UTEC
UNIVERSIDAD DE INGENIERÍA Y TECNOLOGÍA

Digital Image Processing

Midterm Exam

Elective Course 2016-2

General Guidelines:

Time: 120 minutes
Number of questions: 6
You are advised to read the entire exam first. Each question has different point values.
The use of books, class notes and a calculator is allowed. Laptops are not allowed.
The development of the exam is strictly individual.

Question 1 (2 pts)

A student has written the following statement:

> "Median filtering may reduce the number of occupied bins in a histogram. It will never increase the number of occupied bins".

Is this statement true? Explain your reasoning.

Solution: The statement is true. Median filtering selects the median value in the neighborhood of each pixel. Therefore, it will not introduce any new values (as opposed to, for example, a mean filter). However, it may reduce the number of occupied bins as it may remove spurious noise values. That is, it may empty the bins as these values are removed.

Question 2 (3 pts)

Consider the spatial filter H given by

$$H = \begin{bmatrix} -1 & -2 & 0 \\ -2 & 0 & 3 \\ 0 & 3 & 1 \end{bmatrix}$$

and assume a 6-bit grayscale image I.

(a) (1 pt) Determine the maximum and minimum possible values that a pixel, to which this spatial filter is applied, can have. Do not apply any type of normalization.

(b) (2 pts) Propose a gray-level transformation function to ensure that any output of this filter will be a standard 6-bit grayscale image.

Solution: Since a 6-bit grayscale image is assumed, the highest value that each pixel can have, per se, is 63, and the lowest value is 0.

(a) After applying the filter, the maximum value is achieved when the negative values of the filter multiply 0's and the positive values of the filter multiply 63's. Then, the maximum achievable value is $v_{max} = (3+3+1)(63) = 441$. Following the same reasoning, the minimum value will be $v_{min} = (-2-2-1)(63) = -315$.

(b) A 6-bit image must have a range from 0 to 63. Since the maximum and minimum possible values for the gray levels are $v_{max} = 441$ and $v_{min} = -315$, respectively, the function will be

$$I_{out} = 63\left(\frac{I_{in} - v_{min}}{v_{max} - v_{min}}\right) = 63\left(\frac{I_{in} + 315}{756}\right)$$

where I_{in} and I_{out} are the input and output images, respectively.

Question 3 (5 pts)

Consider an 8-bit image I_{in}, with coordinates (v, w), and the following horizontal and vertical shear transformation T, which is applied point by point:

$$T = \begin{bmatrix} 1 & s_h & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $s_h = 0.5$ and $s_v = 1.2$. We want to apply this point transformation to I_{in} in order to generate a sheared image I_{out} with coordinates (x, y). To do so, we use the so-called inverse mapping technique: we scan every pixel of I_{out} and find the correspondence in I_{in}. Suppose we are currently at $(x, y) = (347, 265)$ in I_{out}.

(a) (2 pts) For this pixel, find the corresponding coordinate (v, w) in I_{in} (when the transformation T applied to I_{in} generates I_{out}).

(b) (3 pts) Find the gray level for this pixel using bilinear interpolation. A part of I_{in}, which you might find useful, is shown below.

	w=227	w=228	w=229	w=230
v=71	60	65	70	77
v=72	66	70	80	88
v=73	70	90	100	130
v=74	90	95	110	150

Solution:

(a) The application of the transformation T can be written in matrix form as

$$\begin{bmatrix} x & y & 1 \end{bmatrix} = \begin{bmatrix} v & w & 1 \end{bmatrix} \begin{bmatrix} 1 & 0.5 & 0 \\ 1.2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

or in equation form as

$$x = v + 1.2w \quad (1)$$
$$y = 0.5v + w. \quad (2)$$

We are interested in the opposite relation. This can be obtained by taking the inverse of matrix T, or by performing some basic elemental algebraic operations on (1) and (2). The result is

$$v = 2.5x - 3y$$
$$w = -1.25x + 2.5y.$$

Using these expressions and the input value $(x, y) = (347, 265)$, the corresponding coordinates in I_{in} are $v = 72.5$, $w = 228.75$.

(b) For the bilinear transformation, we need to use the pixels surrounding (v, w). Considering $\tilde{v} = \text{floor}(v)$, and $\tilde{w} = \text{floor}(w)$. The general expression is given by

$$f(\tilde{v}+a, \tilde{w}+b) = (1-a)(1-b)f(\tilde{v}, \tilde{w}) + a(1-b)f(\tilde{v}+1, \tilde{w}) + (1-a)bf(\tilde{v}, \tilde{w}+1) + abf(\tilde{v}+1, \tilde{w}+1).$$

Replacing the coordinate values $\tilde{v} = 72, \tilde{w} = 228$, and the values of $a = 0.5$ and $b = 0.75$, we have:

$$f(72.5, 228.75) = 0.5(0.25)f(72, 228) + 0.5(0.25)f(73, 228) + 0.5(0.75)f(72, 229) + 0.5(0.75)f(73, 229).$$

The required values can be obtained from the given matrix. That gives: $f(72, 228) = 70$, $f(73, 228) = 90$, $f(72, 229) = 80$, $f(73, 229) = 100$. Replacing all, the desired gray level is 87.5.

Question 4 (3 pts)

An 8-bit digital image has a histogram where the gray levels are equally distributed in the range from 160 to 220 (uniform distribution). Sketch the new histograms for each operation as well as the transformation functions for each, and describe the produced effect on the image contrast and brightness.

(a) (1 pt) Calculation of the image negative

(b) (1 pt) Addition of 50 to all pixel gray levels

(c) (1 pt) Application of a thresholding function where the threshold is selected as gray level 128.

Solution:

(a) The new image will have the order of the gray levels reversed. All the pixels which were 160 will become 255-160=95, and all the pixels which were 220 will be 35. The image will be the inverse (the brighter areas will be darker, and vice-versa). The image will still have low contrast but now it will be darker. The new histogram will have values from 35 to 95. The transformation function is a line with negative slope.

(b) This will lighten the image while maintaining a poor contrast as only a limited range of gray scale values are used. It will force all pixels which previously had a grayscale values between 206 and 220 to grayscale 255. This will make more of the pixels have value 255, reducing the contrast. The histogram ranges from 210 to 254, uniformly, but 255 now has more values. The transformation is a line starting at point (0,50) ending at (205,255) and keeps a 255 value after 205.

(c) This will make the entire image white as the threshold is set below every gray scale value in the image. The new histogram will have values only at 255. The transformation function is a typical threshold.

Question 5 (2 pts)

The following figure presents the result of applying a filter to an image.

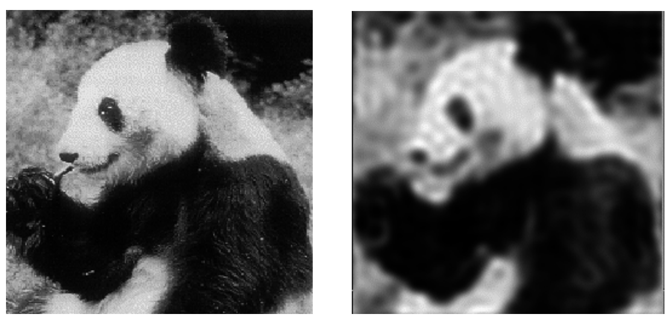

Figure 1: Image before filtering (left) and after filtering(right)

(a) (1 pt) What type of filter do you think was used? Justify your answer

(b) (1 pt) Propose at least one way to improve the result.

Solution:

(a) The filter was an ideal lowpass filter since the image is blurred and presents the characteristic ringing effect.

(b) One way is to use another lowpass filter, such as the Butterworth (with a low order, e.g. not greater than 2) or the Gaussian lowpass filter.

Question 6 (5 pts)

You are given the following spatial-domain filter

0.15	0.35	0	-0.35	-0.15

(a) (3 pts) Compute the spectrum and phase of this filter

(b) (1 pt) What do you think the effect of applying this filter to an image will be? In other terms, what type of filter do you think it is? Justify your answer

(c) (1 pt) How would you modify the filter to achieve exactly the opposite effect? Provide the steps you would follow. You do not need to find the actual filter that will achieve this effect.

Solution:

(a) The DFT in this case will be

$$F(u) = \sum_{x=0}^{4} f(x) e^{-j2\pi ux/N}$$
$$F(u) = 0.15e^{j0} + 0.35e^{-j2\pi u/5} - 0.35e^{-j6\pi u/5} - 0.15e^{-j8\pi u/5}$$

Replacing each of the terms of the filter:

$$F(0) = 0.15 + 0.35 - 0.35 - 0.15 = 0$$
$$F(1) = 0.15 + 0.35e^{-j2\pi/5} - 0.35e^{-j6\pi/5} - 0.15e^{-j8\pi/5} = 0.495 - 0.6813j$$
$$F(2) = 0.15 + 0.35e^{-j4\pi/5} - 0.35e^{-j12\pi/5} - 0.15e^{-j16\pi/5} = -0.12 + 0.039j$$
$$F(0) = 0.15 + 0.35e^{-j6\pi/5} - 0.35e^{-j18\pi/5} - 0.15e^{-j24\pi/5} = -0.12 - 0.039j$$
$$F(0) = 0.15 + 0.35e^{-j8\pi/5} - 0.35e^{-j24\pi/5} - 0.15e^{-j32\pi/5} = 0.495 + 0.6813j$$

Taking the magnitude of these complex numbers, the spectrum is

$$|F| = \begin{bmatrix} 0 & 0.84 & 0.126 & 0.126 & 0.84. \end{bmatrix}$$

Taking the angle of these complex numbers, the phase is

$$\phi = \begin{bmatrix} 0 & -54^\circ & 162^\circ & -162^\circ & 54^\circ \end{bmatrix}$$

(b) It can be seen that there is a zero dc-term and a finite magnitude at higher frequencies. Then, this can be considered a highpass filter. The effect of applying this filter will be to enhance the large variations.

(c) In this case, a lowpass filter is desired using the previous filter. One way is to use the fact that $H_{LP} = 1 - H_{HP}$ but considering a previous normalization of the values of the filter.