

Homework #7-1

Problem 1: a) consider a 4mF capacitor voltage $V(t) = Ae^{-100t} + Be^{-600t}$ V for $t \geq 0$ where A and B are constants. Assume initial voltage $V(0) = 50\text{V}$ and initial current $i_c(0) = 20\text{A}$. Determine A, B, and $i_c(t)$

$$V(t) = Ae^{-100t} + Be^{-600t}, \quad t \geq 0$$

$$V(0) = A + B = 50$$

$$i_c = C \frac{dV}{dt}$$

$$i_c(t) = C \left[-100Ae^{-100t} - 600Be^{-600t} \right]$$

$$C = 4 \cdot 10^{-3} \text{ F} \Rightarrow i_c(t) = -0.4Ae^{-100t} - 2.4Be^{-600t}$$

$$i_c(0) = 0.4A - 2.4B$$

$$i_c(0) = 20 \Rightarrow -0.4A - 2.4B = 20$$

$$A + 6B = -50$$

$$B = 50 - A \\ 50 - 70 = \boxed{-20 = B}$$

$$A + 6(50 - A) = -50$$

$$-5A + 300 = -50$$

$$\boxed{-5A = -350} \\ \boxed{A = 70}$$

$$i_c(t) = -0.4Ae^{-100t} - 2.4Be^{-600t} \quad \boxed{A = 70}$$

b)

$$V_c = \frac{1}{C} \int i_c(t) dt \Rightarrow \frac{1}{0.5} \int (6 - 6e^{-t}) dt + k$$

$$V_c(t) = 2(6t + 6e^{-t}) + k$$

$$V_c(0) = 0\text{V}$$

$$V_c(0) = 2(0) + k = 0 \Rightarrow 12 + k = 0$$

$$\boxed{k = -12}$$

$$V_c(t) = 12(6t + 6e^{-t} - 1) \Rightarrow V_c(2) = 12(2 + 6e^{-2} - 1) = 12(1 + e^{-2}) \quad \textcircled{1}$$

$$i_c(2) = 6(1 - e^{-2}) \text{ A}$$

$$i_d(2) = V_c(2) \cdot i_c(2) \Rightarrow 72(1 + e^{-2})(1 - e^{-2}) \\ = 72(1 - e^{-4}) = 70.681$$