial of a probability experiment.
A sample space is the set of all possible outcomes of a probability experiment.

Example 1. Find the sample space for rolling two dice.

Solution

each die can land in six different ways, and two dice are rolled, the sample space can be presented by a rectangular array.

Die one	Die two					
	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,2)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,2)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,2)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,2)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Example 2. Find the sample space for the gender of the children if a family has three children.

Solution

Use B for boy and G for girl,there are eight possibilities.
BBB BBG BGB GBB GGG GGB GBG BGG

There are three basic interpretations of probability:

1. Classical probability.

2. Empirical or relative frequency probability.
3. Subjective probability.

Classical Probability

Classical probability uses sample spaces to determine the numerical probability that an event will happen,it assumes that all outcomes in the sample space are equally likely to occur
Equally likely events are events that have the same probability of occurring.
The probability of any event E is

$$\frac{Number\ of\ outcomes\ in\ E}{Total\ number\ of\ outcomes\ in\ the\ sample\ space}$$

This probability is denoted by

$$P(E) = \frac{n(E)}{n(S)}$$

Example 3. Find the probability of getting a black 10 when drawing a card from a deck.

Solution

Since there are 4 suits (hearts, clubs, diamonds, and spades) and 13 cards for each suit (ace through king), there are 52 outcomes in the sample space, and there are two black 10s the 10 of spades and the 10 of clubs. Hence the probability of getting a black 10 is
$P(black\ 10) = \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$

Example 4. If a family has three children, find the probability that two of the three children are girls.

The sample space for the gender of the children for a family that has three children has eight outcomes, that is
BBB, BBG, BGB, GBB, GGG, GGB, GBG, BGG.
there are three ways to have two girls, GGB, GBG, and BGG,
$P(two\ girls) = \frac{n(E)}{n(S)} = \frac{3}{8}$

Example 5. A card is drawn from an ordinary deck. Find these probabilities

1. Of getting a jack
2. Of getting the 6 of clubs
3. Of getting a 3 or a diamond
4. Of getting a 3 or a 6

Solution

1. There are 52 cards in a deck,There are 4 jacks so there are 4 outcomes in event E and 52 possible outcomes in the sample space.
   $P(jack) = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

2. Since there is only one 6 of clubs in event E, the probability of getting a 6 of clubs is
   $P(6\, of\ clubs) = \frac{n(E)}{n(S)} = \frac{1}{52}$

3. There are four 3s and 13 diamonds, but the 3 of diamonds is counted twice in this listing. Hence, there are 16 possibilities of drawing a 3 or a diamond, so
   $P(3\, or\ diamond) = \frac{n(E)}{n(S)} = \frac{16}{52} = \frac{4}{13}$
   we call this *inclusive or*

4. Since there are four 3s and four 6s,
   $P(3\, or\ 6) = \frac{n(E)}{n(S)} = \frac{8}{52} = \frac{2}{13}$
   we call this *exclusive or*

There are four basic probability rules

1. The probability of any event E is a number (either a fraction or decimal) between and including 0 and 1. This is denoted by $0 \leqslant P(E) \leqslant 1$
2. If an event E cannot occur (the event contains no members in the sample space), its probability is 0.
3. If an event E is certain, then the probability of E is 1.
4. The sum of the probabilities of all the outcomes in the sample space is 1.

Example 6. When a single die is rolled, what is the probability

1. of getting a 9.
2. of getting a number less than 7.

Solution

1. it is impossible to get a 9, since the space doesn't contain it,So, the probability is
   $P(9) = 0$.
2. Since all outcomes 1, 2, 3, 4, 5, and 6—are less than 7, the probability is
   $P(number\ less\ than\ 7) = \frac{n(E)}{n(S)} = \frac{6}{6} = 1$

Complementary Events

The complement of an event E is the set of outcomes in the sample space that are not included in the outcomes of event E. The complement of E is denoted by$\overline{E}$.
The outcomes of an event and the outcomes of the complement make up the entire sample space.

Example 7. Find the complement of each event.

1. Rolling a die and getting a 4
2. Selecting a day of the week and getting a weekday

Solution

1. Getting a 1, 2, 3, 5, or 6
2. Getting Friday or Saturday

The rule for complementary events can be stated
$P(\overline{E}) + P(E) = 1$

Empirical Probability

empirical probability relies on actual experience to determine the likelihood of outcomes.
Given a frequency distribution, the probability of an event being in a given class is This probability is called empirical probability and is based on observation.
$P(E) = \frac{frequency\ for\ the\ class}{total\ frequencies\ in\ the\ distribution} = \frac{f}{n}$
This probability is called empirical probability and is based on observation.

Example 8. in a frequency distribution for a travel survey find the probability that a person will travel by airplane over the holiday.

Method	Frequency
Drive	41
Fly	6
Train or bus	3
	50

Solution

$P(E) = \frac{f}{n} = \frac{6}{50} = \frac{3}{25}$

Example 9. In a sample of 50 people, 21 had type O blood, 22 had type A blood, 5 had type B blood, and 2 had type AB blood. Set up a frequency distribution and find the following probabilities.

1. A person has type O blood.
2. A person has type A or type B blood.
3. A person has neither type A nor type O blood.
4. A person does not have type AB blood

Solution

Type	Frequency
A	22
B	5
AB	2
O	21
	Total=50

1. $P(O) = \frac{f}{n} = \frac{21}{50}$
2. $P(A\ or\ B) = \frac{22}{50} + \frac{5}{50} = \frac{27}{50}$.
3. $P(neither\ A\ nor\ O) = P(B\ or\ AB) = \frac{5}{50} + \frac{2}{50} = \frac{7}{50}$
4. $P(not\ AB) = 1 - p(AB) = 1 - \frac{2}{50} = \frac{48}{50} = \frac{24}{25}$

Subjective Probability

Subjective probability uses a probability value based on an educated guess or estimate, employing opinions and inexact information.

The Addition Rules for Probability

Two events are ***mutually exclusive*** events if they cannot occur at the same time

Example 10. Determine which events are mutually exclusive and which are not, when a single die is rolled

1. Getting an odd number and getting an even number
2. Getting a 3 and getting an odd number
3. Getting an odd number and getting a number less than 4
4. Getting a number greater than 4 and getting a number less than 4

Solution

1. The events are mutually exclusive, since the first event can be 1, 3, or 5 and the second event can be 2, 4, or 6.
2. The events are not mutually exclusive, since the first event is a 3 and the second can be 1, 3, or 5. Hence, 3 is contained in both events.

Addition Rules

1. When two events A and B are mutually exclusive, the probability that A or B will occur is
   $P(AorB) = P(A) + P(B)$
2. If A and B are not mutually exclusive, then
   $P(AorB) = P(A) + P(B) - P(AandB)$

Example 11. A day of the week is selected at random. Find the probability that it is a weekend day.

Solution

there are two weekend days, A is the events of the selecting Friday, B is the event of selecting Saturday and the two evens are mutually exclusive, hence,
$P(Saturday\, or\, Sunday) = P(Friday) + P(Saturday) = \frac{1}{7} + \frac{1}{7} = \frac{2}{7}$

Example 12. In a hospital unit there are 8 nurses and 5 physicians; 7 nurses and 3 physicians are females. If a staff person is selected, find the probability that the subject is a nurse or a male.

Solution

The sample space is

Staff	Females	Males	Total
Nurses	7	1	8
Physicians	3	2	5
Total	10	3	13

A is the events of the selecting a nurse, B is the event of selecting male and the two evens are not mutually exclusive, hence,
$P(nurse\,or\,male) = P(nurse) + P(male) - P(nurse\,and\,male) = \frac{8}{13} + \frac{3}{13} - \frac{1}{13} = \frac{10}{13}$

The probability rules can be extended to three or more events. For three mutually exclusive events A, B, and C,
$P(AorBorC) = P(A) + P(B) + P(C)$
For three events that are not mutually exclusive,
$P(AorBorC) = P(A) + P(B) + P(C) - P(AandB) - P(AandC) - P(BandC) + P(AandBandC)$

Example 13. If one card is drawn from an ordinary deck of cards, find the probability of getting the following.

1. A king or a queen or a jack
2. A king or a queen or a diamond
3. An ace or a diamond or a heart
4. A 9 or a 10 or a spade or a club

Solution

1. The events in (1) are mutually exclusive ,hence
   $P(AorBorC) = P(A) + P(B) + P(C)$
   $P(kingoraqueenorajack) = P(king) + P(queen)P(jack) = \frac{4}{52} + \frac{4}{52} + \frac{4}{52} = \frac{12}{52} = \frac{3}{13}$

2. The events in (2) are not mutually exclusive , since a card can be king and a diamond or queen and a diamond
   $P(AorBorC) = P(A) + P(B) + P(C) - P(AandC) - P(BandC)$
   $P(Akingoraqueenoradiamond) = P(aking) - P(aqueen) + P(adiamond) - P(King\ and\ diamond) - P(queen\ and\ diamond)$
   $= \frac{4}{52} + \frac{4}{52} + \frac{13}{52} - \frac{1}{52} - \frac{1}{52} = \frac{19}{52}$

*Note:*the symbol $\cup$ represents the union of two sets, and $A \cup B$ corresponds to A or B. The symbol $\cap$ represents the intersection of two sets, and $A \cap B$ corresponds to A and B.