

关于反三角函数





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## 一. 反三角函数的含义

1. 本质: 三角函数的反函数

2.  $y = \sin x \Rightarrow x = \arcsin y$  <sup>→ - 了整体!</sup> 习惯上写成:  $y = \arcsin x$

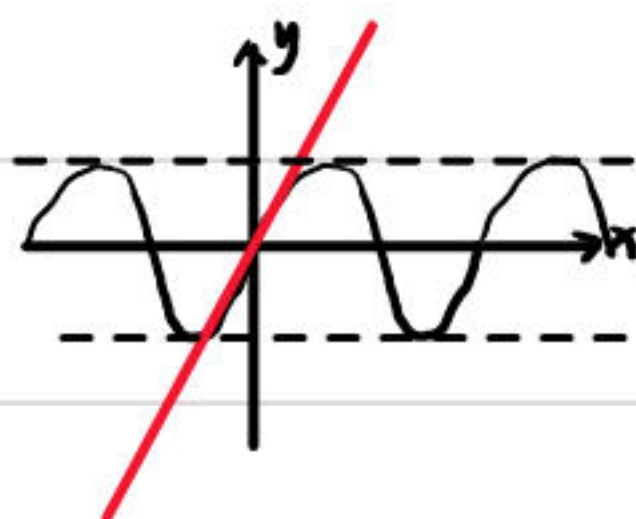
$$y = \cos x \Rightarrow y = \arccos x$$

例:  $\arcsin \frac{1}{2} = ?$   
即:  $\sin(?) = \frac{1}{2} \Rightarrow ? = \frac{\pi}{6}$

$$y = \tan x \Rightarrow y = \arctan x$$

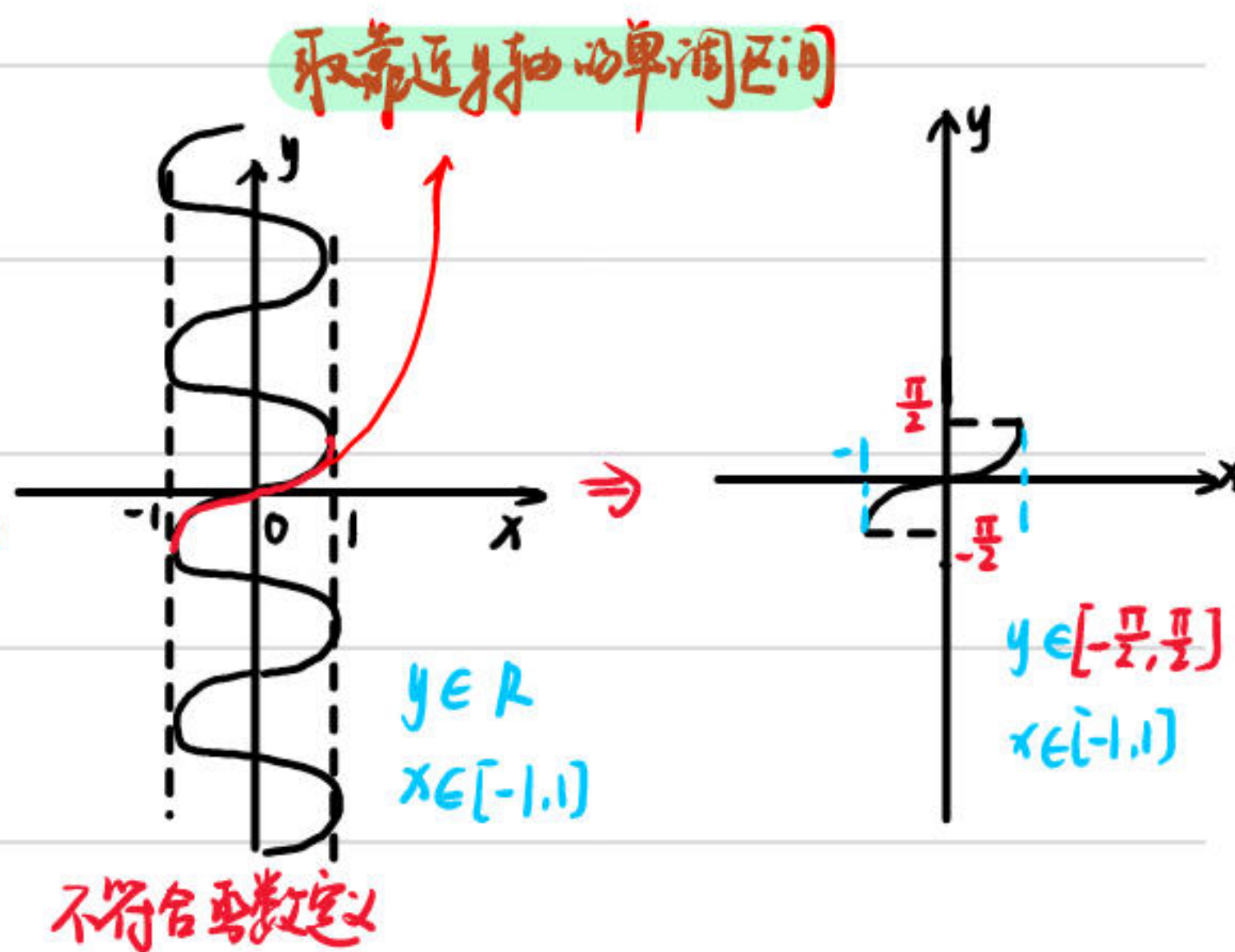
## 二. 反三角函数的图像

1.  $y = \sin x$   
 $\begin{cases} x \in \mathbb{R} \\ y \in [-1, 1] \end{cases}$



$y = \arcsin x$

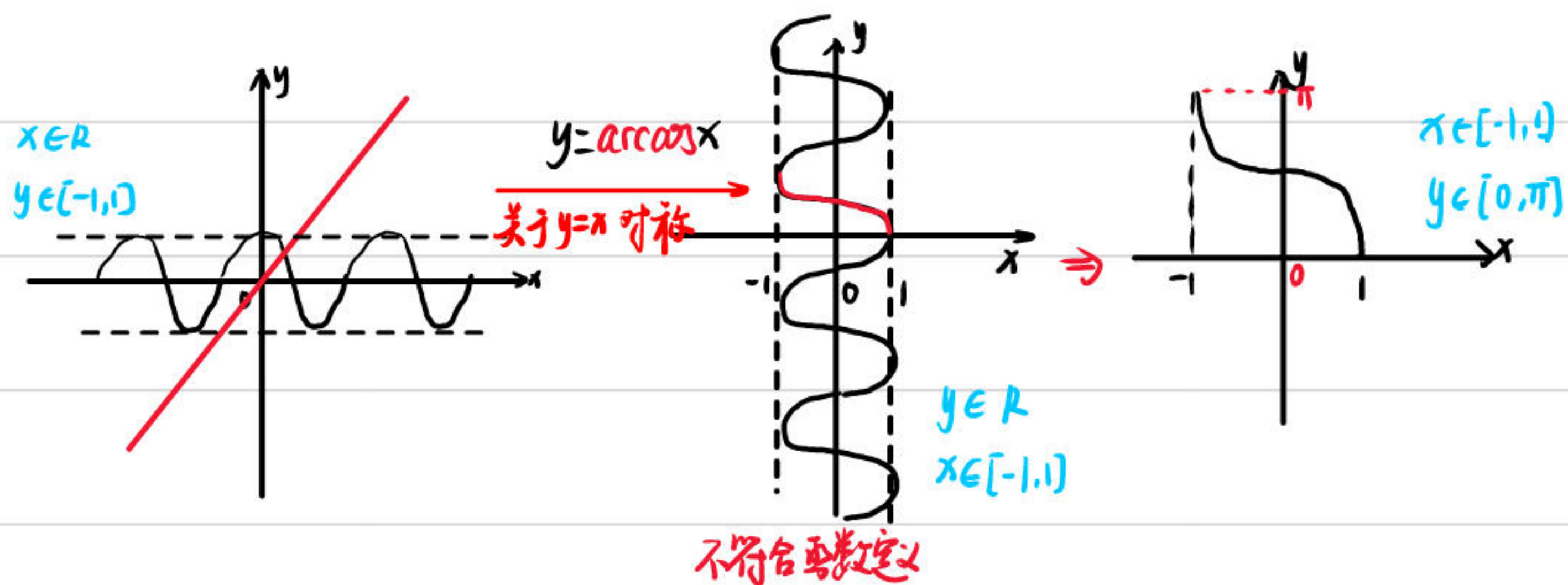
关于  $y = x$  对称



故:  $y = \arcsin x$  值域为  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ , 定义域为  $[-1, 1]$

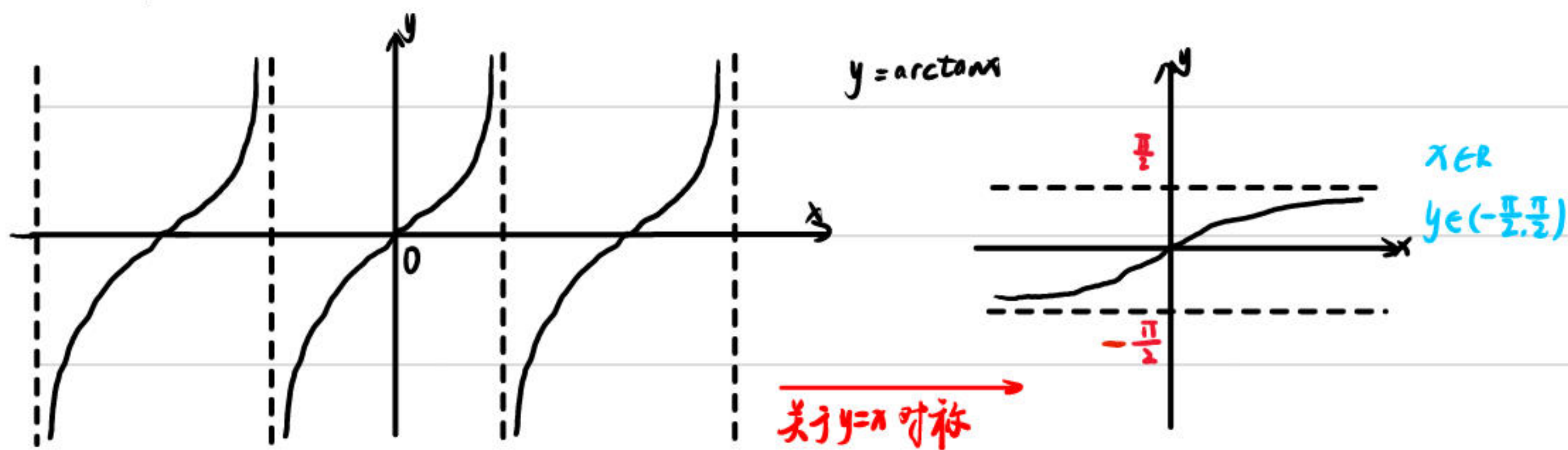
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2.  $\cos x$



$\therefore y = \arccos x$  的值域为  $[0, \pi]$ , 定义域为  $[-1, 1]$

3.  $y = \tan x$

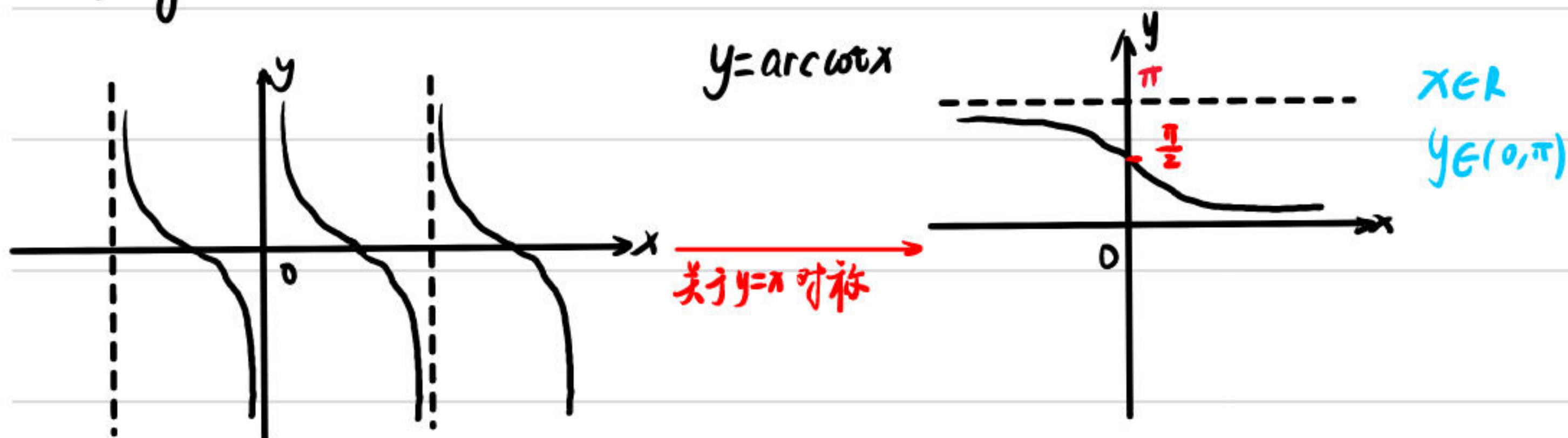


$\therefore \arctan x$  定义域为  $\mathbb{R}$ , 值域为  $(-\frac{\pi}{2}, \frac{\pi}{2})$



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4.  $y = \cot x$



$\therefore y = \operatorname{arccot} x$  值域为  $[0, \pi]$ , 定义域为  $\mathbb{R}$

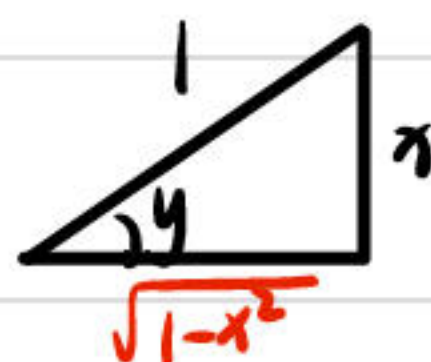
### 三. 反三角函数的求导与积分

(一) 求导

1.  $x = \sin y, y = \arcsin x$ , 求  $(\arcsin x)' = ?$

$$(\arcsin x)' = \frac{1}{(\sin y)'} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}} \quad \star$$

表达式是  $x$  的式子

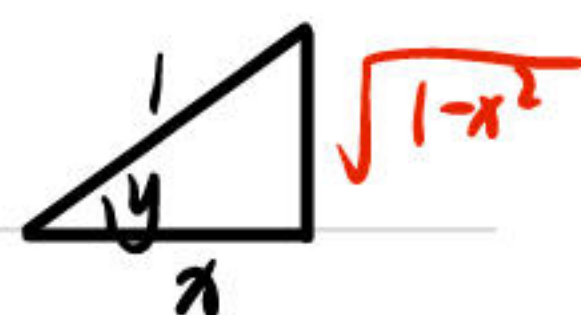


2.  $(\arccos x)' = ?$

$$(\arccos x)' = \frac{1}{(\cos y)'} = -\frac{1}{\sin y} = -\frac{1}{\sqrt{1-x^2}} \quad \star$$

$$y = \arccos x$$

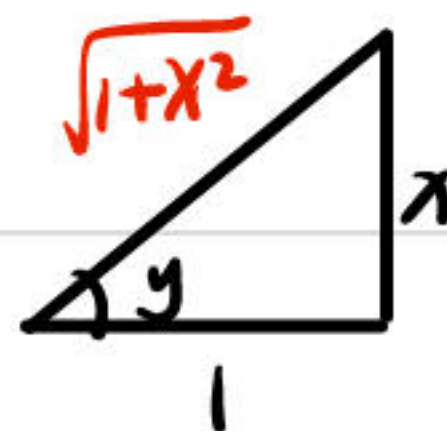
$$\Rightarrow x = \cos y$$



3.  $(\arctan x)' = ?$

$$y = \arctan x \quad (\arctan x)' = \frac{1}{(\tan y)'} = \frac{1}{\frac{1}{\cos^2 y}} = \cos^2 y = \frac{1}{1+x^2} \quad \star$$

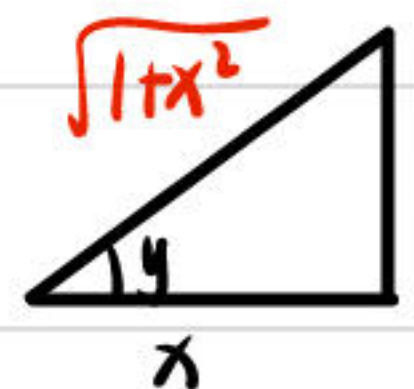
$$x = \tan y$$



4.  $(\operatorname{arccot} x)' = ?$

$$y = \operatorname{arccot} x \quad (\operatorname{arccot} x)' = \frac{1}{(\cot y)'} = \frac{1}{-\frac{1}{\sin^2 y}} = -\sin^2 y = -\frac{1}{1+x^2} \quad \star$$

$$x = \cot y$$



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## (二) 积分 (分部积分!)

1.  $\int \arcsin x \cdot dx = ?$

$$\int \arcsin x \cdot dx = x \cdot \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx = x \arcsin x + \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} d(1-x^2)$$

$$= x \arcsin x + \sqrt{1-x^2} + C$$

2.  $\int \arccos x \cdot dx = ?$

$$\int \arccos x \cdot dx = x \arccos x - \int x d \arccos x = x \arccos x + \int \frac{x}{\sqrt{1-x^2}} dx$$

$$= x \arccos x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} d(1-x^2) = x \arccos x - \sqrt{1-x^2} + C$$

3.  $\int \arctan x \cdot dx = ?$

$$\int \arctan x \cdot dx = x \arctan x - \int x \cdot d(\arctan x) = x \arctan x - \int \frac{x}{1+x^2} dx$$

$$= x \arctan x - \frac{1}{2} \int \frac{1}{1+x^2} d(1+x^2) = x \arctan x - \frac{1}{2} \ln(1+x^2) + C$$

4.  $\int \operatorname{arccot} x \cdot dx = ?$

$$\int \operatorname{arccot} x \cdot dx = x \operatorname{arccot} x + \frac{1}{2} \ln(1+x^2) + C$$



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#### 四. 反三角函数的特殊公式

$$1. \sin(\arcsin x) = x \quad \cos(\arccos x) = x \quad \tan(\arctan x) = x$$

$$2. \arcsin x + \arccos x = \frac{\pi}{2}$$

$$\begin{aligned} \text{证明: } \sin(\arcsin x + \arccos x) &= \sin(\arcsin x) \cos(\arccos x) + \cos(\arcsin x) \sin(\arccos x) \\ &= x^2 + (\sqrt{1-x^2})^2 = 1 \end{aligned}$$

$$\text{而 } \arcsin x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

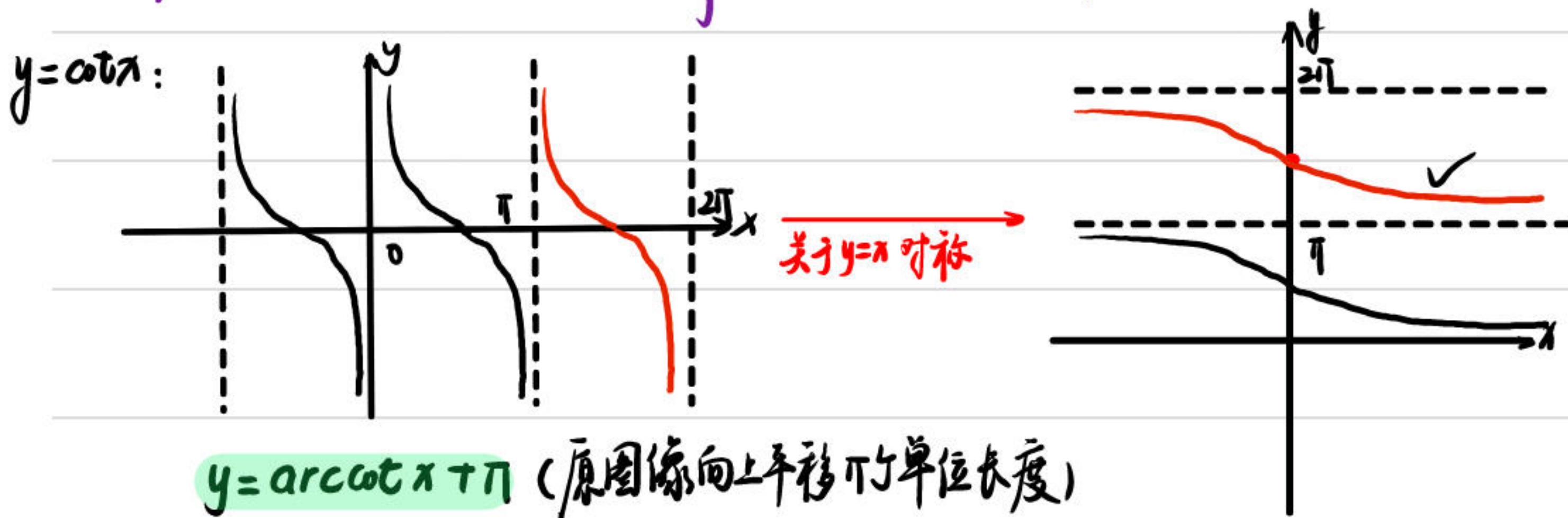
$$\arccos x \in [0, \pi]$$

$$\therefore \arcsin x + \arccos x \in \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$$

$$\therefore \arcsin x + \arccos x = \frac{\pi}{2}$$

#### 五. 题型: 在不同区间上构造反三角函数

例: (2020·上海交大期中考试题)  $f(x) = \cot x$  ( $\pi < x < 2\pi$ ) 的反函数是



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