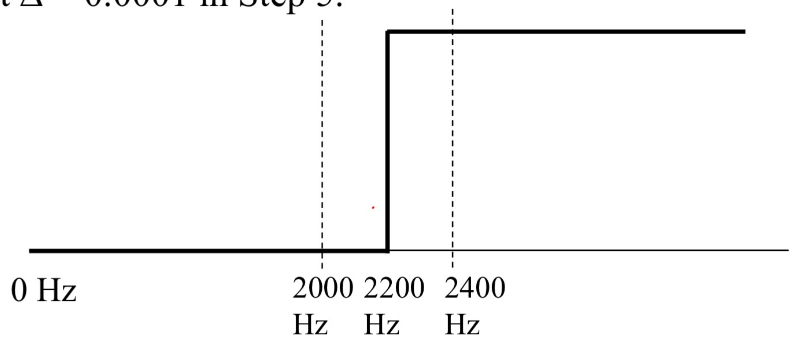
(1) Design a Mini-max **highpass** FIR filter such that (40 scores)

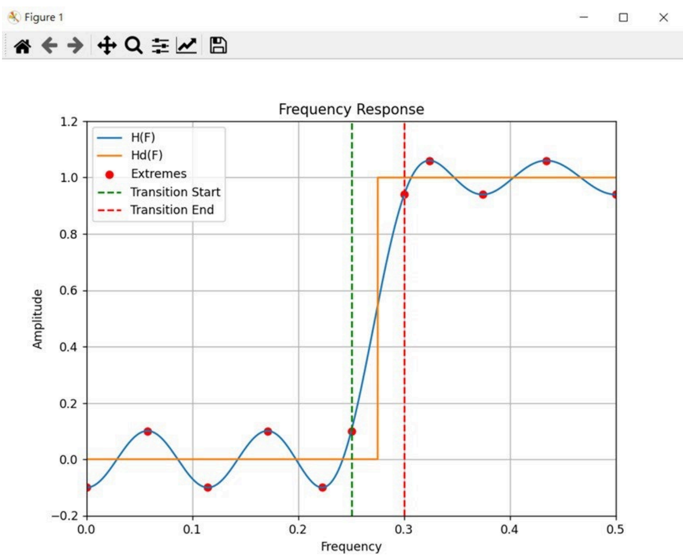
① Filter length = 19, ② Sampling frequency $f_s = 8000$Hz,

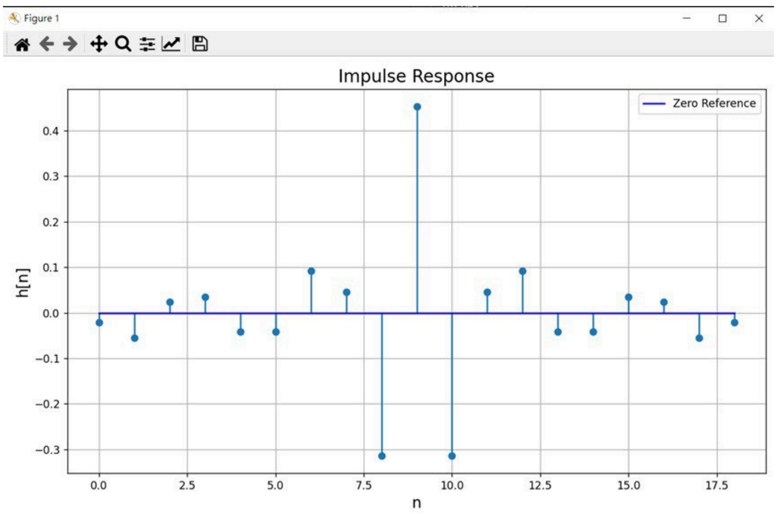
③ Stop Band 0~2000Hz ④ Transition band: 2000~2400 Hz,

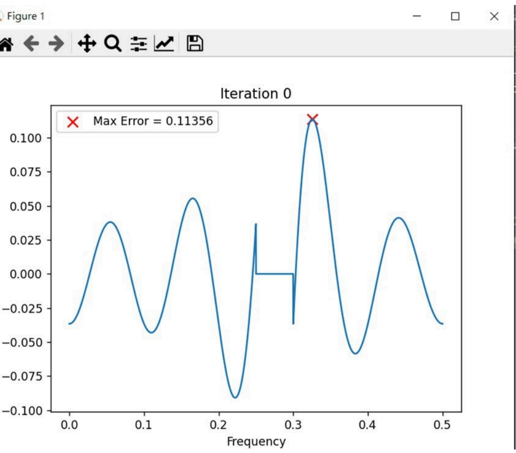
⑤ Weighting function: $W(F) = 1$ for passband, $W(F) = 0.6$ for stop band .

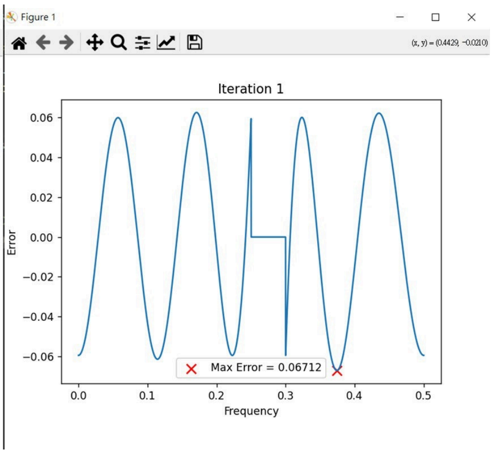
⑥ Set $\Delta = 0.0001$ in Step 5.

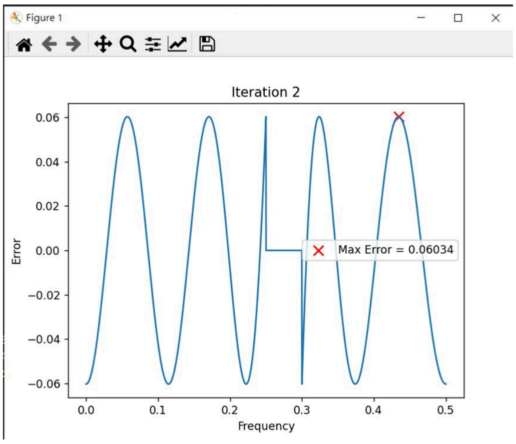


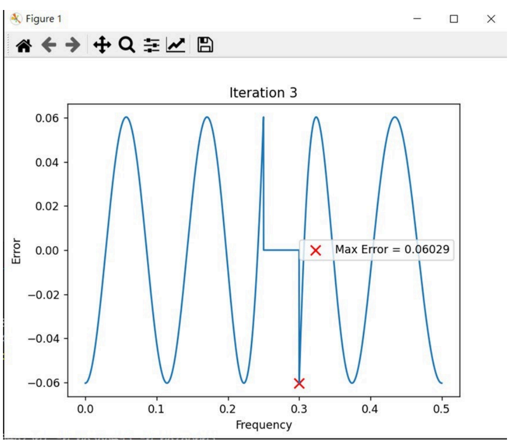










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Iteration 0 max_err is : 0.11356
Iteration 1 max_err is : 0.06712
Iteration 2 max_err is : 0.06034
Iteration 3 max_err is : 0.06029
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```python
if __name__ == "__main__":
    filter_design = MiniMaxFIRFilter(N=19, fs=8000, delta=0.0001,cutoff=2200,f1=2000,f2=2400)
    filter_coefficients = filter_design.design_filter()
    print(filter_coefficients)
```

(2) (a) How do we convert convolution into an addition operation?

(b) What are two main advantages of the FT in engineering?

(c) From the view point of implementation, what are the disadvantages of the discrete Fourier transform? (15 scores)

(a) $y[n] = x[n] * h[n] \xrightarrow{F.T} Y(f) = X(f)H(f)$

$\xrightarrow{\log} \log(Y(f)) = \log(X(f)) + \log(H(f))$

Using logarithm to convert convolution into addition.

(b) 1. spectrum analysis.

2. It can tranfer convolution into multiplication.

eg. $y(t) = x(t) * h(t) \xrightarrow{F.T.} Y(f) = X(f)H(f)$

$= \int x(t-\tau)h(\tau)d\tau$

(c) 1. 包含複數虛部運算

2. 無理數

⇒ 運算複雜度高，實現複雜，也較耗能。

$e^{j\theta} = \cos\theta + j\sin\theta$

$DFT(x[x]) = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi mn}{N}}$

⇒ 可使用 Walsh Transform 改善

(3) Discuss how to implement $y[n] = x[n] * h[n]$ efficiently where

$h[n] = (0.7^n + (-0.6)^{n+1})u[n]$, $u[n]$: unit step function (10 scores) $|z|>|a|$

$y[n] = x[n] * (0.7^n u[n] + (-0.6)^{n+1} u[n])$

Z.T. $Y(z) = X(z)H(z)$

$= X(z)\left[\frac{1}{1-0.7z^{-1}} + \frac{-0.6}{1-(-0.6)z^{-1}}\right]$

$= X(z)\frac{0.4+1.02z^{-1}}{(1-0.7z^{-1})(1-(-0.6)z^{-1})}$

$\Rightarrow Y(z)(1-0.7z^{-1})(1-(-0.6)z^{-1}) = X(z)(0.4+1.02z^{-1})$

z^{-1} $y[n] - 0.1y[n-1] - 0.42y[n-2] = 0.4x[n] + 1.02x[n-1]$

$y[n] = 0.4x[n] - 1.02x[n-1] + 0.1y[n-1] + 0.42y[n-2]$ #

Pf. $Z\{a^n u[n]\}$

$= \sum_{n=0}^{\infty} a^n z^{-n} = \sum_{n=0}^{\infty}\left(\frac{a}{z}\right)^n$

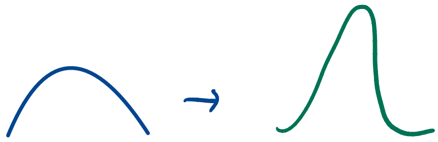
$= \frac{1}{1-az^{-1}}$

(4) Why (a) the step invariance method and (b) the bilinear transform can reduce or avoid the aliasing effect in IIR filter design? (10 scores)

(a) － 開始就用 step function 對 input 做 convolution，

$$h_{u,t}(t) = h_a(t) * u(t) = \int_{-\infty}^{\infty} h_a(\tau)u(t-\tau)d\tau = \int_{-\infty}^{t} h_a(\tau)d\tau$$

$$\Rightarrow H_{a,u}(f) = \frac{H_a(f)}{j2\pi f} \Rightarrow \text{推論}$$



$f \downarrow \; H_{a,u}(f) \uparrow$

$f \uparrow \; H_{a,u}(f) \downarrow$

⇒ 壓低高頻，aliasing effect 降低 #

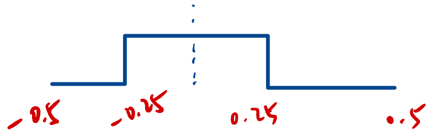
(b) 把 $(-\infty, \infty)$ 無限長的 filter mapping 到 $(-\frac{f_s}{2}, \frac{f_s}{2})$

如此一來，$f_{new} = \frac{f_s}{\pi} \operatorname{atan}(\frac{2\pi}{c} f_{old})$ ⇒ 可消除 aliasing effect。#

(5) Design the 7-point FIR filter in the MSE sense where the ideal filter is

$H_d(F) = 1$ for $|F| < 0.25$, $H_d(F) = 0$ for $0.25 < |F| < 0.5$ (10 scores)

(Cont.)



$$s[0] = \int_{-0.5}^{0.5} H_d(F)\,dF = \int_{-0.25}^{0.25} dF = 0.5$$

$$s[n] = 2\int_{-0.5}^{0.5} \cos(2\pi nF) H_d(F)\,dF$$

$$= 2\int_{-0.25}^{0.25} \cos(2\pi nF)\,dF$$

$$= \frac{1}{\pi n}\left(\sin(2\pi nF)\Big|_{-0.25}^{0.25}\right)$$

$$= \frac{2}{\pi n}\sin\left(\frac{\pi n}{2}\right)$$

$s[1] \approx 0.6366$

$s[2] \approx 0$

$s[3] \approx -0.2122$

$h[0] = s[3]/2 = -0.1061$

$h[1] = s[2]/2 = 0$

$h[2] = s[1]/2 = 0.3183$

$h[3] = s[0] = 0.5$

$h[4] = s[1]/2 = 0.31831$

$h[5] = s[2]/2 = 0$

$h[6] = s[3]/2 = -0.1061$ #

(6) (a) Write two reasons why the transition band plays a critical role in Minimax filter design.

(b) Estimate the pass and stop band ripples if filter length = 21, $\Delta_t = 0.0002$, and the transition band is 1950~2050 Hz.

(c) Estimate the pass and stop band ripples if filter length = 31, $\Delta_t = 0.0001$, and the transition band is 1750~ 2250 Hz.

In (b)(c), suppose that pass and stop band ripples are equal. (15 scores)

(a) 1. transition band 過窄：訊號變動劇烈，誤差會變大，所需長度越長。

transition band 過寬：誤差較小，但會影響濾波頻寬。

$$N = \frac{2}{3}\frac{1}{\Delta F}\log_{10}\left(\frac{1}{10\delta_1\delta_2}\right), \quad \begin{array}{l} \Delta F \downarrow \; \delta_1\delta_2 \text{ 固定} \Rightarrow N \nearrow \\ \Delta F \nearrow \; N \text{ 固定} \Rightarrow \delta\delta_2 \downarrow \end{array}$$

(b) $\Delta F = (2050 - 1950) \times 0.0002 = 0.02$

$$\delta^2 = 10^{\left(\frac{-3N\Delta F}{2} - 1\right)/2} = 10^{\left(\frac{-3 \cdot 21 \cdot 0.02}{2} - 1\right)/2} \approx 0.1531 = 1.531 \times 10^{-1}$$

(C) $\Delta F = (2250 - 1950) \times 0.0001 = 0.05$

$$\delta^2 = 10^{\frac{3N\Delta F}{2} - 1} = 10^{\left(-\frac{3 \cdot 31 \cdot 0.05}{2} - 1\right)/2} \approx 0.02175$$

$$= 2.175 \times 10^{-2}$$