

MATH 131: Quiz #5

For what values of the weights w_0 and w_1 the quadrature formula $Q(f) = w_0 f(-1/3) + w_1 f(1/3)$ integrates exactly in the interval $[-1, 1]$ polynomials of degree 1?

$$Q(f) = w_0 f(-1/3) + w_1 f(1/3)$$

$$f(x) = a_0 + a_1 x$$

interval $[-1, 1]$

$$G(f) = \int_{-1}^1 f(x) dx \Rightarrow g(F) = \int_{-1}^1 (a_0 + a_1 x) dx$$

$$3a_0 = w_0 \left(a_0 - \frac{a_1}{3}\right) + w_1 \left(a_0 + \frac{a_1}{3}\right)$$

$$\Rightarrow \left[a_0 + a_1 \left(\frac{x}{3}\right)^2 \right]_{-1}^1$$

$$3a_0 = w_0 \left(w_0 + w_1\right) + a_1 \cdot \left(-\frac{w_0}{3} + \frac{w_1}{3}\right)$$

$$\Rightarrow G(f) = 3a_0 + \frac{a_1}{3}(0)$$

$$w_0 + w_1 = 3 \quad ; \quad -\frac{w_0}{3} + \frac{w_1}{3} = 0$$

$$\Rightarrow G(f) = 3a_0$$

$$-w_0 + w_1 = 0$$

$$w_0 + w_1 = 3$$

$$-w_0 + w_1 = 0$$

$$w_1 = 1$$

$$w_0 = w_1$$

$$w_0 = 1$$