泰勒公式

日期: /

$\cos x = 1 - \frac{1}{2}x^2 + o(x^2)$

1. $\lim_{x\to 0}\frac{\sin x}{x} = 1$ $\quad \lim_{x\to 0}\frac{\cos x}{1-\frac{1}{2}x^2} = 1$

(1) e.g. $f(x)=\sin x$

c_0:

$x\to 0$ 时, $R(x) = c_0 + c_1 x$

$f(0) = R(0) \Rightarrow c_0 = 0$

$0 = c_0$

c_1: $f'(0) = \cos x\big|_{x=0} = 1$

$R'(0) = c_1 = 1 \Rightarrow c_1 = 1$

$\therefore R(x) = 0 + 1\cdot x = x$

(2) e.g. $f(x)=\cos x$

$f(0) = \cos 0 = 1$

$\frac{d\cos x}{dx}\big|_{x=0} = -\sin(0) = 0$

$\frac{d^2\cos x}{dx^2}\big|_{x=0} = -\cos(0) = -1$

$\frac{d^3\cos x}{dx^3}\big|_{x=0} = \sin(0) = 0$

$\frac{d^4\cos x}{dx^4}\big|_{x=0} = \cos(0) = 1$

导数值, 切线斜率 $f'(x)$, $f''(x)$, $f'''(x)$

导数信息 → 函数信息

$\frac{df}{dx}$	$\frac{d^2f}{dx^2}$	$\frac{d^3f}{dx^3}$	$\frac{d^4f}{dx^4}$
↑	↑	↑	↑
0	$-\frac{1}{2}$	0	$2\frac{1}{4}$

$R(x) = 1 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \cdots$

$R(0) = c_0 = 1$

$R'(0) = 1\cdot c_1 + 2c_2 x\big|_{x=0} = c_1 = 0$

$R''(0) = 2c_2 = -1 \Rightarrow c_2 = -\frac{1}{2}$

$R'''(0) = 3\times 2\times 1\times c_3 = 0 \Rightarrow c_3 = 0$

$R^{(4)}(0) = 4\times 3\times 2\times 1\times c_4 = 1 \Rightarrow c_4 = \frac{1}{24}$

函数值, 切线斜率 $f'(x)$, $f''(x)$, $f'''(x)$

2. $n!$?

$R(x)$ $\qquad 8!$

$\frac{d^8}{dx^8}(c_8 x^8) = 8\times 7\times 6\times 5\times 4\times 3\times 2\times 1\times x^0\times c_8 = \frac{d^8 f}{dx^8} \Rightarrow c_8 = \frac{1}{8!}\cdot\frac{d^8 f}{dx^8}\big|_{x=0}$

$\frac{d^n}{dx^n}(x^n c_n) = c_n n! x^0 = \frac{d^n f}{dx^n} \Rightarrow c_n = \frac{1}{n!}\frac{d^n f}{dx^n}\big|_{x=0}$

3. $x = x_0$ 处近似

$x=0$: $R(x) = c_0 + c_1 x + c_2 x^2 + \cdots + c_n x^n + \cdots$

$x = x_0$: $R(x) = c_0 + c_1(x-x_0) + c_2(x-x_0)^2 + \cdots + c_n(x-x_0)^n + \cdots$

4. $c_0, c_1, c_2 \cdots c_n$:

$c_1 \to \frac{df}{dx}$ $\quad c_2 \to \frac{d^2f}{dx^2}$ $\quad c_3 \to \frac{d^3f}{dx^3}$ $\cdots$ $\quad c_n \to \frac{d^n f}{dx^n}$

$\frac{dR}{dx} = c_1$ $\quad \frac{d^2R}{dx^2} = 2\cdot c_1$ $\quad \frac{d^3R}{dx^3} = 3\times 2\times 1\times c_3$ $\quad \frac{d^nR}{dx^n} = n!\, c_n$

c_0: $f(x_0)$ $\quad c_1 = \frac{1}{1!}\frac{df}{dx}\big|_{x=x_0}$ $\quad c_2 = \frac{1}{2!}\frac{d^2f}{dx^2}\big|_{x=x_0}$ $\quad c_3 = \frac{1}{3!}\frac{d^3f}{dx^3}\big|_{x=x_0}$

5. $R(x) = f(x_0) + \frac{1}{1!}f'(x_0)(x-x_0) + \frac{1}{2!}f''(x_0)(x-x_0)^2 + \cdots + \frac{1}{n!}f^{(n)}(x_0)(x-x_0)^n$

日期: /

e^x $x=0$ $\frac{d^n(f)}{dx^n}\Big|_{x=0}=1$ $e=2.7\cdots$

$e^x \approx R(x) = 1+\frac{1}{1!}x+\frac{1}{2!}x^2+\frac{1}{3!}x^3+\cdots+\frac{1}{n!}x^n$

$e^1 = R(1) = 1+1+\frac{1}{2!}\times 1+\frac{1}{3!}\times 1+\cdots$

日 期:　　/

日 期:　　/