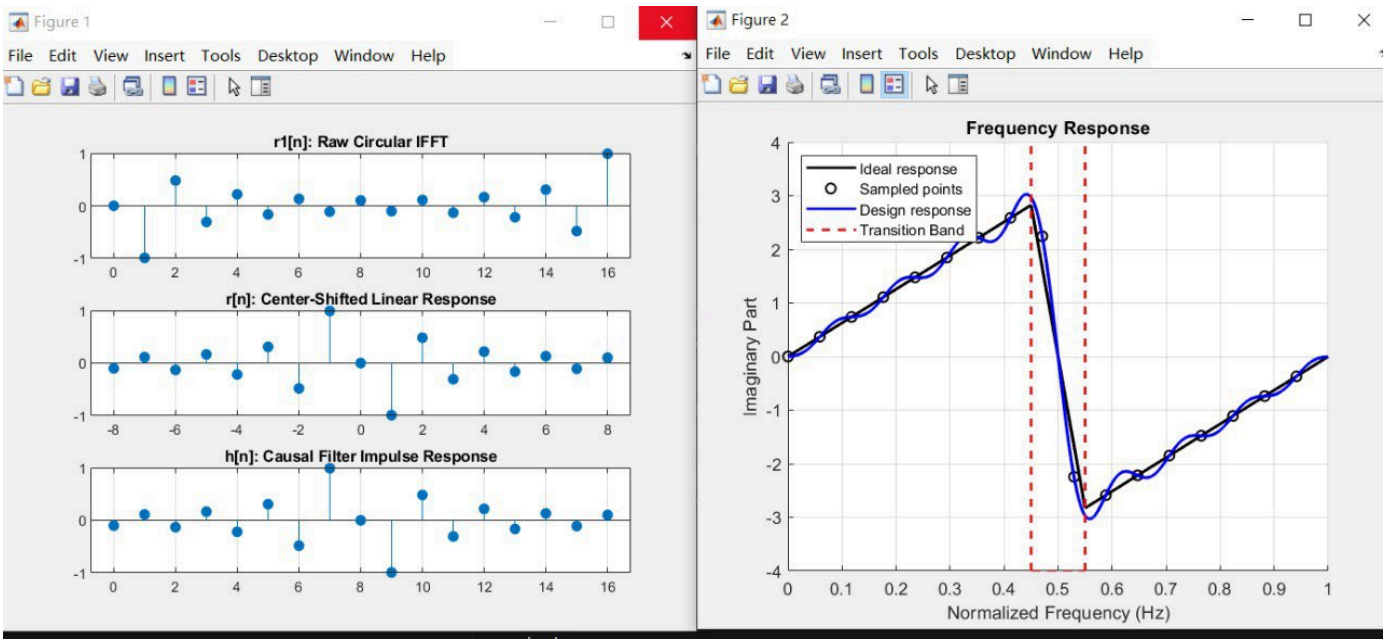
ADSP HW2 M11302149

(1) Write a Matlab or Python code that uses the frequency sampling method to design a $(2k+1)$-point discrete differentiation filter $H(F) = j2\pi F$ when $-0.5 < F < 0.5$ (k is an input parameter and can be any integer). (25 scores)

The transition band is assigned to reduce the error (unnecessary to optimize). (i) The impulse response and (ii) the imaginary part of the frequency response (DTFT of $r[n]$, see pages 112 and 113) of the designed filter should be shown in the homework. The code should be handed out by NTU Cool.

(i) $k=8$ (ii)



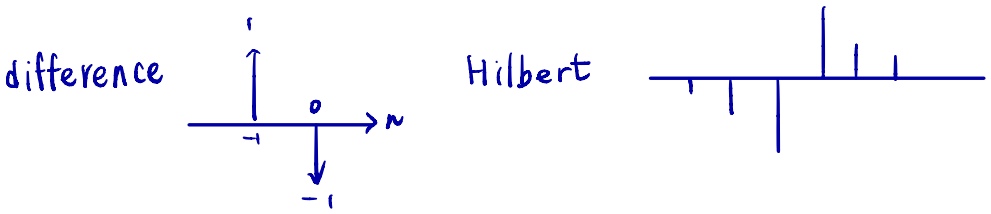
$2\pi j$

(2) (a) What are the two main advantages of the minimum phase filter compared to other IIR filters?
(b) What is the advantage of the Hilbert transform compared to the difference for edge detection?
(c) What is the advantage of the Wiener filter compared to pass-stop band filters for noise removal?
(d) What are the two advantages of the cepstrum compared to the equalizer for multipath problems? (20 scores)

(a) (1) 能量集中在 n=0 附近，其他部分快速衰減為 0

(2) 所有 poles 和 zeros 都在單位圓裡，
forward 跟 inverse transform 穩定。 ※

(b) 可以更好區分 noise 和 edge，Hilbert 考慮相位變化 #



(c) 能保留訊號完整度外，更好的去除 noise，
Wiener filter 不需要給 pass-stop band 而是用机率統計找到 signal 和 noise 的關係。 #

(d) cepstrum : T domain * ⟹ Cepstrum +

1. 不需先計算出路徑參數(eg. delay)，直接在 Cepstrum 上濾除。
2. Equlizer 的 H(Z) 可能 unstable，Cepstrum 則不會。
(放大 noise) #

(3) Why it is improper to use *IFFT*(*FFT*(*x*[*n*])*H*(*F*)) for filter design in practice? (5 scores)

如果直接把 Input 做 DFT，再設定 H[m] 的 passband
stopband，再做 IDFT ⇒ $O(N\log N)$ 計算複雜度很高 #

(4) (a) What is the role of the weight function in FIR filter design? (b) Can the techniques of the weight function be applied in the FIR filter designed by (i) the MSE method and (ii) the frequency sampling method?

(10 scores)

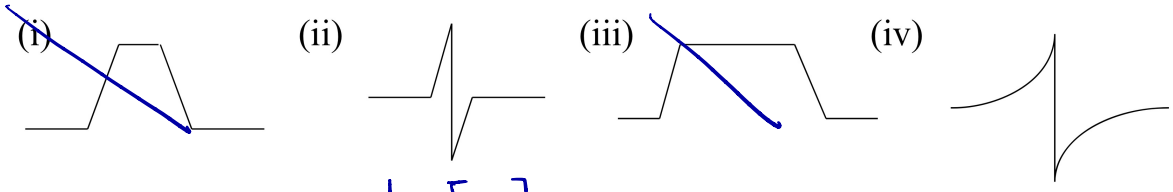
(a) 求解最佳化，程式迭代時，對不同頻率區間使用 Weight function 強調部分的設計精度 #

(b) (i) 可以。 $MSE = \int_{-\pi}^{\pi} W(w) \cdot |H_d(w) - H(w)|^2 dw$ #

(ii) 不行。 Frequecy Sampling Method 是在固定頻率點指定濾波器 response 然後用 IFFT 求 Time domain 係數，不是最佳化問題，無法使用 Weight function #

(5) The following figures are the impulse responses of some filters. Which one is a suitable edge detector when we want to extract (a) small scaled features? (b) large scaled features? Also illustrate the reasons. (10 scores)

(i) (ii) (iii) (iv)



① $h[n] = h[-n]$

② $|h[n_1]| \leq h_2[n_2]$ if $|h_1| > |h_2|$

(a) (ii) 因為對於細微邊緣敏感，且能量隨 |n| 遞減很快 #

(b) (iv) (iv) 是平滑再微分，對 noise 不敏感。能把大尺度的特徵提取出來。

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(6) If the z-transform of $h[n]$ is $H(z)=\dfrac{2z^3-2z^2-3z-2}{z^2-0.7z+0.1}$

(a) Convert the IIR filter into the minimum phase filter.

(b) Determine the cepstrum of $h[n]$. (20 scores)

(a)

$$H(z)=\frac{2z^3-2z^2-3z-2}{z^2-0.7z+0.1}=\frac{2(z-2)(z^2+z+1/2)}{(z-0.5)(z-0.2)}$$

$$\Rightarrow \frac{2(z-2)(z+(0.5+0.5j))(z+(0.5-0.5j))}{(z-0.5)(z-0.2)}\cdot 2\frac{z-1/2}{z-2}$$

$$=\frac{4(z-0.5)(z+(0.5+0.5j))(z+(0.5-0.5j))}{(z-0.5)(z-0.2)}$$

(b)

$$H(z)=\frac{-4z^2(1-0.5z)(1-(-0.5-0.5j)z^{-1})(1-(0.5+0.5j)z^{-1})}{(1-0.5z^{-1})(1-0.2z^{-1})}$$

zero, zero, zero; pole, pole

$$\hat{H}[n]=\begin{cases}\log(-4)=\log(4)+j\pi, & n=0\\ \dfrac{0.5^n+0.2^n-(-0.5+0.5j)^n-(-0.5-0.5j)^n}{n}, & n>0\\ \dfrac{0.5^n}{n}, & n<0\end{cases}$$

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(7) Suppose that the cepstrum of a signal $x[n]$ is

$\hat{x}[2]=0.8,\quad \hat{x}[n]=0$ *otherwise.*

Determine $x[n]$ using the Z transform and exp( ). (10 scores)

$$\log X(z)=\sum_{n=-\infty}^{\infty}\hat{x}[n]z^{-n}$$

$$\log X(z)=0.8z^{-2}$$

$$X(z)=\exp(0.8z^{-2})=\sum_{k=0}^{\infty}\frac{(0.8z^{-2})^k}{k!}=\sum_{k=0}^{\infty}\frac{0.8^k}{k!}z^{-2k}$$

$$\xrightarrow{Z^{-1}} x[n]=\begin{cases}\dfrac{0.8^k}{k!}, & n \text{ is even.}\\ 0, & n \text{ is odd.}\end{cases}$$

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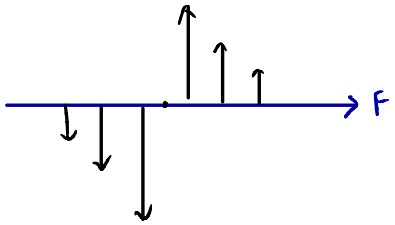
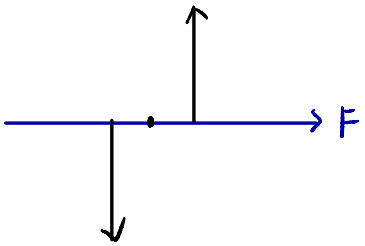
Bonus：（学号9）除了積分、微分器，還有哪些是 odd 對稱？

① Hilbert filter：



② Difference



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