An Entropy Approach for Dynamic Value at Risk Quantile Adjustment

Abstract

This research aims to demonstrate that the entropy of a given system, in this case a wide group of futures, can be a useful predictor to dynamically adjust the quantile in a Value at Risk approach. In this case the entropy is expressed with respect to the standardized volume of every asset considered. The idea comes from the intuition of statistical mechanic concept of Grand Canonical Ensemble. The energy of the system in the financial case can be viewed as the volume that every day is exchanged on a given instrument. By the fact that there are several random configurations of the system, is possible to compute the entropy and use it as an indicator to modify the alpha in the VaR context. The idea that the entropy of the Volume can be useful in risk management comes from the hypothesis that the randomness of the exchanged quantity for several instrument has some significant autocorrelation in the past and that can be useful to reduce the loss of the VaR approach. This methodology is also compared with an indicator based on Realized Variance that will be overperformed. The real achievement of this research is the overperformance of the Entropy Adjusted VaR over the classical VaR without using sophisticated or intensive computation techniques.

1 Introduction

As the computational capabilities of hardware increases, also the technique used by practitioners will move to more sophisticated models. The classical econometrics must face the challenges of machine learning and AI, working as a benchmark to the new way of thinking. The more time passes, the more the economic science is getting contaminated by physical approach like ABM to simulate the behavior of economic agents on the market. The capability of data collection is also increasing, allowing models, in particular case for the high frequency trading ones, to work with tens of terabytes of data. In this research there will be shown how, with a free data-provider, is possible to build an easy and free systemic indicator that outperforms similar indicator that comes from classical econometric.

2 Literature

The conceptual basis of the research is rooted in different papers and intuition over the time. The first is the work of the RiskMetrics group (*Roundtable: The Limits of VaR)* that put the foundations of the modern risk management approach and elects the VaR as an extremely useful tool to manage portfolio risk, confirmed also several years later during Basel II (Jorion, Philippe,2006). In the following research the non-parametric estimation of the Value at Risk (Markovic, 2007; Novak, 2011) realized with historical data collected thanks to the Yahoo Finance API will be the method of estimation for the VaR.

Another key work that is used as reference is the result of Barndorff-Nielsen and Shephard (Econometric analysis of realized volatility and its use in estimating stochastic volatility models, 2002) where the concept of realized variance is defined and put in a framework of high frequency tool for econometric analysis. The approach, historically speaking, responded to the ever-increasing size of data available for financial analyst. The underlying concept of summing up the returns of a given day to obtain the daily realized variance is the base for the entropy computation that is performed using a similar intuition. To define Entropy the main reference is the Boltzmann's Entropy as referred by Planck (The theory of heat radiation equation, 1914) with the universally known formulation that links together the probability of a state and the entropy with the log transformation of the probability. This is widely used in physics more than economics for the characteristics of the system considered in the discipline, for example a pot of boiling water to explain the behavior of the particles while the heat increases, while it's harder to visualize such type of concepts in a non-physical environment such as the financial market. The efficiency of entropy in the analysis of components of financial market is also highlighted by the work of Grilli & Santoro (Forecasting financial time series with Boltzmann entropy through neural networks, 2022) in which is shown that, on four different stocks, a NN based on entropy has an overall good forecasting ability not only for great and complex system but also for small database using a LSTM Model.

3 Hypothesis

To build the VaR from historical simulation is necessary to specify a distribution for the returns. In this case the distribution will be a t-student due to the leptokurtic behavior of financial returns that are also lightly skewed on the left. The underlying model for returns is a t-GARCH(1,1). For the intraday data is also hypothesized that, when not available due API limitation, the missing values of the futures are filled with a forward propagation because is supposed that the volume keeps constant until the new observed value is available. The basic VaR is also considered to be at 95%. The instruments used to create the system are the constituents of the "Futures" page of Yahoo Finance to demonstrate that also a toy-model, based on a simplified environment, is capable to capture the main feature of the entropy. The hypothesis to verify is that the entropy-adjusted VaR of certain instruments outperforms both the basic VaR at 95% and a Realized-Variance-Adjusted VaR in terms of total losses and expected shortfall. One last hypothesis, that will be proved right in the Results section, is the fact that entropy exhibit autocorrelations at some point and that using this information from a significant lag can give back a model that overperforms the classical one.

4 Methodology

The dataset is retrieved downloading hourly data of all the 32 instruments which the quotes are available on Yahoo Finance. Due to the limitations of the API the maximum range for the download consists in the last two years of data for the intraday hourly data. The sample starts in the 2022-08-25 and ends in the 2024-08-10. It contains 8993 observations for each of the 32 futures with missing values for the volumes treated as explained before. This is for both the returns that will be used to extract realized variance and for the volumes that will be used to compute the entropy.

4.1 Entropy Computation

The first operation to ensure a valid methodology and interpretable results is the standardization of the volume of every asset subtracting the mean of the asset's volume and dividing that difference by the standard deviation of the asset's volume itself. This ensures that all values are scaled and comparable with each other. The second step is the definition of a probability space. This is realized by dividing into bins the volume for every asset. Every bin contains a 5% of all the observations, taking as extremes the lowest and highest observed value in the standardized volume. The length of the bins is selected according to two criteria: 1) more bins would have required a higher number of observations and 2) the 5% is the same proportion that there is in the VaR while considering the returns. After that, all the values are counted and assigned to a different bin with respect to their value. The 20 bins contain the count for all the asset's volume corresponding to that level. This disposition makes possible to compute the probability of a given state, in this case the standardized volume being a value X that falls in a bin. The relative frequency can be obtained and so the probability. However, this procedure creates an indicator for the entropy that takes in to account the whole time series of volume that has no use in the research context. To make it useful, one should apply this same reasoning daily, compute and fill the bins with the intraday data for volume. The entropy is computed, according to the Boltzmann's formulation, as:

$$\boldsymbol{E} = -\sum_{j=1}^{20} \boldsymbol{p}(\boldsymbol{b}_j) \log\left[\boldsymbol{p}(\boldsymbol{b}_j)\right]$$

Equation 1*: The following equation express the entropy as the minus sum of the probability of each bin times its logarithm.*

This procedure, repeated every day, gives back a time series as shown in *Figure 1* in the Appendices A. The values are then rescaled another time such that the highest entropy day will have a value of 1 and the lowest entropy day will have a value of 0, as shown in the second graph of *Figure 1*.

4.2 Realized Variance

The next passage is to compute an alternative measure, based on volatility of the returns, to confront the adjustment based on entropy of the volume. The hourly returns, for each asset, are grouped by day, elevated to the second power and summed up. The daily value obtained will be the sum of the intraday returns squared. The hourly interval is also useful to prevent microstructural noises that one can observe if is using high frequency data. The resulting bias will be asymptotically close to 0 due to the absence of the bid-ask spread at that level of frequency. The formula used is the following:

$$RV_t^{(m)} = \sum_{i=1}^{24} (p_{it} - p_{i-1,t})^2$$

Equation 2*: This sums all the 24, one per hour, price difference squared during a particular day, giving back the realized variance.*

This procedure creates one value per day and as results, applying it for every day *t*, a time series for every financial instrument. Once that the series is obtained it can be rescaled between 0 and 1 to make it comparable with the entropy mentioned before.

4.3 Value at Risk

The Value at Risk is obtained from the historical simulation procedure. With a train dataset, a *t-GARCH(1,1)* estimates the one-step-ahead volatility prediction for every asset, moving forward with a rolling procedure in the test database. The procedure is done with daily data to match the length of the normalized entropy and returns and gives back the conditional volatility $\boldsymbol{h}_{t+1|t}$ for every reiteration. The conditional volatility is the used to compute the VaR according to the standard formulations:

$$VaR_{t+1} = h_{t+1|t}\, \varphi(p)$$

Equation 3*: Standard VaR one-step-ahead. The p* will be defined according to the model taken in consideration.

The main modeling part here is referred to how is defined the p. The normalization serves this goal. The value p, for the entropy adjusted VaR will be defined as:

$$p_e = 0.93 + 0.04 * E_{n,t-5}$$

Equation 4*: p is defined as a fixed amount 0.93 plus 0.04 times the normalized entropy of the same day in last week. The average value of the product will be around 0.02.*

And for the Realized Variance-Adjusted VaR will be defined as:

$$p_{rv} = 0.93 + 0.04 * RV_{n,t-1}$$

Equation 5*: p is defined as a fixed amount 0.93 plus 0.04 times the normalized realized variance. The average value of the product will be around 0.02.*

The base VaR will be computed always at a 95% level. The formulation of the two p, given the fact that the average of the normalized entropy is 0.4931 and the several mean values for the futures normalized returns are very close to 0.5, creates a fair confrontation between the three models that, on average, will have a confidence interval of 95%, so in the end one will be fixed and two of them will react to the past information.

4.4 Comparing the results

The three different VaR are compared under several aspects. At first there will be highlighted the number of violations and the total loss, subsequently the respective expected shortfall and MSE. The expected shortfall is defined as:

$$ES_{p,t+1} = \frac{1}{l}\int_{o}^{l} VaR(p)dp$$

Equation 6*: The average of the VaR at the quantile level p over the proportion of losses l that exceed the VaR*

The difference between the VaR will be verified using the Diebold-Mariano test to check if there are significative differences in the accuracy of the models and resumed in *Table 1* in Appendices B. The Diebold-Mariano test statistic is expressed as:

$$F = \frac{e_i{}'e_i}{e_j{}'e_j}$$

Where *i* and j refers to the errors vector (Tx1) of the two models. The tabulation of the critical values is available in the original article in the Reference part of the *Appendices C*. Performing this test, can be also viewed as testing equal forecast variance if: *"(1) loss is quadratic and (2) the forecast errors are (a) zero mean, (b) Gaussian, (c) serially uncorrelated, or (d) contemporaneously uncorrelated"*.

5 Results

The first result that is worth of notice is the autocorrelation plot of the entropy series represented in *Figure 2* in the *Appendices A*. The entropy of a given day exhibits a significantly high autocorrelation with the entropy at lag 5,10,15, 20 and so on. That means, given the structure of the data, that the entropy of the system today is correlated with the entropy of the past week in the same day. This is traduced in a correlation between the randomness of the exchanged volume over time, indicating that is reasonable to expect that, for example all the Mondays, will have a similar activity on the market in terms of buy and sell quantity. This is reasonable and confirmed by the fact that also volumes have a precise repartition in the week, with days that have more activity in terms of volume with respect to the others. This behavior of the entropy time series is the reason why the considered lag is 5 in the VaR adjustment, instead of 1. A further confirmation for this can be viewed in the *Figure 3* in *Appendices A* where the periodogram of the entropy has its peak at around 0.2 highlighting that 5 is the dominant period, with all the other frequency being noise, and that the autocorrelation of the entropy today and one week ago is significant.

Moving ahead is possible to notice, as expressed in *Table 1* in the *Appendices B,* that the adjustment not only overperforms the basic VaR with a fixed quantile but also overperforms the one built on the Realized Variance to adjust the alpha. The Diebold-Mariano test statistics also confirms that there is significant difference between the accuracy of the two models in terms of MSE. In specific terms is possible to notice that, for different instruments the Entropy Adjusted VaR gives back a lower number of violations, lower total loss and a lower Expected Shortfall. In *Figure 4-to-6* is also possible to notice the distribution and the frequency of violations for every model into account for the three different cases corresponding to the different assets. The systemic approach offered by entropy synthetize the randomness of the exchanged volume into a number and, with the existence of strong autocorrelations between the value today and one week ago, offers a better performing model in terms of risk management compared to the standard one and the non-systemic counterpart represented by the Realized Variance.

Looking at the VaR graph in the *Appendices A* is at first possible to notice that the estimation will be different according to the volatility clustering characteristics of the instruments. In fact, the CL=F(Crude oil future Sep. 24, *Figure 5*) has a way less smooth VaR with respect to the NQ=F(Nasdaq 100 Sep. 24, *Figure 4*) and HG=F(Copper Sep. 24, *Figure 6*). Clearly also the Realized Variance and the Entropy Adjusted will have paths like the Basic thanks to the common estimation part regarding the conditional volatility one step ahead. For what regards the losses of the three models, in the case of the futures mentioned before is possible to see that, using the information of the volume of one week ago is possible to have on average a lower loss with respect to other models. In terms of distribution of the losses appears clear that the Entropy-Adjusted has more concentration in the values near the 0, where the frequency of losses is greater. This traduces in a lower concentration of losses in levels where these are heavier, resulting in a both lower number of violation and lower total loss. The expected shortfall, in these three examples is always lower using the Entropy-Adjusted and, as will be shown, is lower in most cases. From the results is possible to notice that the number of violations is lower in every case for the Entropy-Adjusted. The same is valid for the total losses, there is no case, in the 32 assets selected as system, where the Base VaR and the Realized Variance-Adjusted VaR can give back a lower total loss. These results alone are not sufficient to state that the Entropy-Adjusted is overall better in every situation. In fact, is possible that, despite being lower violations and lower number of total losses, the average loss incurred as soon as the VaR is violated can be higher. In fact, this happens with some instruments. The futures GC=F(Gold Dec. 24) have a lower Expected Shortfall using the Realized Variance approach. The futures PA=F(Palladium Sep. 24), RB=F(RBOB Gasoline Sep. 24), ZO=F(Oat Dec. 24), HO=F(Heating Oil Sep. 24), SB=F(Sugar Oct. 24), ZR=F(Rough Rice Nov. 24), ZS=F(Soybean Nov. 24) and ZB=F(U.S. Treasury Bond futures Sep. 24) have a lower Expected Shortfall using the Base VaR. There is also a significant difference in forecast accuracy in terms of Mean Squared Error verified with the Diebold-Mariano test. In all the other 23 cases the Entropy-Adjusted VaR performs better than both Realized Variance than the Base VaR with the only exception being the RTY=F(Mini Russel future) where the Null hypothesis of the Diebold-Mariano is not rejected suggesting equal accuracy of the three. The highest Expected Shortfall, as previously said, is caused by the fact that even having in any possible case a lower number of violations and total losses, the single loss will, on average be heavier than the other models. It's also possible to notice that in *Figure 7*, where the distribution of the losses for the different assets that overperform the Entropy-Adjusted VaR, that in some cases there are extreme negative returns like the case of HO=F where it has a negative return of -7%. Another noticeable case is the one of RB=F where highest negative return is not as high as HO=F but has way more frequency with respect to other cases.

6 Conclusions

What was presented in the previous paragraphs can be viewed as the starting point for a more advanced analysis realized using high-frequency data. Although the unavailability of that kind of data, due to the constraints of free data providers, it did not prevent significant results from emerging. Hourly data provided a substantial number of observations, without the risk of observing microstructural noises. Instead, with the usage of minute-by-minute data, the dataset would have been limited to just one month(due to API's limitation), which is insufficient to effectively apply a t-GARCH(1,1) model and accurately estimate VaR. However, using higher-frequency data, such as minute-by-minute or tick-by-tick but on a timeframe longer than one month, could potentially provide even more precise and informative results especially in the context of high-frequency risk management(this is true only if the problem of the bid-ask spread is addressed or does not effectively manifest). The Entropy-Adjusted is effectively demonstrated always lower than any of the other two models in terms of violations and total loss, and in 23 cases over 32 has also a lower expected shortfall, resulting consistently in an overall better approach.

The higher performance of the Entropy-Adjusted VaR is achieved without risking over-parameterization, thanks to the non-parametrical nature of the historical estimation.

The strong autocorrelation observed at a lag 5, along with the periodogram analysis, clearly shows that the randomness in trading volume throughout the week is influenced by the corresponding day in the previous week. This finding suggests that is possible to extract signals from the weekly market cycle, according to the cyclical nature of supply and demand in the market.

By creating a single indicator for the traded volume of a group of futures, this approach aims to show that it's possible to improve risk management practices without relying on overly complex or computationally intensive methods. Enhancing this approach could involve, in addition to using tick-by-tick data, developing a system that includes all instruments traded on a specific exchange, like the CME or NYSE, to better simulate a physical environment. In such a scenario, the model's performance could be even better due to the higher quality of data and the greater relevance of entropy in this context. Despite some limitations, the results presented here strongly support the use of entropy as an effective and straightforward method for advancing risk management in the increasingly complex landscape of financial markets.

Appendices

A Figures

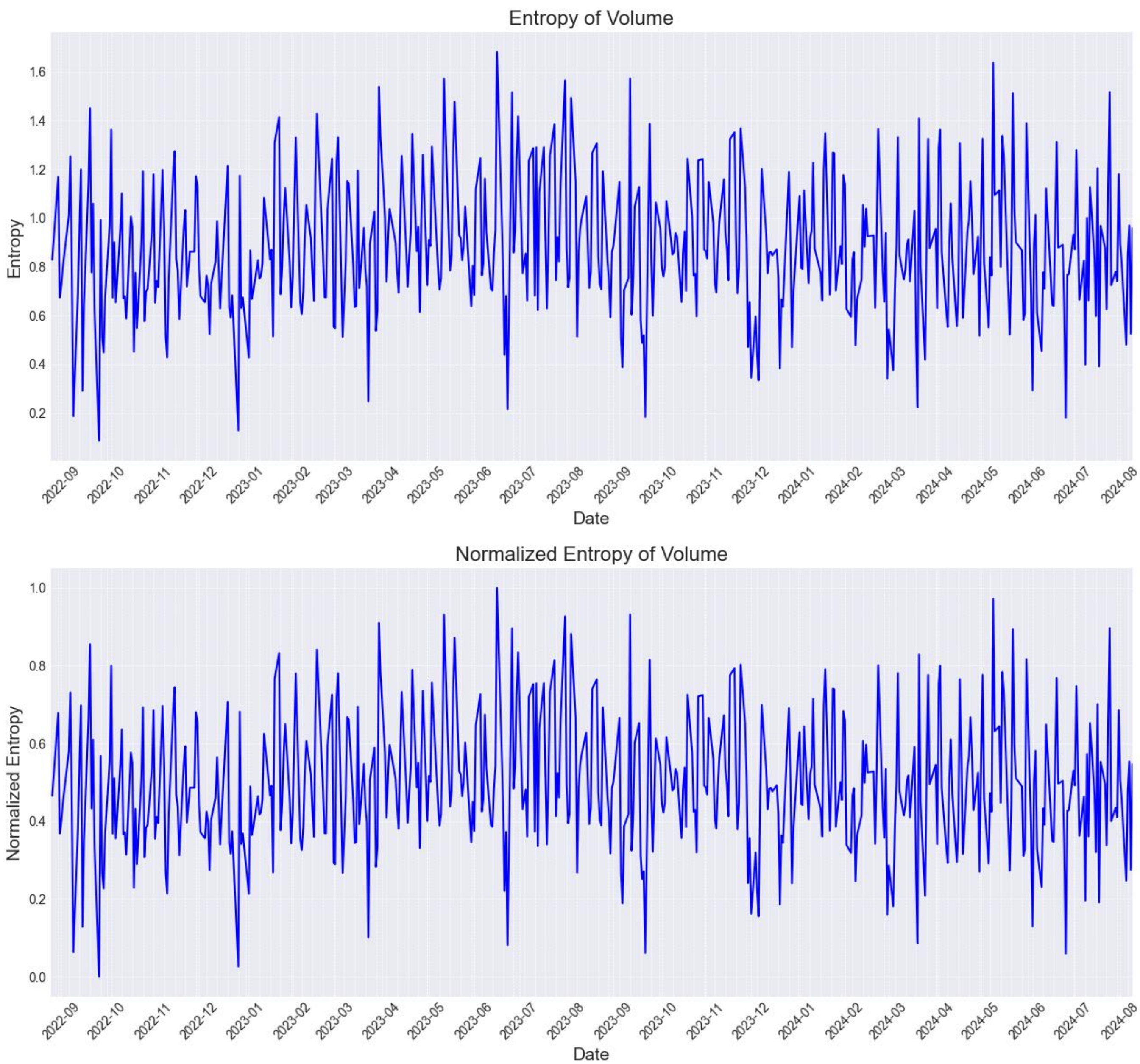

Figure 1: The time series of the Entropy of the system and its normalized version scaled between 0 and 1.

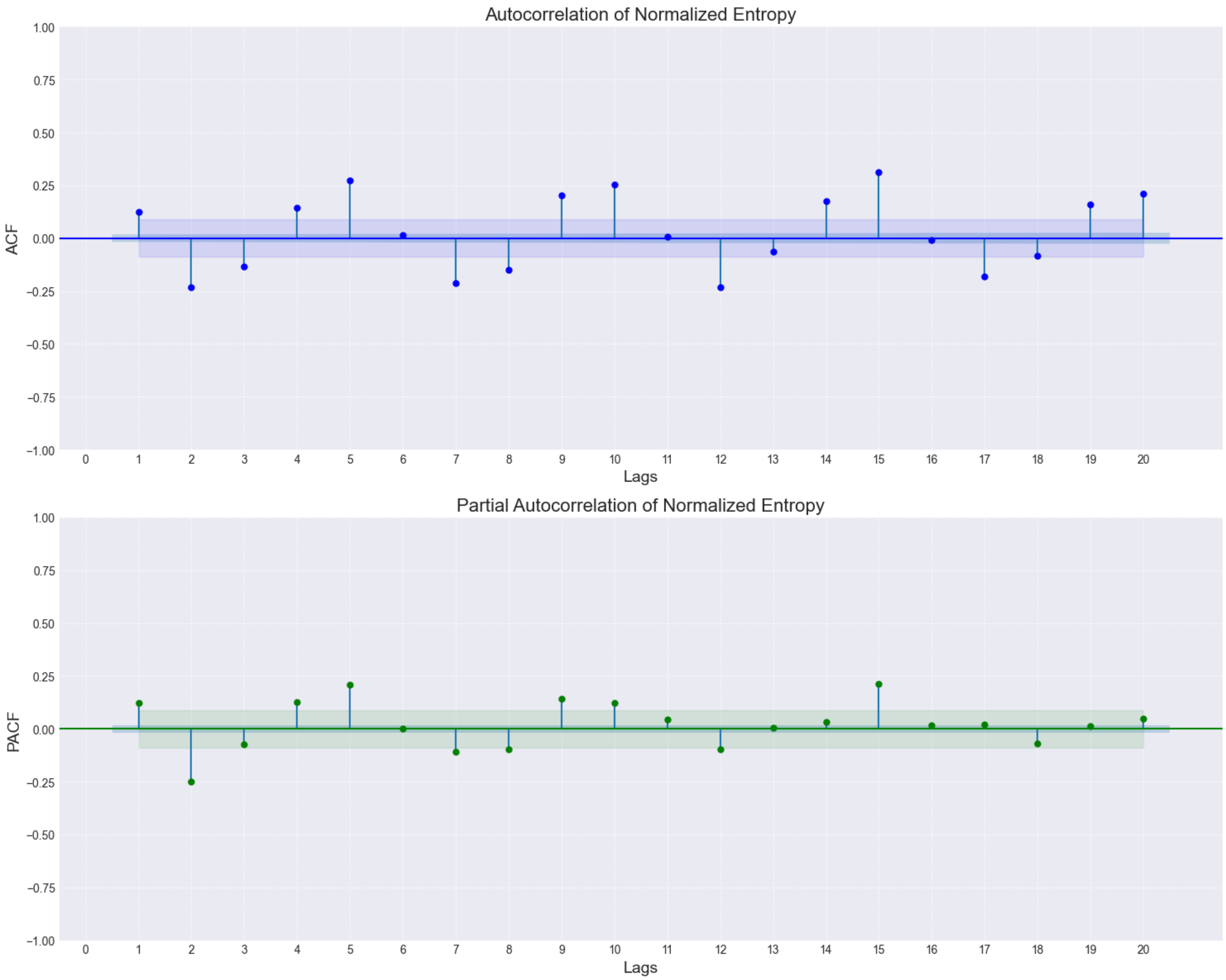

Figure 2: Plots of the ACF and PACF of the Entropy. It's clearly visible the significance of the autocorrelation, with a particular focus on Lag 5.

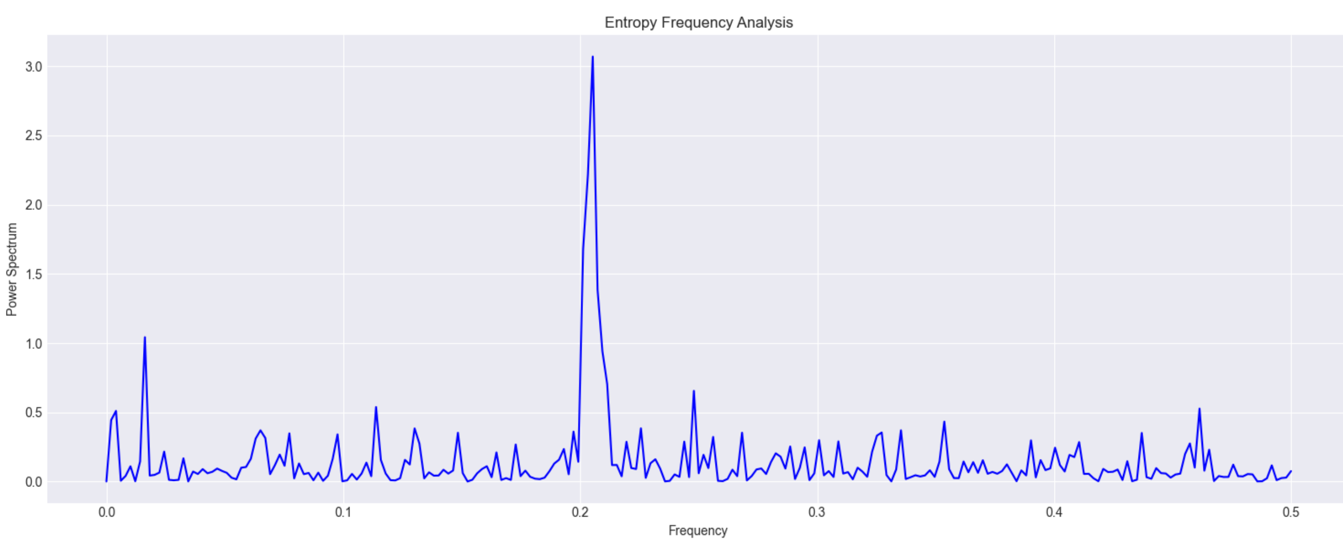

Figure 3: Periodogram of the Entropy, the dominant frequency is near 0.2 as previously mentioned and proving the weekly cyclicity of the entropy for the volume, the smallest peaks represent the noise.

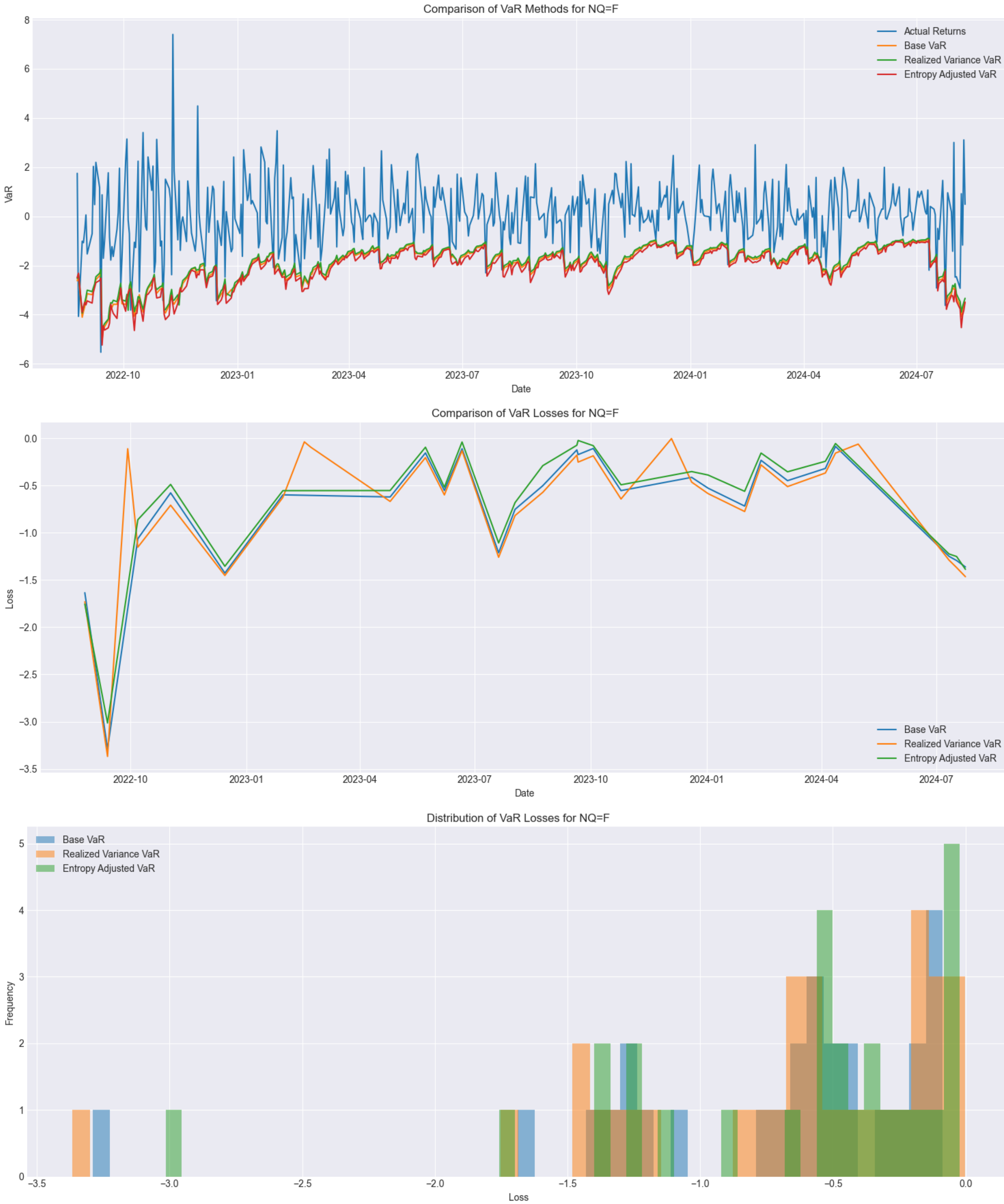

Figure 4: Graphical representation of VaR, losses suffered and distribution of the losses for the future NQ=F.

Figure 5: Graphical representation of VaR, losses suffered and distribution of the losses for the future CL=F.

Figure 6: Graphical representation of VaR, losses suffered and distribution of the losses for the future HG=F.

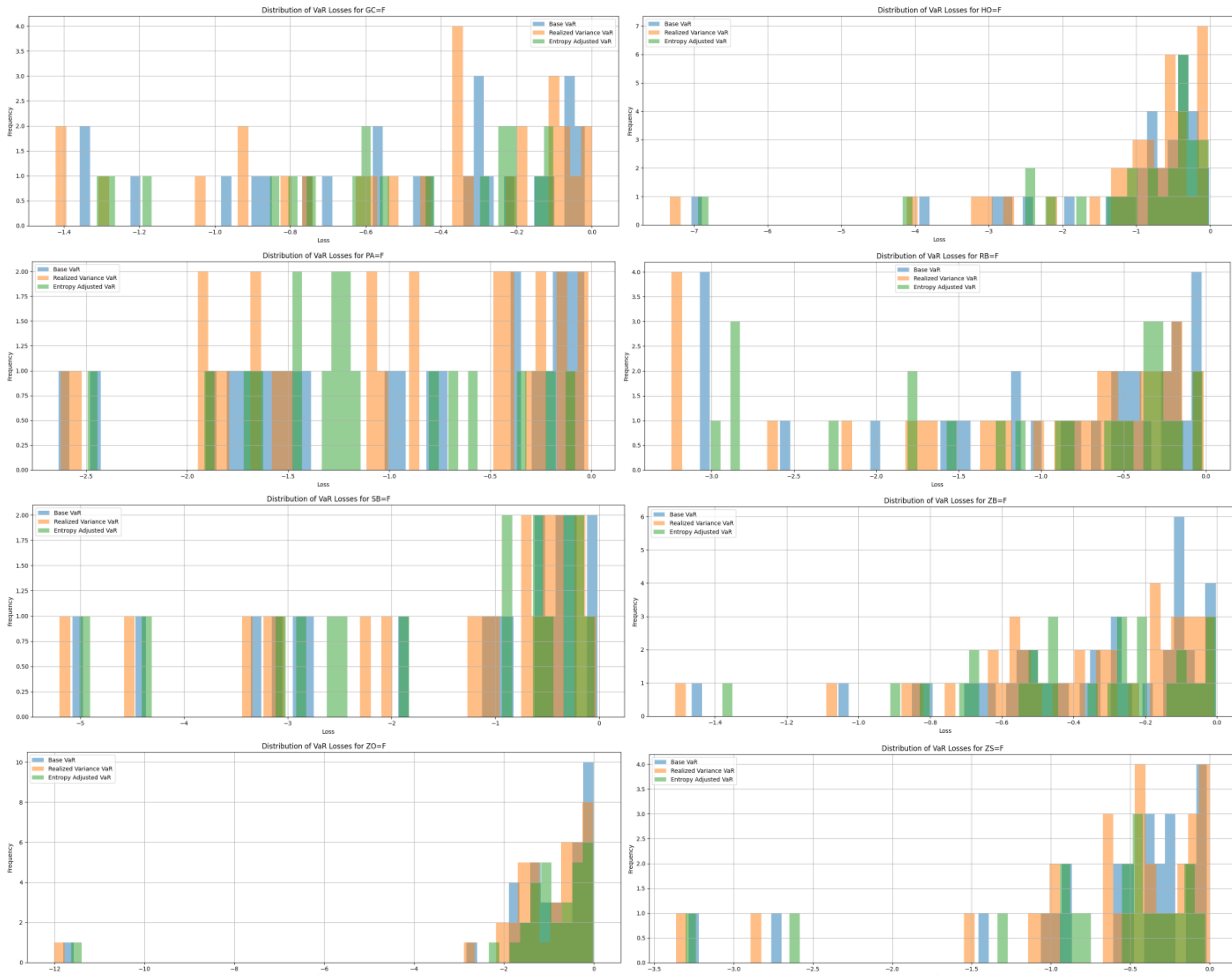

Figure 7: *The distribution of the losses for Base VaR(Blue), Realized Variance Adjusted Var(Orange) and Entropy Adjusted VaR(Green) for: (1)GC=F, (2) HO=F, (3) PA=F, (4) RB=F, (5) SB=F, (6) ZB=F, (7) ZO=F, (8) ZS=F*

B Tables

Table 1: Results compared of the three different methods

a) Base Value at Risk Results

Asset	Number of Violations	Total Losses	Expected Shortfall	DM Test for MSE (Base vs RV)	DM Test for MSE (Base vs Entropy)
NQ=F	**27**	**-20.0957893938**	**-0.7442884960**	**-**	**-13.1382846276*** **
CL=F	**29**	**-24.5153614247**	**-0.8453572905**	**-**	**-18.0645907767*** **
HG=F	**27**	**-16.9110111090**	**-0.6263337447**	**-**	**-17.4334189975*** **

b) Realized Variance Adjusted Value at Risk Results

Asset	Number of Violations	Total Losses	Expected Shortfall	DM Test for MSE (RV vs Base)	DM Test for MSE (RV vs Entropy)
NQ=F	**32**	**-22.1091907149**	**-0.6909122098**	**-**	**-16.0347129168*** **
CL=F	**33**	**-28.5600788083**	**-0.8654569335**	**-**	**-20.6615689939*** **
HG=F	**28**	**-19.5819726922**	**-0.6993561675**	**-**	**-21.7461606209*** **

c) Entropy Adjusted Value at Risk Results

Asset	Number of Violations	Total Losses	Expected Shortfall	DM Test for MSE (Entropy vs Base)	DM Test for MSE (Entropy vs RV)
NQ=F	**27**	**-17.9462170769**	**-0.6646747065**	**-13.1382846276*** **	**-16.0347129168*** **
CL=F	**23**	**-19.2610808122**	**-0.8374382961**	**-18.0645907767 *** **	**-20.6615689939*** **
HG=F	**21**	**-12.9156660412**	**-0.6150317162**	**-17.4334189975*** **	**-21.7461606209*** **

Note: The three asterisk *** indicates that the null hypothesis is rejected at *10%,5%* and *1%*. Under the Null hypothesis of the Diebold-Mariano test the two model have the same accuracy. In this case the null is always rejected stating that the model based on entropy outperforms in a significant way both the realized variance one and the basic model. The test of RV vs Basic (or vice versa) is not performed for obvious reasons. As previously mentioned, the only case in which the null is not rejected is the RTY=F case (p-value is *0.31838769000514144*) at all the three significance level suggesting that the accuracy of the three model, for that case is the same.

C References

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