Mode 1 (on-treatment) flow₁:

$$\frac{dx}{dt} = \left(\alpha_x \left(k_1 + \frac{(1 - k_1)z}{z + k_2} \right) - \beta_x \left(k_3 + \frac{(1 - k_3)z}{z + k_4} \right) - m_1 \left(1 - \frac{z}{z_0} \right) \right) x$$

$$\frac{dy}{dt} = m_1 \left(1 - \frac{z}{z_0} \right) x + \left(\alpha_y \left(1 - d \frac{z}{z_0} \right) - \beta_y \right) y$$

$$\frac{dz}{dt} = \frac{-z}{\tau}$$

$$\frac{\partial}{\partial t} = m_1 \left(1 - \frac{1}{z_0} \right) x + \left(\alpha_y \left(1 - d \frac{1}{z_0} \right) - \beta_y \right) y$$

$$\frac{\partial z}{\partial t} = \frac{-z}{\tau}$$

$$\frac{dz}{dt} = \frac{-z}{\tau}$$

$$\frac{dv}{dt} = \left(\alpha_x \left(k_1 + \frac{(1-k_1)z}{z+k_2}\right) - \beta_x \left(k_3 + \frac{(1-k_3)z}{z+k_4}\right) - m_1 \left(1 - \frac{z}{z_0}\right)\right) x$$

$$+ m_1 \left(1 - \frac{z}{z_0}\right) x + \left(\alpha_y \left(1 - d\frac{z}{z_0}\right) - \beta_y\right) y$$

$$\begin{aligned}
\text{jump}_{1\to 2} : & \text{jump}_{2\to 1} : \\
x + y \le r_0 \land \frac{dx}{dt} + \frac{dy}{dt} < 0 & x + y \ge r_1
\end{aligned}$$

$$+ y \le r_0 \wedge \frac{dx}{dt} + \frac{dy}{dt} < 0$$

$$x + y \ge r_1 \wedge \frac{dx}{dt} + \frac{dy}{dt} > 0$$

flow₂: **Mode 2 (off-treatment)**

$$\frac{dx}{dt} = \left(\alpha_x \left(k_1 + \frac{(1-k_1)z}{z+k_2}\right) - \beta_x \left(k_3 + \frac{(1-k_3)z}{z+k_4}\right) - m_1 \left(1 + \frac{(1-k_3)z}{z+k_4}\right) - m_2 \left(1 + \frac{(1-k_3)z}{z+k_4}\right) - m_3 \left(1 + \frac{(1-k_3)z}{z+k_4}\right) - m_4 \left(1 + \frac{(1-k_3)z}{z+k_4}\right) - m_5 \left(1 + \frac{$$

 $+ m_1 \left(1 - \frac{z}{z_0}\right) x + \left(\alpha_y \left(1 - d\frac{z}{z_0}\right) - \beta_y\right) y$

$$\frac{dx}{dt} = \left(\alpha_x \left(k_1 + \frac{(1 - k_1)z}{z + k_2}\right) - \beta_x \left(k_3 + \frac{(1 - k_3)z}{z + k_4}\right) - m_1 \left(1 - \frac{z}{z_0}\right)\right) x$$

$$\frac{dy}{dt} = m_1 \left(1 - \frac{z}{z_0}\right) x + \left(\alpha_y \left(1 - d\frac{z}{z_0}\right) - \beta_y\right) y$$

$$\frac{dz}{dt} = \frac{z_0 - z}{z_0}$$

$$\frac{dy}{dt} = m_1 \left(1 - \frac{z}{z_0} \right) x + \left(\alpha_y \left(1 - d \frac{z}{z_0} \right) - \beta_y \right) y$$

$$\frac{dz}{dt} = \frac{z_0 - z}{\tau}$$

$$\frac{dv}{dt} = \left(\alpha_x \left(k_1 + \frac{(1 - k_1)z}{z + k_2} \right) - \beta_x \left(k_3 + \frac{(1 - k_3)z}{z + k_4} \right) - m_1 \left(1 - \frac{z}{z_0} \right) \right) x$$