problem 3 HW#9-3

a) use the linearity property to find the Laplace transform of

$f(t) = A\cos(\beta t) \Rightarrow x(t) = \frac{1}{2}\left[e^{i\beta t}\mu(t) + e^{-i\beta(t)}\mu(t)\right]$

$$X(s) = \frac{s + \cancel{i\beta} + s - \cancel{i\beta}}{2(s^2 - (i\beta)^2)} = \boxed{\left(\frac{s}{s^2+\beta^2}\right)}$$

b) $$\int_0^t V_A e^{-\alpha x}\,dx = \frac{1}{(s+\alpha)s}$$

c) $i(t) = 30e^{-1200t}u(t)\ \text{mA}$ $\qquad v(t) = 0.1\frac{di(t)}{dt}\ [V]$

$$I(t) = 30e^{-1200t}u(t) \Rightarrow I(s) = \frac{30}{(s+1200)}$$

$$v(t) = 0.1\frac{d(t)}{dt} \Rightarrow v(s) = 0.1 \cdot s I(s)$$

$$v(s) = \frac{0.1 s \cdot 30}{s + 1200} = \boxed{\left(\frac{3s}{s+1200}\right)}$$

Problem 4:

a) $$\frac{dv(t)}{dt} + 6v(t) = 4u(t);\ v(0^-) = -3V$$

$$sv(s) - v(0^-) + 6v(s) = \frac{4}{s}$$

$$v(s)\cdot(6+s) + 3 = \frac{4}{s} \Rightarrow v(s) = \frac{\frac{4}{s} - 3}{s+6} \Rightarrow \boxed{\frac{4-3s}{s(s+6)}}$$