Probability
Lecture 9

Lecture objectives

- Find the total number of outcomes in a sequence of events, using the fundamental counting rule.
- Find the number of ways that r objects can be selected from n objects, using the permutation rule and combination rule.
- Find the probability of an event, using the counting rules.

The Counting Rules

- In a sequence of n events in which the first one has k_1 possibilities and the second event has k_2 and the third has k_3, and so forth, fundamental rule say the total number of possibilities of the sequence will be $k_1.k_2.k_3...k_n$
- The number of ways of linearly arranging n distinct objects when the order of arrangement matters $= n!$.
- The arrangement of n objects in a specific order using r objects at a time is called a permutation of n objects taking r objects at a time ${}_nP_k = \frac{n!}{(n-r)!}$.
- The number of ways of choosing r distinct objects from n distinct objects when the order of selection is important $= n(n-1)...(n-r+1)$.
- The number of ways of choosing r distinct objects from n distinct objects when the order of selection is not important${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$.
- The number of ways of choosing r objects from n distinct objects if the same object could be chosen repeatedly $= n^r$.
- The number of ways of distributing n distinct objects into k distinct categories when the order in which the distributions are made is not important and n_i objects are to be allocated to the ith category $\frac{n!}{n_1!n_2!...n_k!}$

Example 1. Suppose a business owner has a choice of 5 locations in which to establish her business. She decides to rank each location according to certain criteria, such as price of the store and parking facilities. How many different ways can she rank the 5 locations?

Solution

There are

$$5! = 5 * 4 * 3 * 2 * 1 = 120$$

different possible rankings. The reason is that she has 5 choices for the first location, 4 choices for the second location, 3 choices for the third location, etc.

Example 2. Suppose the business owner in last example wishes to rank only the top 3 of the 5 locations. How many different ways can she rank them?

Solution

Using the fundamental counting rule, she can select any one of the 5 for first choice, then any one of the remaining 4 locations for her second choice, and finally, any one of the remaining locations for her third choice.

$$5 * 4 * 3 = 60$$

Example 3. The manager of a department store chain wishes to make four-digit identification cards for her employees. How many different cards can be made if she uses the digits 1, 2, 3, 4, 5, and 6 and repetitions are permitted

Solution

Since there are 4 spaces to fill on each card and there are 6 choices for each space, the total number of cards that can be made is $6 * 6 * 6 * 6 = 1296$.

Example 4. A coin is tossed and a die is rolled. Find the number of outcomes for the sequence of events.

Solution

Since the coin can land either heads up or tails up and since the die can land with any one of six numbers showing face up, there are $2 * 6 = 12$ possibilities.

Example 5. The advertising director for a television show has 7 ads to use on the program. If she selects 1 of them for the opening of the show, 1 for the middle of the show, and 1 for the ending of the show, how many possible ways can this be accomplished?

Solution

Since order is important, the solution is ${}_7p_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = 210$

Example 6. A newspaper editor has received 8 books to review. He decides that he can use 3 reviews in his newspaper. How many different ways can these 3 reviews be selected?

Solution

$\binom{8}{3} = \frac{8!}{3!*5!} = 56$

Example 7. In a club there are 7 women and 5 men. A committee of 3 women and 2 men is to be chosen. How many different possibilities are there?

Solution

first we select 3 women from 7 women, which can be done in ${}_7C_3$
second 2 men must be selected from 5 men, which can be done in ${}_5C_2$
Finally, by the fundamental counting rule, the total number of different ways is ${}_7C_3 *_5 C_2 = 35 * 10 = 350$

Probability and Counting Rules

Example 8. Find the probability of getting 4 aces when 5 cards are drawn from an ordinary deck of cards.

Solution

There are ${}_{52}C_5$ ways to draw 5 cards from a deck. There is ${}_4C_4 = 1$ way to get 4 aces , but there are 48 possibilities to get the fifth card. Therefore, there are 48 ways to get 4 aces and 1 other card. Hence,
$P(4\, aces) = \frac{{}_4C_4 * 48}{{}_{52}C_5} = 0.000018$

Example 9. A box contains 24 transistors, 4 of which are defective. If 4 are sold at random, find the following probabilities.

1. Exactly 2 are defective.
2. None is defective.
3. All are defective.
4. At least 1 is defective.

Solution

There are ${}_{24}C_4$ ways to sell 4 transistors

1. Two defective transistors can be selected as ${}_4C_2$ and two nondefective ones as ${}_{20}C_2$
   $P(exactly\, 2\, defectives) = \frac{{}_4C_2 * {}_{20}C_2}{{}_{24}C_4} = 0.107$
2. The number of ways to choose no defectives is ${}_{20}C_4$
   $P(no\, defectives) = \frac{20C_4}{{}_{24}C_4} = 0.456$
3. The number of ways to choose 4 defectives from 4 is ${}_4C_4$, or 1.
   $P(all\, defectives) = \frac{1}{{}_{24}C_4} = 0.00009$
4. To find the probability of at least 1 defective transistor, find the probability that there are no defective transistors, and then subtract that probability from 1.

Example 10. A combination lock consists of the 26 letters of the alphabet. If a 3-letter combination is needed, find the probability that the combination will consist of the letters ABC in that order. The same letter can be used more than once. (Note: A combination lock is really a permutation lock.)

Solution

- Since repetitions are permitted, there are $26 \star 26 \star 26 = 17,576$ different possible combinations.
- And since there is only one ABC combination, the probability is

$$P(ABC) = 1/26^3 = 1/17,576.$$

Example 11. There are 8 married couples in a tennis club. If 1 man and 1 woman are selected at random to plan the summer tournament, find the probability that they are married to each other.

Solution

Since there are 8 ways to select the man and 8 ways to select the woman, there are $8 * 8 = 64$ ways to select 1 man and 1 woman. Since there are 8 married couples,
$P(married\, couple) = \frac{8}{84} = 0.125$