



University of California Merced  
School of Engineering  
Department of Electrical Engineering

## **ENGR 065 Circuit Theory**

### **Lab #6: Superposition**

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#### **Section**

Thursday 9:00 am - 11:50

ENGR065-03L

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## Objectives

- Learn the principle of superposition.
- Measure the currents and voltages of the given circuit.

### 1. Introduction

The purpose of this lab is to learn the main idea of superposition. For that, we will analyze a circuit with theoretical calculations first. The theoretical value will be calculated and the value will then be compared to the measured value from the activity. We will see that the values are maybe identical to the calculations, showing that the principle of superposition is valid.

### 2. Methods & Procedures

The superposition theorem allows us to view a circuit with multiple inputs. The first step is to suppress all but one input. To suppress a voltage, you must replace it with a short circuit. To suppress current, you have to replace it with an open circuit which should be in a linear sense. The response of any element should also be the sum of each response for each input.

#### **Superposition Principle:**

- **Homogeneity:**  $y(ax) = ay(x)$

- **Additivity:**  $y(x_1 + x_2 + \dots + x_n) = y(x_1) + y(x_2) + \dots + y(x_n)$

[If a circuit (system) is linear (homogeneity + additivity) , we have

$$y(ax) = ay(x) \quad y(x_1 + x_2 + \dots + x_n) = y(x_1) + y(x_2) + \dots + y(x_n)]$$

#### **Steps in Applying Superposition Principle**

1. If there are various independent sources in a circuit, keep ONE independent source in the circuit and remove all the other independent sources. The removed voltage sources are replaced with short circuits and current sources are replaced with open circuits.
2. Find the response that was made by the kept independent source.
3. Repeat both steps 1 & 2 for each independent source.
4. When doing it algebraically add the responses driven by all the independent sources found in the steps 1 to 3 above.

### 3. Results & Discussion

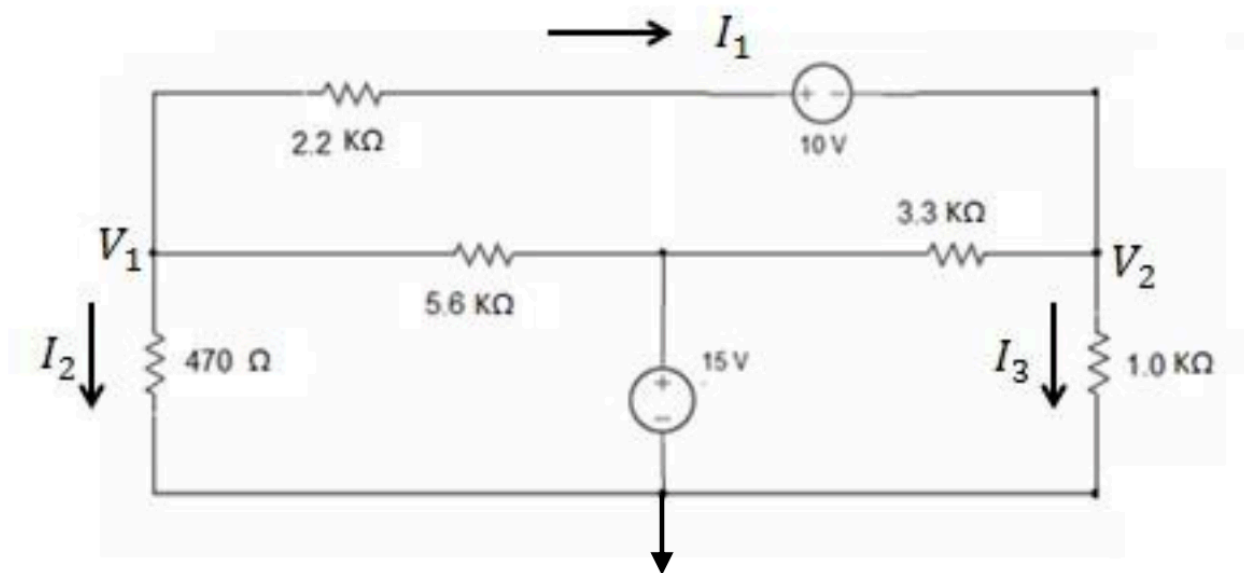
#### Experimental Procedure

In the lab we were asked to recreate the circuits that are shown, in doing so we are making sure to include a voltage sensor to keep track of the voltage across the circuit. We then make tests where having both the current source and voltage source be present in the circuit. We then conduct a test with only the voltage source included in the circuit, and then a test where only the current source is included.

#### Analysis of Experimental Data

We provide our results in Tables 1-3 for the circuit below, where each was tested for the condition mentioned in the Procedure section.

#### Circuit:



**Table 1:**

With the circuit built with voltages placed correctly we were able to calculate for the theoretical value of doing calculations of the known variables as well as the measured value of doing the lab physically.

|                   | $I_1$    | $I_2$     | $I_3$    | $V_1$   | $V_2$   |
|-------------------|----------|-----------|----------|---------|---------|
| Theoretical Value | 0.0139 A | -0.0122 A | -0.011 A | 5.734 V | 11 V    |
| Measured Value    | 2.71 mA  | 4.34 mA   | 0.52 mA  | 2.723 V | 0.702 V |

**Table 2:**

In Table 2 we remove the the 10 voltage source from the circuit to measure and see the differences of taking it out. Once calculated and measured we get the resulting table.

|                   | $I_1$   | $I_2$   | $I_3$   | $V_1$  | $V_2$  |
|-------------------|---------|---------|---------|--------|--------|
| Theoretical Value | 0.33 mA | 3.28 mA | 3.24 mA | 1.54 V | 3.24 V |
| Measured Value    | 0.51 mA | 2.31 mA | 2.21 mA | 1.45 V | 2.96 V |

**Table 3:**

In this lastly table the 10 voltage source is put back into its original place. From there we remove the 15 voltage source and short circuit the source to the wire. Once everything is connected and put properly we measure the voltages and currents from the circuit to get the resulting table.

|                   | $I_1$   | $I_2$   | $I_3$    | $V_1$   | $V_2$  |
|-------------------|---------|---------|----------|---------|--------|
| Theoretical Value | 0.68 mA | 0.74 mA | 0.52 mA  | 0.34 V  | 0.52 V |
| Measured Value    | 2.2 mA  | 2.04 mA | -1.68 mA | 3.538 V | 0 V    |

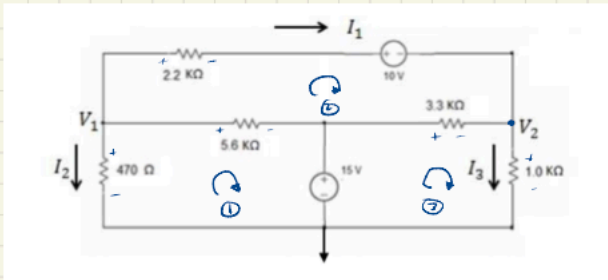
#### 4. Conclusion and Recommendations

Furthermore, as we created our circuit using superposition we were aware where the voltage source and the current source were affecting our circuit. We know that the superpositions are linear systems which we see in our lab that take in more than one independent energy sources. We as well see the different paths being used with different independent energy sources.

The superposition principle is a method that can be used to find voltages of a circuit that has multiple power sources, be it current or voltage sources. When isolating each source of power, and finding the respective voltage that source produces in a circuit, we are able to sum up each individual voltage and find the voltage of the original circuit. This would only work under the certainty of the precondition principles, that of the elements from homogeneity and additivity.

## Theoretical Work:

3.)



$$\begin{aligned} \text{KVL (1): } & -470I_2 + 5.6(I_2 - I_1) + 15 = 0 \\ & -470I_2 + 5.6I_2 - 5.6I_1 + 15 = 0 \\ & 5130I_2 - 5.6kI_1 = -15 \\ & -5.6kI_1 = -15 - 5130I_2 \Rightarrow I_1 = \frac{-15 - 5130I_2}{-5.6k} \end{aligned}$$

$$\begin{aligned} \text{KVL (2): } & -5.6k(I_2 - I_1) + 2.2I_1 + 10 - 3.3k(I_2 - I_3) = 0 \\ & -5.6kI_2 + 5.6kI_1 + 2.2I_1 + 10 - 3.3kI_2 + 3.3kI_3 = 0 \\ & -8.9kI_2 + 7.8kI_1 + 3.3kI_3 = -10 \end{aligned}$$

$$\begin{aligned} \text{KVL (3): } & -15 + 3.3k(I_2 - I_3) + 1kI_3 = 0 \\ & -15 + 3.3kI_2 - 3.3kI_3 + 1kI_3 = 0 \\ & -2.3kI_3 + 3.3kI_2 = 15 \\ & I_2 = \frac{15 + 2.3kI_3}{3.3k} \end{aligned}$$

$$\begin{aligned} \text{123} \quad & -8.9kI_2 + 7.8k\left(\frac{15 + 5130I_2}{5.6k}\right) + 3.3kI_3 = -10 \\ & -8.9kI_2 + 20.8929 + 71453571I_2 + 3.3kI_3 = -10 \\ \text{(4)} \quad & 20.829 - 1754.6429I_2 + 3.3kI_3 = -10 \end{aligned}$$

$$\begin{aligned} \text{324} \quad & 20.829 - 1754.6429\left(\frac{15 + 2.3kI_3}{3.3k}\right) + 3.3kI_3 = -10 \\ & 20.829 - 7.9756 - 1222.9329I_3 + 3.3kI_3 = -10 \end{aligned}$$

$$\begin{aligned} 2077.0671I_3 + 12.8534 &= -10 \\ I_3 &= -0.011 \text{ A} \end{aligned}$$

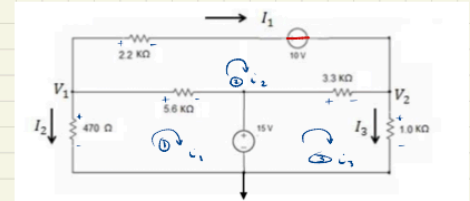
$$I_2 = \frac{15 + 2.3k(-0.011)}{3.3k} = -0.0122 \text{ A}$$

$$I_1 = \frac{-15 - 5130(-0.0122)}{-5.6k} = 0.01386 \text{ A}$$

$$\begin{aligned} V_1 &= 470(0.0122) \\ V_1 &= 5.734 \text{ V} \end{aligned}$$

$$\begin{aligned} V_2 &= 1k(0.011) \\ &= 11 \text{ V} \end{aligned}$$

4.)



$$\begin{aligned} \text{KVL (1): } & -470I_1 + 5.6(I_1 - I_2) + 15 = 0 \\ & -470I_1 + 5.6I_1 - 5.6I_2 + 15 = 0 \\ & 5130I_1 - 5.6kI_2 = -15 \\ & I_1 = \frac{5.6kI_2 - 15}{5130} \end{aligned}$$

$$\begin{aligned} \text{KVL (2): } & -5.6k(I_1 - I_2) + 2.2k(I_2 - I_3) - 3.3k(I_3 - I_2) = 0 \\ & -5.6kI_1 + 5.6kI_2 + 2.2kI_2 - 3.3kI_3 + 3.3kI_2 = 0 \\ & -5.6kI_1 + 11.1kI_2 - 3.3kI_3 = 0 \end{aligned}$$

$$\begin{aligned} \text{KVL (3): } & -15 + 3.3k(I_3 - I_2) + 1kI_3 = 0 \\ & -15 + 3.3kI_3 - 3.3kI_2 + 1kI_3 = 0 \\ & 4.3kI_3 = 3.3kI_2 + 15 \\ & I_3 = \frac{3.3kI_2 + 15}{4.3k} \end{aligned}$$

$$\begin{aligned} \text{122} \quad & -5.6k\left(\frac{5.6kI_2 - 15}{5130}\right) + 11.1kI_2 - 3.3kI_3 = 0 \\ & 6113.0604I_2 + 16.3743 + 11.1kI_2 - 3.3kI_3 = 0 \\ & 163743 + 17213.0604I_2 - 3.3kI_3 = 0 \end{aligned}$$

$$16.3743 - 3.3 \left( \frac{3.3k i_2 + 15}{4.3k} \right) = -17213.0604 i_2$$

$$16.3743 - 2532.5581 i_2 - 11.5116 = -17213.0604 i_2$$

$$4.8627 = -14680.5023 i_2$$

$$i_2 = -0.00033 \text{ A}$$

$$i_1 = I_2$$

$$i_3 = I_3$$

$$i_3 = \frac{3.3k(-0.00033) + 15}{4.3k}$$

$$i_3 = -0.00324 \text{ A}$$

$$i_1 = \frac{5.6k(-0.00033) - 15}{5130}$$

$$i_1 = -0.00328 \text{ A}$$

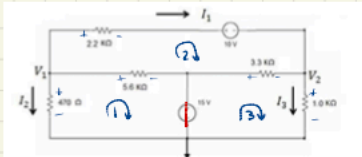
$$V_1 = 470(0.00328)$$

$$V_1 = 1.542$$

$$V_2 = 1k(0.00324)$$

$$V_2 = 3.24 \text{ V}$$

5.)



$$i_1 = I_2, \quad i_3 = I_3$$

$$i_2 = I_1$$

$$\text{KVL(1): } -470i_1 + 5.6k(i_1 - i_2) = 0$$

$$-470i_1 + 5.6ki_1 - 5.6ki_2 = 0$$

$$5130i_1 = 5.6ki_2 \Rightarrow i_1 = \frac{5.6ki_2}{5130}$$

$$\text{KVL(2): } -5.6ki_1 + 5.6ki_2 + 2.2ki_2 + 10 - 3.3k(i_3 - i_2) = 0$$

$$-5.6ki_1 + 5.6ki_2 + 2.2ki_2 + 10 - 3.3ki_3 + 3.3ki_2 = 0$$

$$-5.6ki_1 + 11.1ki_2 - 3.3ki_3 + 10 = 0$$

$$\text{KVL(3): } 3.3k(i_3 - i_2) + 1ki_3 = 0$$

$$3.3ki_3 - 3.3ki_2 + 1ki_3 = 0$$

$$4.3ki_3 = 3.3ki_2 \Rightarrow i_2 = 1.3030i_3$$

$$\underline{182} \quad -5.6k \left( \frac{5.6ki_2}{5130} \right) + 11.1ki_2 - 3.3ki_3 + 10 = 0$$

$$6113.0604i_2 + 11.1ki_2 - 3.3ki_3 + 10 = 0$$

$$17213.0604i_2 - 3.3ki_3 + 10 = 0$$

$$17213.0604(1.3030i_3) - 3.3ki_3 + 10 = 0$$

$$22429.1341i_3 - 3.3ki_3 + 10 = 0$$

$$19129.1341i_3 = -10$$

$$i_3 = -0.00052 \text{ A}$$

$$i_2 = 1.3030(-0.00052)$$

$$i_2 = -0.00068 \text{ A}$$

$$i_1 = \frac{5.6k(-0.00068)}{5130}$$

$$i_1 = -0.00074 \text{ A}$$

$$V_1 = 470(0.00074)$$

$$V_1 = 0.3478 \text{ V}$$

$$V_2 = 1k(0.00052)$$

$$V_2 = 0.52 \text{ V}$$