Automatic Change-Point Detection

Using Piecewise Support Vector Quantile Regression

Authors: Jing-Rung Yu, Sheng-Yu Tsai, Chun-Yen Tsai, Donald Lien

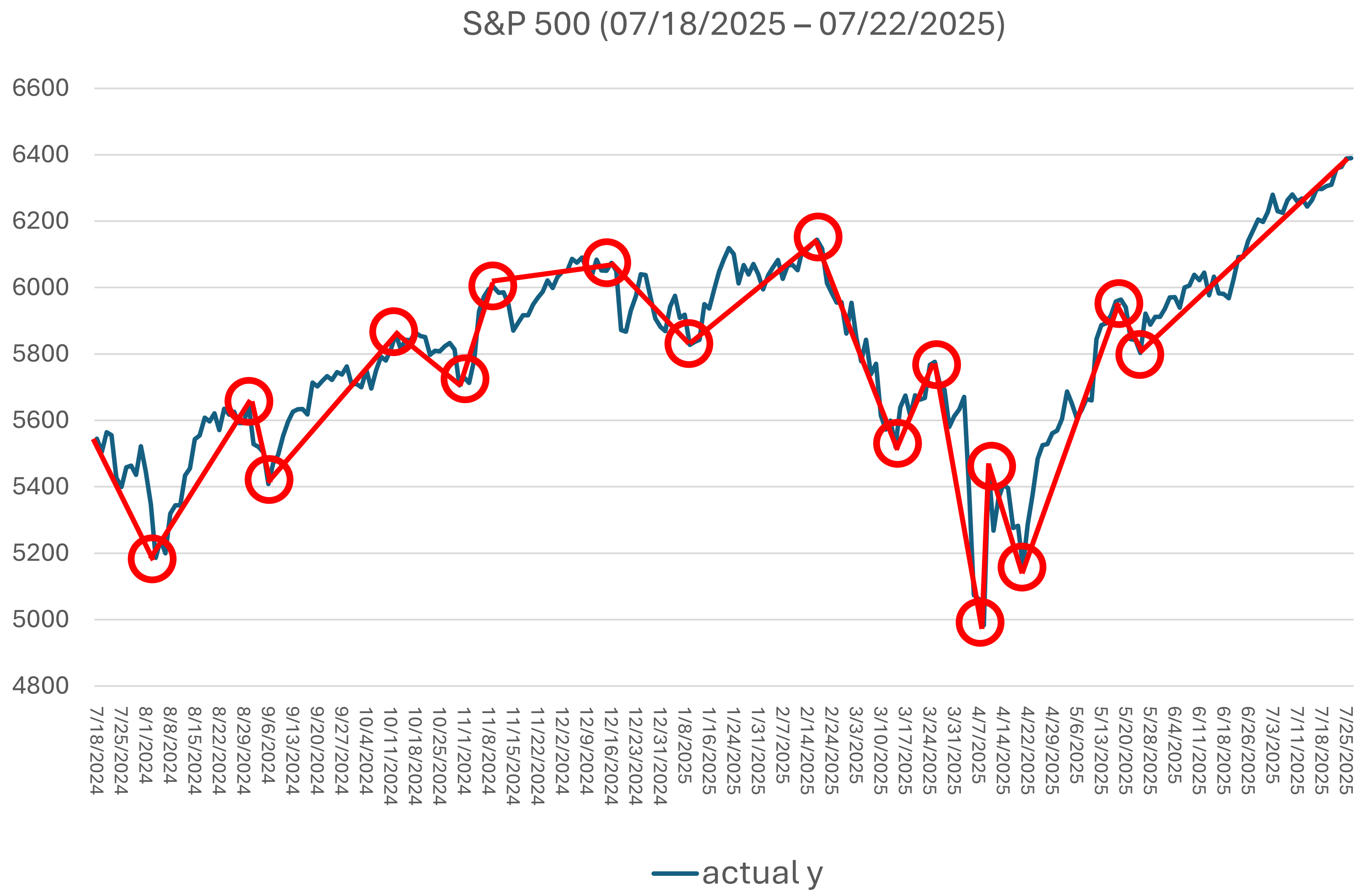
Speaker: Sheng-Yu Tsai (Justin)

National Chi Nan University, Taiwan

Outline

- Motivations
- Framework
- Preliminary Experiments
- Conclusions

Pattern Change Detection (PCD)



PCD Statistical Methods (Gupta, Wadhvani, & Rasool, 2024)

1. Statistical Methods	CUSUM, PLR
2. Bayesian Methods	BCPA, BOCD
3. Time-Frequency Methods	WCPD、SDCPD
4. Machine Learning Methods	OC-SVM、LSTM
5. Hybrid Methods	LSCUSUM、Dual-LSTM

PCD Statistical Methods (Gupta, Wadhvani, & Rasool, 2024)

1. Statistical Methods
2. Bayesian Methods
3. Time-Frequency Methods
4. Machine Learning Methods
5. Hybrid Methods

The performance depends on the characteristics of datasets

PCD Statistical Methods (Gupta, Wadhvani, & Rasool, 2024)

1. Statistical Methods
2. Bayesian Methods
3. Time-Frequency Methods
4. Machine Learning Methods
5. Hybrid Methods ⟵ **Often outperform others!**

Key Challenges (Gupta, Wadhvani, & Rasool, 2024)

Detecting multiple change-points

Handling high-dimensional or missing data

Processing large datasets efficiently

Note: Prediction bands can provide valuable, actionable information

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Prediction bands

PSVQR Framework

1

2

3

Support Vector Regression

Piecewise Linear Model

Quantile Loss Function

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kernel free

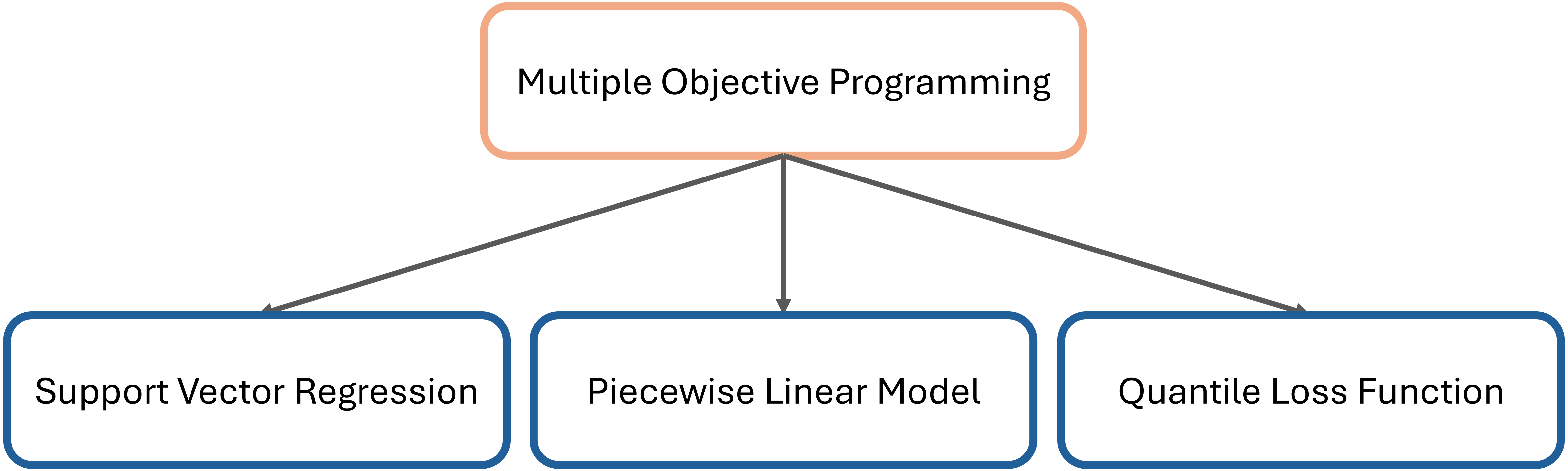
Piecewise regression

Detect multiple CP

Different quantile tau

Provide prediction intervals

PSVQR Framework



Multiple Objective Programming

Support Vector Regression

Piecewise Linear Model

Quantile Loss Function

Key Features

1. Robust to Noise or Outliers
2. Multiple Change Point Detection
3. Interval Prediction

The Proposed Model

l1-norm *Quantile loss* *Change Point*

$$\min_{w_j^+, w_j^-, \xi, \rho} \boxed{\sum_{j=1}^{n} (w_j^+ + w_j^-)} + \boxed{C \sum_{i=1}^{m} \left((1-\tau)\xi_i^- + \tau \xi_i^+\right)} + \boxed{C_2 \sum_{j=1}^{n} \rho_j}$$

Derived from SVR

$$s.t. \quad y_i - \left(a + \sum_{j=1}^{n} (w_j^+ - w_j^-) x_{ij} + \boxed{\sum_{j=1}^{n} \sum_{t=1}^{m} (b_{jt}^+ - b_{jt}^-)\left(|x_{ij} - p_{jt}| + x_{ij} - p_{jt}\right)/2}\right) \leq \varepsilon + \xi_i^+$$

Piecewise Terms

$$\left(a + \sum_{j=1}^{n} (w_j^+ - w_j^-) x_{ij} + \boxed{\sum_{j=1}^{n} \sum_{t=1}^{m} (b_{jt}^+ - b_{jt}^-)\left(|x_{ij} - p_{jt}| + x_{ij} - p_{jt}\right)/2}\right) - y_i \leq \varepsilon + \xi_i^-$$

w_j: the hyperplane weight vector

m: the size of sliding window

ε : the ε-insensitive tube width

C_1, C_2: the penalties

a : intercept term

ρ_j: numbers of change points of x_j

b_{jt}: coefficient of change p_{jt}

ξ_i : residuals outside the ε-insensitive tube

Experiment Design

Dataset: S&P 500 From 2024/07/18 to 2025/07/22

PSVQR-f1: use 1 feature (index)

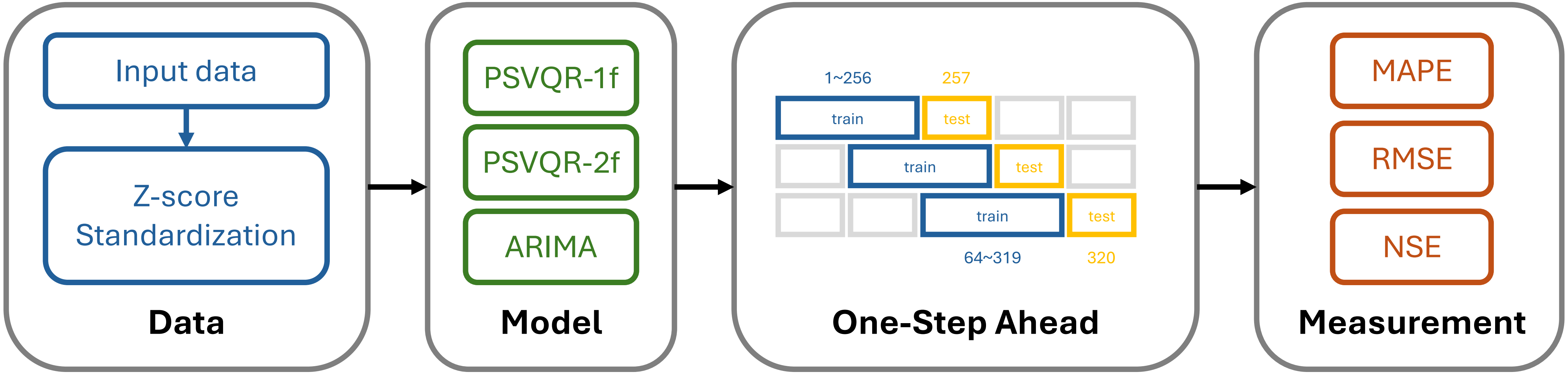
PSVQR-f2: use 2 features (index, VIX)

Prediction y:

- price

Features:

- Sequential Index
- VIX (Volatility Index)



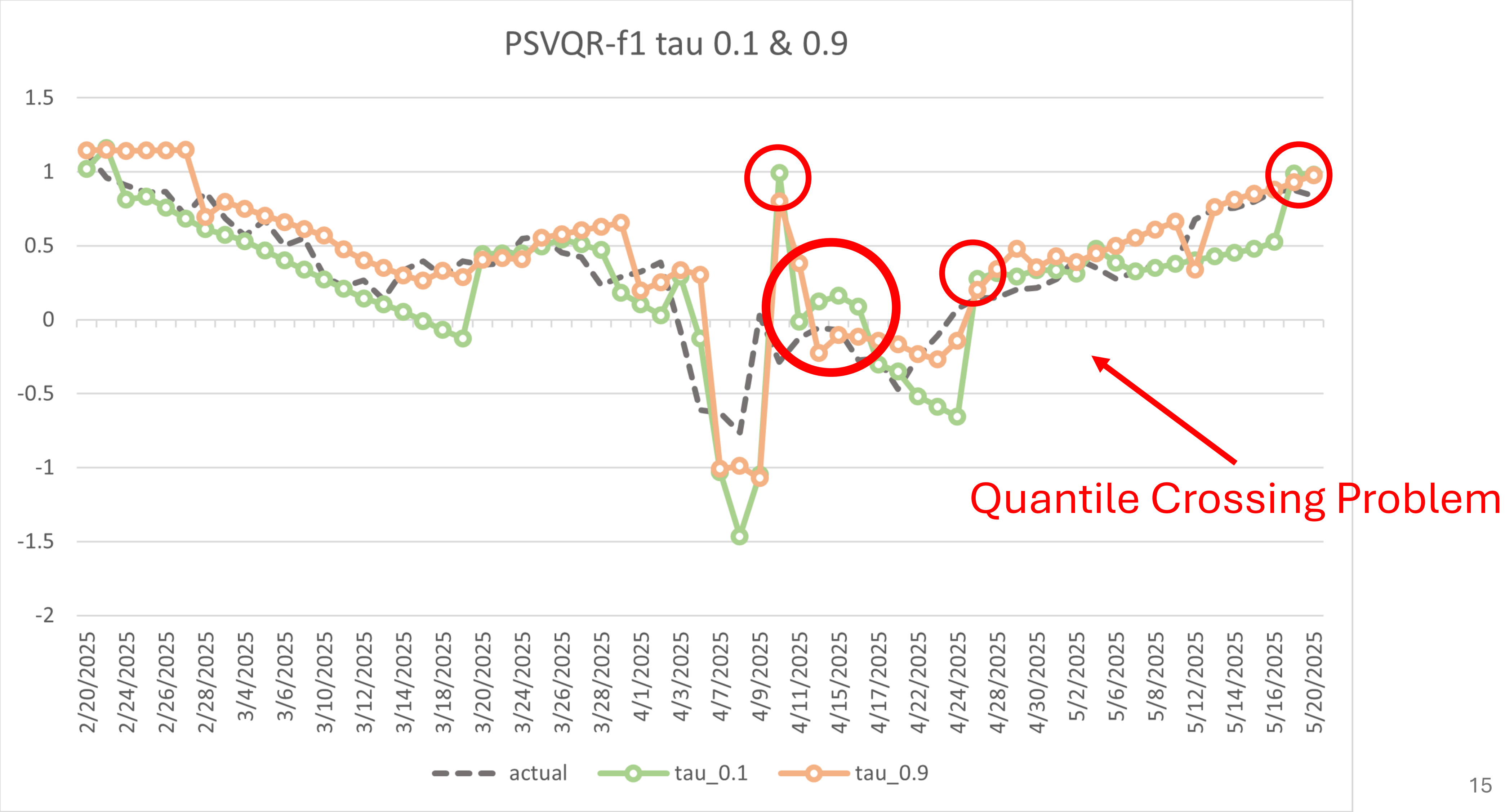
Prediction Results (Using Index)

Model	MAPE	RMSE	NSE
ARIMA	0.6023	0.1815	0.8093
PSVQR-1f, tau=0.1	1.9936	0.3265	0.5620
PSVQR-1f, tau=0.3	0.8064	0.2530	0.7098
PSVQR-1f, tau=0.5	1.7960	0.2454	0.7255
PSVQR-1f, tau=0.7	0.4981	0.2524	0.7300
PSVQR-1f, tau=0.9	0.4769	0.3039	0.8183

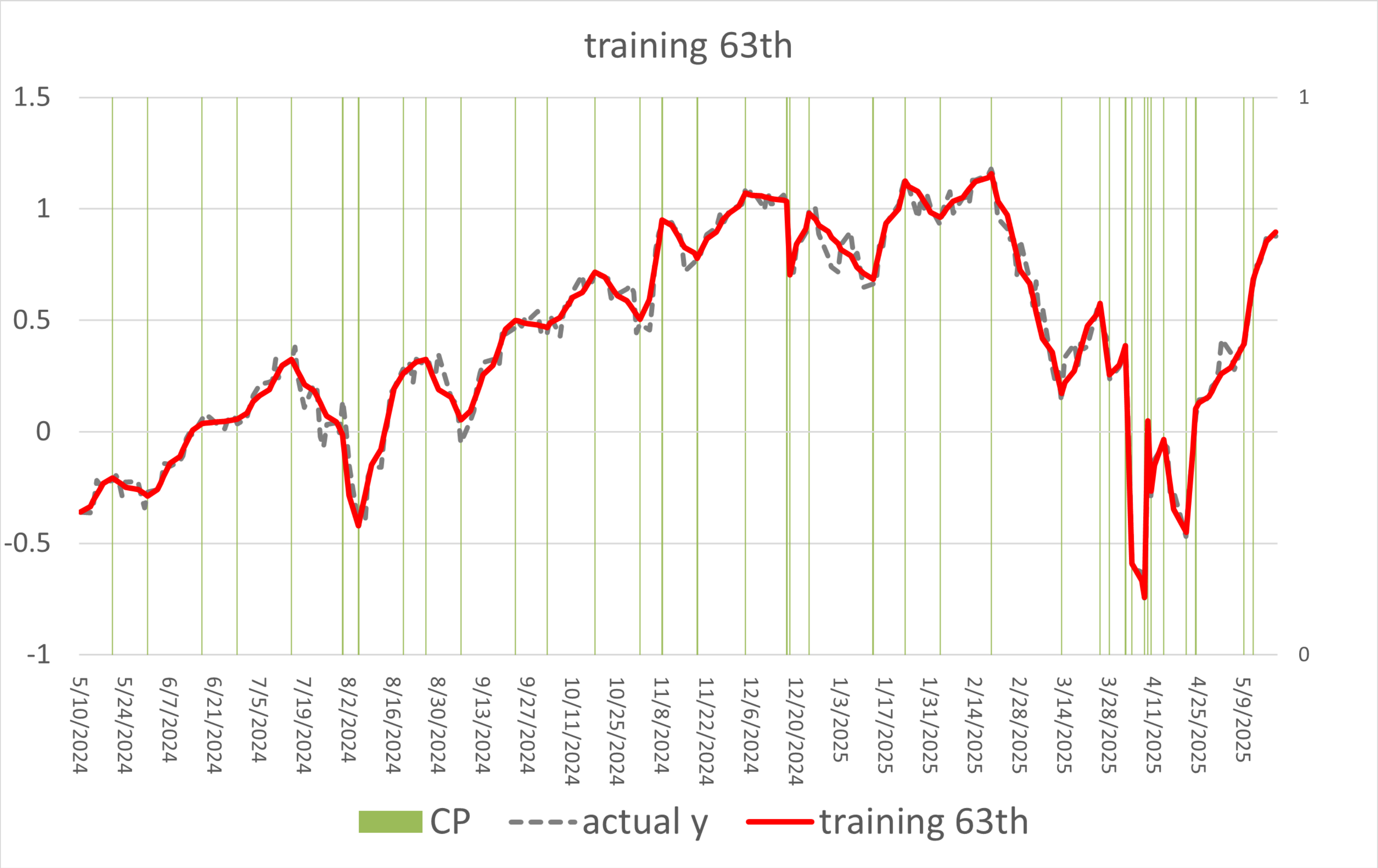
Features:
- sequential index

Prediction y:
- price

Prediction Intervals (PSVQR-f1 tau=0.1, 0.9)



CPD Results (PSVQR-f1 tau=0.5)



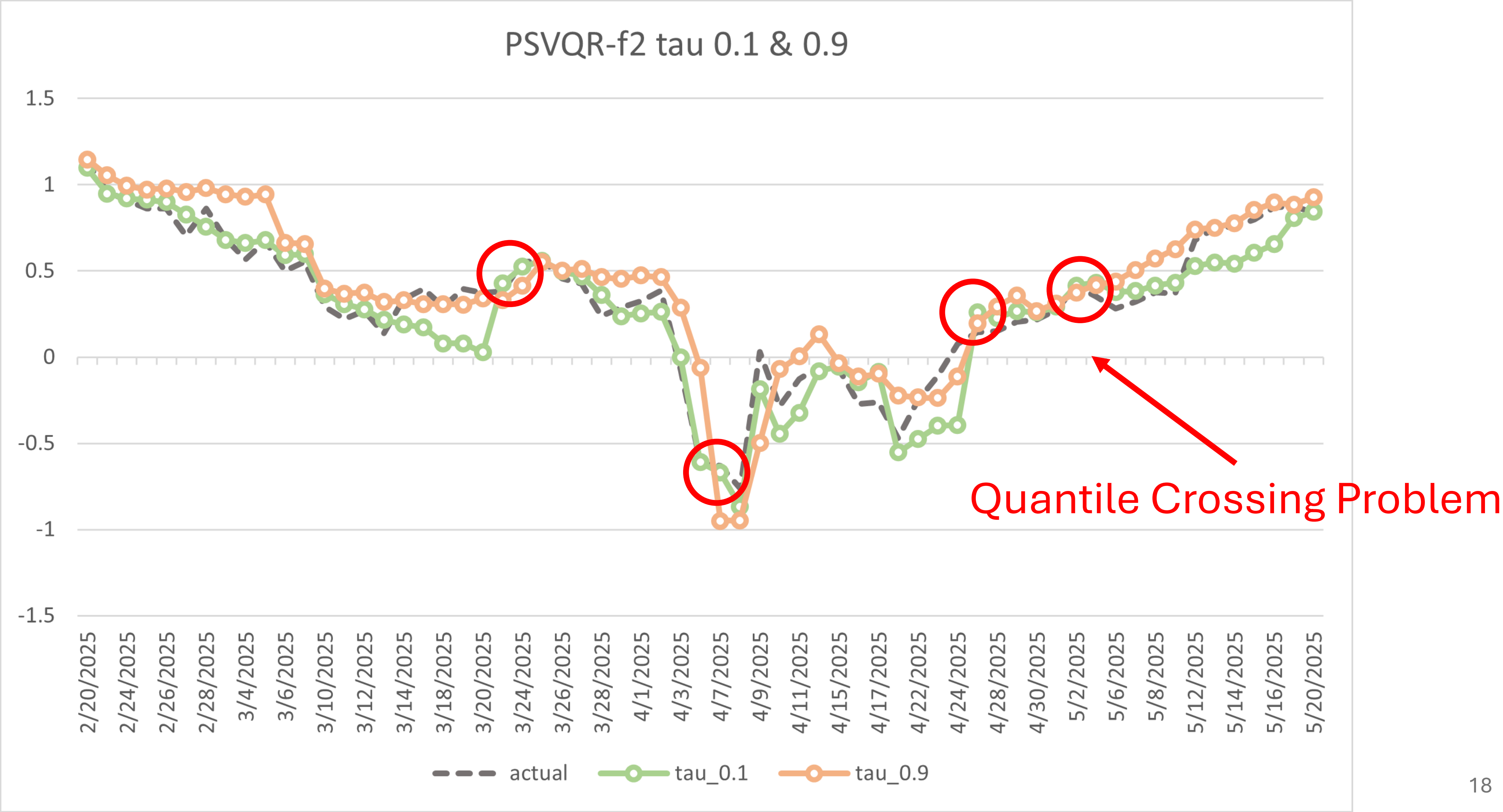
Prediction Results (Using Index and VIX)

Model	MAPE	RMSE	NSE
ARIMA	0.6023	0.1815	0.8093
PSVQR-2f, tau=0.1	0.9996	0.1399	0.8981
PSVQR-2f, tau=0.3	0.3770	0.1419	0.8994
PSVQR-2f, tau=0.5	0.9267	0.1520	0.8786
PSVQR-2f, tau=0.7	0.3982	0.1549	0.8667
PSVQR-2f, tau=0.9	1.059	0.1786	0.8450

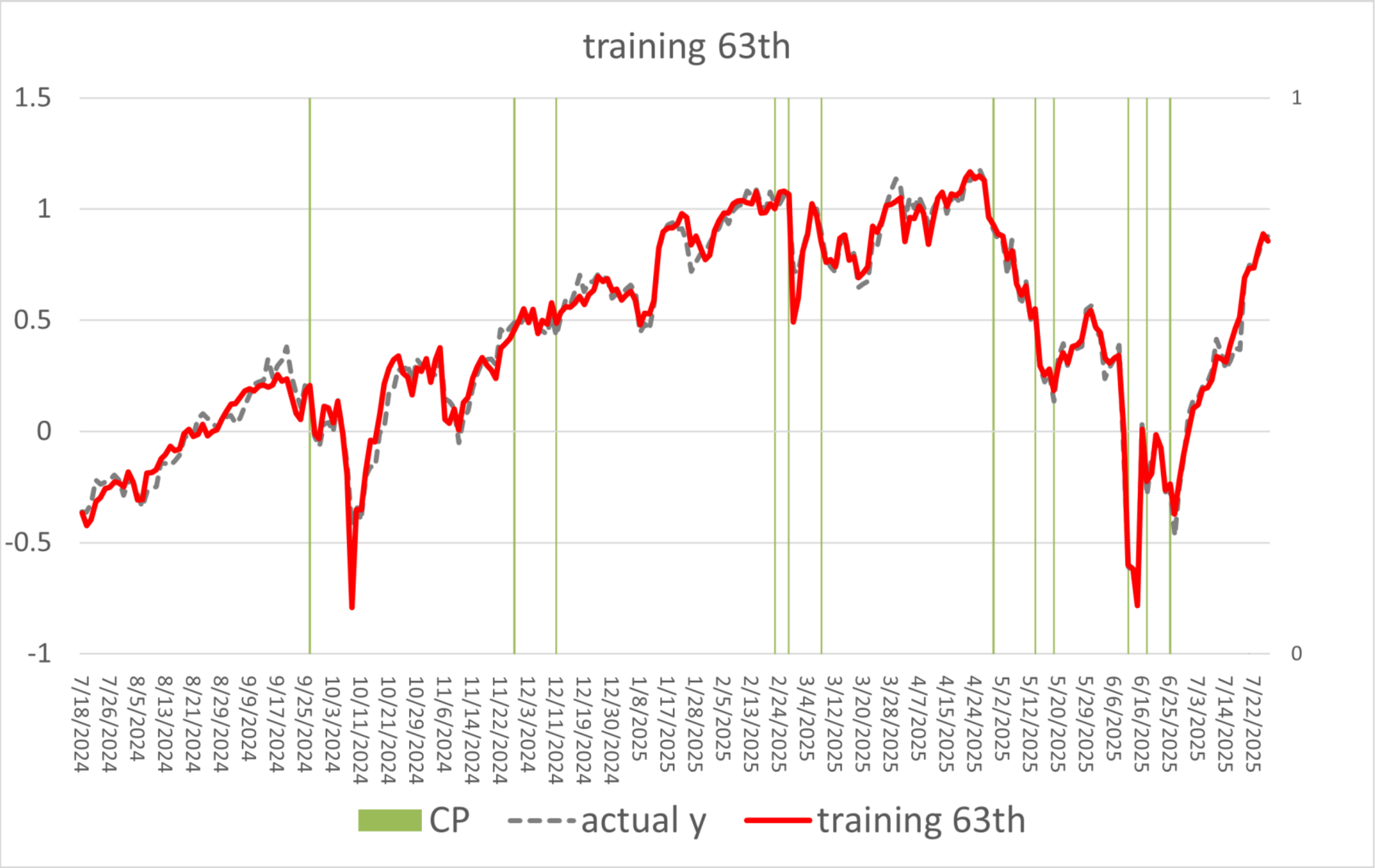
Features:
- sequential index
- VIX (Volatility Index)

Prediction y:
- price

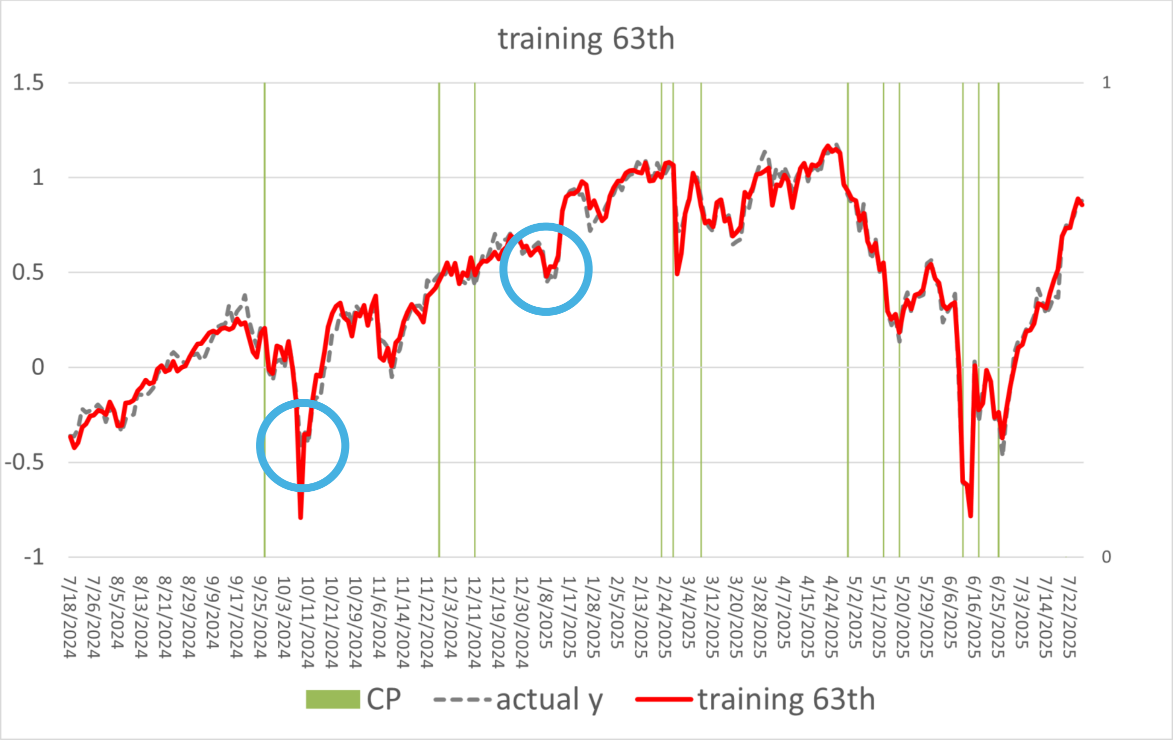
Prediction Intervals (PSVQR-f2 tau=0.1, 0.9)

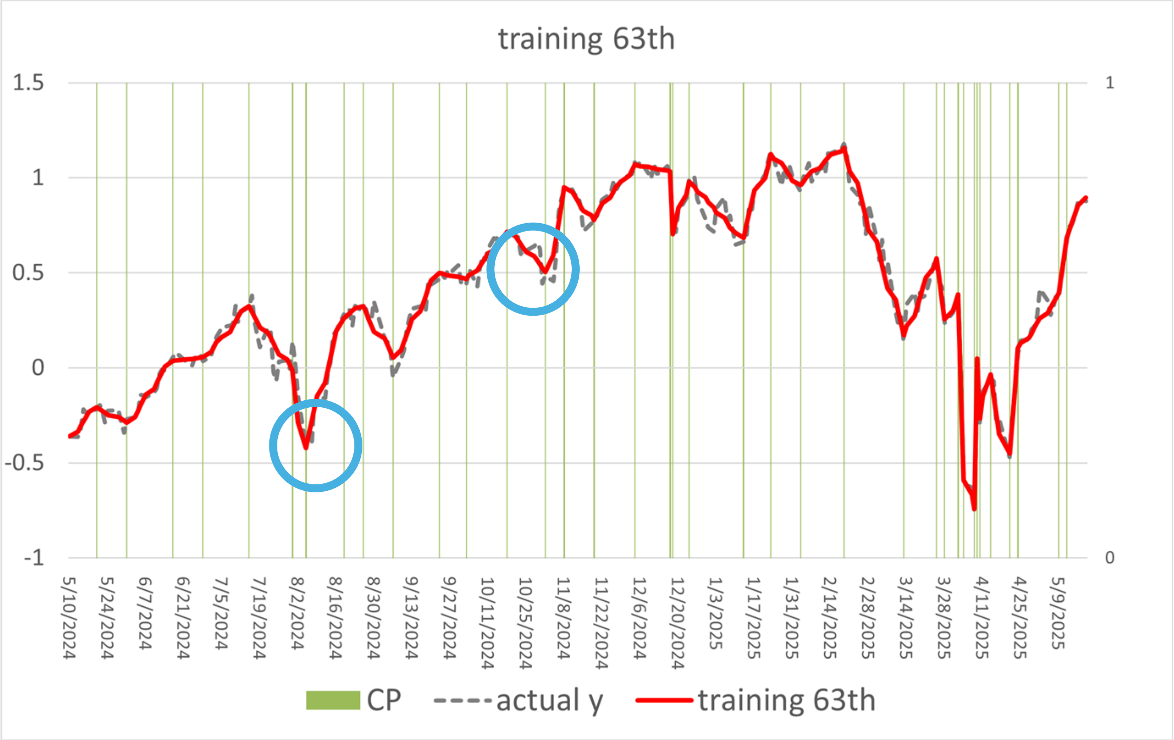


CPD Results (PSVQR-f2 tau=0.5)



Experiment Results (PSVQR-f2 vs PSVQR-f1)

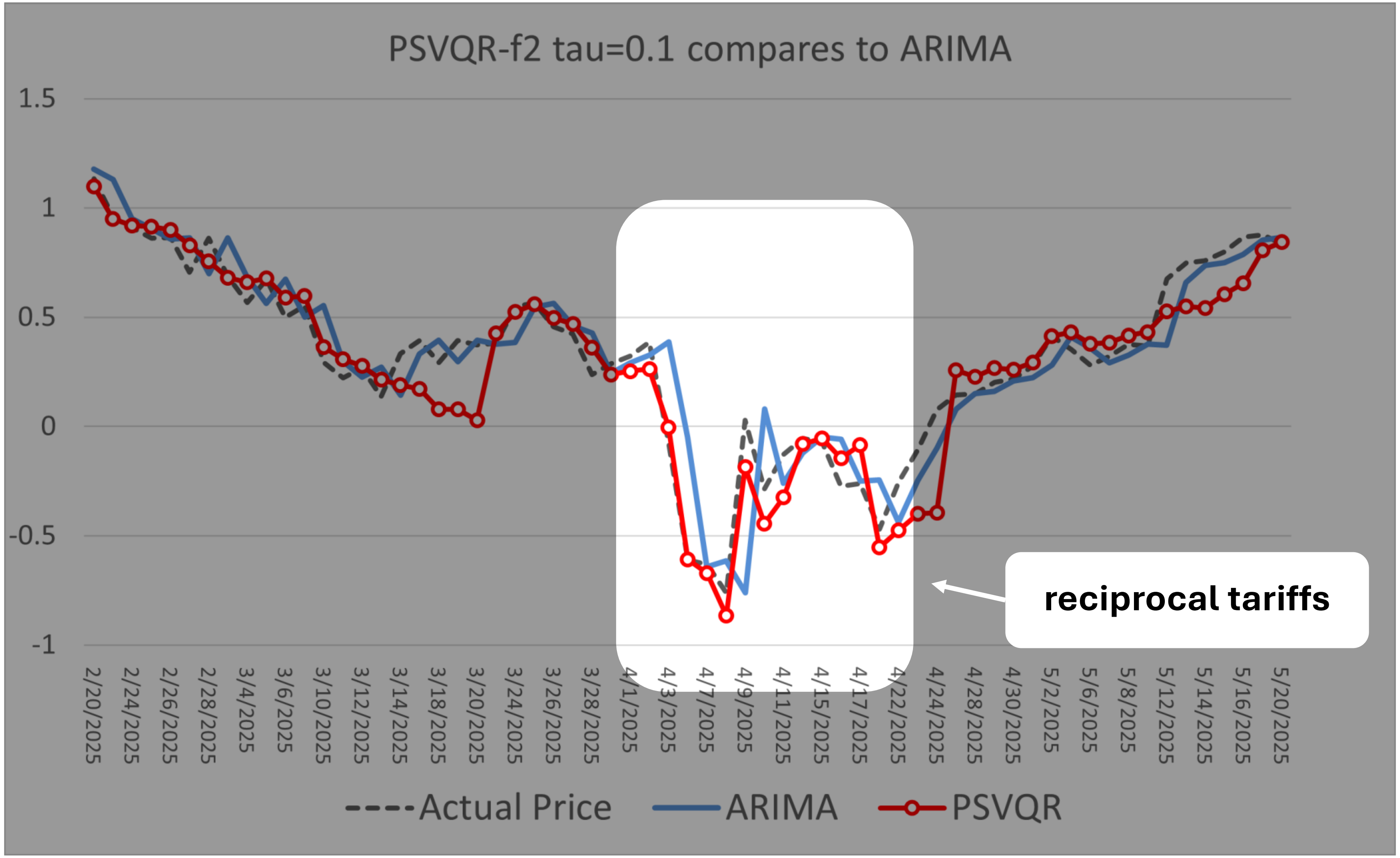




Use 2 features

Use 1 features

Compare to Benchmark



Conclusion

- PSVQR-f2 outperforms ARIMA model in S&P 500 predictions.
- PSVQR-f1 is better for CPD, while PSVQR-f2 is better for prediction.
- PSVQR has crossing quantile problem.

Future Work

- Find a proper method to decide the penalty of CP amounts.
- Solve the quantile crossing problem (e.g., Dai, Kuosmanen, & Zhou, 2023).
- Transfer CPU-based solver to GPU solutions.
- Add more features and compare it with other PCD methods on different datasets.

References

- Gupta, M., Wadhvani, R., & Rasool, A. (2024). Comprehensive analysis of change-point dynamics detection in time series data: A review. Expert Systems with Applications, 123342.
- Bassett Jr, G., & Koenker, R. (1982). An empirical quantile function for linear models with iid errors. *Journal of the American Statistical Association*, *77*(378), 407-415.
- Dai, S., Kuosmanen, T., & Zhou, X. (2023). Non-crossing convex quantile regression. *Economics letters*, *233*, 111396.

Thanks!

Contact:

Sheng-Yu Tsai (Justin Tsai)

justintsai77@gmail.com

六、 相關文獻參考 (Supporting Literature)

以下是一些關於「分位數交叉」問題的經典及相關文獻，可以在您報告的 Q&A 環節或未來研究中提供支持。

1.Koenker, R. (2005). *Quantile Regression*. Cambridge University Press.

1. **重要性：** 這是分位數迴歸領域最權威的「聖經」。書中詳細討論了分位數交叉的現象及其理論基礎。引用這本書代表您對該領域有著根本性的理解。

2.Bondell, H. D., Reich, B. J., & Wang, H. (2010). Noncrossing quantile regression curve estimation. *Biometrika*, 97(4), 825-838.

1. **重要性：** 這是一篇處理非交叉問題的關鍵論文。它提出了一種在模型估計過程中就加入限制的方法，來確保不同分位數的預測線不會交叉。這篇是您提出「未來工作」方向時最有力的參考。

3.Takeuchi, I., & Furuhashi, T. (2005). Non-crossing quantile regression with support vector machines. *2005 SICE Annual Conference*.

1. **重要性：** 這篇論文與您的研究方法**高度相關**。它專門探討如何在**支持向量機 (SVM)** 的框架下解決分位數交叉問題。在 Q&A 時若被問到您的模型該如何改進，引用這篇文獻會非常精準且具說服力。

祝您報告順利！